Gaseous Detectors

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Summary of First Lecture

- Overview: old and new detectors
- Detection principles
- Interaction of charged particles with matter
  - Inelastic collisions with atomic electrons ⇨ ionization energy loss
  - Emission of Cherenkov radiation
  - Emission of Transition radiation

    ⇨ unified treatment in **PAI model**
- Mean energy loss: Bethe formula and friends
- Energy loss distributions (straggling functions): Landau et al.
- Multiple scattering
  - important for smaller momenta
  - contribution to momentum resolution

Detection of Ionization

Principle: collection of electrons and ions (holes) produced in detector medium by ionizing radiation

Detector material:

- gas ⇨ fast collection of e^- and ions, e.g. Ne, Ar
- liquid ⇨ higher density, e.g. liquid Ar
- solid ⇨ higher density, self-supporting, e.g. semiconductor

Setup:

- vessel with two electrodes and thin entrance window
- filled with active medium ⇨ creation of electron-ion (hole) pairs
- electric field between anode and cathode
  - separation of e^- and ions (holes), drift and diffusion
  - signal induction
  - collection at anode/cathode

Plan of the Lecture

1. Introduction
2. Interactions of charged particles with matter
3. Drift and diffusion of charges in gases
4. Avalanche multiplication of charge
5. Signal formation and processing
6. Ionization and proportional gaseous detectors
7. Track reconstruction and momentum measurement

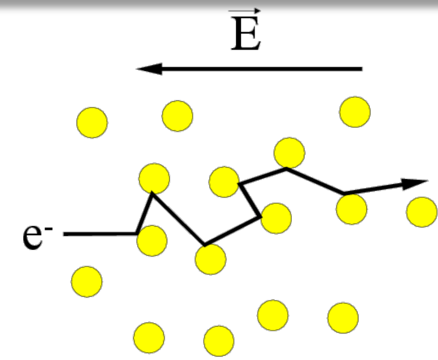
3 Drift and Diffusion of Charges in Gases

3.1 Drift of charge carriers: equation of motion

3.2 Microscopic picture

3.3 Diffusion

3.1 Drift of Charge Carriers



Microscopically:

External electric field ⇨ acceleration

Collisions ⇨ slowing down

Macroscopically: drift motion with drift velocity $\boldsymbol{u}$

Ansatz: Langevin equation

$$m\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = e\boldsymbol{E} + e(\boldsymbol{u}\times\boldsymbol{B}) - K\boldsymbol{u}$$

Ku = frictional force

$\tau = \frac{m}{K}$ = characteristic time

Solution for $t \gg \tau$: steady state for which $\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = 0$

$$\boldsymbol{u} = \frac{e}{m}\tau\,|\boldsymbol{E}|\,\frac{1}{1+\omega^2\tau^2}\left[\hat{\boldsymbol{E}} + \omega\tau\left(\hat{\boldsymbol{E}}\times\hat{\boldsymbol{B}}\right) + \omega^2\tau^2\left(\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}}\right)\hat{\boldsymbol{B}}\right]$$

$\omega = \frac{eB}{m}$ = cyclotron frequency

⇨ Drift velocity $\boldsymbol{u}$ dominated by dimensionless parameter $\omega\tau$

Drift of Charge Carriers

$$\boldsymbol{u} = \frac{e}{m}\tau\,|\boldsymbol{E}|\,\frac{1}{1+\omega^2\tau^2}\left[\hat{\boldsymbol{E}} + \omega\tau\left(\hat{\boldsymbol{E}}\times\hat{\boldsymbol{B}}\right) + \omega^2\tau^2\left(\hat{\boldsymbol{E}}\cdot\hat{\boldsymbol{B}}\right)\hat{\boldsymbol{B}}\right]$$

Drift velocity $\boldsymbol{u}$ dominated by dimensionless parameter $\omega\tau$ (carries charge sign):

- $\omega\tau = 0 \Rightarrow \mathbf{u} \parallel \mathbf{E}$

  $\mathbf{u} = \frac{e}{m}\tau\mathbf{E} = \mu\mathbf{E},$ $\quad \mu = \frac{e}{m}\tau$ (scalar mobility, carries charge sign)

- with magnetic field:

  ⇨ generalized mobility: $\mu = \frac{e}{m}M^{-1}$ (tensor of 2nd order)

- $\omega\tau \gg 1,\ e > 0$
  - $\hat{\mathbf{E}}\cdot\hat{\mathbf{B}} \neq 0 \Rightarrow \mathbf{u} \parallel \mathbf{B}$
  - $\hat{\mathbf{E}}\cdot\hat{\mathbf{B}} = 0 \Rightarrow \mathbf{u} \parallel (\mathbf{E}\times\mathbf{B})$

Drift of Charge Carriers

Case of ***E*** orthogonal to ***B***:

$$\hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{B}} = 0\,, \quad \boldsymbol{E} = \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$$

$$\Rightarrow u_x = \frac{(e/m)\tau}{(1+\omega^2\tau^2)} |\boldsymbol{E}|$$

$$u_y = -\frac{(e/m)\tau}{(1+\omega^2\tau^2)} \omega\tau |\boldsymbol{E}|$$

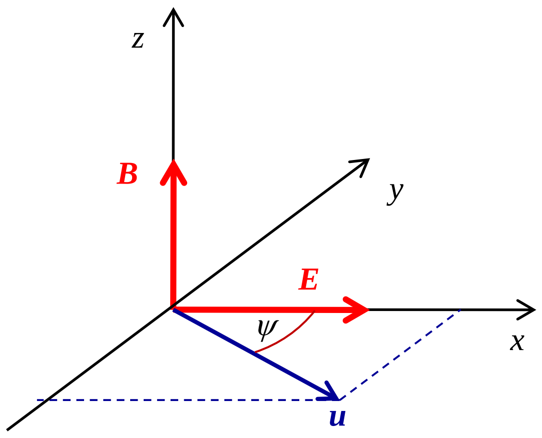
$$u_z = 0$$

$$\tan\psi \equiv \frac{u_y}{u_x} = -\omega\tau$$

Drift of Charge Carriers

With $\tan\psi \equiv \frac{u_y}{u_x} = -\omega\tau$

Lorentz angle



$$\Rightarrow |\boldsymbol{u}| = \frac{(e/m)\tau}{\sqrt{1+\omega^2\tau^2}}|\boldsymbol{E}| = \frac{e}{m}\tau\underbrace{|\boldsymbol{E}|\cos\psi}$$

component of $\boldsymbol{E}$ in drift direction

$$\cos\psi = \frac{u_x}{|\boldsymbol{u}|} = \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

i.e. magnitude of drift velocity is determined by component of electric field in drift direction

Drift of Charge Carriers

Numerical example:

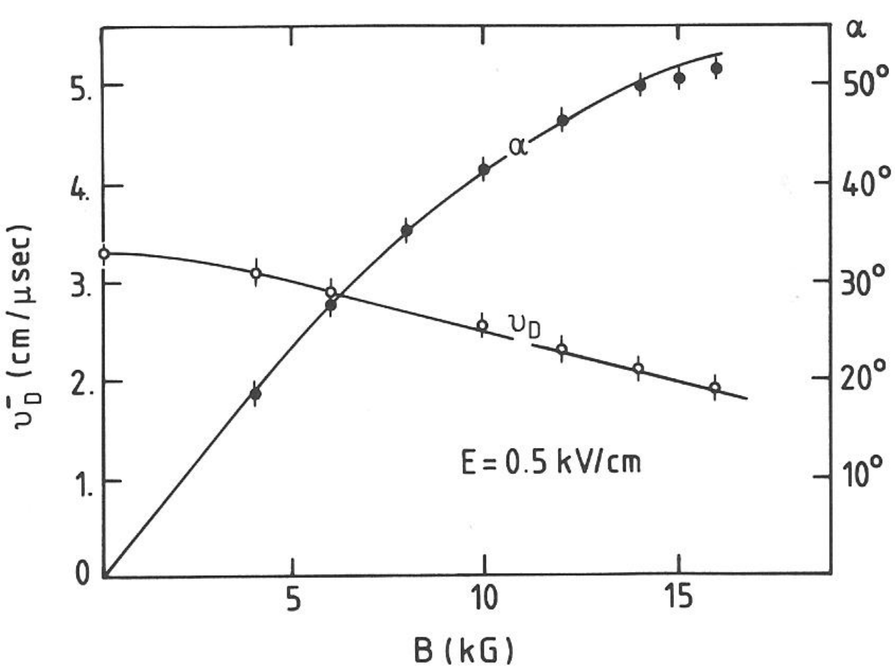
$\mu = 10^4 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ for electrons

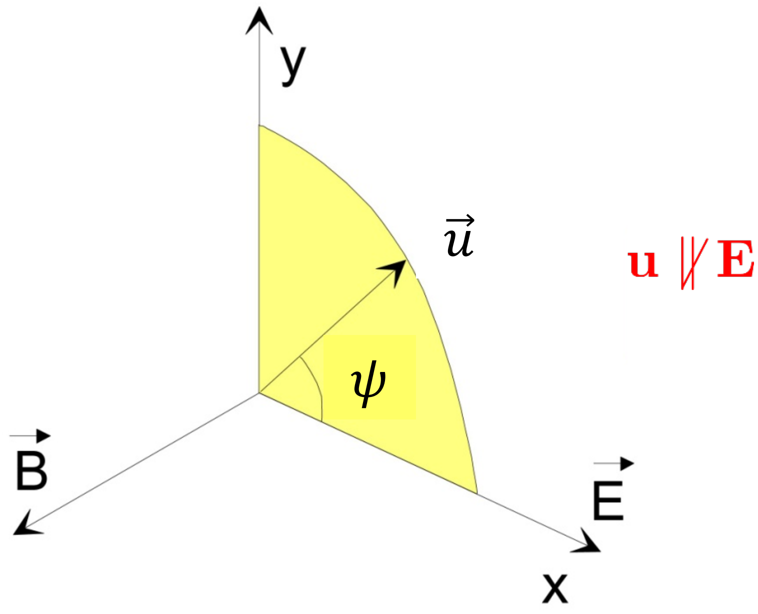
$\mu = 1 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ for ions

$B = 1\text{T} = 10^{-4} \text{ Vs cm}^{-2}$ typical

$$\Rightarrow \omega\tau = B\mu \sim \begin{cases} 10^{-4} & \text{for ions} \\ 1 & \text{for electrons, i.e. } \psi = 45° \end{cases}$$

⇨ effect on ion drift negligible





Electron drift velocity and Lorentz angle
(E orthogonal to B)

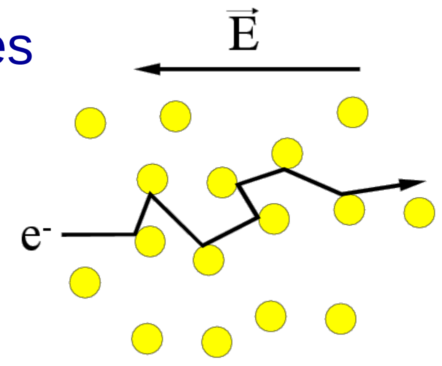
[A. Breskin et al., NIM 124, 189 (1975)]

3.2 Microscopic Picture

Drifting electrons and ions are scattered on gas molecules

⇨ Direction of motion randomized in each collision

⇨ „stop & go“ motion of individual particles



On average:

- constant drift velocity $\boldsymbol{u}$ in the direction given by $\boldsymbol{E}$ (or $\boldsymbol{E}$ and $\boldsymbol{B}$)

Goal: derive basic relations between

- Microscopic quantities of instantaneous velocity c, mean time τ between collisions, and fractional energy loss Λ
  - ⇨ distributed according to distribution functions!
  - ⇨ here: use suitable averages
- Macroscopic quantities of drift velocity u and isotropic diffusion coefficient D and

3.2.1 Drift of Electrons

Microscopic picture: electron between two collisions

$$m\frac{d\boldsymbol{v}}{dt} = e\boldsymbol{E} \quad \Rightarrow \quad \boldsymbol{v}(t) = \frac{e\boldsymbol{E}}{m}t + \boldsymbol{v}(0)$$

Average in time:

$$\langle \boldsymbol{v}(t) \rangle \equiv \boldsymbol{u} = \frac{e\boldsymbol{E}}{m}\langle t \rangle + \langle \boldsymbol{v}(0) \rangle$$

$$\langle t \rangle = \tau \quad \text{since} \quad \langle t \rangle = \int_0^\infty t\, e^{-t/\tau} \frac{dt}{\tau} = \tau$$

average time since last collision = average time between collisions

For $m \ll M$ ⇨ electron scatters isotropically, i.e. no preferential direction after collision

$$\Rightarrow \langle \boldsymbol{v}(0) \rangle = 0$$

$$\Rightarrow \boldsymbol{u} = \frac{e\boldsymbol{E}}{m}\tau$$

- extra velocity gained by e- in electric field between collisions, in addition to its instantaneous, randomly oriented velocity
- corresponds macroscopically to drift velocity

Drift of Electrons

Next encounter of gas molecule ⇨ extra energy is lost (on average),
i.e. there is a balance between energy picked up and collision losses

Drift distance x: average number of collisions $N = \frac{t}{\tau} = \frac{x}{u} \cdot \frac{1}{\tau}$

$$\Rightarrow eEx = \frac{x}{u\tau} \cdot \Lambda \cdot \varepsilon_E$$

Λ = average relative energy loss per collision

ε_E = equilibrium energy (only part due to electric field, not thermal energy)

With $\frac{1}{\tau} = n\sigma c$ and $\varepsilon = \frac{1}{2}mc^2 = \varepsilon_E + \frac{3}{2}kT \approx \varepsilon_E$ $\quad \varepsilon_E \gg \frac{3}{2}kT$ usually fulfilled for e- in detectors

follows: $u^2 = \frac{eE}{mn\sigma}\sqrt{\frac{\Lambda}{2}}$ drift velocity $\quad \Rightarrow u \propto \sqrt{E}$ if Λ and σ const. $\Rightarrow \mu \propto \frac{1}{\sqrt{E}}$

$c^2 = \frac{eE}{mn\sigma}\sqrt{\frac{2}{\Lambda}}$ average instantaneous velocity

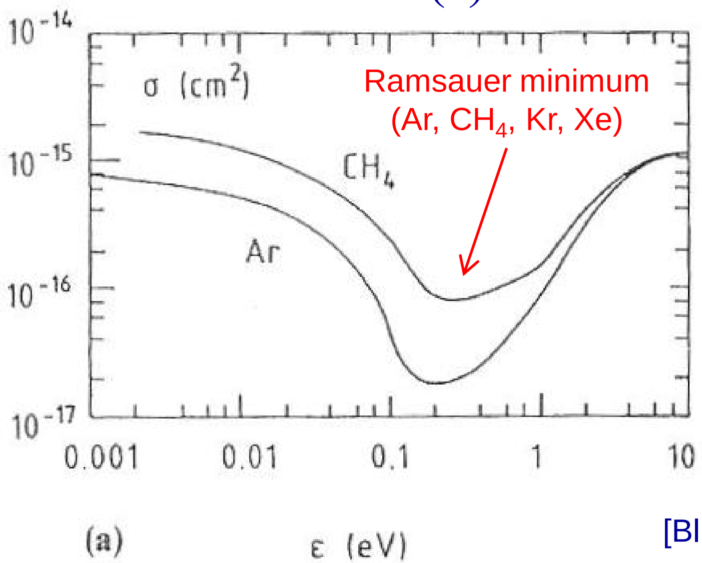
Drift of Electrons

- rapid acceleration in electric field
- small energy loss in elastic collisions with atoms
- e^- momentum randomized in collisions
- energy gain in electric field is mainly in random motion >> thermal energy

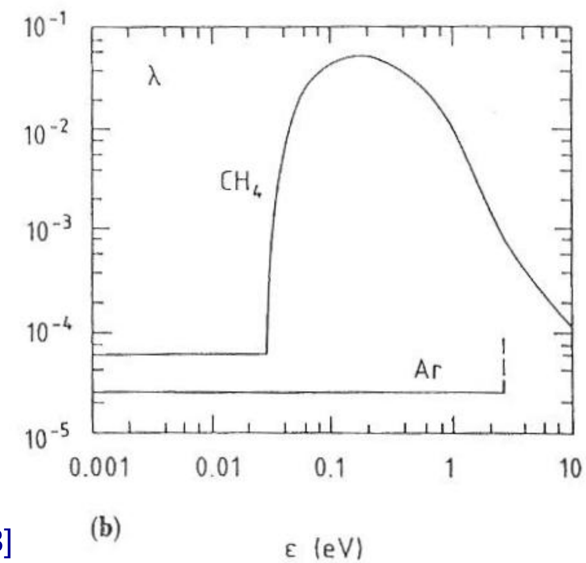
⇨ drift velocity $$u^2 = \frac{eE}{mn\sigma}\sqrt{\frac{\Lambda}{2}} \ll c^2 = \frac{eE}{mn\sigma}\sqrt{\frac{2}{\Lambda}}$$ average velocity

$\Lambda = \Lambda(\varepsilon)$ average fractional energy loss per collision

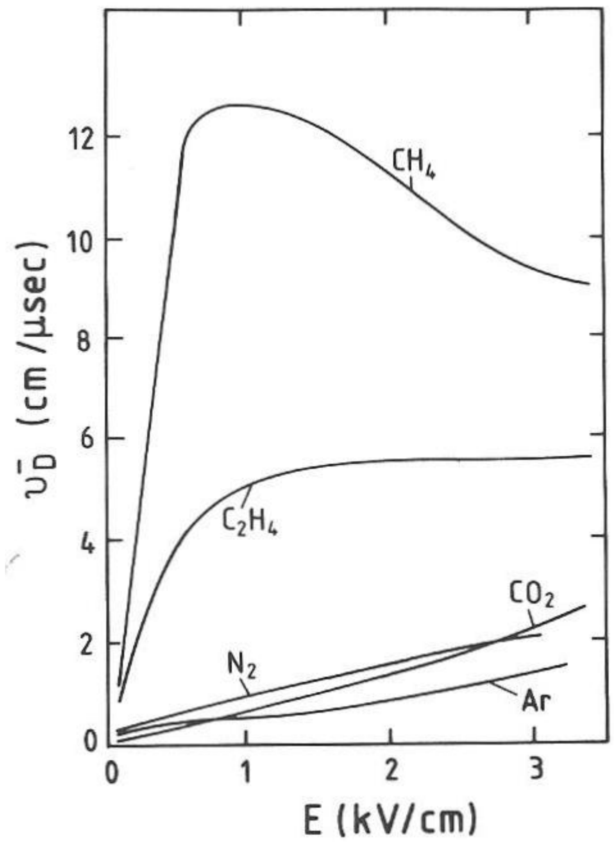
$\sigma = \sigma(\varepsilon)$ collision cross section

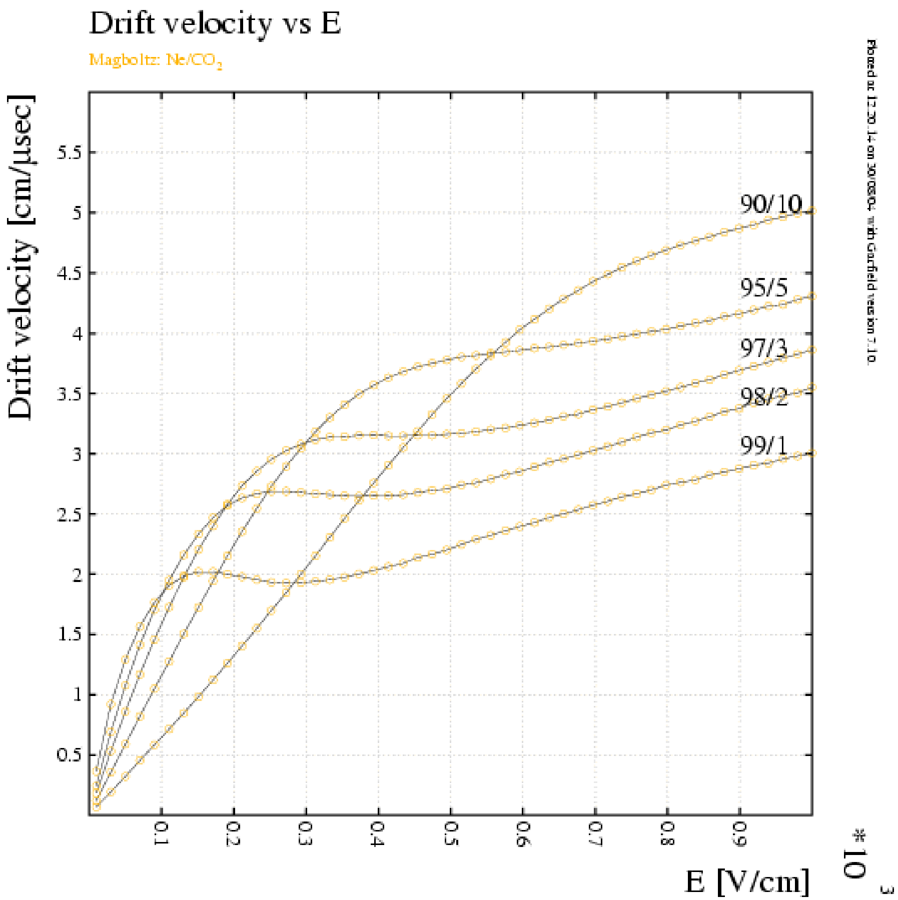


[Blum,Rolandi, Springer, 1993]



Drift Velocity of Electrons





Ne/CO_2 Mixtures

3.2.2 Drift of Ions

- Only elastic collisions with other gas atoms/molecules
- Two limiting cases:
  - Low field ⇨ ion random velocity is thermal (never reached for e-)

$$\Rightarrow u = \left(\frac{1}{m} + \frac{1}{M}\right)^{1/2} \left(\frac{1}{3kT}\right)^{1/2} \frac{eE}{n\sigma} \propto E$$

  - High field ⇨ neglect thermal motion (this is the general case for e-)

$$\Rightarrow u = \left(\frac{eE}{mn\sigma}\right)^{1/2} \left[\frac{m}{M}\left(1 + \frac{m}{M}\right)\right]^{1/2} \propto \sqrt{E}$$

for ions ($\sigma \sim \mathrm{const.}$,
Λ only from elastic collisions)

3.3 Diffusion of Charge Carriers

Field-free gas: quick thermalization of charge carriers in collisions

⇨ Maxwell distribution of velocities (thermal equilibrium):

$$F(c)\mathrm{d}c = 4\pi n \left(\frac{m}{2\pi kT}\right)^{3/2} c\, e^{-mc^2/(2kT)}\mathrm{d}c \qquad \bar{c} = \sqrt{\frac{8kT}{\pi m}}$$

$$\overline{E}_{\mathrm{kin}} \equiv \varepsilon = \frac{m}{2}\overline{c^2} = \frac{3}{2}kT$$

Definition of temperature

Point-like charge cloud at t=0 ⇨ Distribution at time t?

Ansatz: $\mathbf{J} = -D\,\nabla n$

$\mathbf{J}$ = flux density
∇n = gradient of particle density
D = diffusion coefficient

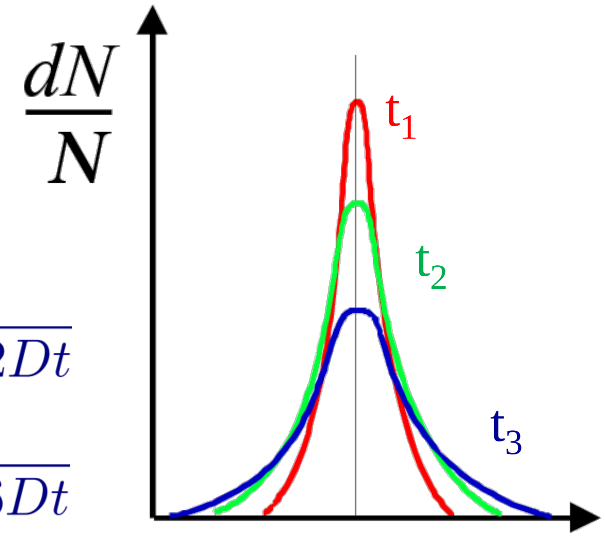
⇨ diffusion equation $\dfrac{\partial n}{\partial t} = D\,\Delta n$

Solution: Gaussian law

$$\frac{\mathrm{d}N}{N} = \frac{n\mathrm{d}z}{N} = \frac{1}{\sqrt{4\pi Dt}} e^{-z^2/4Dt}\mathrm{d}z$$

$$\sqrt{\langle z^2\rangle} \equiv \sigma_z = \sqrt{2Dt}$$

$$\sqrt{\langle r^2\rangle} \equiv \sigma_r = \sqrt{6Dt}$$



Drift and Diffusion

Microscopic picture: consider electron/ion starting at $t = 0$ with velocity c (1-D)

After $N = t/\tau$ collisions (statistically independent) ⇨ displacement x

$$x = \sum_{i=1}^{N} \xi_i \qquad \overline{x} = \sum_{i=1}^{N} \overline{\xi_i} = 0 \qquad \overline{x^2} = N\overline{\xi_i^2}$$

ξ_i = x-component of ith displacement

since $\overline{\xi_i} = 0$ (isotropic scattering, fulfilled for e- and ions for very small fields)

since probability distribution is equal for each step

With $\xi = c_x t$ follows $\overline{\xi^2} = \overline{c_x^2}\,\overline{t^2} = \frac{1}{3}\overline{c^2}\,\overline{t^2}$

(symmetry)

$$\overline{t^2} = \int t^2 e^{-t/\tau} \frac{\mathrm{d}t}{\tau} = 2\tau^2 \qquad \Rightarrow \overline{\xi^2} = \frac{2}{3}\overline{c^2}\,\tau^2$$

$$\Rightarrow \overline{x^2} = N\overline{\xi^2} = \frac{2}{3}\overline{c^2}\,\tau t \equiv \sigma_x^2$$

Drift and Diffusion

Comparison with solution of diffusion equation: $\sigma_x = \sqrt{2Dt}$

$$\Rightarrow D = \frac{\overline{c^2}\tau}{3} = \frac{2}{3}\frac{\varepsilon}{m}\tau \qquad \text{with} \qquad \varepsilon = \frac{1}{2}m\overline{c^2}$$

With $\mu = \frac{e}{m}\tau$ follows $\varepsilon = \frac{3}{2}\frac{De}{\mu}$ (valid for ions and electrons!)

⇨ determination of average e^- kinetic energy by measurement of D/μ

Diffusing body in thermal equilibrium with environment: $\varepsilon = \frac{3}{2}kT$

(fulfilled for ions at low fields only!)

$$\Rightarrow \frac{D}{\mu} = \frac{kT}{e}$$

Nernst-Townsend formula

Drift and Diffusion

Diffusion depends on average kinetic energy: $\varepsilon = \frac{3}{2}\frac{De}{\mu}$ from kinetic gas theory

Width of electron cloud after drifting distance L: $\sigma_x^2 = 2Dt = \frac{2DL}{\mu E} = \frac{4\varepsilon L}{3eE}$

⇨ small e$^-$ energies at high drift fields required for small diffusion

Thermal limit:

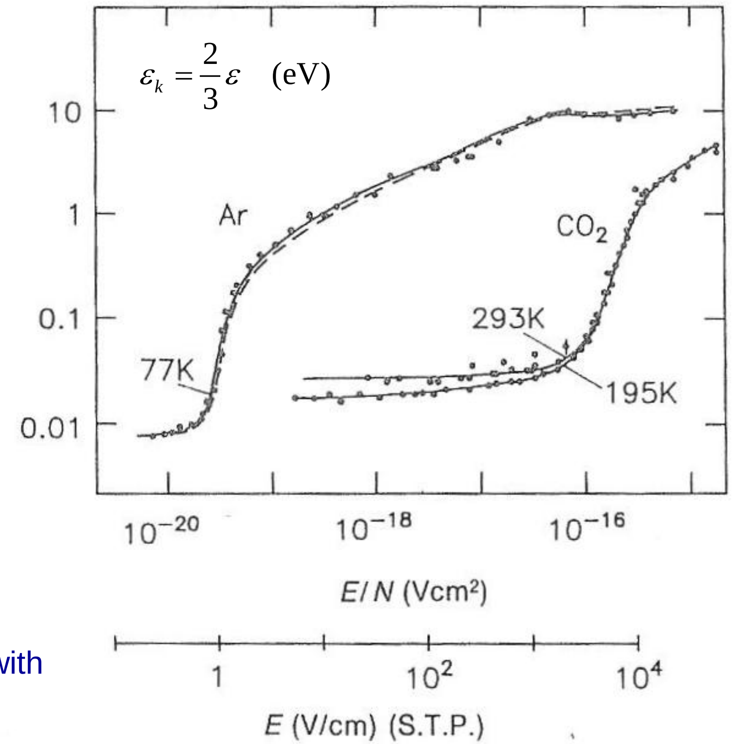
$$\varepsilon = \frac{3}{2}kT \Rightarrow \sigma_x^2 = \frac{2kTL}{eE}, \sigma_x \propto \sqrt{T}$$

Argon: $\varepsilon_k = \frac{2}{3}\varepsilon \gg kT \quad \text{for} \quad E > 1\,\mathrm{V/cm}$

⇨ “hot gas”

CO_2: $\varepsilon_k \gg kT \quad \text{for} \quad E > 2\,\mathrm{kV/cm}$

⇨ “cold gas”



[Blum,Rolandi: Particle Detection with Drift Chambers, Springer, 1993]

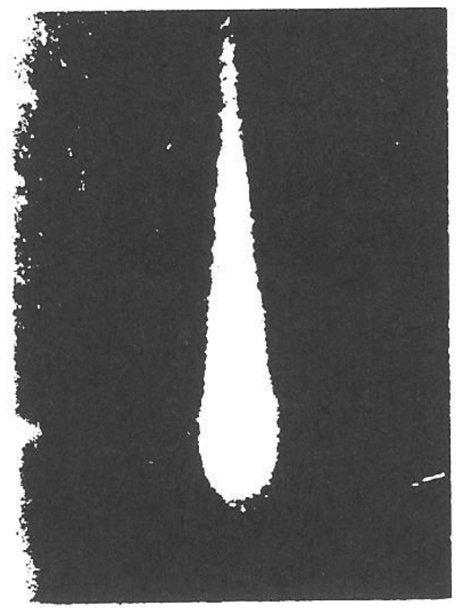
4 Avalanche Multiplication of Charge

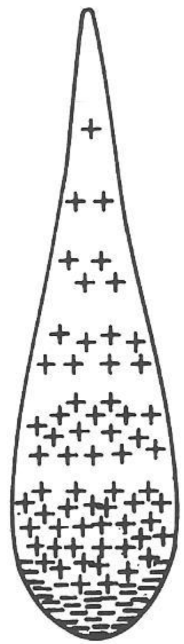
4.1 Gas amplification
4.2 Choice of detector gas
4.3 Statistical fluctuations of gain

Increase electric field in gaseous detector up to several kV/cm

⇨electrons gain sufficient energy between collisions to ionize gas molecules

⇨avalanche generation





Drift velocity $u_e \gg u_{ion}$

Diffusion

⇨ drop-like distribution of charges (in homogeneous field)

- e- in the front
- tail of positive ions

[L.B. Loeb, Basic Processes of Gaseous Electronics, Univ. Cal. Press (1961)]

Probability of ionization per unit path length: 1. Townsend coefficient

$$\alpha = \frac{1}{\lambda_{\text{ion}}} = n\sigma_{\text{ion}}$$

λ_{ion} = mean free path for secondary ionizing collision
σ_{ion} = cross section for ionizing collision
n = density of gas molecules

Avalanche development: N electrons in avalanche

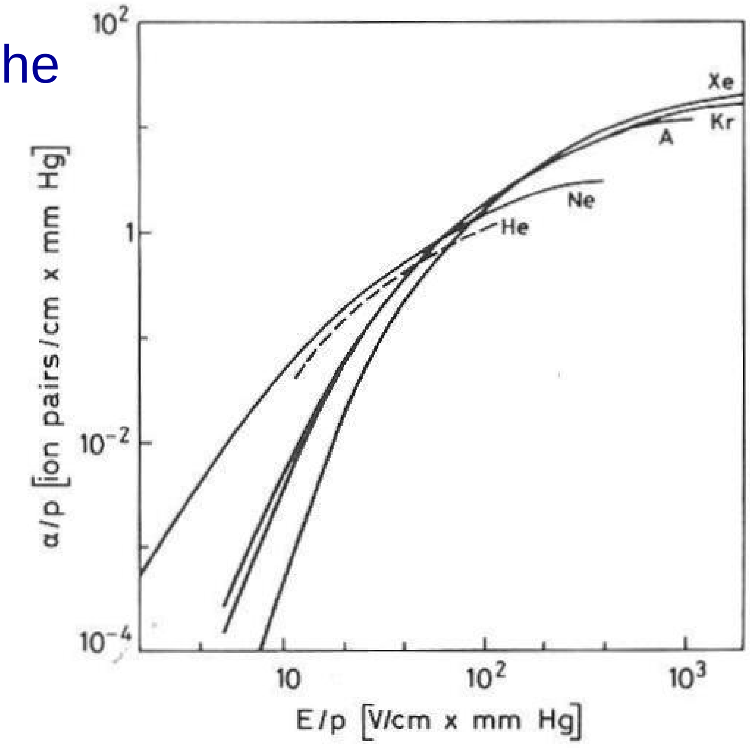
- Homogeneous electric field:

$$\mathrm{d}N = N\alpha\mathrm{d}s \Rightarrow N(s) = N_0 e^{\alpha s}$$

Gain: $G = \frac{N}{N_0} = e^{\alpha s}$

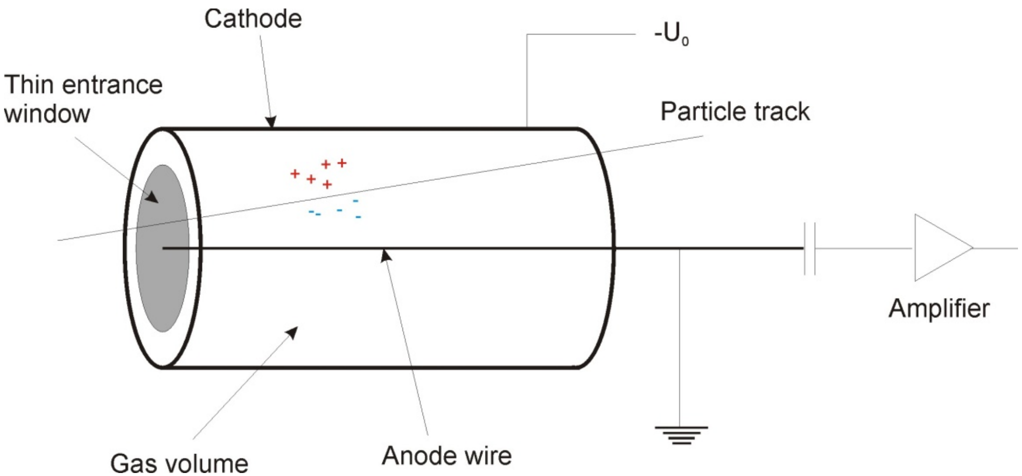
- Inhomogeneous electric field: $\alpha = \alpha(s)$

$$G = \exp\left[\int_{s_1}^{s_2} \alpha(s)\mathrm{d}s\right] = \exp\left[\int_{E_1}^{E_2} \frac{\alpha(E)}{\mathrm{d}E/\mathrm{d}s}\mathrm{d}E\right]$$



[S.C. Brown, Basic Data of Plasma Physics, MIT Press (1959)]

Proportional Counter

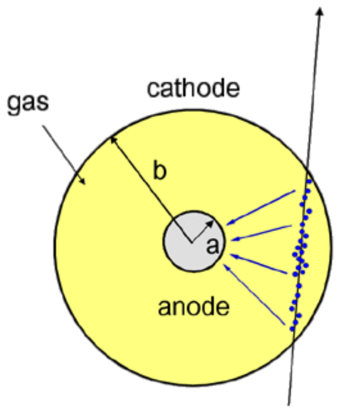


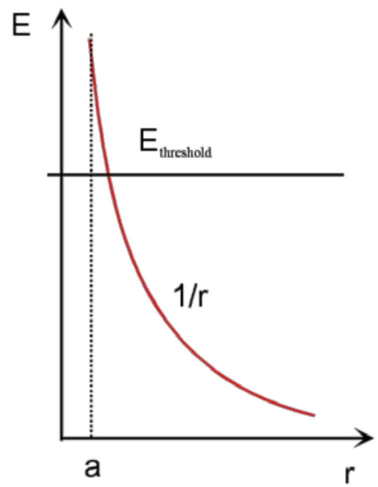
Charge gain:

$$G = \frac{N}{N_0} \approx e^{\text{const}\cdot U}$$

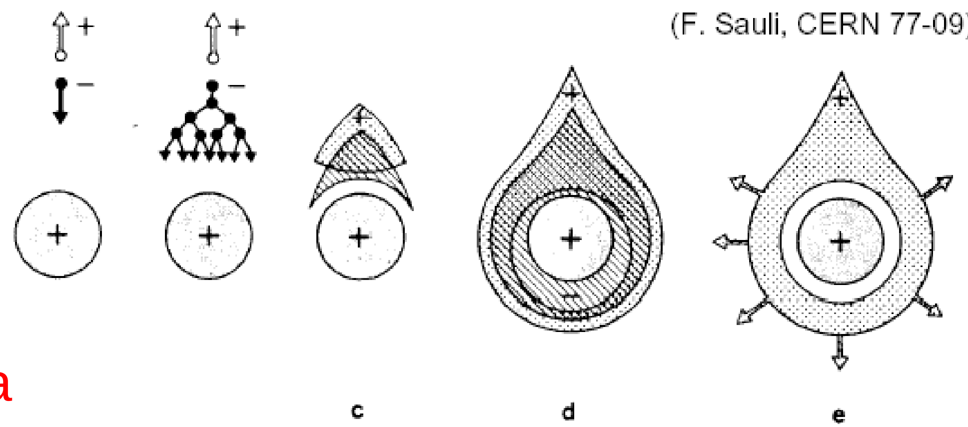
Typical values:

- Proportional wire counter: $G \sim 10^4 - 10^6$
- GEM: $G \sim 10^3 - 10^5$





(F. Sauli, CERN 77-09)



Avalanche multiplication starts at r~5a

⇨ negligible position dependence of pulse height

4.2 Choice of Detector Gas

In principle: avalanche formation occurs in every gas,
depending on the electric field

Determines substantially the properties of the detector

Experimental requirements:

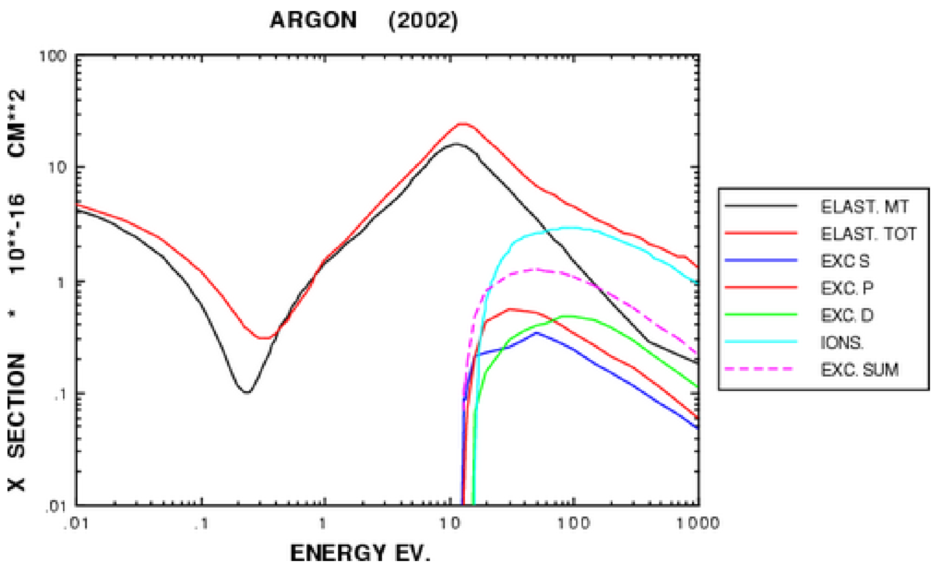
- low operating voltage
- high gain
- good proportionality
- high rate capability
- long lifetime, no aging

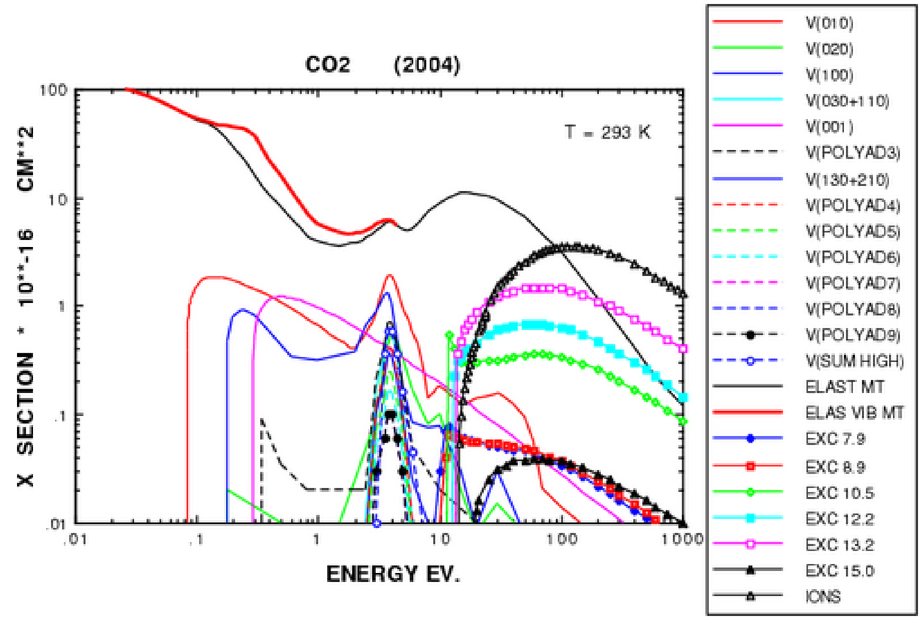
⇨ partly contradicting each other

⇨ no ideal gas meeting all requirements

Noble gases: main component

- avalanche multiplication occurs at much lower fields than in complex molecules
- energy dissipation mostly through ionization, in contrast to many non-ionizing energy dissipating modes in polyatomic molecules





Choice of Detector Gas

Choice of noble gas ⇨ high specific ionization by MIPs (particle physics)

	Xe	Kr	Ar	Ne	He
Z	54	36	18	10	2
W (eV)	22	24	26	36	41
n_T (ip/cm)	307	192	94	39	7.8

Xe, Kr: expensive, multiple scattering ⇨ deflection of incoming particle

He: high leak rate

Ne: ~8× more expensive than Ar ⇨ closed gas system necessary

Ar: commonly used

But: continuous discharge for $G \geq 10^3 - 10^4$!

Choice of Detector Gas

Reason: formation of excited and ionized noble gas atoms in avalanche

a) Excited noble gas atoms:

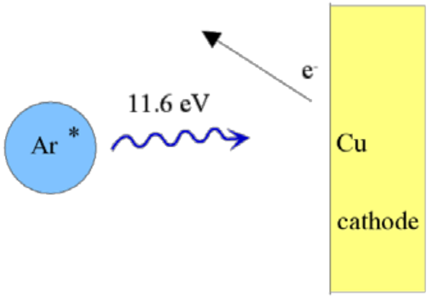
- deexcitation through radiative transitions

  ⇨ emission of photons with $h\nu = E_{ex}$ (11.6 eV for Ar, 16.6 eV for Ne)

- electrodes (metal): $h\nu > E_W$ (4.4 eV for Cu)

  ⇨ photons can extract photoelectrons from electrodes

  ⇨ new avalanches at different locations in detector



Choice of Detector Gas

b) Ionized noble gas atoms:

- drift to cathode where they are neutralized
  by extracting an electron
- conservation of energy
  - emission of photon
  - extraction of another electron from cathode (secondary emission)

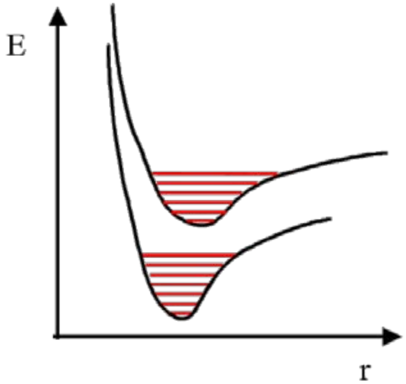
  ⇨ further avalanches, delayed with respect to original event

Solution: addition of quench gas, i.e. polyatomic molecules,

e.g. CH_4, CO_2, BF_3, …

a)

- absorption of photons in a wide energy range (7.9 – 14.5 eV for CH4) through excitation of rotational and vibrational levels
- energy dissipation via dissociation or elastic collisions



b)

- charge exchange noble gas → quench gas very efficient due to low ionization potential of molecules
- drift of ionized molecule to cathode, neutralisation
- emission of secondary radiation at cathode unlikely
  - dissociation
  - polymerisation (formation of larger molecular complexes)

5 Creation of the Signal

5.1 Signal Formation by Moving Charges

5.2 Ramo-Shockley Theorem

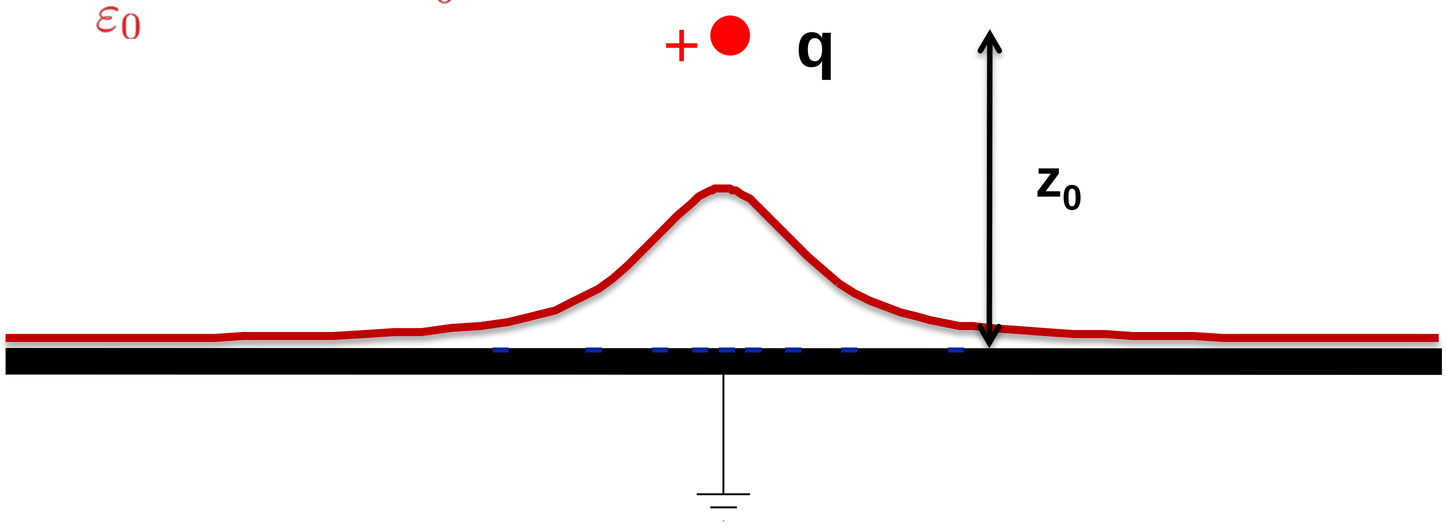
5.3 Planar Detector

5.4 Cylindrical Wire

Consider charge q above a grounded electrode

- electric field is perpendicular to conductor at the surface
- changes take place only on surface
- surface charge density σ and electric field E on the surface are related by Gauss' law

$$EA = \frac{1}{\varepsilon_0}\sigma A \Rightarrow \sigma = \varepsilon_0 E$$



Signal Formation

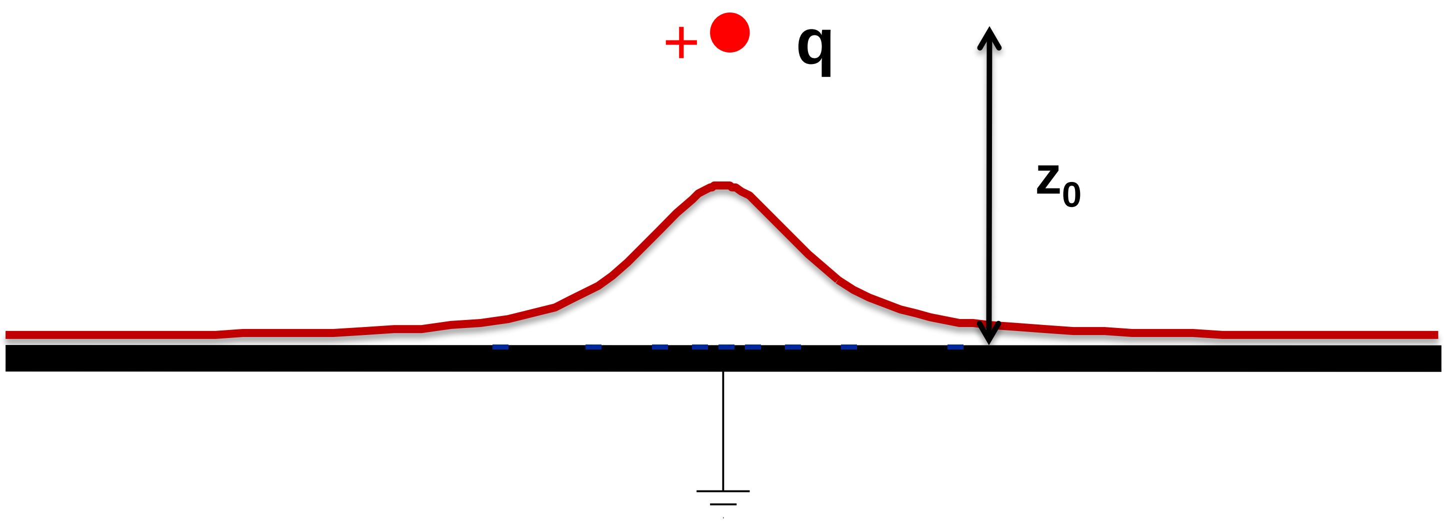
In order to find the charge induced on an electrode, we have to

a) solve the Poisson equation with boundary condition $\phi = 0$ on the conductor surface

$$\Delta\varphi = -\frac{\rho}{\varepsilon_0}, \quad \mathbf{E} = -\boldsymbol{\nabla}\varphi$$

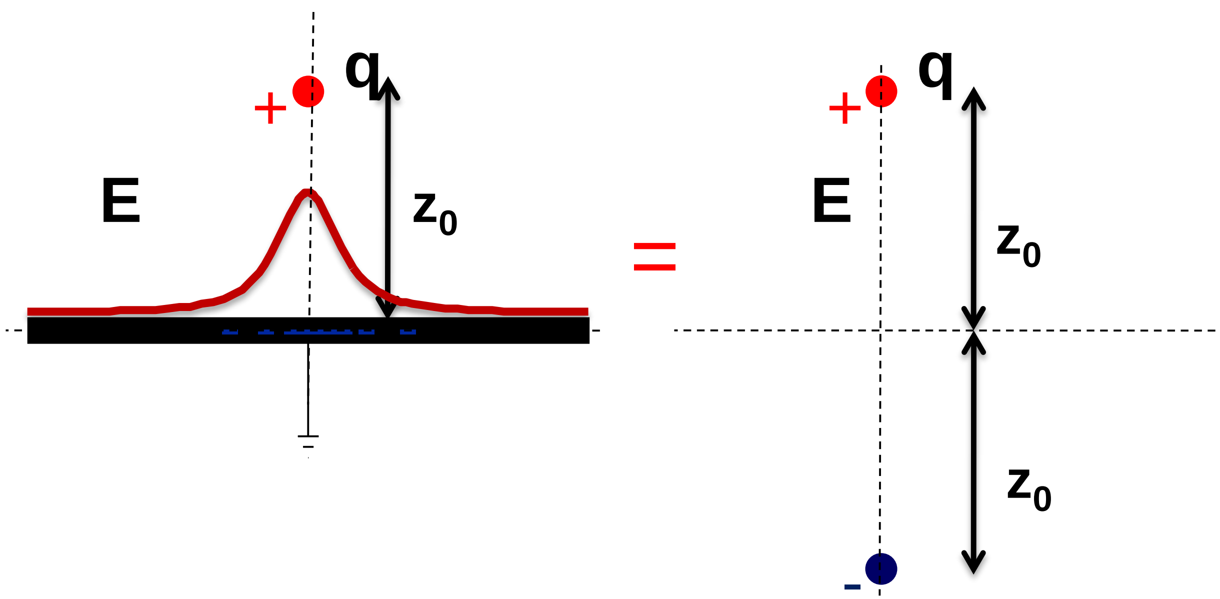
b) calculate the electric field E on the surface of the conductor

c) integrate $\varepsilon_0 E$ over the surface of the electrode



For this particularly simple setup with one electrode

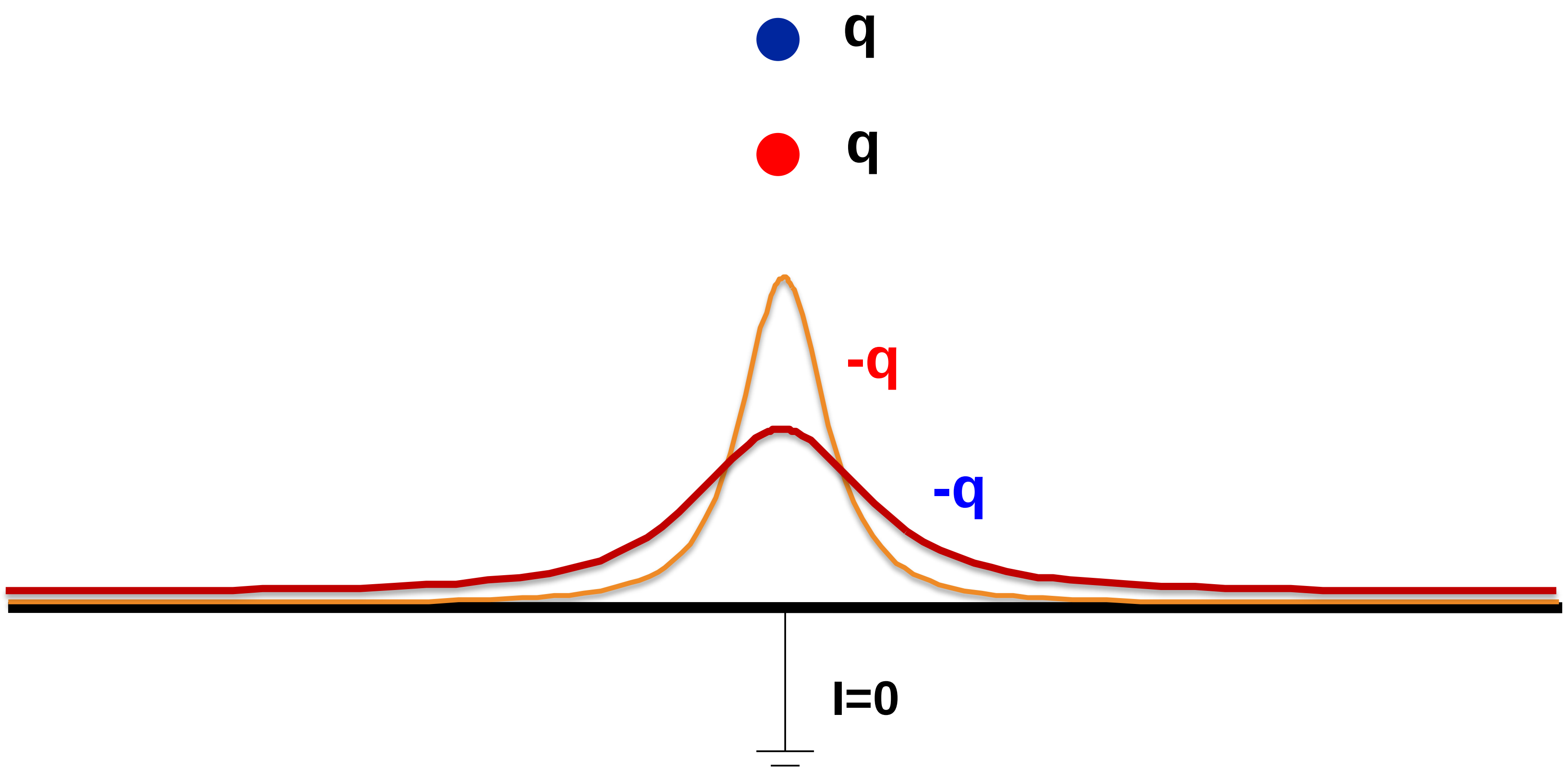
⇨ use mirror charge



Signal Formation

Movement of charge q (no external field needed!)

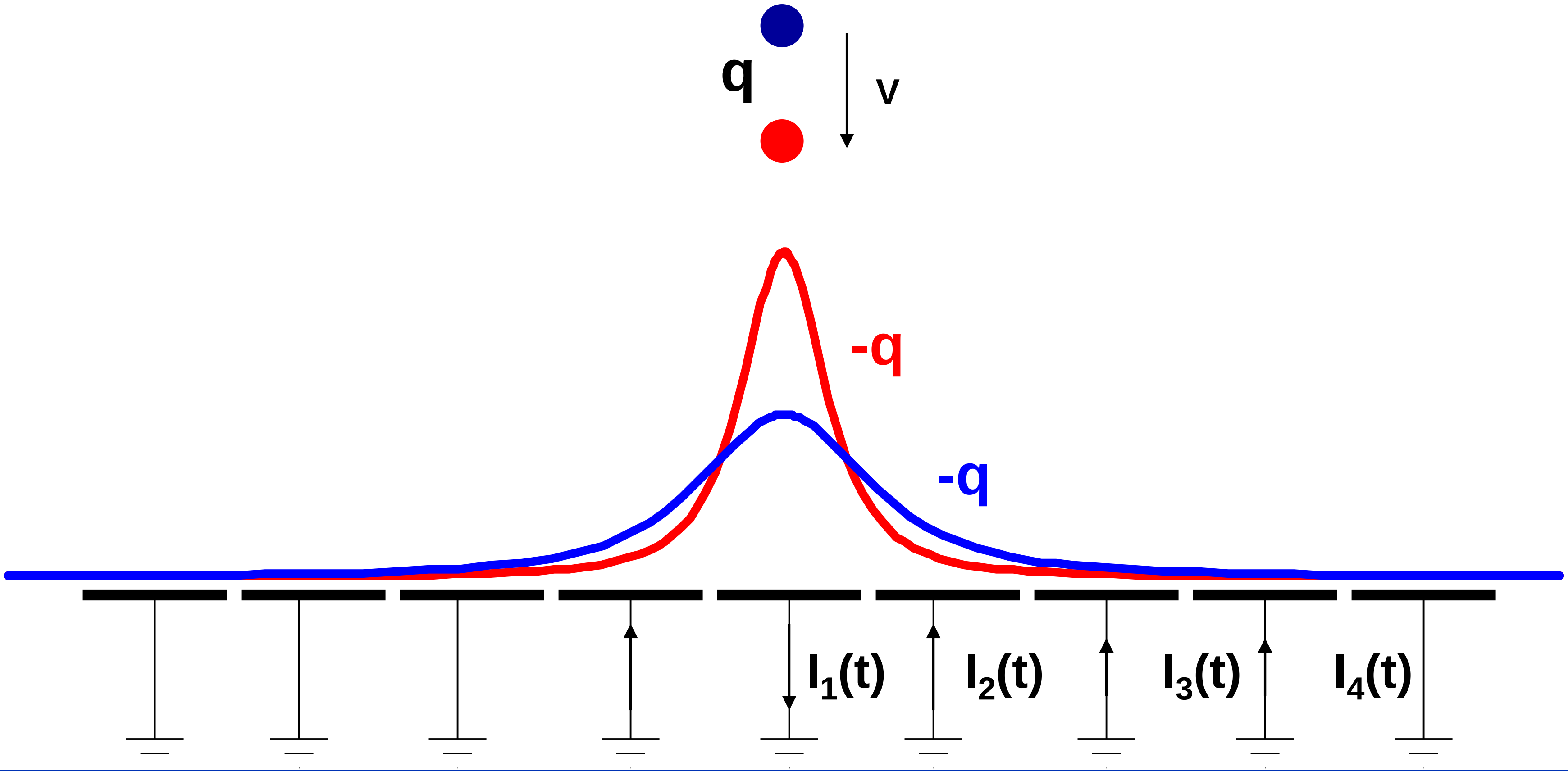
⇨ change of induced surface charge on electrode



Signal Formation

Movement of charge q (no external field needed!)

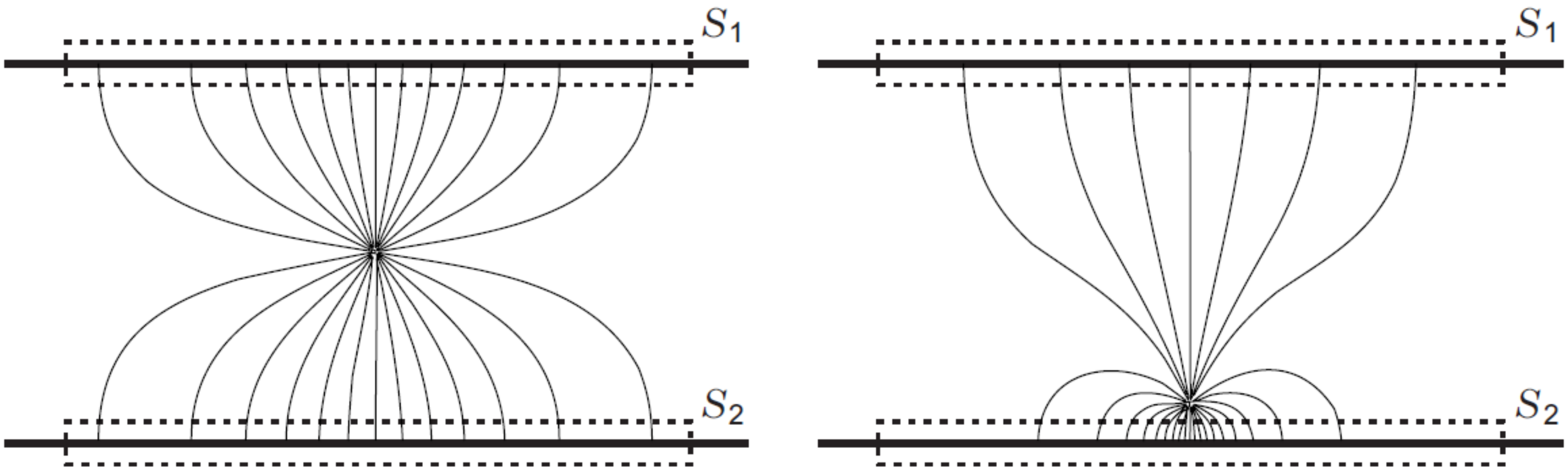
- ⇨ change of induced surface charge on electrode
- ⇨ current on segmented electrodes



Signal Formation

How to calculate the induced signal?

- solve Poisson equation with moving charge
- use mirror charges to get rid of electrodes
- use Ramo-Shockley theorem



[H. Spieler, Semiconductor detector systems, Oxford, 2005]

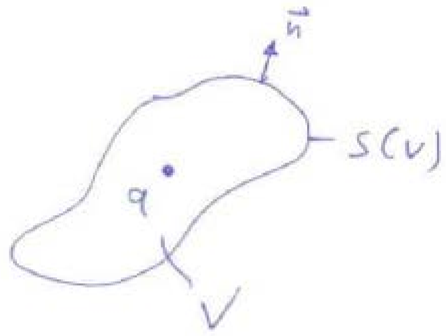
5.2 Ramo-Shockley Theorem

[W. Shockley, J. Appl. Phys. 9, 635 (1938), S. Ramo, Proc. IRE 27, 584 (1939)]

Calculation of signals induced on grounded electrodes:

- Gauss' Law: point charge inside closed surface

$$\oint \mathbf{E} \cdot \mathbf{n} \mathrm{d}a = \frac{1}{\varepsilon_0} \int_V \rho(\mathbf{x}) \mathrm{d}^3 x$$



- Green's 2[nd] theorem:

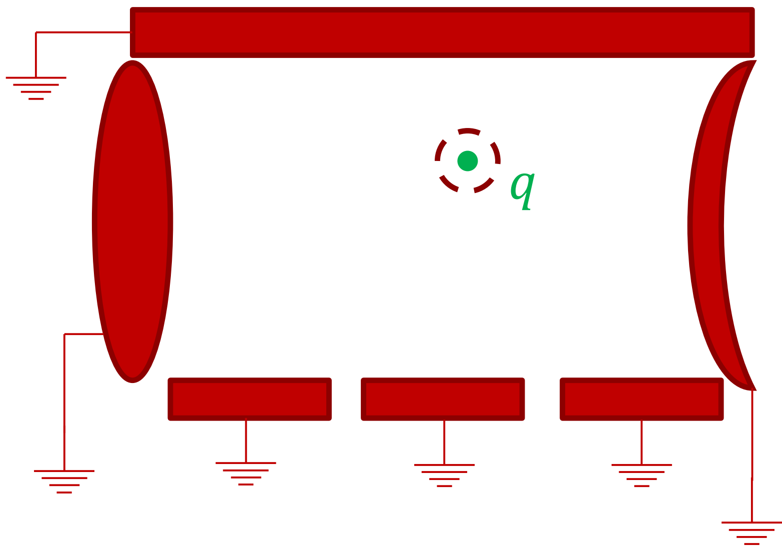
$$\int_V (\phi \Delta \psi - \psi \Delta \phi)\, \mathrm{d}^3 x = \oint_S \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right) \mathrm{d}a$$

with $\frac{\partial \phi}{\partial n} \equiv \boldsymbol{\nabla} \phi \cdot \mathbf{n} \,[= -\mathbf{E} \cdot \mathbf{n}]$

Ramo-Shockley Theorem

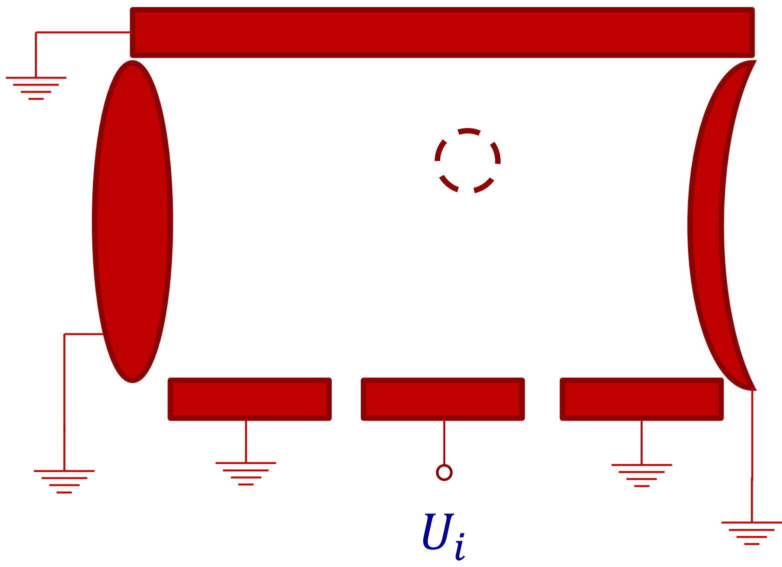
Consider detector volume delimited by grounded electrodes:

- Real situation: charge q at position $\boldsymbol{x}_0$, all electrodes grounded, i.e. $U_i = 0, i = 1,2, \dots$ ⇨ solution $\phi(\boldsymbol{x})$



Ramo-Shockley Theorem

Consider detector volume delimited by grounded electrodes:



- Real situation: charge q at position $\boldsymbol{x}_0$, all electrodes grounded, i.e. $U_i = 0, i = 1,2, \ldots$ ⇨ solution $\phi(\boldsymbol{x})$
- Auxiliary situations: charge q removed, all electrodes grounded except electrode i, i.e. $U_j = 0, j \neq i$ ⇨ solutions $\phi_i(\boldsymbol{x})$ with $\phi_i(\boldsymbol{x}) = U_i$ at surface of electrode i

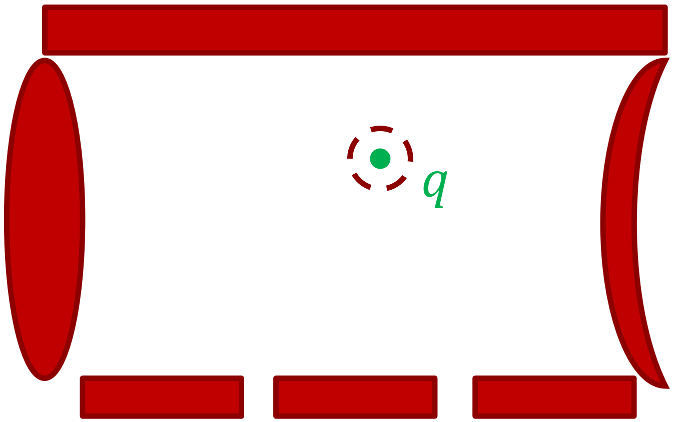
Space between electrodes and charge q free of charges

$$\Rightarrow \Delta\phi(\mathbf{x}) = 0$$

$$\Delta\phi_i(\mathbf{x}) = 0$$

Ramo-Shockley Theorem

Apply Green's 2nd theorem to volume V delimited by $S(V)$:



$$\underbrace{\int_V (\phi\Delta\phi_i - \phi_i\Delta\phi)\,\mathrm{d}^3x}_{=0} = \oint_{S(V)} \left(\phi\frac{\partial\phi_i}{\partial n} - \phi_i\frac{\partial\phi}{\partial n}\right)\mathrm{d}a$$

for every i

Solve surface integral by splitting it up into 3 parts

Ramo-Shockley Theorem

$$\Rightarrow Q_i = -q\frac{\phi_i(\mathbf{x}_0)}{U_i}$$

induced charge on electrode i by charge q at position x_0 when all electrodes are grounded

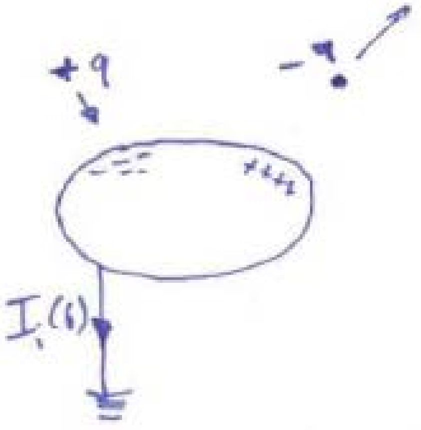
$\phi_i(x_0) =$ potential at point x_0 when point charge q is removed, electrode i is put to potential U_i and all other electrodes are grounded

weighting potential of electrode i

Point charge moving along trajectory $x_0(t)$

⇨ time-dependent induced charge on electrode i

⇨ current $I_i(t) = -\frac{\mathrm{d}Q_i(t)}{\mathrm{d}t}$



Sign convention: positive current points away from electrode

Ramo-Shockley Theorem

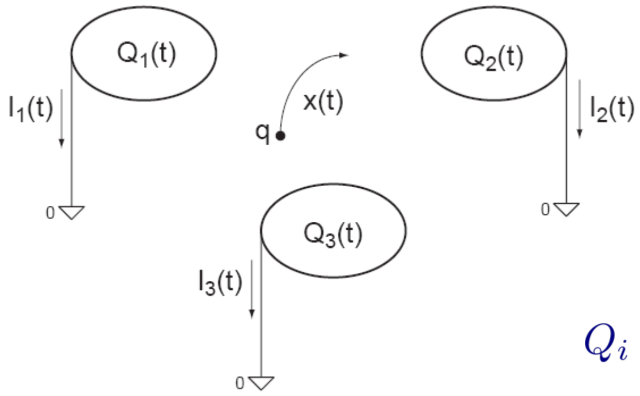
$$I_i(t) = -\frac{\mathrm{d}Q_i(t)}{\mathrm{d}t} = \frac{q}{U_i}\frac{\mathrm{d}}{\mathrm{d}t}\phi\left[\mathbf{x}_0(t)\right]$$

$$I_i(t) = \frac{q}{U_i}\boldsymbol{\nabla}\phi_i\left[\boldsymbol{x}_0(t)\right]\cdot\frac{\mathrm{d}\boldsymbol{x}_0(t)}{\mathrm{d}t} = -\frac{q}{U_i}\boldsymbol{E}_i\left[\boldsymbol{x}_0(t)\right]\cdot\boldsymbol{v}(t)$$

Ramo-Shockley Theorem

The current induced on a grounded electrode by a point charge q moving along a trajectory $\boldsymbol{x}_0(t)$ is $I_i(t)$, where $\boldsymbol{E}_i(\boldsymbol{x}_0)$ is the electric field in the case where the charge q is removed, electrode i is set to voltage U_i, and all other electrodes are grounded.

Ramo-Shockley Theorem

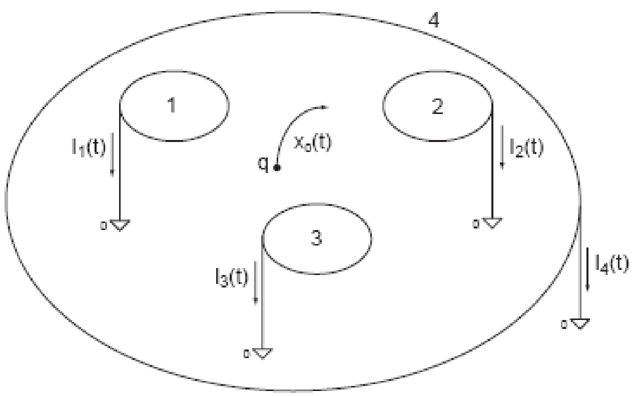


Consequences:

1. Charge induced on electrode i by a charge q moving from point 1 to 2 is

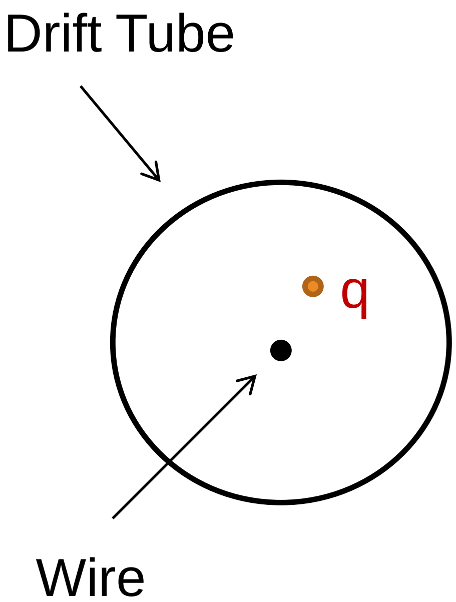
$$Q_i = \int_{t_1}^{t_2} I_i(t)\mathrm{d}t = -\frac{q}{U_i}\int_{t_1}^{t_2} \boldsymbol{E}_i[\boldsymbol{x}(t)]\,\dot{\boldsymbol{x}}(t)\mathrm{d}t = \frac{q}{U_i}\left[\phi_i(\boldsymbol{x}_1) - \phi_i(\boldsymbol{x}_2)\right]$$

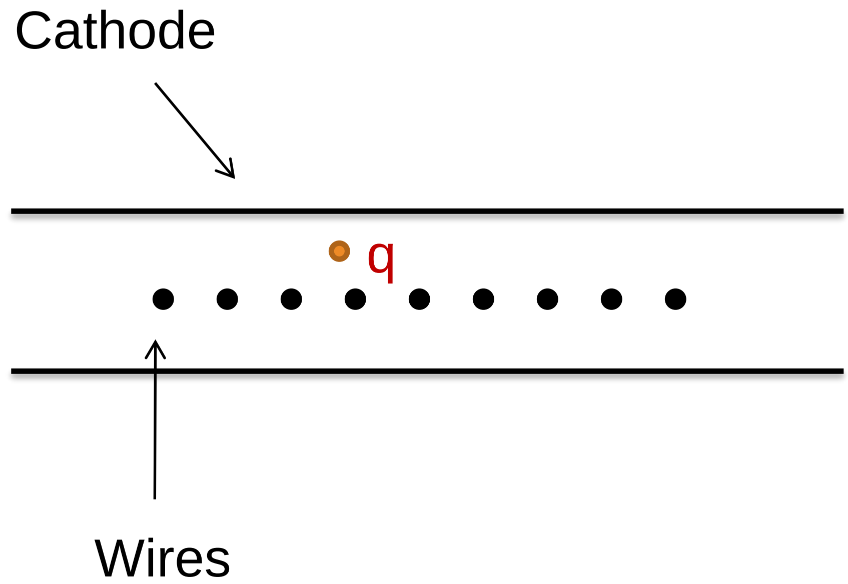
   and is independent of the actual path



2. Once all charges have arrived at the electrodes, the total induced charge in a given electrode is equal to the charge that has arrived at this electrode
3. In case there is one electrode enclosing all others, the sum of all induced currents is zero at any time

Ramo-Shockley Theorem



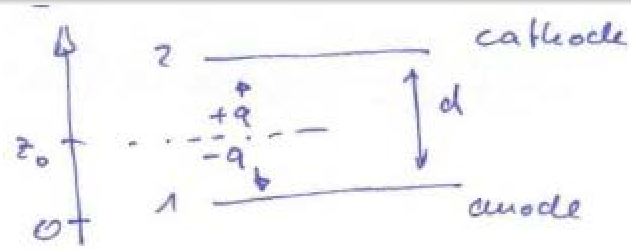


The sum of all induced currents is therefore zero at any time !

5.3 Examples

- Planar ionization chamber
- Planar strip detector
- Proportional wire counter

Planar Ionization Chamber



Weighting potential of anode

⇒ set anode to potential U_1

and ground cathode

$$\phi_1(z=0) = U_1$$

$$\phi_1(z) = \frac{U_1}{d}(d-z)$$

$$E_1(z) = \frac{U_1}{d}$$

Ramo: $I_1(t) = -\frac{q}{U_1} \cdot E_1[z(t)] \cdot \dot{z}(t)$

$$\dot{z} = u = \mu E$$

$$\frac{dz}{dt} = \mu \frac{U_1}{d}, \quad z(t=0) = z_0 \quad \Rightarrow \quad z(t) = \mu \frac{U_1}{d} \cdot t + z_0$$

Planar Ionization Chamber

Jons and e^- contribute to signal! , $u_{ion} \ll u_e$

Three regions in time:

- $t < t_e = \frac{z_0}{u_e}$ $\Rightarrow$ $z_{ion}(t) = z_0 + u_{ion}\, t$, $\dot{z}_{ion} = u_{ion}$

  $z_e(t) = z_0 - u_e\, t$, $\dot{z}_e = -u_e$

$$\Rightarrow I_1(t) = I_1^{ion}(t) + I_1^{e}(t) = -\frac{q}{U_1} \cdot \frac{U_1}{d} \cdot u_{ion} - \frac{(-q)}{U_1} \cdot \frac{U_1}{d} \cdot (-u_e)$$

$$= -\frac{q}{d}\left(u_{ion} + u_e\right)$$

- $t_e < t < t_{ion} = \frac{d - z_0}{u_{ion}}$ $\Rightarrow$ $z_{ion}(t) = z_0 + u_{ion}\, t$, $\dot{z}_{ion} = u_{ion}$

  $z_e(t) = 0$, $\dot{z}_e = 0$

$$\Rightarrow I_1(t) = -\frac{q}{d}\, u_{ion}$$

- $t > t_{ion}$ $\Rightarrow$ $I_1(t) = 0$

Planar Ionization Chamber

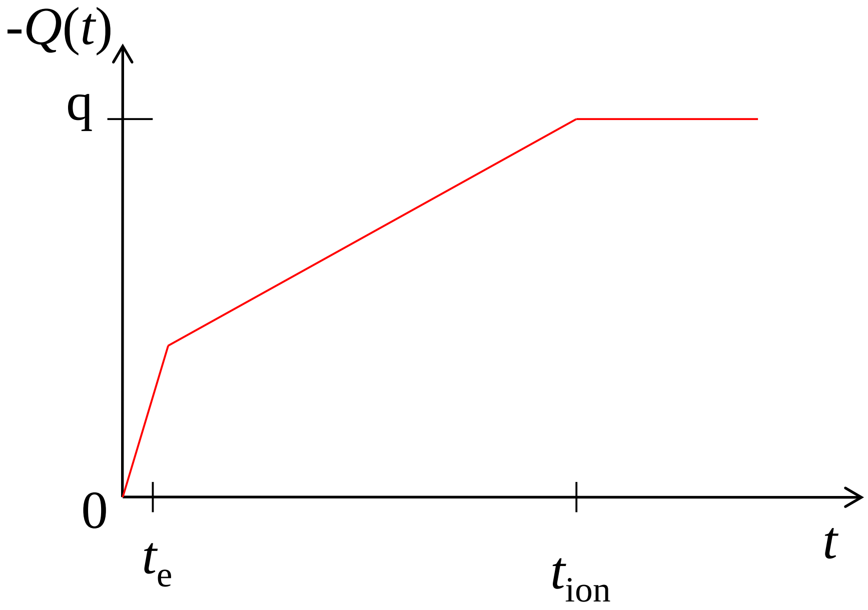
universitätbonn

Charge induced on anode :

$$Q_1(t) = \int_0^t I_1(t')\,dt'$$

- $t < t_e \Rightarrow Q_1(t) = -\frac{q}{d}(u_{ion} + u_e) \cdot t$
- $t_e < t < t_{ion} \Rightarrow Q_1(t) = -\frac{q}{d}(u_{ion} t + z_0)$
- $t > t_{ion} \Rightarrow Q_1(t) = -\frac{q}{d}(d + 0) = -q$ total induced charge on anode

Planar Ionization Chamber



- Steep linear rise until e- reach anode
- Slow linear rise until ions reach cathode, contribution from e- constant
- Constant for $t > t_{\text{ion}}$

Example: Ar at NTP

$$d = 5\,\text{cm}$$
$$E = 500\,\text{V/cm}$$

$$\Rightarrow t_e = 12.5\,\mu s$$
$$t_{\text{ion}} = 6\,\text{ms}$$

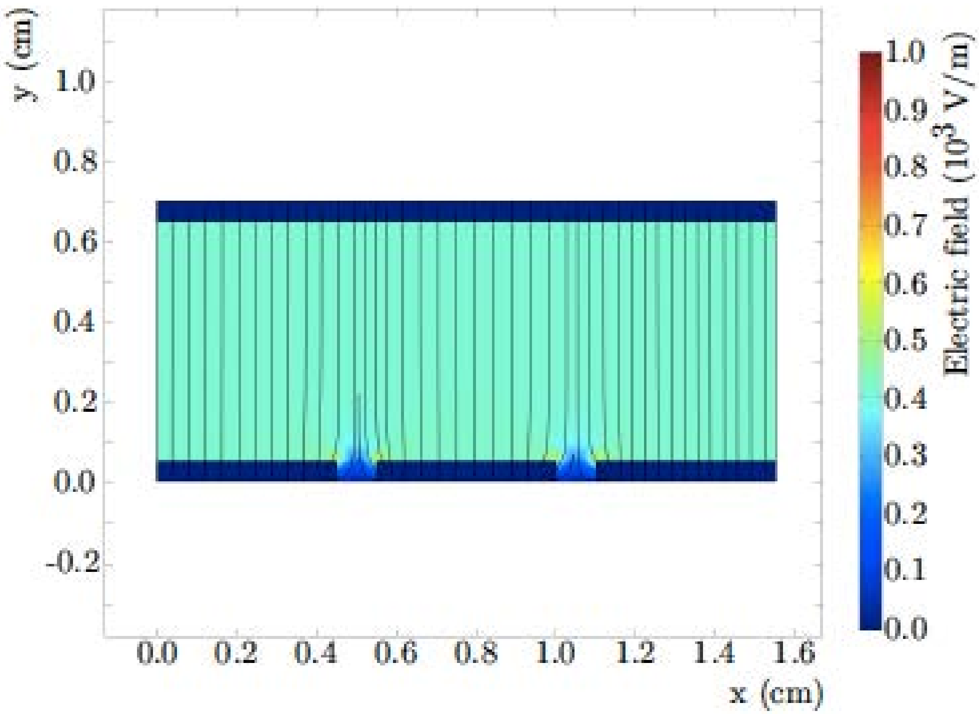
Planar Strip Detector

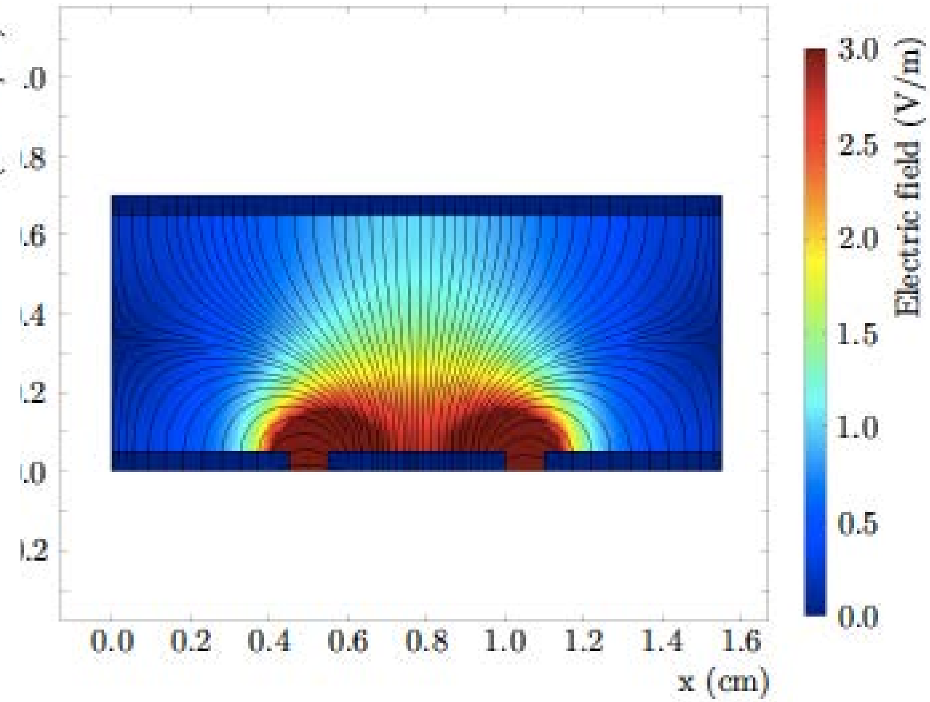
How do the real field and the weighting field look like?

Planar Strip Detector

Real field

Weighting field





M. Berger, PhD, Development Commissioning and Spatial Resolution Studies of a GEM based TPC, Munich, 2015]

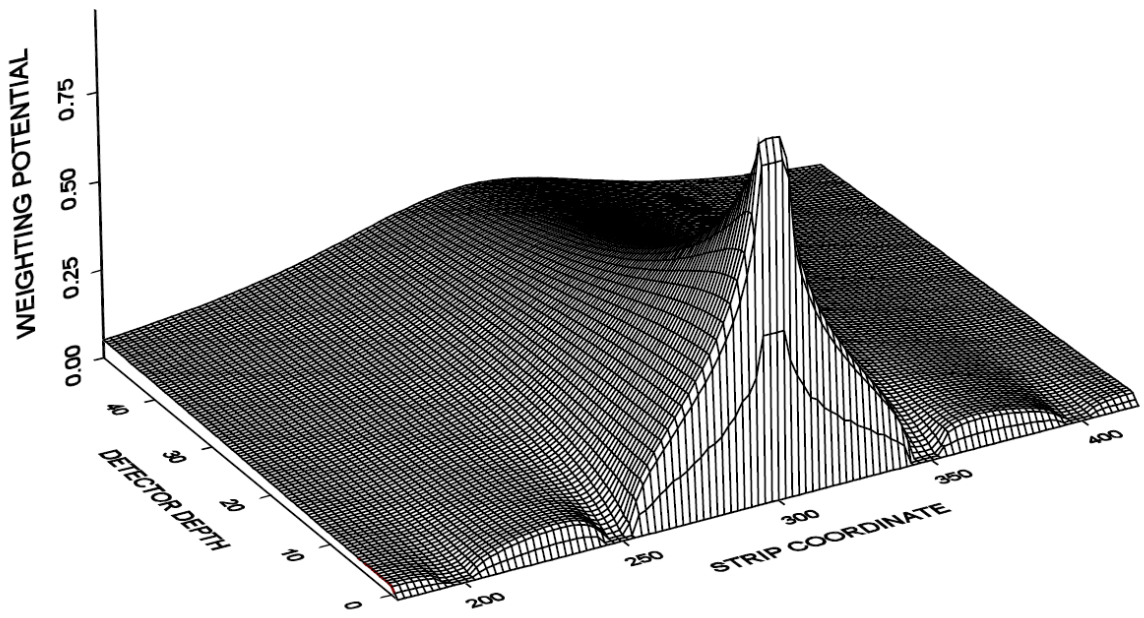
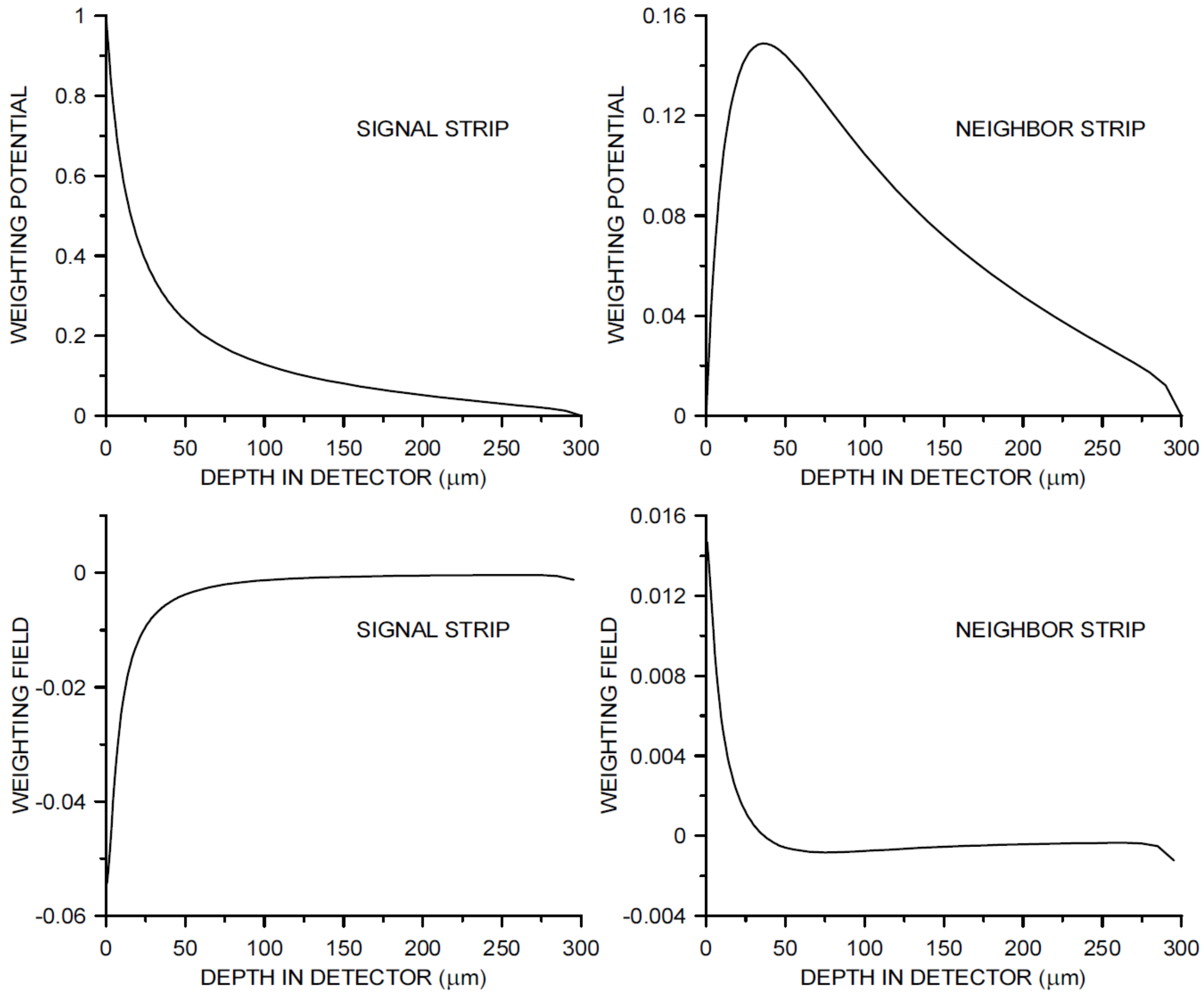


FIG. 2.29. Weighting potential for a 300 μm thick strip detector with strips on a pitch of 50 μm. The central strip is at unit potential and the others at zero. Only 50 μm of depth are shown.

[H. Spieler, Semiconductor detector systems, Oxford, 2005]

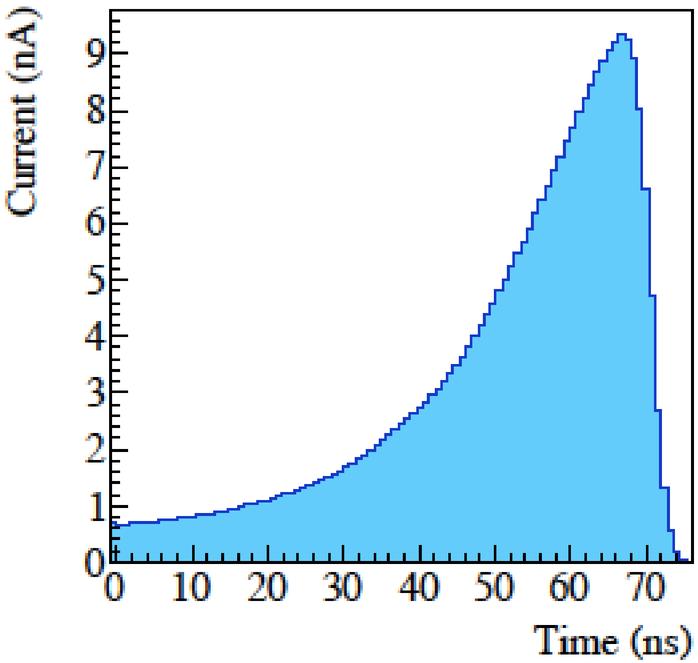
Planar Strip Detector



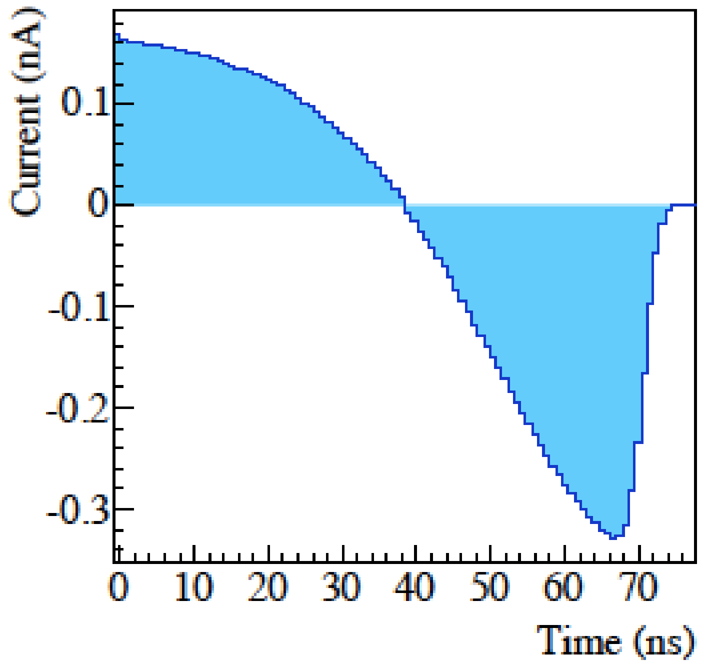
[H. Spieler, Semiconductor detector systems, Oxford, 2005]

Currents according to Ramo-Shockley theorem

Signal strip



Neighbouring strip



M. Berger, PhD, Development Commissioning and Spatial Resolution Studies of a GEM based TPC, Munich, 2015]

Cylindrical Wire

Analogously to plane ionization chamber: drifting charges induce currents

$$\phi(r) = \frac{\mathcal{C}U}{2\pi\varepsilon_0} \ln\left(\frac{b}{r}\right) \qquad \text{for} \qquad a < r < b,\ \mathcal{C} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

$$\phi(b) = 0$$

Ramo's theorem:

- weighting potential of wire ⇨ set to potential U_1, ground tube
- here as for parallel plate counter: weighting field ist equal to real field!

$$\phi_1(r) = \frac{U_1 \ln\left(\frac{b}{r}\right)}{\ln\left(\frac{b}{a}\right)} \qquad E_1(r) = \frac{U_1}{r \ln\left(\frac{b}{a}\right)}$$

Signal Shape of Cylindrical Wire

Contribution of e^- and ions: multiplication at r'

- electrons: $I_1^e(t) = -\frac{(-q)}{U_1} \cdot \frac{U_1}{r \ln(b/a)} \cdot \dot{r}_e(t)$

$$Q_1^e = \int_{a+r'}^{a} I_1^e(t)\,dt = \int_{a+r'}^{a} \frac{q}{r\ln(b/a)}\,dr$$

$$= -\frac{q}{\ln(b/a)} \ln\left(\frac{a+r'}{a}\right)$$

- ions: $I_1^{ion}(t) = -\frac{q}{U_1} \cdot \frac{U_1}{r\ln(b/a)} \cdot \dot{r}_{ion}(t)$

$$Q_1^{ion} = \int_{a+r'}^{b} I_1^{ion}(t)\,dt = -\frac{q}{\ln(b/a)} \ln\left(\frac{b}{a+r'}\right)$$

Ratio of contributions:

$a = 10\,\mu m$, $b = 10\,mm$, $r' = 1\,\mu m$ $\Rightarrow$ $\frac{Q_1^{e}}{Q_1^{ion}} < 1\%$

$\Rightarrow$ signal almost entirely generated by ions,

e^- contribution negligible

Time development: assume all ions are produced at $r=a$
(i.e. at the wire)

Ion trajectory: $u = \mu E$

$$\frac{dr(t)}{dt} = \mu \frac{U}{r(t)\ln(b/a)} \Rightarrow r(t) = a\sqrt{1+\frac{t}{t_0}}, \quad t_0 = \frac{a^2 \ln(b/a)}{2\mu U}$$

with $r(0) = a$

$$\dot{r}(t) = \frac{1}{2} a \left(1+\frac{t}{t_0}\right)^{-1/2} \cdot \frac{1}{t_0}$$

$$\Rightarrow I_1(t) = -\frac{q}{U_1} \cdot \frac{U_1}{r\ln(b/a)} \cdot \dot{r} =$$

$$= -\frac{q}{2\ln(b/a)} \cdot \frac{1}{t+t_0}$$

$\Rightarrow$ negative wire signal

hyperbolic form with characteristic time constant $t_0 \sim$ few ns

Signal Shape of Cylindrical Wire

Induced charge at time t:

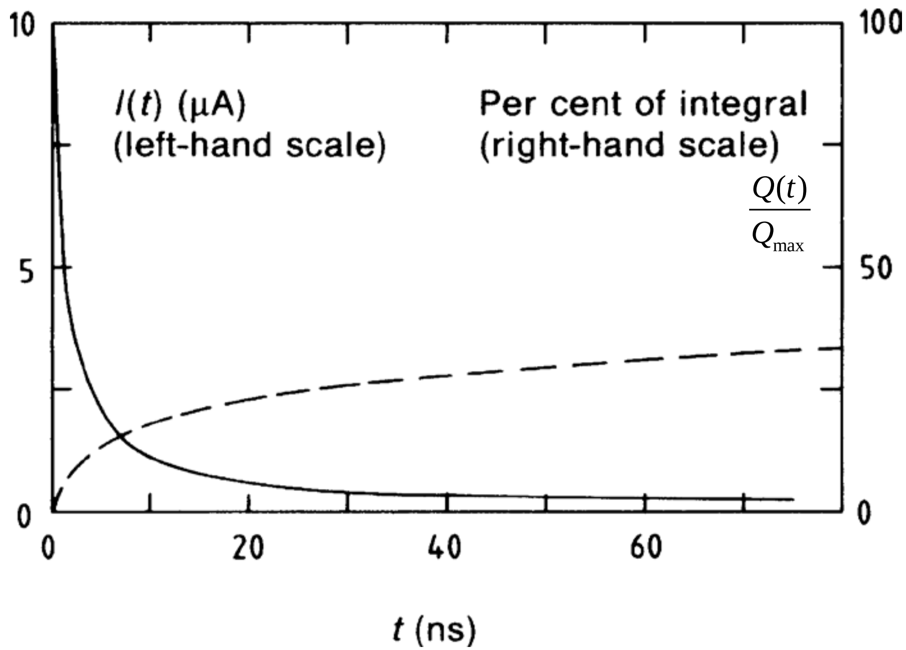
$$Q_1(t) = \int_0^t I_1(t')\,dt' = -\frac{q}{2\ln(b/a)} \cdot \ln\left(1 + \frac{t}{t_0}\right)$$

Once ions have arrived at tube wall (at $t = t_{max}$):

$$\Rightarrow Q_1(t_{max}) = -q$$

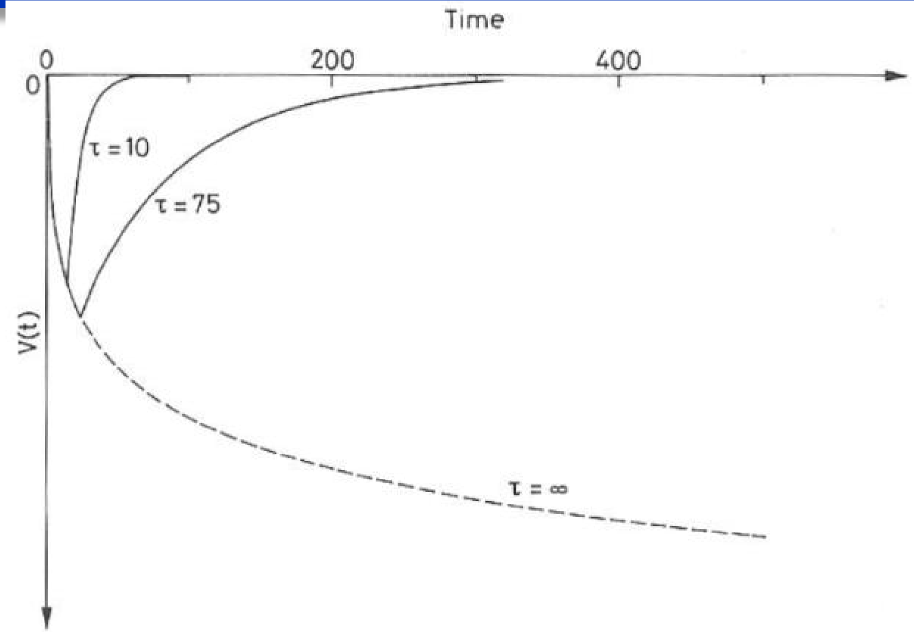
Current induced on cathode: $I_2(t) = -I_1(t)$

Signal Shape



$t_0 = 1.25$ ns
b/a = 500
$q = 10^6$

[W. Blum et al., Particle Detection with Drift Chambers, Springer (2008)]



Q(t), U(t) with RC element

[W.R. Leo, Techniques for Nuclear and Particle Physics Experiments, Springer (1994)]

Arguing with Energy?

Many textbooks deduce the signal shape using energy conservation

- Signal is calculated using energy balance:

  Energy gained by charge in electric field =

  change of energy stored in capacitor
- In some special cases, this argument gives the correct result, e.g. for a 2-electrode system because there the weighting field and the real field are equal.
- But the argument is very misleading:
  - An induced current signal has nothing to do with energy. In a gas detector the electrons are moving at constant speed in a constant electric field, so the energy gained by the electron in the electric field is lost into collisions with the gas, i.e. heating of the gas.
  - In absence of an electric field, the charge can be moved across the gap without using any force and currents are flowing.

Summary: Signal Calculation

1. Calculate particle trajectory $x_0(t)$ in the „real" electric field
2. Remove all impedance elements, ground the electrodes and calculate currents induced by moving charge on grounded electrodes

$$I_i(t) = \frac{q}{U_i} \nabla \phi_i \left[\boldsymbol{x}_0(t) \right] \cdot \frac{\mathrm{d}\boldsymbol{x}_0(t)}{\mathrm{d}t} = -\frac{q}{U_i} \boldsymbol{E}_i \left[\boldsymbol{x}_0(t) \right] \cdot \boldsymbol{v}(t)$$

3. Place these currents as ideal current sources on a circuit where the electrodes are simple nodes and the mutual electrode capacitances are added between the nodes. They are calculated from the weighting field by

$$c_{nm} = \frac{\varepsilon_0}{V_w} \oint_{\boldsymbol{A}_n} \boldsymbol{E}_m(\boldsymbol{x}) d\boldsymbol{A}$$

$$C_{nn} = \sum_m c_{nm} \qquad C_{nm} = -c_{nm} \quad n \neq m$$