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Dilaton and PseudoDilaton

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Chapter 3 (pp. 59-75) in
At the Frontier of Spacetime – Scalar-Tensor Theory, Bells Inequality, Mach's Principle, Exotic Smoothness
Dedicated to the 80th birthday of Carl Brans, Springer, 2016
also in arXiv 1512.01360 gr-qc

Reduced Planckian Unit system

$$c = \hbar = M_{\mathrm{P}}(= (8\pi G)^{-1/2}) = 1$$

Units of length, time, energy re-expressed in conventional units

$$8.10 \times 10^{-33}\mathrm{cm}, \quad 2.70 \times 10^{-43}\mathrm{sec}, \quad 2.44 \times 10^{18}\mathrm{GeV}$$

Today's age of the Universe

$$t_0 \approx 1.37 \times 10^{10}\mathrm{y} \approx 10^{60}.$$

Discovery of **Accelerating Universe**

Rises et al (98), Perlmutter et al (99)

Toward the end of the last century, they newly discovered **Dark-Energy DE**, with the density $\rho_{\rm de}$, in addition to an ordinary matter density $\rho_{\rm m}$ (baryonic, leptonic, ...), in the conventional representation

$$3H^2 = \Lambda_{\rm de}\rho_{\rm de} + \Lambda_{\rm m}\rho_{\rm m} \quad \text{with}$$

$$\rho_{\rm de} \sim \rho_{\rm m} \sim t_0^{-2}, \quad \text{but} \quad \Lambda_{\rm de} \sim 0.3, \quad \Lambda_{\rm m} \sim 0.7;$$

in place of the older one, $3H^2 = \rho_{\rm m}$, with $\Lambda_{\rm de} = 0, \Lambda_{\rm m} = 1$. The condition $\Lambda_{\rm de} > 0$ might be the most simplified indication of the extra acceleration, but is far from reaching the far deaper content of the concept DE. I will re-interpret the issues in the language of the **Scalar-Tensor Theory, STT.**

Scalar-Tensor-Theory (STT) with $\Lambda \sim 1$ added

Jordan(55), Brans and Dicke(61)

Two **conformal frames**, **Jordan and Einstein Frs** ;

- **Jframe** $\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_{\rm m} - \Lambda\right)$

  Scalar field ϕ, **Nonminimal coupling**, related to Dirac's *variable G* , and $\epsilon = \pm 1$.

- **Eframe** $\mathcal{L} = \sqrt{-g_*}\left(\frac{1}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + L_{*\rm m} - \Lambda\exp(-4\hat{\zeta}\sigma)\right)$

  Einstein term with a *constant G*, also **Canonical** scalar field σ.

Applying the **Conformal trsf**

$$g_{\mu\nu} = \Omega(x)^{-2}g_{*\mu\nu}, \quad \phi = \xi^{-1/2}\Omega, \quad \text{with} \quad \Omega = \exp(\hat{\zeta}\sigma)$$

supplemented by $\zeta = 6 + \epsilon\xi^{-1}$, to $\mathcal{L}$ in JF turns out to re-express the same Lagrangean in the form in EF.
Hats inidicate the correct dimensionality of the constants;

$$\hat{\xi} = \xi M_{\rm P}^{-2}, \quad \hat{\zeta} = \zeta M_{\rm P}^{-1}$$

An example of Λ-**cosmology** (Dolgov; Fujii and Maeda); applied to simplified *asymptotic* solution in Robertson-Walker metric, for the assumed *spatially uniform* and radiation-dominated Universe in JF:

$$6\varphi H^2 = \frac{1}{2}\epsilon\dot{\phi}^2 + \Lambda + \rho_{\rm m} - 6H\dot{\varphi}; \quad \text{where } H = \dot{a}/a;\ \varphi = \frac{1}{2}\xi\phi^2,$$

$$\ddot{\varphi} + 3H\dot{\varphi} = 4\zeta^2\Lambda$$

$$\dot{\rho}_{\rm m} + 4H\rho_{\rm m} = 0$$

Possible solutions and parameters

$$H=0, a=\text{const}; \quad \phi=\xi^{-1/2}\Omega=\sqrt{\frac{4\Lambda}{6\xi+\epsilon}}t; \quad \rho_{\rm m}=\text{const}=-3\Lambda\frac{2\xi+\epsilon}{6\xi+\epsilon}$$

Then we may derive the Accelerating Univ $3H_*^2 \sim \mathcal{O}(\Lambda)^2 t_*^{-2}$ in EF. But the realistic $\rho_{\rm m}$ may not be purely matter-empty, but may provide us with a small *seed* of the matter particles, as represented by a massive scalar field Φ, for example;

$$-\mathcal{L}_m = -\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2\right), \quad m = \text{const}$$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2 = \Omega^{-2} ds_*^2 = \Omega^{-2}\left(-dt_*^2 + a_*^2 d\vec{x}^2\right)$$

$-dt_*^2$ is defined as the time-component of ds_*^2.

$$dt = \Omega^{-1} dt_* \Rightarrow t dt \sim dt_* \Rightarrow t_* \sim t^2,$$

$$a_* = \Omega a \sim t \sim t_*^{1/2} \Longrightarrow$$ We have moved to **live in EF** !

We find

$$-\sqrt{-g} g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi = -\Omega^{-4}\sqrt{-g_*}\Omega^2 g_*^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi$$

$$\approx \sqrt{-g_*} g_*^{\mu\nu}\partial_\mu(\Omega^{-1}\Phi)\partial_\nu(\Omega^{-1}\Phi) = -\sqrt{-g_*} g_*^{\mu\nu}\partial_\mu \Phi_* \partial_\nu \Phi_*$$

where

$$\Phi_* \approx \Omega^{-1}\Phi \quad \text{or} \quad \Phi_* \approx \Omega^{1-d}\Phi, \quad d = D/2$$

Then

$$\sqrt{-g} m^2 \Phi^2 \approx \sqrt{-g_*}\Omega^{-4} m^2 \Omega^2 \Phi_*^2 \equiv \sqrt{-g_*} m_*^2 \Phi_*^2$$

$$\Longrightarrow \quad m_* = m\Omega^{-1} \sim m a_*^{-1} \sim t_*^{-1/2}$$

A *DECREASING* particle mass is totally INCONSISTENT with today's astronomy, measuring the cosmological distances in**UNITS** of microscopic inverse mass. We have no way of knowing how the units themselves change (**Own-Unit Insensitivity Principle**).

No change of Units implies *Constant* **UNITS** — Rydberg const

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$$g_{\mu\nu} = \Omega^{-2} g_{*\mu\nu}, \quad \sqrt{-g} = \Omega^{-4}\sqrt{-g_*}, \quad \Omega \sim \sqrt{\Lambda} t$$

Another effort to make the theoretical ideas to *meet the real world*. Instead of a mass introduced in JF, consider an **interaction** term with a dimensionLESS coupling constant h;

$$-\mathcal{L}_{\rm int} = \frac{1}{2}\sqrt{-g}\,h\phi^2\Phi^2,$$

— Absence of dimensional constants is called *scale invariance*,
though not precisely in the same as in usual 3-dim terminology

which is sent to EF;

$$-\mathcal{L}_{\rm int} = \frac{1}{2}\sqrt{-g_*}\Omega^{-4}h\hat{\xi}^{-1}\Omega^2\Omega^2\Phi_*^2 \equiv \frac{1}{2}\sqrt{-g_*}m_*^2\Phi_*^2,$$

with $m_*^2 = h\hat{\xi}^{-1}$

The newly defined m_* now stays CONSTANT due to a complete cancellation among Ω's. – **Spontaneously created mass** – of a more general nature, due to **spontaneously broken scale (or dilatation) invariance.**

Noether current for Scale trsf of $\mathcal{L}$ in JF is obtained from

$$\ell J^{\mu} = \sum_{u} \frac{\partial \mathcal{L}}{\partial \partial_{\mu} u} \delta u; \quad \delta g_{\rho\sigma} = 2\ell g_{\rho\sigma}, \quad \delta\phi = \ell\phi, \quad \delta\Phi = -\ell\Phi$$

yielding (Fujii and Maeda, CambBook, App M)

$$J^{\mu} = \frac{1}{2}\sqrt{-g} g^{\mu\nu} \partial_{\nu} \left(\xi\zeta^{-2}\phi^{2} + \Phi^{2} + \Lambda \right)$$

Moving to EF and imposing the conservation law $\partial_{\mu} J^{\mu} = 0$, we reach

$$\zeta^{-1} \Box_{*} \sigma = -\left(g_{*}^{\mu\nu} \partial_{\mu} \Phi_{*} \partial_{\nu} \Phi_{*} + \Lambda \right) \to \sigma = -\Box_{*}^{-1} \zeta \left(\ldots\ldots \right)$$

indicating σ as a **massless Nambu-Goldstone Boson**
—**"Dilaton"** —— unique to **DE**-related phenomena

Naturally extending to the **Higgs Sector** (in the Standard Model) by adding the quartic term, in *D dimensions* in EF, also following Nambu-Jona-Lasinio's *suggestion*

$$-\mathcal{L}_{\rm H} = \sqrt{-g}\left(\frac{1}{2}h\phi^2\Phi^2 + \frac{\lambda}{4!}\Phi^4\right) = \sqrt{-g_*}\Omega^{D-4}\left(\frac{1}{2}\tilde{m}^2\Phi_*^2 + \frac{\lambda}{4!}\Phi_*^4\right),$$

substituted from

$$\sqrt{-g} = \sqrt{-g_*}\Omega^{-D}, \tilde{m}^2 = h\hat{\xi}^{-1/2}, \phi = \hat{\xi}^{-1/2}\Omega^{d-1}, \Phi = \Omega^{d-1}\Phi_*,$$
$$\Omega = \exp\left(\hat{\zeta}\sigma\right)$$

Also shifting $\tilde{\Phi} = \Phi_* - v$ such that no term linear in $\tilde{\Phi}$, we obtain

$$-\mathcal{L}_{\rm H} = \exp\left(2\hat{\zeta}(d-2)\sigma\right)\sqrt{-g_*}\mathcal{V}$$

$$\mathcal{V} = \frac{1}{2}m^2\tilde{\Phi}^2 + \frac{1}{2}m\sqrt{\frac{\lambda}{3}}\tilde{\Phi}^3 + \frac{\lambda}{4!}\tilde{\Phi}^4, \quad \text{with } m^2 = -2\tilde{m}^2 = \frac{\lambda}{3}v^2.$$

$$m(=m_*) = 126\mathrm{GeV}$$

The mass term of σ from $\sim \sigma^2$ in

$$\exp\left(2\hat{\zeta}(d-2)\sigma\right) \approx 1 - 2\hat{\zeta}(d-2)\sigma + 2\hat{\zeta}^2(2-d)^2\sigma^2$$

The **1-loop diagram** then arises as a product with the term in $\mathcal{V}$ which provides with (**Dimensional Regularization**)

$$\int d^D \frac{1}{(k^2+m^2)^2} = i\pi^2 (m^2)^{d-2} \frac{\Gamma(d)\Gamma(2-d)}{\Gamma(2)} \approx i\pi^2 \Gamma(2-d),$$

leading to

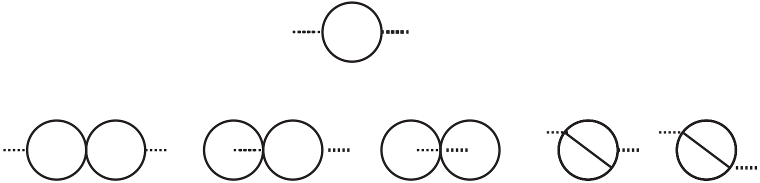
$$\mathcal{I}_1 \sim (2-d)^2 i\pi^2 m^4 \Gamma(2-d) \sim 2-d \to 0,$$

where use has been made of **Classical-Quantum Interplay** or **Anomaly**

$$(2-d)\Gamma(2-d) \to 1, \quad \text{as} \quad d \to 2,$$

or Peaceful use of Quantum Divergences — Y. Takahashi

Figure



(a) (b) (c) (d) (e)

2-loop amplitudes

$$\mathcal{I}_a = i(2\pi)^{-4}\lambda m^4 \left[2\hat{\zeta}(2-d)\sigma\right]^2 \int d^D k \frac{1}{(k^2+m^2)^2} \int d^D k' \frac{1}{(k'^2+m^2)^2}$$

$$= -i\frac{1}{4}\lambda\hat{\zeta}^2 m^4\sigma^2$$

$$\mathcal{I}_2 = \mathcal{I}_a + 2\mathcal{I}_b + \mathcal{I}_c + 2\mathcal{I}_d + 2\mathcal{I}_e = -4i\lambda\hat{\zeta}^2 m^4\sigma^2 \equiv -i(2\pi)^4\mu^2\sigma^2$$

$$\mu = \frac{\sqrt{\lambda}}{2\pi^2}\hat{\zeta} m^2 \quad \textbf{Pseudo-Dilaton mass}$$

Conventional perturbation terms

An example of a single-loop inserted between the two loops of (a)

$$\mathcal{R} \equiv -i(2\pi)^4\lambda^2[i(2\pi)^4]^{-2}\int d^D k\frac{1}{(k^2+m^2)^2} \approx -\frac{\lambda^2}{16\pi^2}\Gamma(-\delta)\left(m^2\right)^{\delta}$$

where $\delta = d - 2$ with

$$\left(m^2\right)^{\delta} = \exp\left(-\frac{\delta}{\delta_m}\right), \quad \delta_m \equiv \frac{1}{-\ln m^2} \approx \frac{1}{75} \approx 0.013$$

The behavior $\Gamma(\delta) \sim \delta^{-1}$ for $\delta \to 0+$ should be re-regularized by mapping the point 0 onto a cutoff δ_c which might be chosen $\sim \delta_m$ yielding (also $\lambda \sim 1$)

$$|\mathcal{R}(\delta_c)| = |\mathcal{R}(\delta_m)| \approx \frac{-\ln m^2}{16\pi^2}e^{-1} \approx \frac{0.48}{2.72} \approx 0.18 \lesssim \mathcal{O}(1)$$

$|\mathcal{R}| \ll 1$ might imply an approximate **CQI-dominance**.

Nambu-Jona-Lasinio formulation re-discovered by requiring D playing the dual role.

Standard Model $\Rightarrow$ $v = 246\text{GeV}, \quad \lambda = 0.787.$

$$\mu = \frac{\sqrt{\lambda}}{2\pi^2}\hat{\zeta} m^2 = \frac{\sqrt{\lambda}}{2\pi^2}\frac{(1.26\times 10^2)}{2.44\times 10^{18}} \approx \frac{\sqrt{\lambda}}{2\pi^2}\times 6.51\zeta\mu\text{eV}.$$

How accurately is ζ determined?

$1/4 < \zeta^2 < \infty$ from $\epsilon = -1$

$\zeta \approx 1.85$ in our fit, Fujii and Maeda, CambBook

$\zeta^2 = 1/2$ for SuperStringModel by Green, Schwarz, Witten

$$\Rightarrow \zeta \approx (0.5 \sim 2.0) \quad \Rightarrow \mu \approx (0.15 \sim 0.59)\mu\text{eV}$$

For the experimental searches for σ, we have proposed (Fujii and Homma) to look into the scattering process of $\gamma\gamma$ implemented by the **exchange** of σ. We start with computing the decay rate of σ into $\gamma\gamma$, by studying the gauge-invariant portion of the photon self-energy part due to the fermion-loop, caused by the electromagnetic coupling

$$\mathcal{L}_{\rm em} = -ieb\left(\bar{\psi} b^{i\mu}\gamma_i\psi\right) A_\mu,$$

or its EF expression in D dimensions,

$$\mathcal{L}_{*\rm em} = -ieb_*\left(\bar{\psi}_* b_*^{i\mu}\gamma_i\psi_*\right) A_{*\mu}\Omega^{d-2},$$

as shown by (6.93) in CambBook, though of course with α coresponding to each quark generation.
We then consider the ψ_* loop leaving the part of A_* and Ω^{d-2} behind separately for the moment, first obtaining

$$\sqrt{-g_*}\frac{-\alpha}{3\pi}\left(k^\mu k^\nu - k^2\eta^{\mu\nu}\right)\Gamma(2-d),$$

also now sandwiched by $A_{*\mu}$ and $A_{*\nu}$, to inherit the effect of the gauge-invariance of the photon self-energy part, together with the divergence as indicated by $\Gamma(2-d)$;

$$\mathcal{M}_{\rm loop} = \sqrt{-g_*} A_{*\mu} \frac{-\alpha_*}{3\pi} \left(k^\mu k^\nu - k^2 \eta^{\mu\nu} \right) A_{*\nu} \Gamma(2-d).$$

At this point we may use an identity

$$A_{*\mu} \left(k^\mu k^\nu - k^2 \eta^{\mu\nu} \right) A_{*\nu} = \frac{1}{4} F_{*\mu\nu} F_*^{\mu\nu}.$$

We next use

$$\Omega^{d-2} = \exp\left(2\hat{\zeta}(d-2)\sigma \right), \quad \text{with} \quad \hat{\zeta} = \zeta M_{\rm P}^{-1},$$

to pick up the linear term $\sim (d-2)$ to be multiplied by $\Gamma(2-d)$ in $\mathcal{M}_{\rm loop}$ to yield

$$\mathcal{L}_{*\sigma\gamma\gamma} = -\sqrt{-g_*} \frac{2\alpha_*}{3\pi M_{\rm P}} \zeta \frac{1}{4} \sigma F_{*\mu\nu} F_*^{\mu\nu},$$

in which the coupling now emerges as a finite, defined uniquely by $\hat{\zeta}$, one of the most fundamental parameters in STT.
Notice also the absence of the mass of the loop fermions, which might result in *the non-reduced* contribution from the higher masses, at the same time, making the leptons relatively enhanced.

Suppose the 2γ system is dominated by σ in the *s*-channel. For sufficiently small CM energies, the scattering amplitude is dominated by the resonance lying at $\mu = m_\sigma$, still the height of the amplitude reaching the (maximized) unitarity limit (*BW formula*), no matter how weak the force might be, leaving only the width *narrowed*. In this sense the scattering amplitude is dominated entirely by unitarity, a unique feature in which further elaborations have been proposed (Homma) by applying the 4-colour mixing technique, also preparing the **induced** beam for the outgoing photons, also for the enhanced results.

This is a place now to stop my talk today. I emphasize two points ;

- I appealed to OUIP and CQI, then met the spontaneous symmetry breaking in terms of Dilaton for the first time in our Cosmlogical studies. I used the spacetime dimensionality further to cause a massless Dilaton $\Longrightarrow$ a massive ps-Dilaton.
- I required CQI, then reaching the unique and finite estimates of the decay PsDilaton $\rightarrow \gamma\gamma$, independent of the loop fermion mass.

I emphasize both points based on our rather elementary analyses of Particle Theory techniques, still reaching so *fundamentally important* aspects in Cosmology then deserving extremely *highly-intensified laser beams* which might, however, *cost* as much as one of the huge high-energy particle accelerators.