MAX-PLANCK-GESELLSCHAFT

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Strong-field QED in tightly focused laser fields

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Outline

- Introduction to strong-field QED and motivation of the present work
- Ultrarelativistic electron states and propagator in tightly focused laser fields
- First-order strong-field QED processes in tightly focused laser fields
- Conclusions

For more details:

1. A. Di Piazza, Phys. Rev. Lett. **113**, 040402 (2014)
2. A. Di Piazza, Phys. Rev. A **91**, 042118 (2015)
3. A. Di Piazza, Phys. Rev. Lett. **117**, 213201 (2016)
4. A. Di Piazza, Phys. Rev. A **95**, 032121 (2017)

Vacuum QED vs strong-field QED

Success of vacuum perturbation theory:

$g_{\mathrm{exp}}/2 = 1.00115965218073(28)$

$g_{\mathrm{the}}/2 = 1.00115965218113(11)(37)(02)(77)$

Strength: $\alpha = e^2/\hbar c = 7.3\times10^{-3}$ (Fine-structure constant)	Energy: $mc^2 = 0.511$ MeV (Electron rest energy)
Length: $\lambda_C = \hbar/mc = 3.9\times10^{-11}$ cm (Compton wavelength)	Field: $E_{cr} = m^2c^3/\hbar\vert e\vert = 1.3\times10^{16}$ V/cm $B_{cr} = m^2c^3/\hbar\vert e\vert = 4.4\times10^{13}$ G (Critical fields of QED)

Challenges of strong-field QED

Experiment: Critical fields of QED are "large"	Theory: Few tractable field configurations

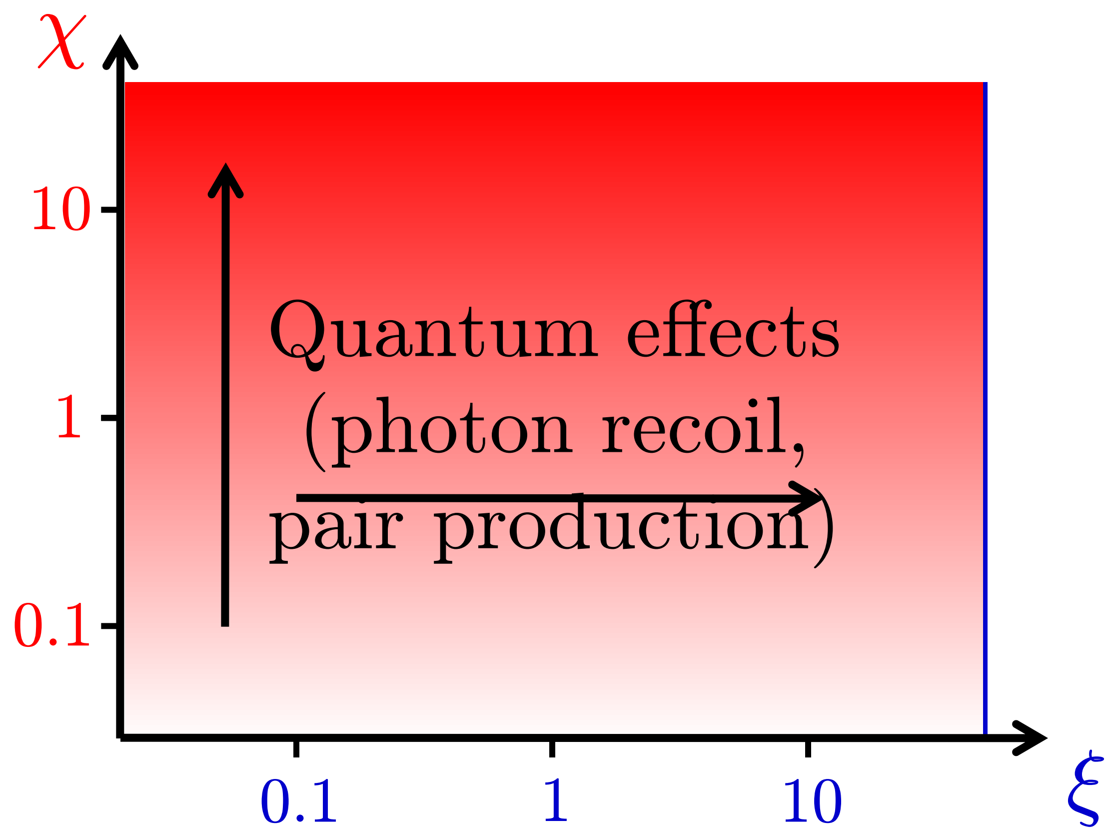
Regimes of QED in a strong laser field

An electron with initial energy ε_0 head-on collides with a plane wave with amplitude E_L and angular frequency ω_L (wavelength λ_L)

Relevant Lorentz- and gauge-invariant parameters (Ritus 1985, Di Piazza et al., 2012):

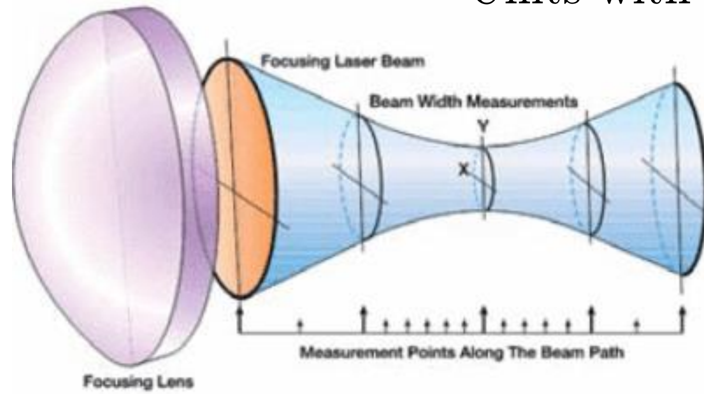
$$\xi = \frac{1}{2\pi}\frac{|e|E_L\lambda_L}{mc^2} = \frac{|e|E_L\lambda_C}{\hbar\omega_L}$$

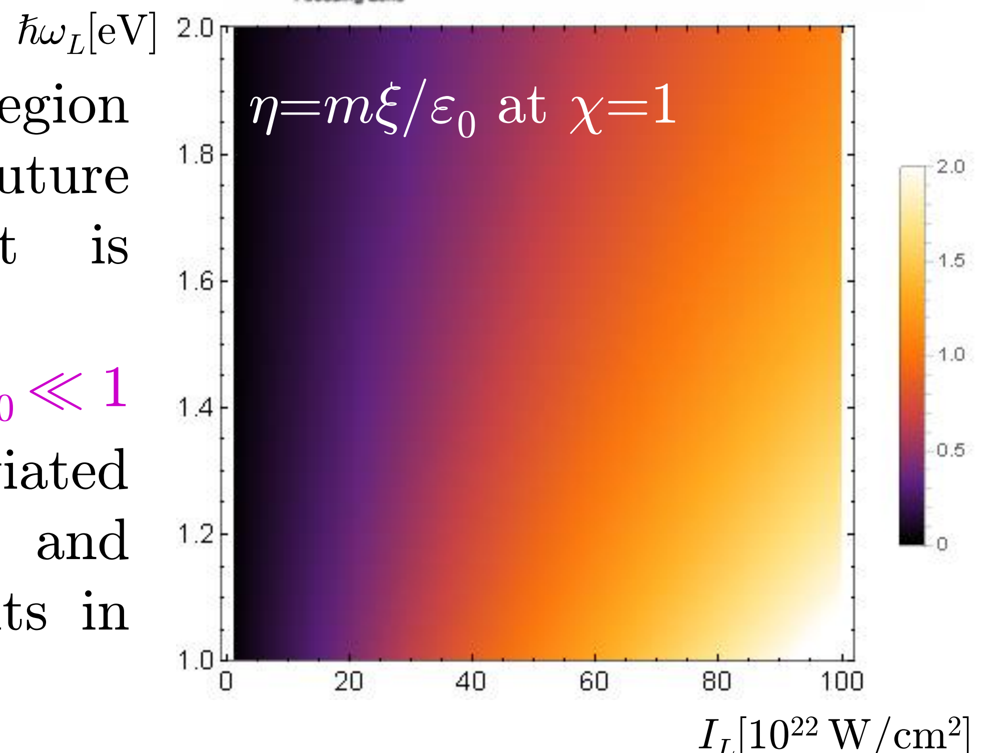
$$\chi = \left.\frac{E_L}{E_{cr}}\right|_{\text{rest frame}} \approx \frac{2\varepsilon_0}{mc^2}\frac{E_L}{E_{cr}}$$

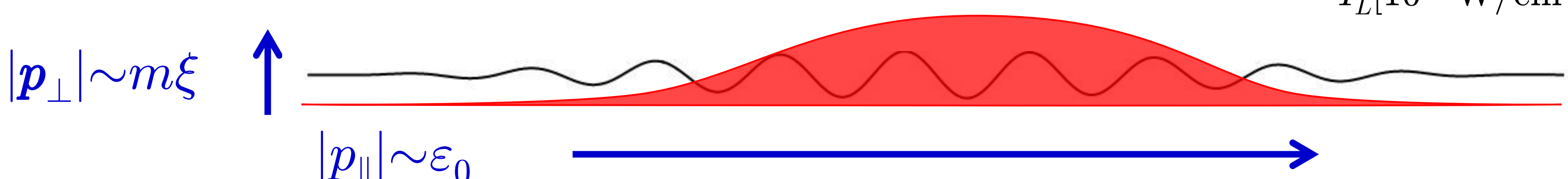


Units with $c=1$

- Nowadays ultrahigh intensities are achieved by compressing the laser energy both in time and space
- Compression in time can be accommodated within the plane-wave description
- The plot shows that in the region of present and near-future experimental feasibility it is $\eta=m\xi/\varepsilon_0 \ll 1$
- Under the condition $\eta=m\xi/\varepsilon_0 \ll 1$ the electron is barely deviated from its initial direction and there are no inversion points in the trajectory:







Quantum motion in the ultrarelativistic regime

- The De Broglie wavelength $\lambda_{DB}=2\pi\hbar/p$ of an ultrarelativistic electron is much smaller than the Compton wavelength $\lambda_C=3.9\times10^{-7}$ μm such that diffraction or "wave-like" effects are negligible at the (μm)-length scales typical of optical lasers
- The motion of ultrarelativistic electrons is "quasiclassical" in an analogous way as the space-time evolution of a light wave can be described by means of "rays" (geometrical optics)
- This is the physical idea behind the Wentzel, Kramers, and Brillouin (WKB) method, which is valid under the condition that $|\nabla\lambda_{DB}|=(|\nabla\lambda_{DB}|/\lambda_{DB})\lambda_{DB}\ll1$ (Landau and Lifshitz 1977)
- For the motion of an electron in a general external electromagnetic field of amplitude F_0 the condition becomes $2\pi(F_0/F_{cr}) \ll \beta^2\gamma^2$, with $F_{cr}=E_{cr},B_{cr}$ (importance of not having points in the trajectory where $\boldsymbol{\beta}=\mathbf{0}$)

Electron wave functions in tightly focused laser beams

- Solve the Dirac equation $[\gamma^\mu(i\hbar\partial_\mu - eA_\mu) - m]\psi = 0$
- By applying the WKB method, look for a solution of the form $\psi(x) = \exp[iS(x)/\hbar]\varphi(x)$ (Pauli 1934)

$$[\gamma^\mu(\partial_\mu S + eA_\mu) + m]\varphi = i\hbar\gamma^\mu\partial_\mu\varphi$$

- At the lowest order in $\hbar$ the bispinor $\varphi(x)$ satisfies the equations

$$[\gamma^\mu p_\mu(x) - m]\varphi_{p,\sigma} = 0$$

$$[\gamma^\mu p_\mu(x) + m]\gamma^\nu\partial_\nu\varphi_{p,\sigma} = 0$$

where $p^\mu(x) = -\partial^\mu S(x) - eA^\mu(x)$ and p,σ are quantum numbers

- The condition $\det[\gamma^\mu p_\mu(x) - m] = 0$ implies that the function $S(x)$ satisfies the Hamilton-Jacobi equation

$$(\partial_\mu S + eA_\mu)(\partial^\mu S + eA^\mu) - m^2 = 0$$

and it can be identified with the classical action

Determination of the classical action

- At $\varepsilon \gg m\xi$ the electron is barely deflected by the laser field
- It is convenient to work with the light-cone coordinates $\phi = x_- = t - \boldsymbol{n}\cdot\boldsymbol{x}$, $T = x_+ = (t + \boldsymbol{n}\cdot\boldsymbol{x})/2$, and $\boldsymbol{x}_\perp = \boldsymbol{x} - (\boldsymbol{n}\cdot\boldsymbol{x})\boldsymbol{n}$, where $\boldsymbol{n}$ can be chosen to be "almost" along the direction of the momentum $\boldsymbol{p}$ of the electron ($|\boldsymbol{n}\times\boldsymbol{p}| \lesssim m\xi$)
- Corresponding light-cone coordinates of the momentum: $p_+ = (\boldsymbol{\varepsilon} + \boldsymbol{n}\cdot\boldsymbol{p})/2$, $\boldsymbol{p}_\perp = \boldsymbol{p} - (\boldsymbol{n}\cdot\boldsymbol{p})\boldsymbol{n}$, and $p_- = \boldsymbol{\varepsilon} - \boldsymbol{n}\cdot\boldsymbol{p} = (m^2 + p_\perp{}^2)/2p_+$
- Momentum hierarchy: $p_+ \approx \varepsilon \gg p_\perp \sim m\xi \gg p_- \sim m^2\xi^{\,2}/\boldsymbol{\varepsilon}$
- By fixing an reference "time" T_0 and by writing the action as $S_p(X;T_0|A) = -(p_+\phi + p_- T - \boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp) + \delta S_p(X;T_0|A)$, the Hamilton-Jacobi equation becomes

$$p_+\left(\frac{\partial\delta S_p}{\partial T} + eA_-\right) + \boldsymbol{p}_\perp\cdot(\boldsymbol{\nabla}_\perp\delta S_p - e\boldsymbol{A}_\perp) - \frac{\partial\delta S_p}{\partial\phi}\frac{\partial\delta S_p}{\partial T} + \frac{1}{2}(\boldsymbol{\nabla}_\perp\delta S_p)^2 - e^2A_-A_+ + \frac{1}{2}e^2\boldsymbol{A}_\perp^2$$
$$- eA_-\frac{\partial\delta S_p}{\partial\phi} - eA_+\frac{\partial\delta S_p}{\partial T} - e\boldsymbol{A}_\perp\cdot\boldsymbol{\nabla}_\perp\delta S_p + p_-\left(\frac{\partial\delta S_p}{\partial\phi} + eA_+\right) = 0$$

- Up to first-order in $1/p_+$ the dependence of the field on the coordinate ϕ can be neglected ($t \approx \boldsymbol{n} \cdot \boldsymbol{x}$) and one obtains

$$S_p(X;T_0|A) = -(p_+\phi + p_- T - \boldsymbol{p}_\perp \cdot \boldsymbol{x}_\perp) - e\int_{T_0}^{T} dT' A_-(\boldsymbol{X}') + \frac{1}{p_+}\int_{T_0}^{T} dT' \left[e\boldsymbol{p}_\perp \cdot \boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X}';T_0) - \frac{1}{2}e^2 \boldsymbol{\mathcal{A}}_\perp^2(\boldsymbol{X}';T_0)\right]$$

where

$$\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0) = \boldsymbol{A}_\perp(\boldsymbol{X}) + \boldsymbol{\nabla}_\perp \int_{T_0}^{T} dT' A_-(\boldsymbol{X}')$$

with $\boldsymbol{X} = (T, \boldsymbol{x}_\perp)$ and $\boldsymbol{X}' = (T', \boldsymbol{x}_\perp)$

- By plugging this expression into the equations

$$[\gamma^\mu p_\mu(x) - m]\varphi_{p,\sigma} = 0$$
$$[\gamma^\mu p_\mu(x) + m]\gamma^\nu \partial_\nu \varphi_{p,\sigma} = 0$$

where $p^\mu(x) = p^\mu(X) = -\partial^\mu S_p(X;T_0|A) - eA^\mu(X)$, one finds

$$\psi_{p,\sigma}(X;T_0|A) = e^{iS_p(X;T_0|A)/\hbar}\left[1 - \frac{e}{2p_+}\gamma_+ \boldsymbol{\gamma}_\perp \cdot \boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0)\right]\frac{u_{p,\sigma}}{\sqrt{2\varepsilon}}$$

with $\gamma_+ = (\gamma^0 + \boldsymbol{n}\cdot\boldsymbol{\gamma})/2$ and $\boldsymbol{\gamma}_\perp = \boldsymbol{\gamma} - (\boldsymbol{n}\cdot\boldsymbol{\gamma})\boldsymbol{n}$

- Apart from the term $g(\boldsymbol{X};T_0)=-e\int_{T_0}^{T}dT'A_-(\boldsymbol{X}';T_0)$ in $S_p(X;T_0|A)$, the state

$$\psi_{p,\sigma}(X;T_0|A)=e^{iS_p(X;T_0|A)/\hbar}\left[1-\frac{e}{2p_+}\gamma_+\boldsymbol{\gamma}_\perp\cdot\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0)\right]\frac{u_{p,\sigma}}{\sqrt{2\varepsilon}}$$

$$S_p(X;T_0|A)=-\left(p_+\phi+p_-T-\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp\right)-e\int_{T_0}^{T}dT'A_-(\boldsymbol{X}')$$
$$+\frac{1}{p_+}\int_{T_0}^{T}dT'\left[e\boldsymbol{p}_\perp\cdot\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X}';T_0)-\frac{1}{2}e^2\boldsymbol{\mathcal{A}}_\perp^2(\boldsymbol{X}';T_0)\right]$$

$$\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0)=\boldsymbol{A}_\perp(\boldsymbol{X})+\boldsymbol{\nabla}_\perp\int_{T_0}^{T}dT'A_-(\boldsymbol{X}')=-\int_{T_0}^{T}dT'[\boldsymbol{E}_\perp(\boldsymbol{X}')+\boldsymbol{n}\times\boldsymbol{B}_\perp(\boldsymbol{X}')]$$

  has the same structure of a Volkov state with $\boldsymbol{A}_\perp(T)\rightarrow\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0)$

- The term in $g(\boldsymbol{X};T_0)$ can be removed by a gauge transformation
- The above states reduce to those found in Akhiezer et al. Sov. J. Part. Nucl. **10**, 19 (1979) and in Blankenbecler et al., Phys. Rev. D **36**, 277 (1987) in a time-independent scalar potential
- The local value of the parameter $\chi(\boldsymbol{X})$ has the same structure as in the plane-wave case with the same substitution rule

$$\chi(\boldsymbol{X})=\frac{1}{mF_{cr}}\sqrt{[\varepsilon\boldsymbol{E}(\boldsymbol{X})+\boldsymbol{p}\times\boldsymbol{B}(\boldsymbol{X})]^2-[\boldsymbol{p}\cdot\boldsymbol{E}(\boldsymbol{X})]^2}=\frac{p_+}{m}\frac{1}{F_{cr}}\left|\frac{\partial\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{X};T_0)}{\partial T}\right|$$

Electron propagator in tightly focused laser beams

- Solve the equation $[\gamma^\mu(i\hbar\partial_\mu - eA_\mu) - m]G(x,x'|A) = \delta(x-x')$
- One makes the ansatz $G(x,\, x'|A) = [\gamma^\mu(i\hbar\partial_\mu - eA_\mu) + m]D(x,\, x'|A)$ and solves the equation for the "square" propagator $D(x,x'|A)$

$$\left[(i\hbar\partial_\mu - eA_\mu(x))(i\hbar\partial^\mu - eA^\mu(x)) - \frac{i}{2}e\hbar\sigma_{\mu\nu}F^{\mu\nu}(x) - m^2\right] D(x,x'|A) = \delta(x-x')$$

- It is convenient first to find the propagator in the scalar case

$$\left[(i\hbar\partial_\mu - eA_\mu(x))(i\hbar\partial^\mu - eA^\mu(x)) - m^2\right] D^{(0)}(x,x'|A) = \delta(x-x')$$

by employing the Schwinger representation

$$D^{(0)}(X,X'|A) = -\frac{i}{\hbar}\int_0^\infty ds e^{i(s/\hbar)[2P_\phi P_T - \boldsymbol{P}_\perp^2 + 2e(A_+ P_T + A_- P_\phi + \boldsymbol{A}_\perp\cdot\boldsymbol{P}_\perp) + e^2A^2 - m^2]}\delta(X-X')$$

in light-cone coordinates

- By means of the operator technique one can exploit that at $\varepsilon \gg m\xi$ the electron is barely deflected by the laser field and disentangle the action on the δ-function of the operators in the exponent

- Final result

$$D^{(0)}(X,X'|A) = -\frac{1}{16\pi^2\hbar^3}\exp\left[-\frac{i}{\hbar}e\int_{T'}^{T}d\tau A_-(\tau)+\frac{i}{\hbar}e\frac{\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp}{T-T'}\cdot\int_{T'}^{T}d\tau\boldsymbol{A}_\perp(\tau)\right]$$
$$\times\int_0^\infty dy\exp\left\langle-\frac{i}{\hbar}\left\{\frac{\Delta X^2}{4}y+\left[m^2+\Delta m^2(\boldsymbol{X},\boldsymbol{X}')\right]\frac{1}{y}\right\}\right\rangle$$

where

$$\Delta m^2(\boldsymbol{X},\boldsymbol{X}') = e^2\left\{\frac{1}{T-T'}\int_{T'}^{T}d\tau\boldsymbol{\mathcal{A}}_\perp^2(\tau;\boldsymbol{X},\boldsymbol{X}')-\left[\frac{1}{T-T'}\int_{T'}^{T}d\tau\boldsymbol{\mathcal{A}}_\perp(\tau;\boldsymbol{X},\boldsymbol{X}')\right]^2\right\}$$

with

$$\boldsymbol{\mathcal{A}}_\perp(\tau;\boldsymbol{X},\boldsymbol{X}') = \boldsymbol{A}_\perp\left(\tau,\boldsymbol{x}_\perp-\frac{T-\tau}{T-T'}(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)\right)$$
$$+\frac{T}{T-T'}\int_{T}^{\tau}d\tau'\left[\boldsymbol{\partial}_\perp A_-\left(\tau',\boldsymbol{x}_\perp-\frac{T-\tau'}{T-T'}(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)\right)\right]$$
$$-\frac{T'}{T-T'}\int_{T'}^{\tau}d\tau'\left[\boldsymbol{\partial}_\perp A_-\left(\tau',\boldsymbol{x}_\perp-\frac{T-\tau'}{T-T'}(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)\right)\right]$$

is the local generalization of the "mass dressing"

- The structure is the same as the Volkov propagator with the generalized mass correction and with the additional exponential term which can be removed with a gauge transformation

First-order strong-field QED processes in a tightly focused laser beam

- The above wave-functions and propagator can be applied to study strong-field QED processes in tightly focused laser beams
- Nonlinear Breit-Wheeler pair production and nonlinear Compton scattering have been investigated
- The amplitude of nonlinear Breit-Wheeler pair production reads

$$S_{fi} = -ie\sqrt{4\pi}\int d^4x\, \bar{\psi}^{(\text{out})}_{p,s}(x)\frac{\hat{e}_{k,l}}{\sqrt{2\omega}}e^{-i(kx)}\psi^{(\text{out})}_{-p',-s'}(x)$$

- Since the wave functions have a complex dependence on three coordinates (T and $\boldsymbol{x}_\perp$) it is convenient to calculate directly the number of produced pairs

$$dN_{BW} = \mathcal{N}_\gamma V\frac{d^3\boldsymbol{p}}{(2\pi)^3}V\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{2}\sum_{l,s,s'}|S_{fi}|^2$$

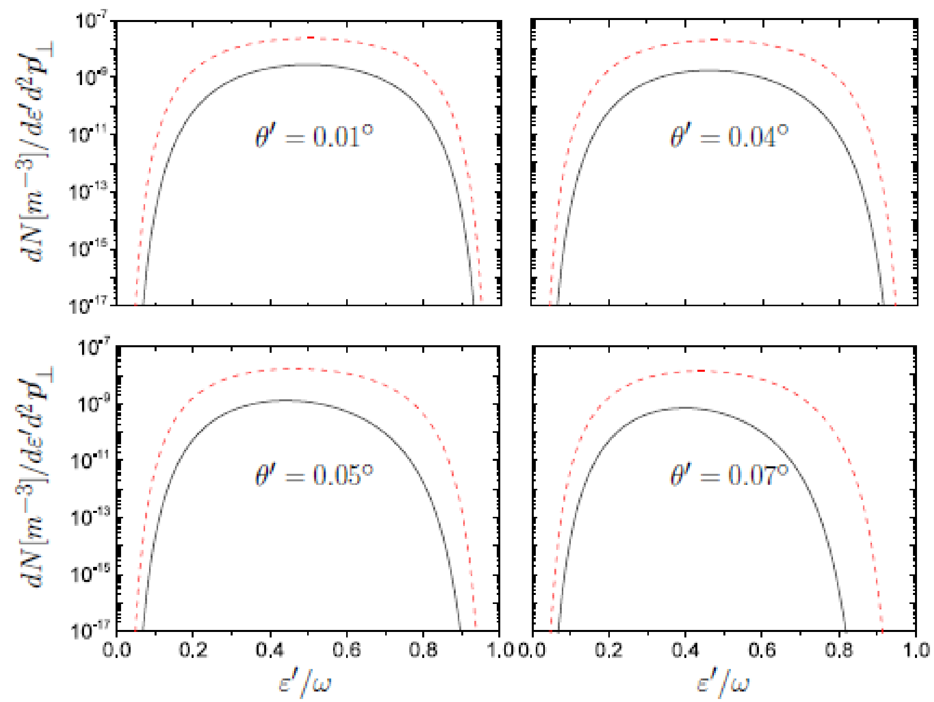
- The dependence of the wave functions on the transverse momenta is the same as of the Volkov states and this allows to perform the corresponding integrals analytically

- The angular resolved positron spectrum reads

$$\frac{dN_{BW}}{d\varepsilon' d\Omega'} = \rho_\Sigma \frac{\alpha}{8\pi^2}\frac{m^2}{\varepsilon^2\omega}\int dT dT' d^2\boldsymbol{x}_\perp e^{i\frac{\omega}{2\varepsilon\varepsilon'}\left\langle T_-\left\{m^2+\left[\boldsymbol{p}'_\perp-\frac{e}{T_-}\int_T^{T'} d\tilde{T}\boldsymbol{\mathcal{A}}_\perp(\tilde{x})\right]^2\right\}-e^2\left\{\frac{1}{T_-}\left[\int_T^{T'} d\tilde{T}\boldsymbol{\mathcal{A}}_\perp(\tilde{x})\right]^2+\int_T^{T'} d\tilde{T}\boldsymbol{\mathcal{A}}^2_\perp(\tilde{x})\right\}\right\rangle}$$
$$\times\left\{\varepsilon'^2+\varepsilon^2+4\varepsilon\varepsilon'+\frac{\omega^2}{m^2}\left[\boldsymbol{p}'_\perp+e\frac{\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{x})+\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{x}')}{2}\right]^2-e^2\frac{(\varepsilon-\varepsilon')^2}{m^2}\left[\frac{\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{x})-\boldsymbol{\mathcal{A}}_\perp(\boldsymbol{x}')}{2}\right]^2\right\}$$

  where ρ_Σ is the number of electrons impinging on the laser field per unit surface and $T_- = T - T'$

- The structure is the same as in a plane wave with the substitution $\boldsymbol{A}_\perp(T) \to \boldsymbol{\mathcal{A}}_\perp(\boldsymbol{x})$ and with the integration over $d^2\boldsymbol{x}_\perp$
- The numerical computation of the angular resolved spectra is also feasible
- Comparison between the results with a Gaussian beam and a plane wave with the same peak power
- The plane-wave model significantly overestimates the positron yield



Conclusions

- A procedure based on the WKB approximation has been proposed for investigating problems in strong-field QED taking into account exactly background laser fields with complex spacetime structure in the ultrarelativistic regime when the energy of the incoming particle is the largest dynamical energy in the problem
- The wave functions and the propagator are relatively simple and show a structure similar to the Volkov states
- The wave functions have been applied for studying analytically and numerically nonlinear Breit-Wheeler pair production and nonlinear Compton scattering in a tightly focused Gaussian beam