A fresh look on the Heisenberg-Euler effective action

[Gies, FK: JHEP **03** 108 (2017)]

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HELMHOLTZ GEMEINSCHAFT
Helmholtz-Institut Jena

LNPC'17 Workshop, Pacifico-Yokohama, 19.-21. April, 2017

HELMHOLTZ ASSOCIATION

Introduction

The Heisenberg-Euler effective action

[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

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$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

$$\left(\begin{array}{l}\mathfrak{E},\mathfrak{B}\quad \text{Kraft auf das Elektron.}\\ |\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\frac{e}{(e^2/mc^2)^2} = \text{„Kritische Feldstärke“.}\end{array}\right)$$

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

Effective self-interactions of prescribed external e.m. field induced by quantum vacuum fluctuations of e⁻/e⁺ and γ.

$$\Gamma_{\text{HE}} = \int d^4x\,\mathcal{L}_{\text{HE}}$$

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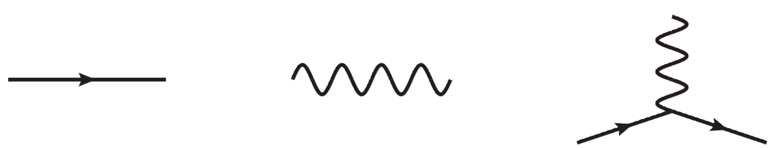
The Heisenberg-Euler effective action [Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

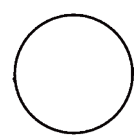
→ the considered microscopic theory is $f^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu$

$$\mathcal{L} \;=\; \bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi \;-\; \frac{1}{4} f_{\mu\nu} f^{\mu\nu} \;+\; e\bar{\psi}\gamma^\mu a_\mu \psi$$

QED



→ typical vacuum diagram



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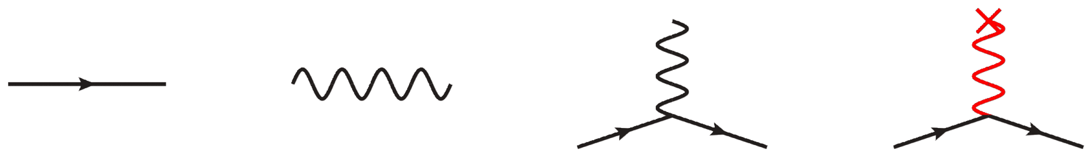
QED + (non-quantized) prescribed external field.



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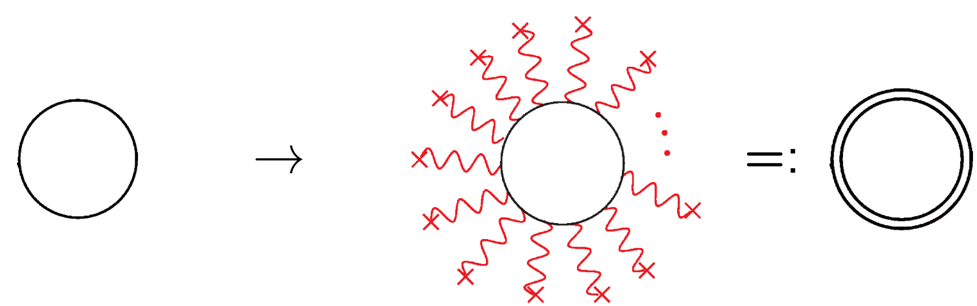

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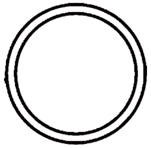
Heisenberg & Euler evaluated



in constant external e.m. fields at one loop.

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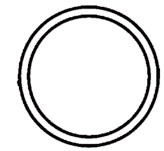


in constant external e.m. fields at one loop.

→ leading contribution in a loop-expansion

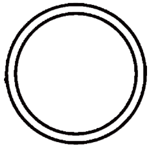
$$\Gamma_{\text{HE}}^{l\text{-loop}} \sim \left(\frac{\alpha}{\pi}\right)^{l-1}.$$

→ to all orders in the coupling to the external field $\sim eA^\mu$.

Introduction

The Heisenberg-Euler effective action (in constant e.m. fields)

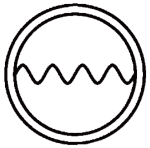
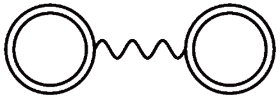
→ at one loop:

$$\Gamma_{\mathrm{HE}}^{\text{1-loop}}[A] =$$ 

[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

[Weisskopf: Kong. Dans. Vid. Selsk., Mat.-fys. Medd. **XIV**, 6 (1936)]

→ at two loops:

$$\Gamma_{\mathrm{HE}}^{\text{2-loop}}[A] =$$  $+$ 

1PI 1PR

[Ritus: Sov. Phys. JETP **42**, 774 (1975)]

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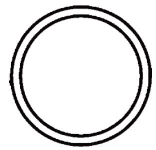
$$\Gamma_{\mathrm{HE}}^{\text{1-loop}}[A] =$$ 

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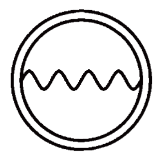
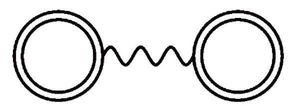
$$\Gamma_{\mathrm{HE}}^{\text{2-loop}}[A] =$$  $+$ 

1PI 1PR

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→ **Lesson 1**: The Heisenberg-Euler effective action is not a standard 1PI effective action Γ, as generically also 1PR diagrams contribute.

[Gies, FK: JHEP **03** 108 (2017)]

A fresh look on the Heisenberg-Euler effective action

Standard 1PI effective action Γ defined via Legendre transform:

$$Z[J] = \int \mathcal{D}\varphi \; \mathrm{e}^{iS[\varphi]+i\int J\varphi}$$

$$\rightarrow \qquad \Gamma[\phi] := \sup_J \left\{ -\int J\phi - i \ln Z[J] \right\} \qquad \phi = \langle \varphi \rangle$$

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Heisenberg-Euler effective action Γ_{HE} :

$$Z[J] = \int \mathcal{D}a \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; \mathrm{e}^{iS[a,\bar{\psi},\psi]+i\int Ja} \qquad\qquad S[a,\bar{\psi},\psi] = S_{\mathrm{QED}}$$

$$\rightarrow \qquad \Gamma_{\mathrm{HE}}[A] := \left\{ -\int J_\mu A^\mu - i \ln Z[J] \right\} \Bigg|_{J=-(\partial F)}$$

i.e., no Legendre transform

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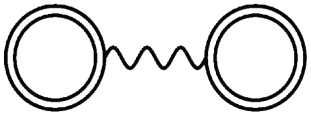
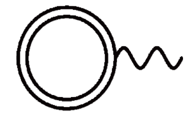
 $\neq 0$ in constant external fields?

Previously: This contribution vanishes, because  $= 0$ in constant fields. [Ritus: Sov. Phys. JETP **42**, 774 (1975)]

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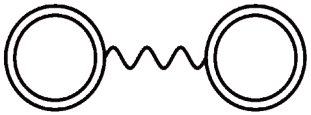
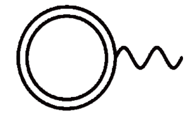
 $\neq 0$ in constant external fields?

Previously: This contribution vanishes, because  $= 0$ in constant fields. [Ritus: Sov. Phys. JETP **42**, 774 (1975)]

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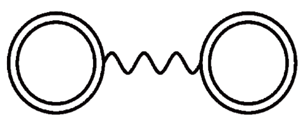
such that  $\sim \int_k k_\mu \frac{g^{\mu\nu}}{k^2} k_\nu \, \delta^2(k) = \int_k \delta^2(k) = \frac{TL^3}{(2\pi)^4}\,.$

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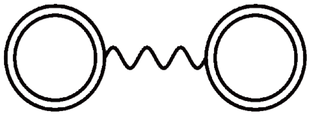
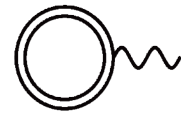
 $\neq 0$ in constant external fields?

Previously: This contribution vanishes, because  $= 0$ in constant fields. [Ritus: Sov. Phys. JETP **42**, 774 (1975)]

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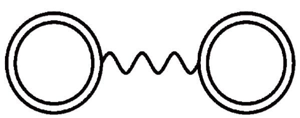
such that  $\sim \int_k k_\mu \frac{g^{\mu\nu}}{k^2} k_\nu \, \delta^2(k) = \int_k \delta^2(k) = \frac{TL^3}{(2\pi)^4}$.

$\rightarrow$ **Lesson 2**: The 2-loop 1PR contribution is finite even in constant external fields. [Gies, FK: JHEP **03** 108 (2017)]

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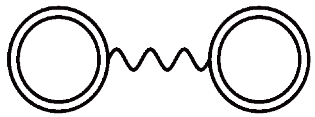
For const. fields  can be determined exactly from $\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}$.

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The result is:

$$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PR}} = \frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial F^{\mu\nu}} \frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial F_{\mu\nu}}$$
$$= \mathcal{F}\left[\left(\frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial \mathcal{F}}\right)^2 - \left(\frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial \mathcal{G}}\right)^2\right] + 2\mathcal{G}\, \frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial \mathcal{F}} \frac{\partial \mathcal{L}_{\mathrm{HE}}^{\text{1-loop}}}{\partial \mathcal{G}}.$$

$\rightarrow$ vanishes for $\vec{E} \perp \vec{B}$, $|\vec{E}| = |\vec{B}|$.

$\rightarrow$ closed form expressions for $\mathcal{G} = 0$ in terms of Hurwitz zeta function.

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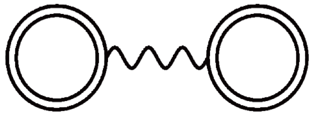
For const. fields  can be determined exactly from $\mathcal{L}_{\rm HE}^{\text{1-loop}}$.

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$\rightarrow$ **Lesson 3**: 1PR diagrams of this type are of course also relevant beyond two loops ↔ not considered so far. [cf. Schubert, et al.]

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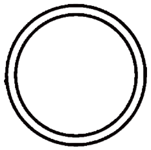
Scaling in weak and strong field limits for purely magnetic field.

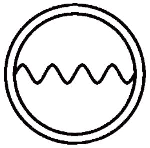
[Gies, FK: JHEP **03** 108 (2017)]

$$\text{weak}\ \left(\frac{eB}{m^2} \ll 1\right)$$

$$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$$



$$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PI}} =$$



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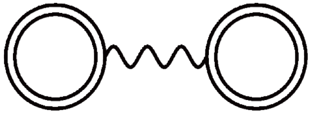

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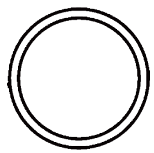
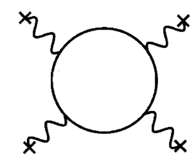
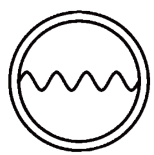
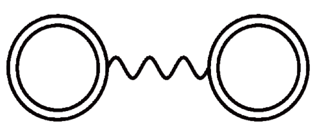
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$ 

$\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$



$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PI}} =$ 

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PR}} =$ 

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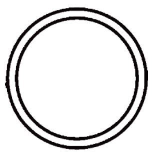
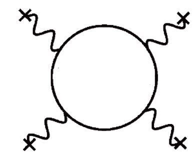
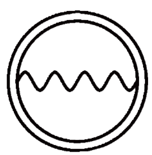
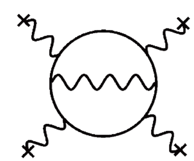
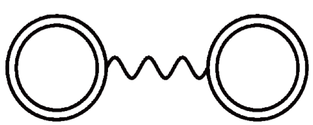
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$  $\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$ 

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PI}} =$  $\times \frac{\alpha}{\pi}\frac{40}{9}$ 

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PR}} =$ 

A fresh look on the Heisenberg-Euler effective action

Scaling in weak and strong field limits for purely magnetic field.

[Gies, FK: JHEP **03** 108 (2017)]

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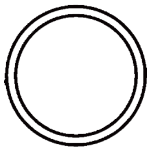
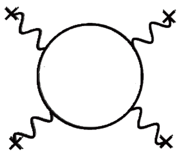
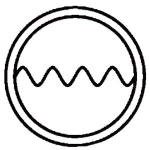
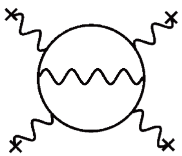
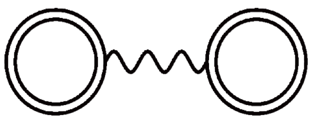
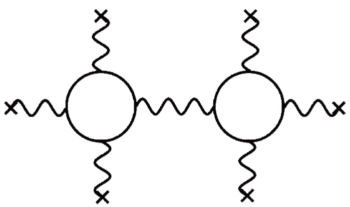
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$  $\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$ 

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PI}} =$  $\times \frac{\alpha}{\pi}\frac{40}{9}$ 

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PR}} =$  $\times \frac{\alpha}{\pi}\frac{4}{45}\left(\frac{eB}{m^2}\right)^2$ 

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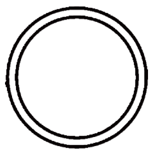
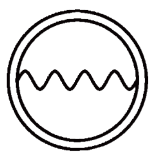
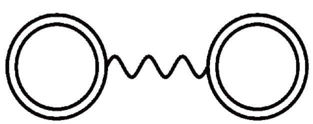
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$  $\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$

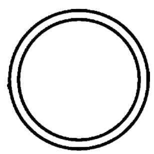
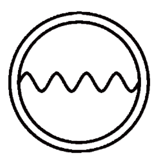
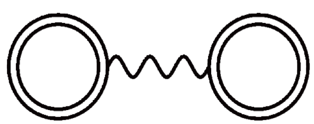
$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PI}} =$  $\times \frac{\alpha}{\pi}\frac{40}{9}$

$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big|_{\mathrm{1PR}} =$  $\times \frac{\alpha}{\pi}\frac{4}{45}\left(\frac{eB}{m^2}\right)^2$

A fresh look on the Heisenberg-Euler effective action

Scaling in weak and strong field limits for purely magnetic field.

[Gies, FK: JHEP **03** 108 (2017)]

	weak $\left(\frac{eB}{m^2} \ll 1\right)$	strong $\left(\frac{eB}{m^2} \gg 1\right)$ fields
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$ 	$\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$	$\frac{m^4}{24\pi^2}\left(\frac{eB}{m^2}\right)^2 \ln\left(\frac{eB}{m^2}\right)$
$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big\vert_{\mathrm{1PI}} =$ 	$\times\frac{\alpha}{\pi}\frac{40}{9}$	
$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big\vert_{\mathrm{1PR}} =$ 	$\times\frac{\alpha}{\pi}\frac{4}{45}\left(\frac{eB}{m^2}\right)^2$	

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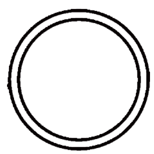
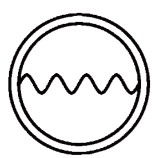
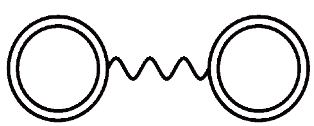
$\mathcal{L}_{\mathrm{HE}}^{\text{1-loop}} =$ 	$\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$	$\frac{m^4}{24\pi^2}\left(\frac{eB}{m^2}\right)^2 \ln\left(\frac{eB}{m^2}\right)$
$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big\rvert_{\mathrm{1PI}} =$ 	$\times \frac{\alpha}{\pi}\frac{40}{9}$	$\times \frac{\alpha}{\pi}\frac{3}{4}$
$\mathcal{L}_{\mathrm{HE}}^{\text{2-loop}}\Big\rvert_{\mathrm{1PR}} =$ 	$\times \frac{\alpha}{\pi}\frac{4}{45}\left(\frac{eB}{m^2}\right)^2$	

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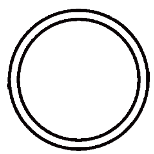
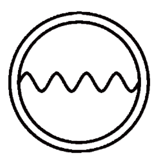
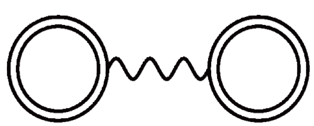
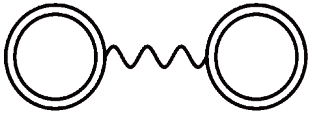
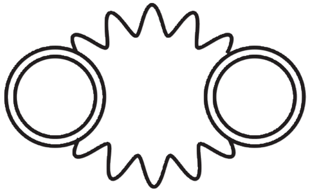
$\mathcal{L}_{\rm HE}^{\text{1-loop}} =$		$\frac{m^4}{360\pi^2}\left(\frac{eB}{m^2}\right)^4$	$\frac{m^4}{24\pi^2}\left(\frac{eB}{m^2}\right)^2 \ln\left(\frac{eB}{m^2}\right)$
$\mathcal{L}_{\rm HE}^{\text{2-loop}}\big\vert_{\rm 1PI} =$		$\times \frac{\alpha}{\pi}\frac{40}{9}$	$\times \frac{\alpha}{\pi}\frac{3}{4}$
$\mathcal{L}_{\rm HE}^{\text{2-loop}}\big\vert_{\rm 1PR} =$		$\times \frac{\alpha}{\pi}\frac{4}{45}\left(\frac{eB}{m^2}\right)^2$	$\times \frac{\alpha}{\pi}\frac{1}{3}\ln\left(\frac{eB}{m^2}\right)$

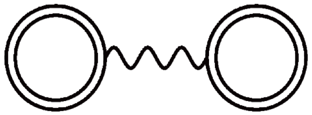
A fresh look on the Heisenberg-Euler effective action

The 1PR diagram  has additional interesting features:

$\rightarrow$ beyond QED, the two loops do not necessarily involve the same particle species

$\rightarrow$ in 1PI sector only possible at >2 loops, e.g.,  .

A fresh look on the Heisenberg-Euler effective action

The 1PR diagram  has additional interesting features:

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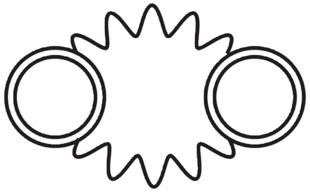
$\to$ in 1PI sector only possible at >2 loops, e.g.,  .

Consider two different species of Dirac fermions with parameters (e, m) and $(\tilde{e}, \tilde{m})$ in a constant magnetic field.

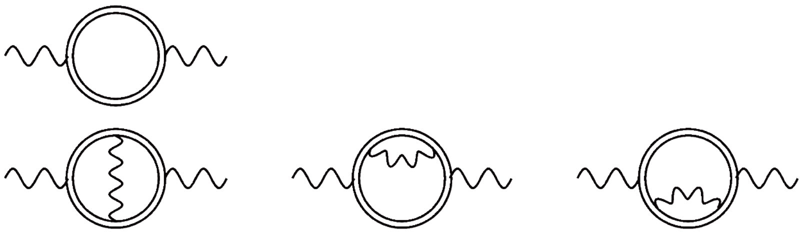
$\to$ $\frac{eB}{m^2} \ll 1\,, \ \frac{\tilde{e}B}{\tilde{m}^2} \gg 1$:

$$\mathcal{L} \sim \frac{m^4\,\tilde{e}^2}{2160\pi^4}\Big(\frac{eB}{m^2}\Big)^4 \ln\Big(\frac{\tilde{e}B}{\tilde{m}^2}\Big).$$

A fresh look on the Heisenberg-Euler effective action

Photon propagation in QED in external electromagnetic fields.

(a) one-particle irreducible: [Gies, FK: JHEP **03** 108 (2017)]



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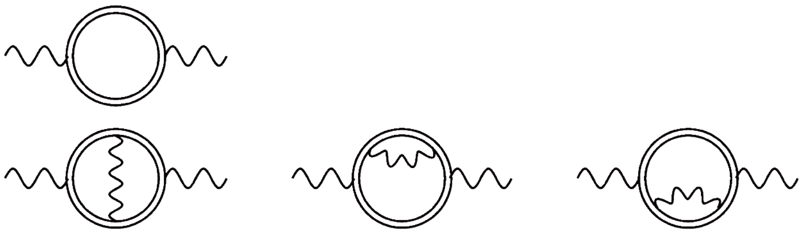
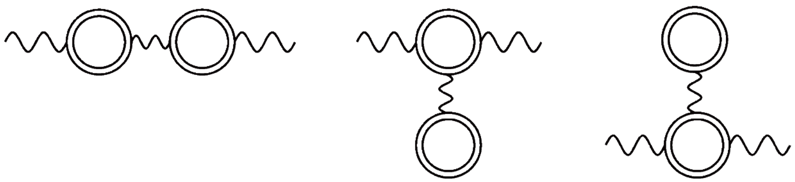


(b) one-particle reducible:



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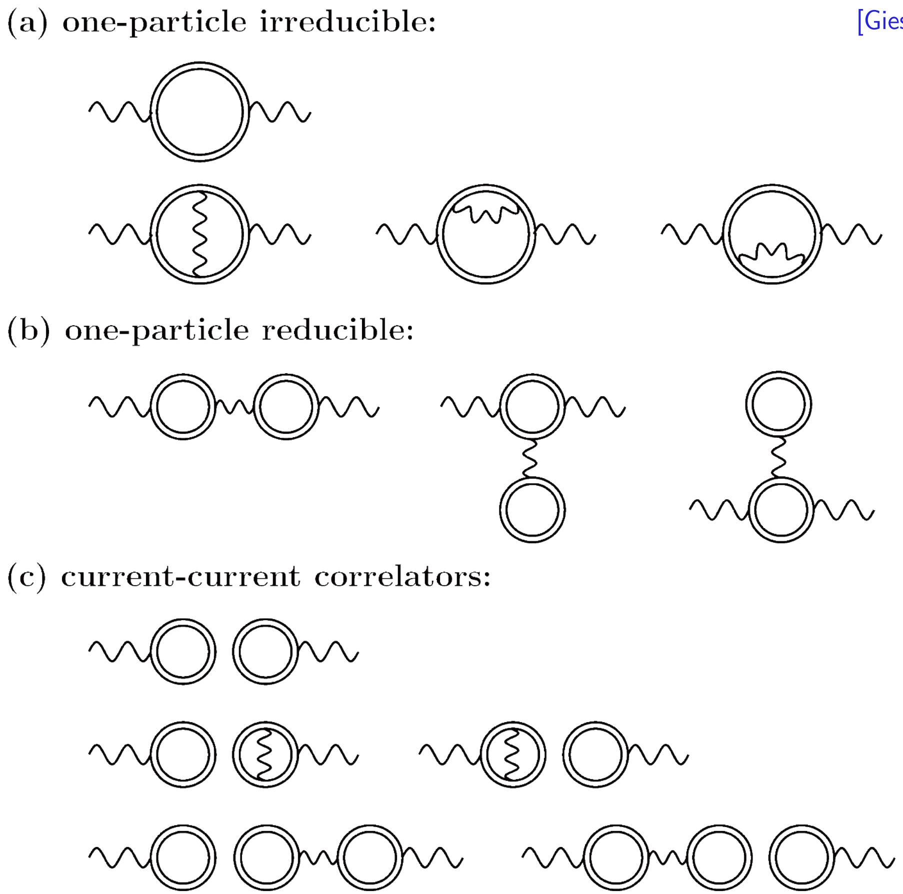
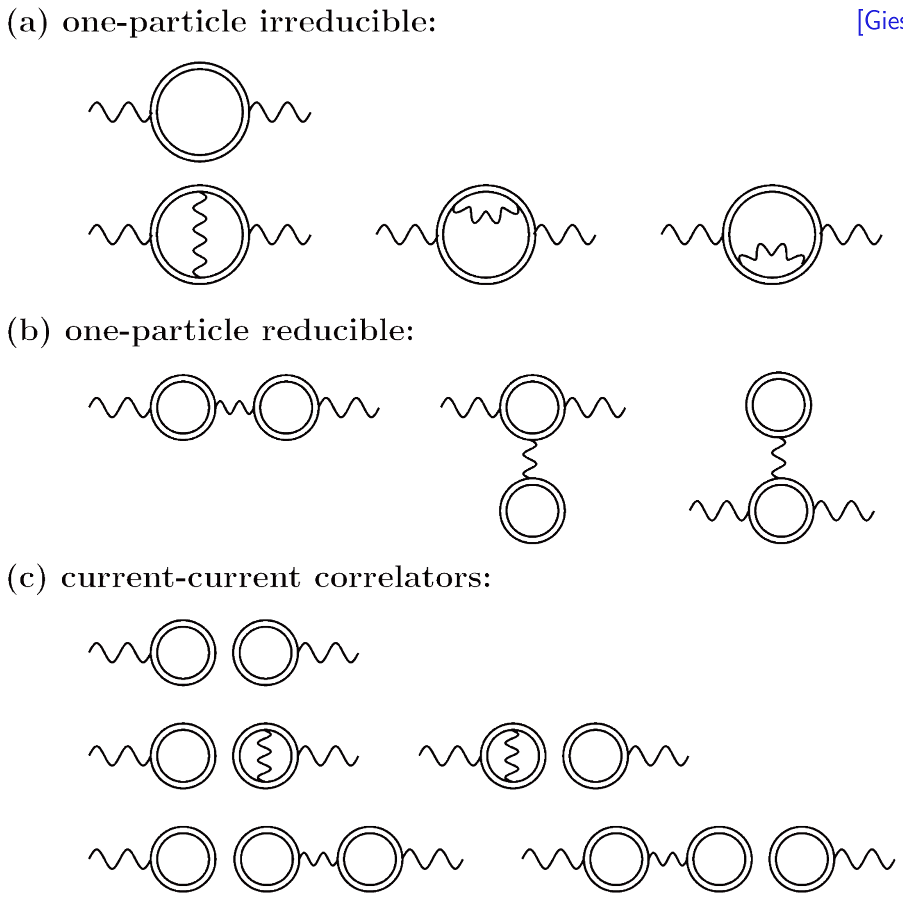


# of loops ↔ powers of α

A fresh look on the Heisenberg-Euler effective action

Photon propagation in QED in external electromagnetic fields.

[Gies, FK: JHEP **03** 108 (2017)]



In low-frequency limit attainable from the Heisenberg-Euler action.

[FK, Shaisultanov: Phys. Rev. D **91** 085027 (2015)]

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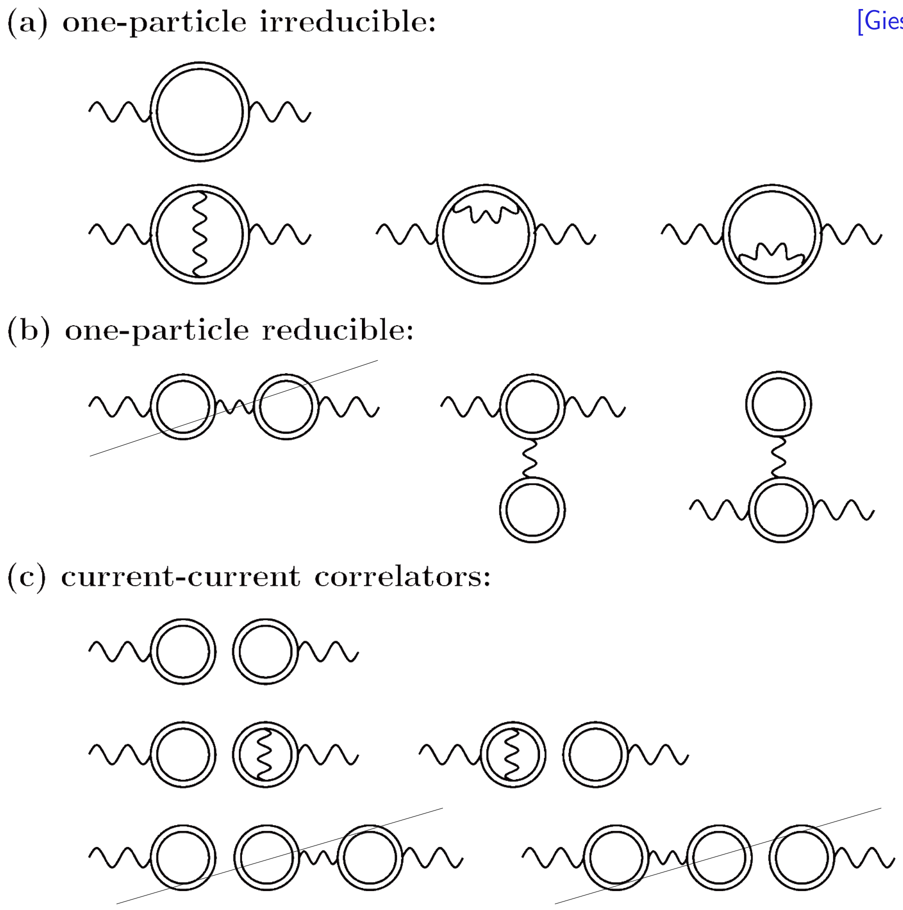

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$$\Pi^{\mu\nu}(k,k'|A)$$

# of loops ↔ powers of α

Conclusions and Outlook

In this talk,

$\rightarrow$ I have highlighted that the Heisenberg-Euler action is not a standard 1PI effective action.

$\rightarrow$ I have shown you that at two loops there is a finite 1PR contribution, previously assumed to vanish in constant fields.

$\rightarrow$ Similar contributions are also relevant beyond two loops.

Conclusions and Outlook

In this talk,

→ I have highlighted that the Heisenberg-Euler action is not a standard 1PI effective action.

→ I have shown you that at two loops there is a finite 1PR contribution, previously assumed to vanish in constant fields.

→ Similar contributions are also relevant beyond two loops.

→ Finally, I briefly discussed photon propagation in external e.m. fields.

→ The corresponding diagrams can be derived from the 1PI sector of the Heisenberg-Euler effective action + photon propergators.

Thank you for your attention!