Resonant photon splitting and photon scattering in a strong electromagnetic field

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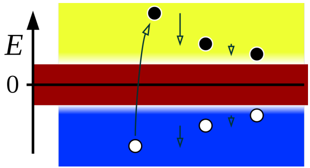
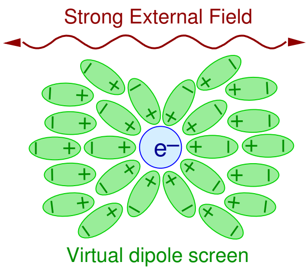
Synopsis

- Motivation
  - Photon splitting (scattering) are rare processes that occur in strong electromagnetic fields
  - Observed in heavy ion collisions, can we use a strong laser?
  - Review calculation methods
  - Use Furry picture for calculation w/out kinematic restrictions
- Furry picture
  - Vertices dressed with Volkov solutions
  - Fourier transform external field modes
- Photon splitting via resonant strong field process
  - Introduce stimulated photon splitting
  - Conditions for resonance
  - Treat photon as unstable particle with decay width
  - Estimate transition rate at resonance
  - Look forward to laboratory tests

Motivation - physics in strong background fields



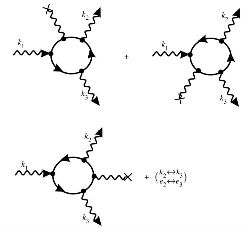
Vacuum polarisation



Bare/Dressed charge

- Strong background field polarises the vacuum
- The screening charge is rearranged, leading to possibly large effects even at modest field strengths
- At Schwinger critical field strength, vacuum decays into real pairs
- New phenomenology results - odd vertex diagrams, resonant propagators, different manifestations of IR divergences
- Careful study allows us to plan experiments to test strong field theory
- Need to investigate experimental signatures within reach using today's and upcoming technology

Photon splitting - techniques and scenarios



Calculation Methods

- Heisenberg-Euler $\omega_i \ll m$ (ω_i initial photon energy)
- Weizsäcker-Williams $\omega_i \gg m$
- Born approximation (exchange of individual equivalent photons from external field)
- Furry picture, no approximations

Scenarios/history

- Strong magnetic fields near astrophysical objects - Adler 1971/96, Mentzel 94...
- Recent astrophysical observations $\to$ vacuum birefringence (Mignani et al, 2016)
- Heavy ion collision $\to$ Strong Coulomb (constant crossed) field
- Observation in HI collisions by Akhmadaliev et al PRL 89, 061802 (2002)
  Review by [Lee et al, Phys Rep 373 p213 (2003)]
- Can we produce in the lab with a strong laser and electrons?

Heisenberg-Euler vs Furry Picture

Heisenberg-Euler constant field approx

$$\mathcal{L}=\mathcal{F}-\frac{1}{8\pi^2}\int_0^\infty \mathrm{d}s\,\frac{e^{-m^2 s}}{s^3}\left[(es)^2\frac{\mathrm{Re}\cosh\left(es\sqrt{2(\mathcal{F})+i\mathcal{G}}\right)}{\mathrm{Im}\cosh\left(es\sqrt{2(\mathcal{F})+i\mathcal{G}}\right)}\mathcal{G}-\frac{2}{3}(es)^2\mathcal{F}-1\right]$$

weak-field limit

$$\mathcal{L}=\frac{1}{2}(E^2-B^2)+\frac{2\alpha^2}{45m^4}$$

- Can be solved exactly for constant field, one loop, renormalised effective action
- Constant field approx $\frac{\Delta E}{E}\leq m_e^{-1}$. Not good for heavy ions, good for strong lasers

Furry Picture

$$\mathcal{L}^{\mathrm{Int}}_{\mathrm{QED}}=\bar\psi(i\partial\!\!\!/-m)\psi-\tfrac{1}{4}(F_{\mu\nu})^2-e\bar\psi(A\!\!\!/^{\mathrm{e}}+A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\mathrm{FP}}_{\mathrm{QED}}=\bar\psi^{\mathrm{FP}}(i\partial\!\!\!/-eA\!\!\!/^{\mathrm{e}}-m)\psi^{\mathrm{FP}}-\tfrac{1}{4}(F_{\mu\nu})^2-e\bar\psi^{\mathrm{FP}}A\!\!\!/\,\psi^{\mathrm{FP}}$$

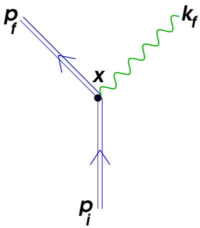
Equations of Motion

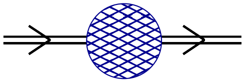
$$(i\partial\!\!\!/-eA\!\!\!/^{\mathrm{e}}-m)\psi^{\mathrm{FP}}=0$$

Wavefunction

$$\psi^{\mathrm{FP}}=\mathbf{E}_p\,e^{-ipx}\,u_p,\quad \mathbf{E}_p=\exp\left[-\tfrac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\mathrm{ext}}k\!\!\!/+i2e(A^e\cdot p)-ie^2A^{\mathrm{ext}\,2}\right)\right]$$

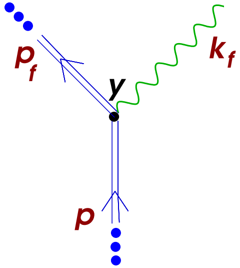
Unstable Strong field particles & resonant transitions





- **Electrons decay in strong field Furry picture**
  - Background field renders vacuum a dispersive medium
  - new effects: Lamb shift, vacuum birefringence, resonant transitions
  - electron has a finite lifetime, Γ and probability of radiation, W
- **Resonant transitions in propagator**
  - required by S-matrix analyticity
  - Optical theorem $W = \text{Im}(\Sigma)$
  - extra propagator poles leading to physically accessible resonances
  - related to energy level structure of vacuum/background field

Similar decay (one photon pair production) and lifetime for photons



- double fermion lines are Volkov-type solutions
- Volkov E_p functions "dress" the vertex

$$\gamma_\mu^{\mathsf{FP}} = \int d^4x\, \bar{E}_{p_f}(x)\gamma_\mu E_p(x)\, e^{i(p_f - p + k_f)\cdot x}$$

- momentum space vertex has contribution nk from external field

$$\gamma_\mu^{\mathsf{FP}}(x) = \sum_{n=-\infty}^{\infty} \int_{-\pi L}^{\pi L} \frac{d\phi}{2\pi L} \exp\left(i\frac{n}{L}\left[\phi - (k\cdot x)\right]\right)\gamma_\mu^{\mathsf{FP}}(\phi)$$

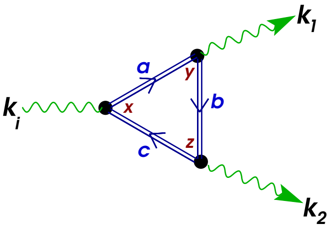
- Fourier transform of circularly polarised field leads to Bessel functions

$$\int_{-\pi}^{\pi} \frac{d\phi_v}{2\pi} \exp\left[ir\phi_v - z\sin\phi_v\right] = \mathrm{J}_r(z)$$

- Fourier transform of circularly polarised field leads to Bessel functions

$$M_{\mathsf{fi}}^{\mathsf{HICS}} = \int \mathsf{d}^4x\, \bar{u}_{\mathsf{fr}}\, \gamma^{\mathsf{FP}}\, u_{\mathsf{is}}\, e^{-i(p_f + k_f - k_i)}, \quad W = \int \frac{\mathsf{d}\vec{p}_{\mathsf{f}}}{2\epsilon_{\mathsf{f}}} \frac{\mathsf{d}\vec{k}_{\mathsf{f}}}{\omega_{\mathsf{f}}} \left|M_{\mathsf{fi}}^{\mathsf{HICS}}\right|^2$$

Photon splitting FP matrix element



- Sums over external field modes at each vertex
- Space-time integrations give momentum conservation at each vertex

$$\delta^{(4)}(a + n_x k - k_i + c), \quad \ldots$$

- integrations over internal (propagator) momenta amalgamate overall momentum conservation

$$\delta^{(4)}(k_1 + k_2 - [n_x + n_y + n_z]\, k - k_i)$$

- shift sums so that there is one "external" momentum contribution and n-1 "internal" sums

$$\sum_{n_x n_y n_z} \to \sum_{n} \delta(k_1{+}k_2{-}nk{-}k_i) \sum_{n_y} \sum_{n_z} f(n, n_y, n_z)$$

- Fourier transform of circularly polarised field leads to Bessel functions

$$\int_{-\pi}^{\pi} \frac{d\phi_v}{2\pi} \exp\left[ir\phi_v - z \sin\phi_v\right] = \mathrm{J}_r(z)$$

Photon splitting decay rates

[Wilke and Wunner PRD 55(2) p997 (1997)]

- Heisenberg-Euler $\omega_i \ll m$
  Magnetic field $B < 0.1 B_{\rm cr}$, $\quad B_{\rm cr} = 4.4 \times 10_9$ T
  Transverse to longitudinal polarisation $(\perp \rightarrow ||\,||)$

$$\Pi^{(3)\rm FP} = \frac{\alpha^3 m}{60\pi^2}\left[\frac{B}{B_{\rm cr}}\right]^6 \left[\frac{\omega_i}{m}\right]^5 \left[\frac{26}{315}\right]^2$$

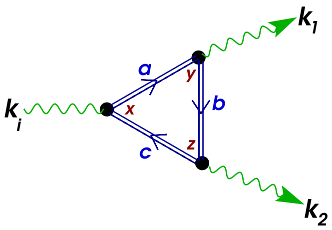
[Papanyan and Ritus JETP 34(6) p1195 (1972)]

- Furry picture, general ω_i, field strength scale dependent coupling constant

$$\Pi^{(3)\rm FP} = 3.7 \times 10^{-5} \alpha^3 \kappa^{f(\kappa)} \frac{m}{\omega_i}, \quad \kappa = \frac{a_0\, k\cdot k_i}{m^2}$$

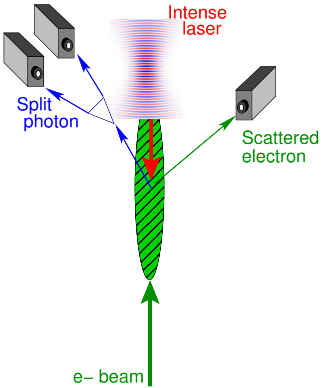
$$f(\kappa) = 6 \text{ for } \kappa \ll 1, \quad f(\kappa) \approx 2/3 \text{ for } \kappa \gg 1$$

$$f(1) = n_{k_{\rm i}}, \text{ number density of initial photons}$$



- For a $\lambda = 1$ μm strong background laser with $a_0 \approx 10$ need 21 GeV initial photons for $\kappa = 1$

Photon splitting - a possible experiment

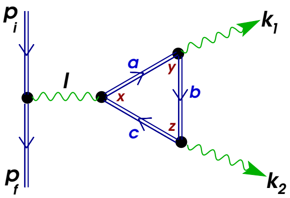


- Relativistic electrons of energy $\gamma = E/m$
- Primary inverse Compton scattering with counter-propagating laser.
- High energy photons radiate forward in a $1/\gamma$ cone and interact with strong laser field again
- Some photons will decay via photon splitting process
- Other decays (one photon pair production) also occur
- Main background will be from ICS photons, so look for distinct photon splitting signature
- Consider the one-step "stimulated" photon splitting (amplitude P_{S})

$$P_{\mathsf{S}} = \bar{\psi}_{\mathsf{f}}^{\mathsf{FP}} \bar{A}_1 \bar{A}_2 \, \Pi^{(3)\mathsf{FP}} D_l \, \psi_{\mathsf{i}}^{\mathsf{FP}}$$

Stimulated photon splitting

$$P_{\mathsf{S}} = \bar{\psi}_f^{\mathsf{FP}} \bar{A}_1 \bar{A}_2 \, \Pi^{(3)\mathsf{FP}} D_l \, \psi_i^{\mathsf{FP}}$$



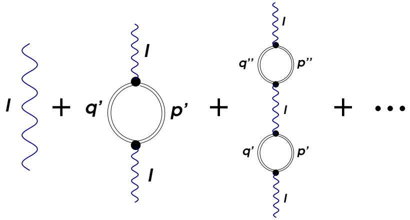
- ψ^{FP} are Volkov-type functions, can be modified to take into account real strong laser pulse
- $A_{1,2}$ are mass shell split photons to be detected
- $\Pi^{(3)\mathsf{FP}}$ is the splitting fermion loop - also has resonance width, but treat it as a ”black box”
- $\psi_f^{\mathsf{FP}} \gamma^\alpha \psi_i^{\mathsf{FP}}$ is the Volkov or dressed current. It provides a resonant photon propagator
- D_l is the photon propagator. Couples to strong field through its self energy. Enables resonances to enhance the splitting process

$$D_l = \int \frac{\mathsf{d}^4 l}{(2\pi)^2} \frac{1}{l^2 + i\Gamma + i\varepsilon} \, e^{il(y-x)}$$

Photon propagator - self energy via resummation

The photon decays in dispersive vacuum with resonance width Γ

$$D^{\mathsf{FP}}_{\mathsf{y,x}} = \int \frac{d^4 l}{(2\pi)^2} \frac{1}{l^2 + i\Gamma + i\varepsilon} e^{il(y-x)}$$



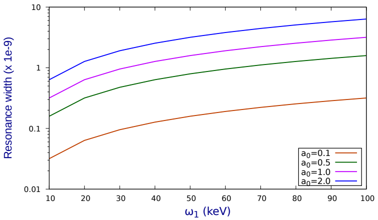
Γ is sum of 1PI diagrams, self energy to leading order

$$\Gamma = \Im \Pi^{(2)\mathsf{FP}}_{\mathsf{p}} + \ldots$$

Parametrisation for small photon energy (Ritus Trudy FIAN 1979)

$$\Gamma \approx 0.23\, \alpha m^2\, \kappa\, e^{-8\kappa/3}, \quad \kappa = a_0\, k \cdot l / m^2$$

$$\Gamma \equiv \Im \Pi^{(2)\mathsf{FP}}_{\mathsf{p}} \equiv W^{\mathsf{OPPP}} \quad \text{(optical theorem)}$$



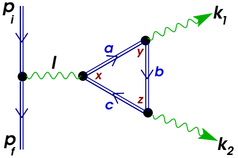
$$W^{\mathsf{OPPP}} = \frac{\pi\alpha}{k\cdot l} \sum_{n>n_0}^{\infty} \int_1^{u_n} \frac{du}{u\sqrt{u(u-1)}} \left[\frac{2}{a_0^2} J_n^2 - \frac{2+2u+u^2}{(1+u)} \left(J_{n-1}^2 + J_{n+1}^2 - 2 J_n^2 \right) \right], \quad n_0 = \frac{m^2(1+a_0^2)}{2k\cdot l}$$

Stimulated Photon splitting - resonance features

$$P_S = \int dxdy\, \bar\psi^{FP}_{fx}\, \Pi^{(3)FP} \frac{dl}{l^2+i\Gamma}\, \psi^{FP}_{ix}\, e^{il(y-x)}, \quad \Pi^{(3)FP} = \sum_s \Pi^{(3)FP}_s$$

$$\to \sum_{rs} \int \bar\psi^{FP}_{fr}\, \Pi^{(3)FP}_s \frac{dl}{l^2+i\Gamma}\, \psi^{FP}_{ir}\, \delta^{(4)}(k_1+k_2+sk-l)\, \delta^{(4)}(l+p_f+rk-p_i)$$

$$\to \sum_{ns} \bar\psi^{FP}_{f(n-s)}\, \Pi^{(3)FP}_s \frac{1}{(k_1+k_2+sk)^2+i\Gamma}\, \psi^{FP}_{i(n-s)}\, \delta^{(4)}(p_f+k_1+k_2+nk-p_i)$$

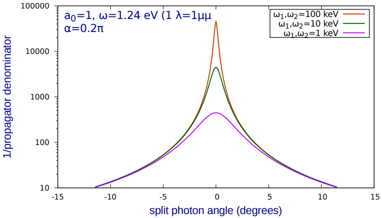


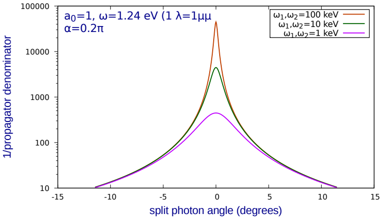
- Photon propagator bridges external current and photon splitting process
- Photon propagator has a decay width Γ_l due to dispersive coupling in Furry picture
- Decompose dressed vertices x, y into Fourier modes of external field contributions
- Multiple resonance conditions

$$(k_1+k_2+sk)^2 = 0 \implies s = \frac{k\cdot(k_1+k_2)}{k_1\cdot k_2} \in \mathcal{Z}$$

Resonance width on the mass shell

$$\frac{1}{(k_1 + k_2 + sk)^2 + \Gamma^2}, \quad \Gamma \approx 0.23\, \alpha m^2\, \kappa\, e^{-8\kappa/3}, \quad \kappa = a_0\, k \cdot (k_1 + k_2)/m^2$$





- Get a (rough) estimate of increase in differential transition rate at resonance
- Compare resonant and non-resonant propagator via an angle scan
- Angle between split photons = θ. Consider the $s = 1$ resonance
- Approximate width due to Ritus - suitable for $\kappa \ll 1$
- 4-5 orders of magnitude increase at resonance for small split angle
- Full study necessary to distinguish signal from background

Stimulated photon splitting rate at resonance

- At resonance, photon propagator on-shell, multiply differential rates of separate processes

$$\mathrm{d}W_{P_\mathrm{S}} = \mathrm{d}W^{\mathrm{HICS}}\, \frac{1}{\Gamma^2}\, \mathrm{d}\Pi^{(3)\mathrm{FP}}$$

- Assume $\lambda = 1\ \mu$m "strong" laser, $\omega_1 = \omega_2,\ k{\cdot}k_1 = k{\cdot}k_2,\ a_0 = 1,\ \kappa \ll 1,\ \varepsilon_i \approx \frac{1}{\omega}$

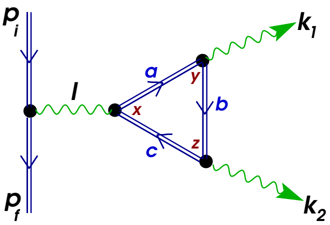
$$\mathrm{d}W^{\mathrm{HICS}} = \alpha \frac{m^2}{\varepsilon_i^2} N$$

$$D|_{\mathrm{resonance}} = \frac{1}{\Gamma^2}, \quad \Gamma \approx 0.23\, \alpha\, \kappa\, e^{-8\kappa/3}$$

$$\mathrm{d}\Pi^{(3)\mathrm{FP}} = 3.7\times 10^{-5} \alpha^3 \kappa^6 \frac{m}{2\omega_1}, \quad \kappa = \frac{a_0\, 2k\cdot k_1}{m^2}$$

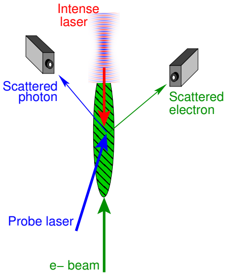
- Assume N=10^{10} electrons per bunch, 50 keV (split) photons

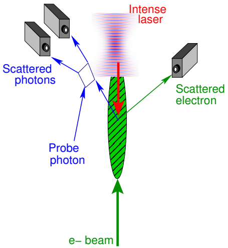
$$\mathrm{d}W_{P_\mathrm{S}} \approx 1 \text{ event per 2 bunch collisions}$$



- Order of magnitude estimate
- Without resonance,
  1 event per 10^4 bunches
- Doable at resonance?
  experimental considerations
- Need detailed calculation

Summary: resonant strong field processes in the lab!

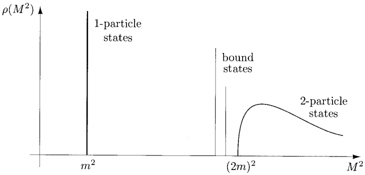




- QFT with background field predicts resonant cross-sections
- Related to Furry picture S-matrix decays of electron and photon
- Exploit resonances to conceivably detect rare processes
- Introduce probe photons to induce other resonant processes
- Resonant photon scattering one more factor of $\alpha = 1/137$ away
- 2nd order stimulated Compton scattering detectable with modest electron energies and laser intensities
- Experimental facilities can already perform tests of the theory
- Collaboration between strong field theorists & experimentalists $\rightarrow$ Design experiment

BACKUP

Can the resonant modes be "summed away" ?



$$\sum_{s=-\infty}^{\infty} \frac{e^{-is\theta}}{s+a} \mathrm{J}_s(z_1)\, \mathrm{J}_{s-n}(z_2)$$

- Internal sums with a circularly polarised background field have no known solution
- Must look for approximations or do sum numerically

$$\sum_{s=-\infty}^{\infty} \frac{1}{s+a} \left(\frac{z_1}{z_2}\right)^n \mathrm{J}_s(z_1)\, \mathrm{J}_{s\text{-}n}(z_2) = ..\ \csc \pi a \ldots$$

- $\csc \pi a$ is divergent for integer a, or resonant peaks when $a = \Re a + i\Gamma$
- Get Kallen-Lehman spectrum by electron self energy in background field
- After regularisation, Wick rotation, etc.. spectrum also contains resonant modes
- Propose resonant modes correspond to bound states
- Bound states in the Furry picture share a common origin to the "mass shift" - quantum and classical effects of the background field