Dynamical evolution of critical fluctuations and its observation in heavy ion collisions

Masakiyo Kitazawa

(Osaka U.)

Sakaida, Asakawa, Fujii, MK, arXiv:1703.08008

See also,

Asakawa, MK, Prog. Part. Nucl. Phys. 90, 299 (2016)

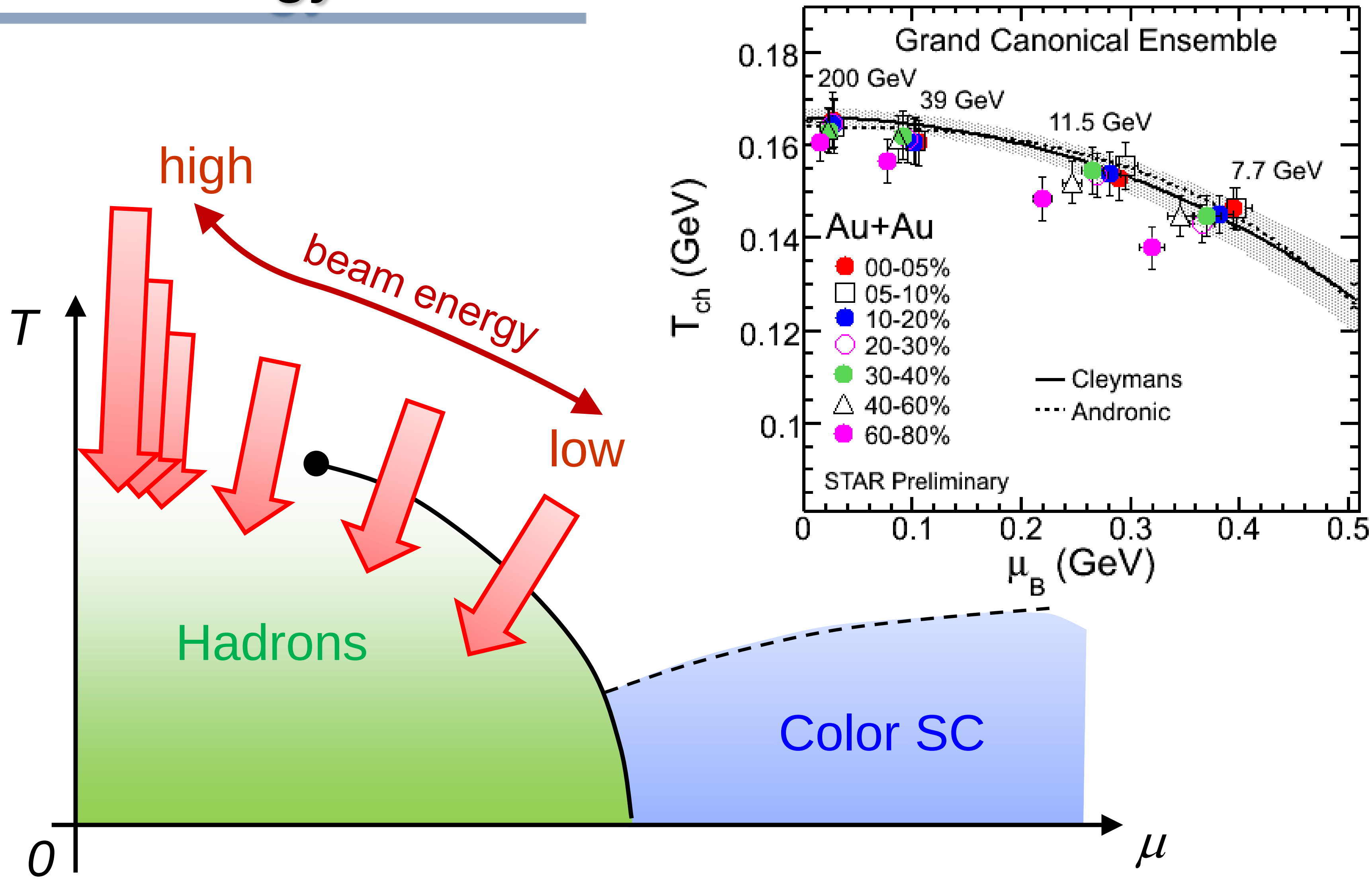
Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)

MK, Nucl. Phys. A942, 65 (2015)

NA61 Theory meeting, Apr., 27, 2017

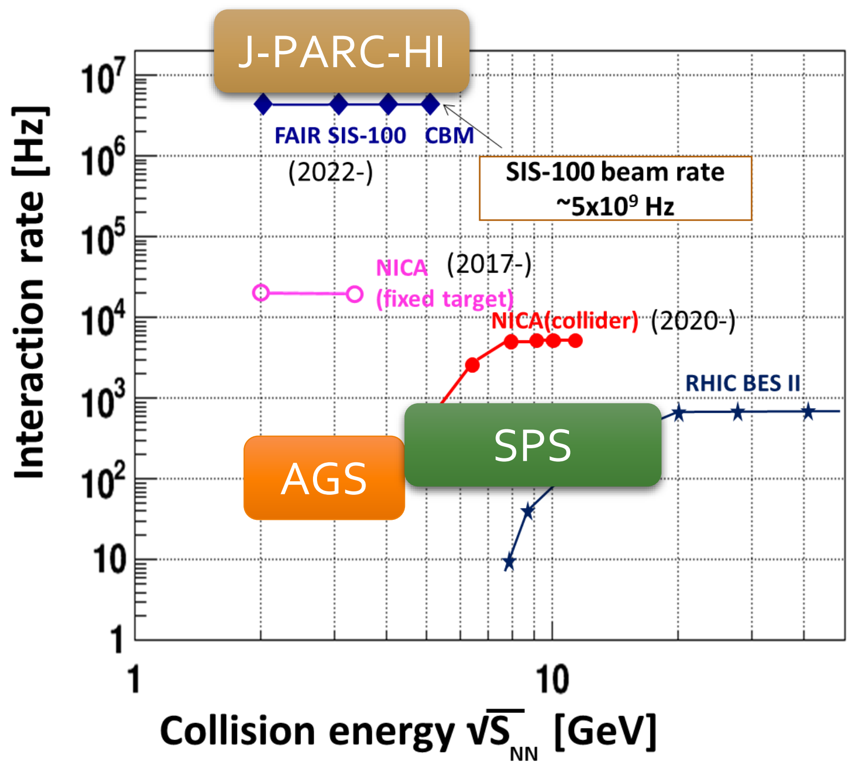
Beam-Energy Scan

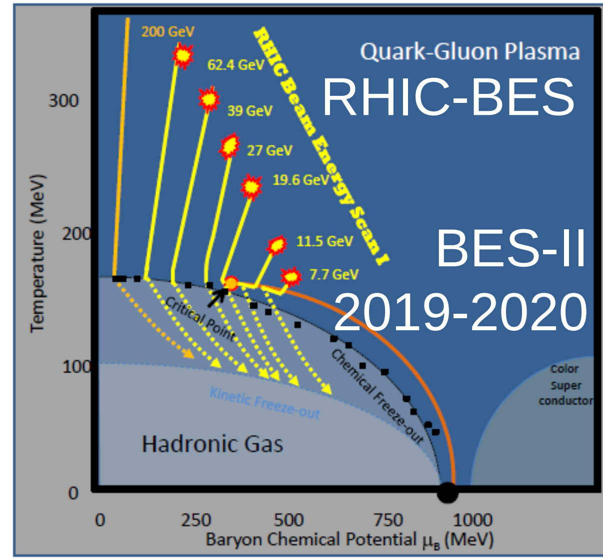
STAR 2012

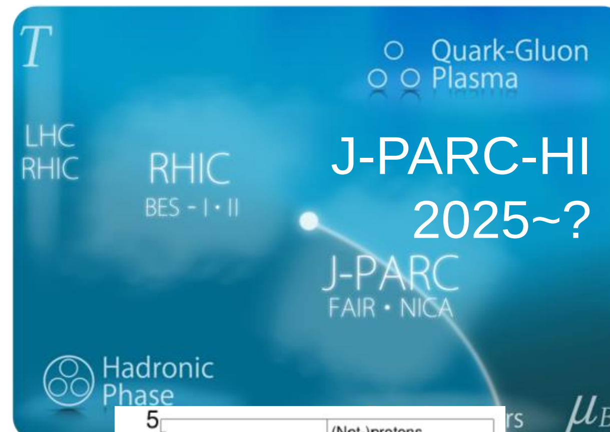


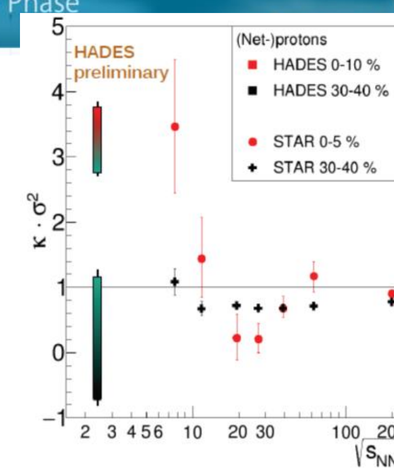
Beam-Energy Scan

Active experimental researches/plans for the beam-energy scan







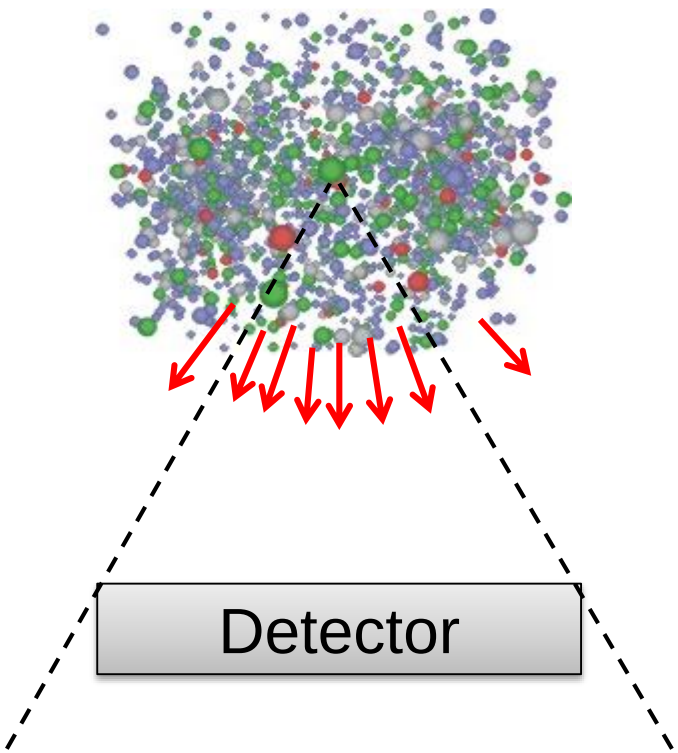


Search for QCD phase structure / critical point

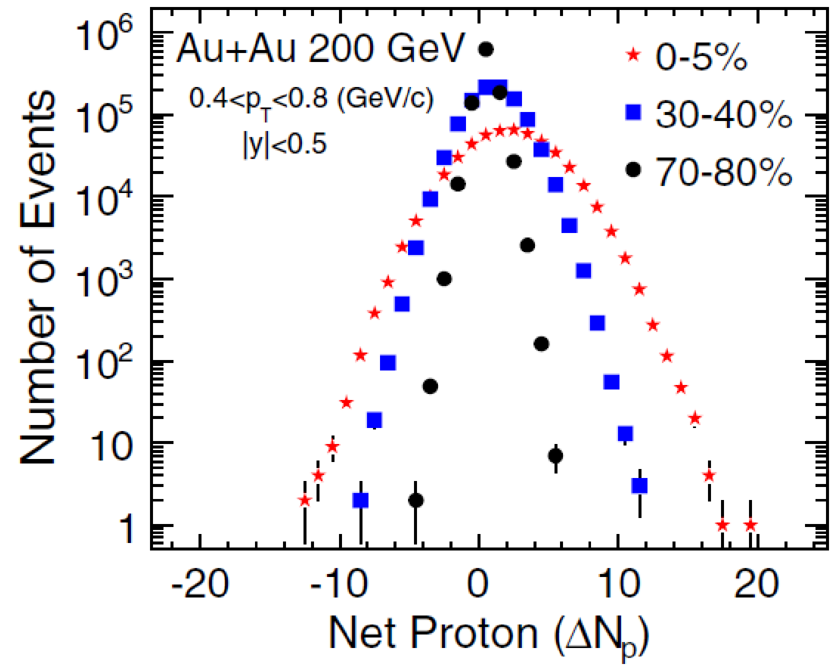
Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



STAR, PRL**105** (2010)

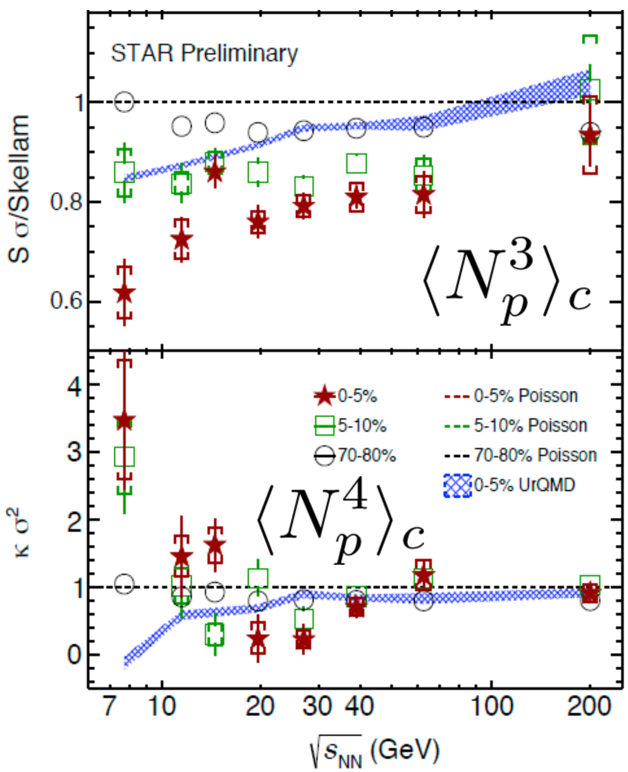


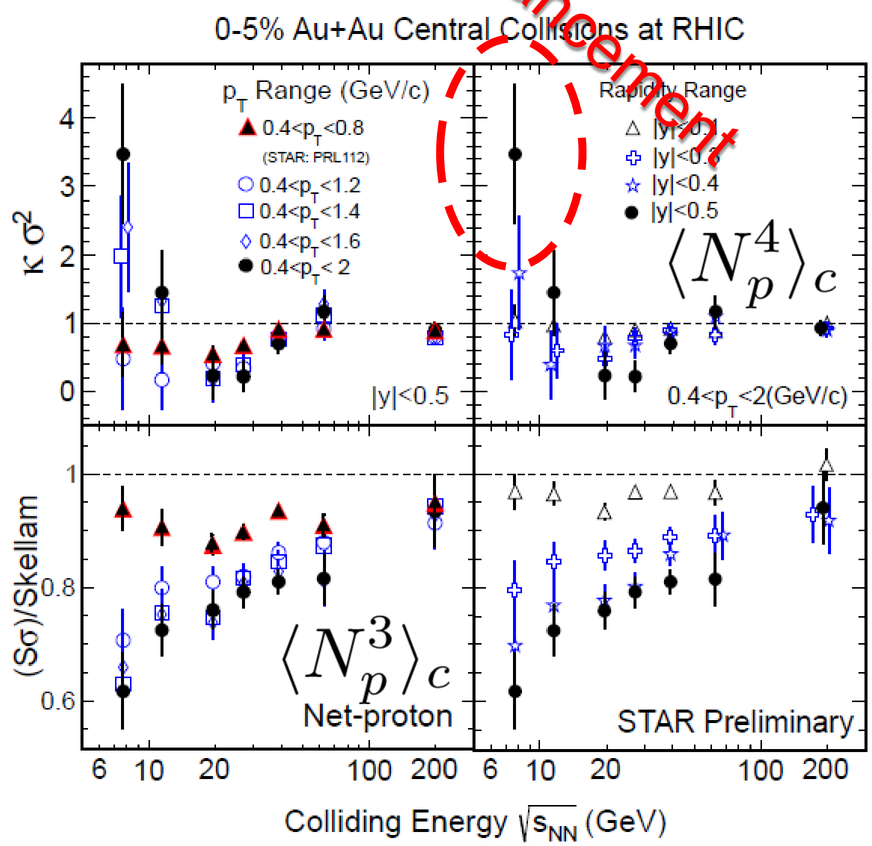
Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Higher-Order Cumulants

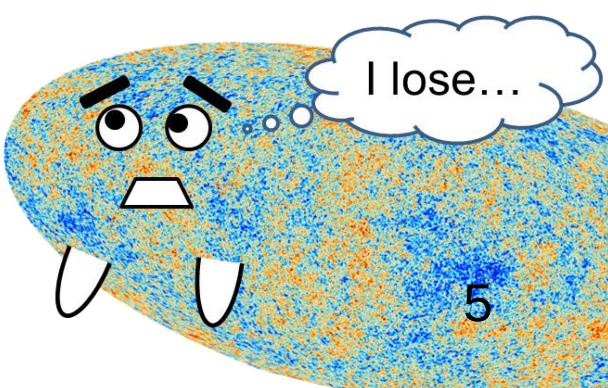
STAR Collab.
2010~





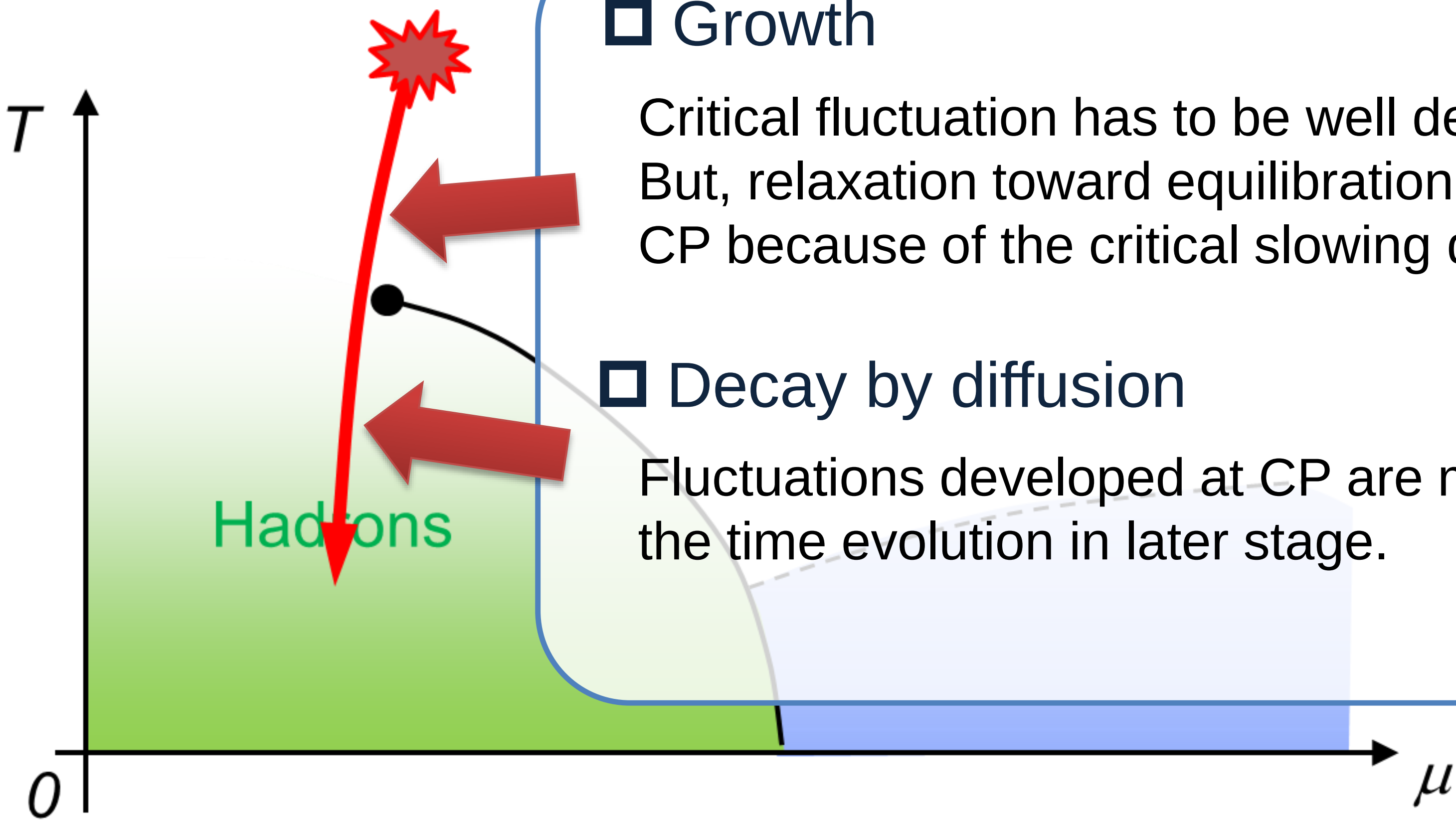
Non-zero **non**-Gaussian cumulants have been established!

Have we measured critical fluctuations?



Remarks on Critical Fluctuation 1

Experiments cannot observe critical fluctuation in equilibrium directly.



☐ Growth

Critical fluctuation has to be well developed.
But, relaxation toward equilibration is slow around CP because of the critical slowing down.

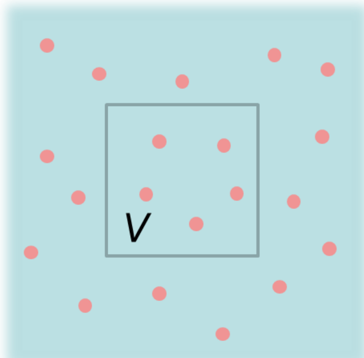
☐ Decay by diffusion

Fluctuations developed at CP are modified by the time evolution in later stage.

Fluctuations: Theory vs Experiment

Theoretical analyses

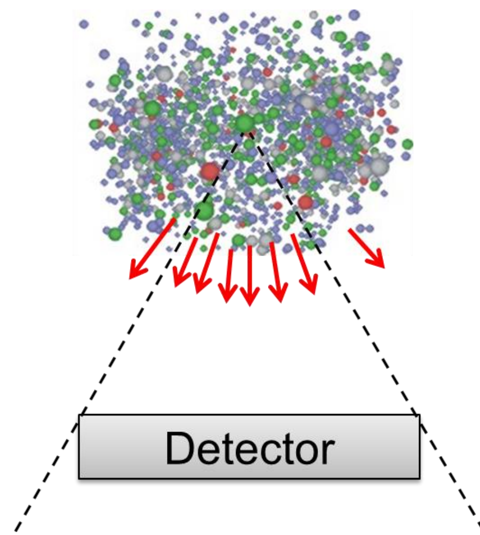
based on statistical mechanics



lattice, critical point,
effective models, …

Fluctuation in a spatial volume

Experiments



Fluctuations in a momentum space

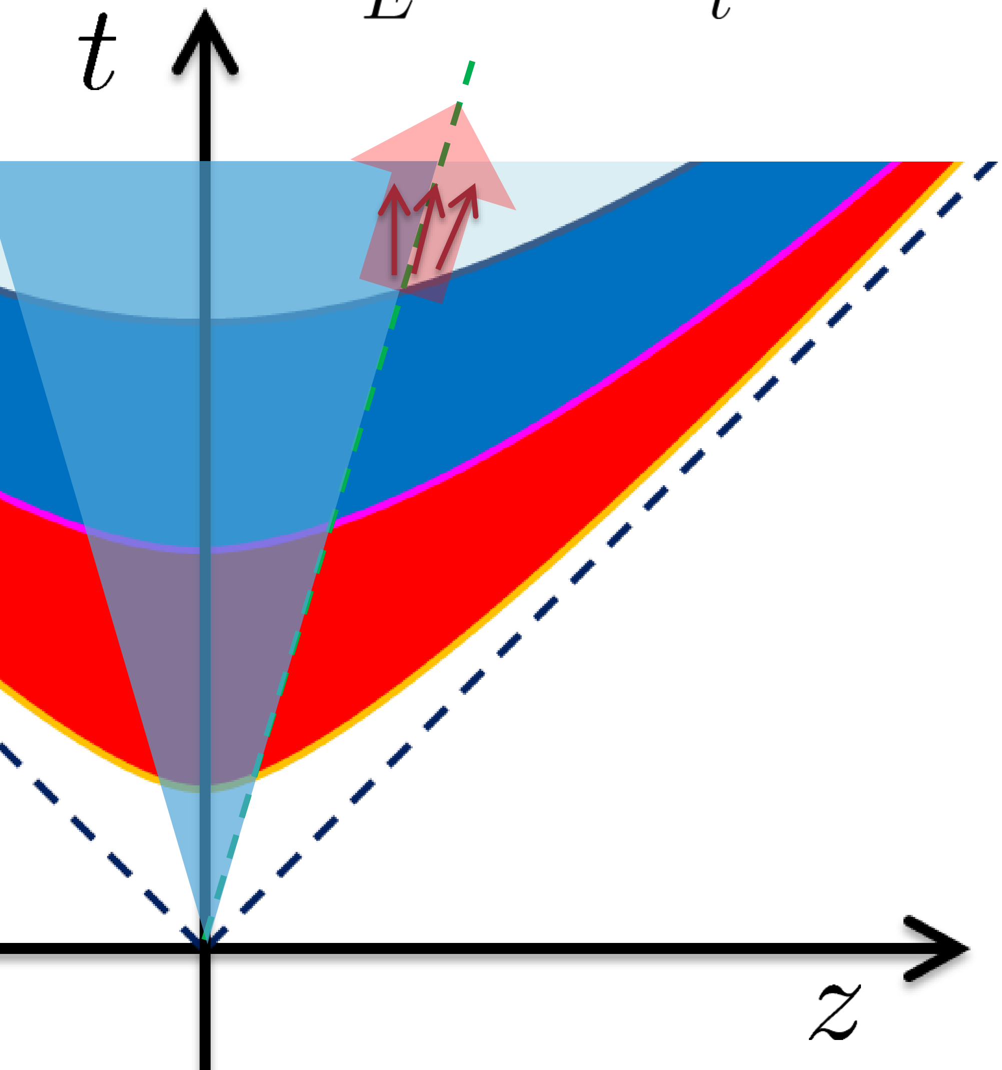
discrepancy in phase spaces

Asakawa, Heinz, Muller, 2000; Jeon, Koch, 2000; Shuryak, Stephanov, 2001

Thermal Blurring

Ohnishi, MK, Asakawa,
PRC94, 044905 (2016)

$$\frac{p_z}{E} = v_z = \frac{z}{t}$$



Under Bjorken picture,

coordinate-space rapidity *Y*

||

momentum-space rapidity *y* of medium

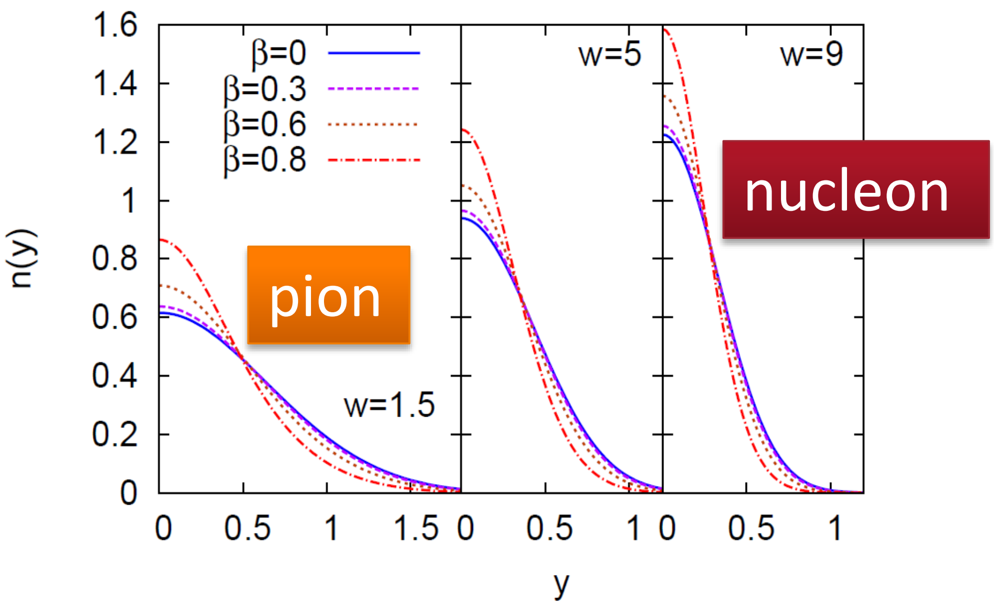
≀

momentum-space rapidity *y* of individual particles

$$\Delta y \simeq \Delta Y$$

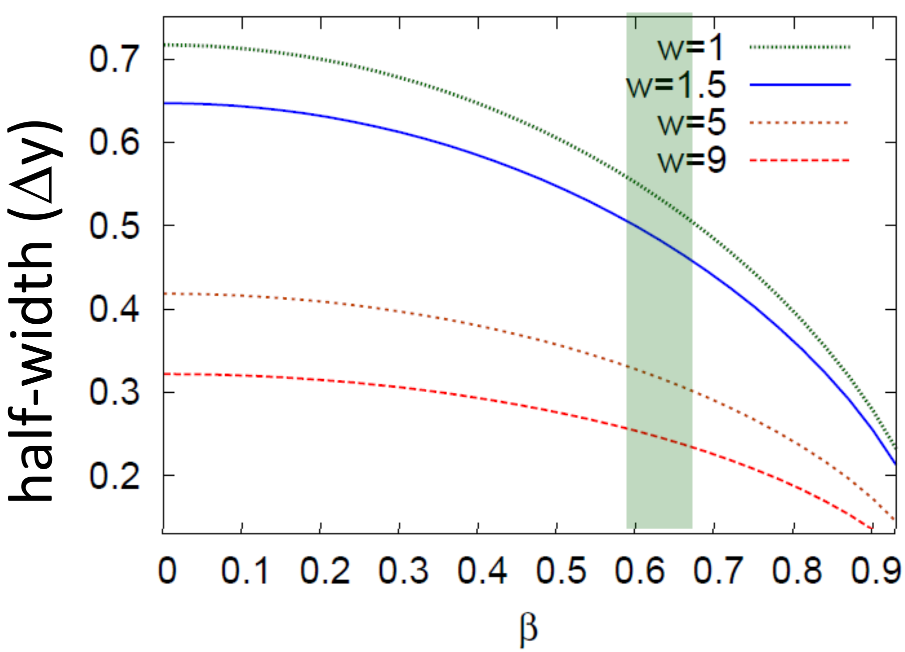
Thermal distribution in y space

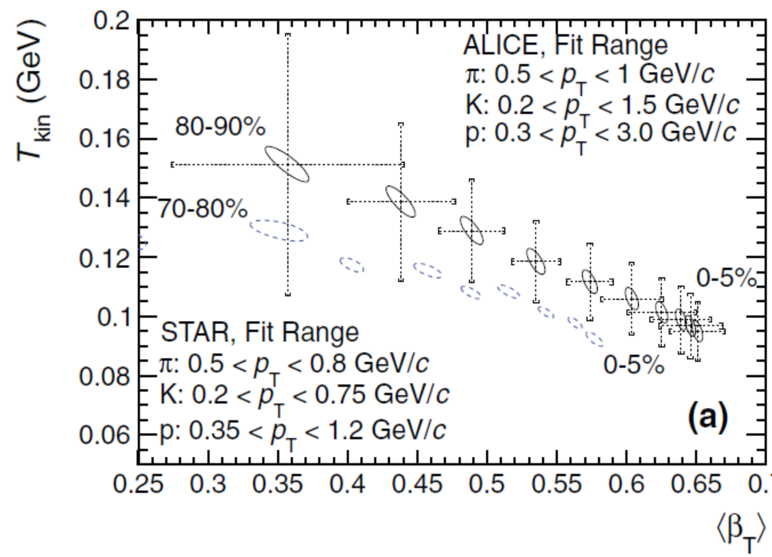
Ohnishi, MK, Asakawa,
PRC94, 044905 (2016)



$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$





- blast wave
- flat freezeout surface

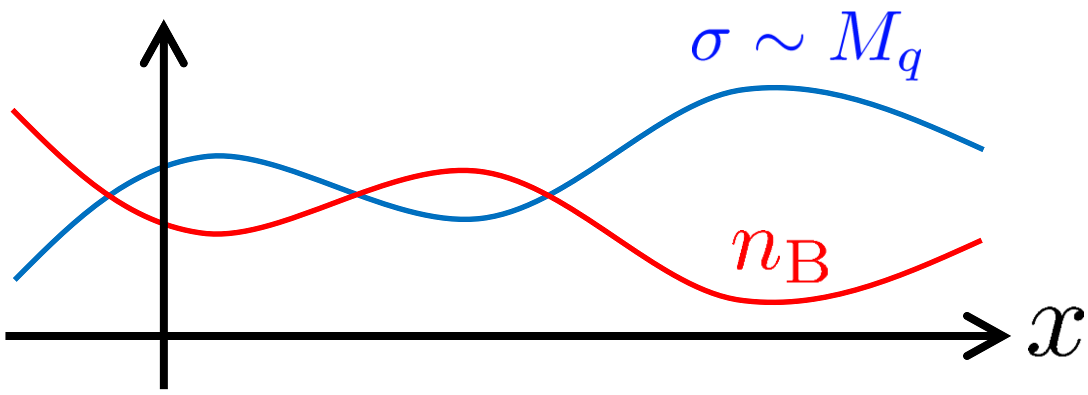
Remarks on Critical Fluctuation 2

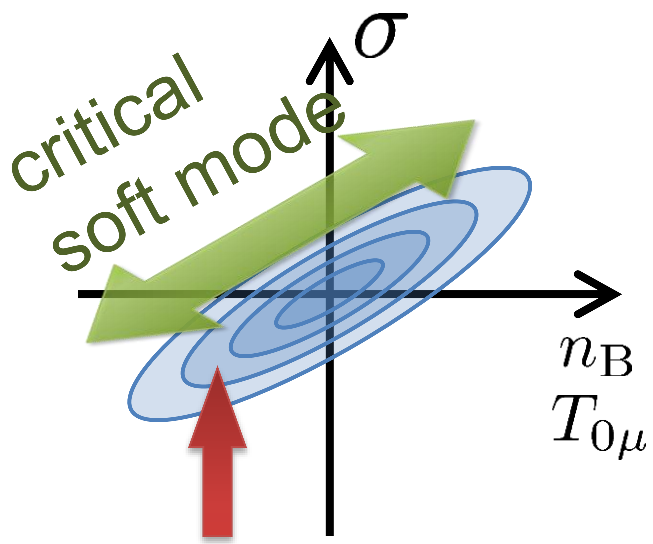
Critical fluctuation is a conserved mode!

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$





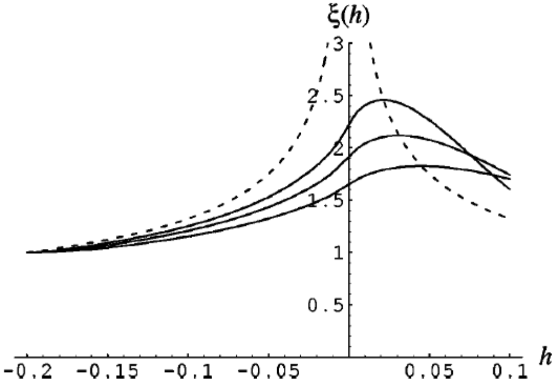
σ: fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

Dynamical Evolution of Critical Fluctuations

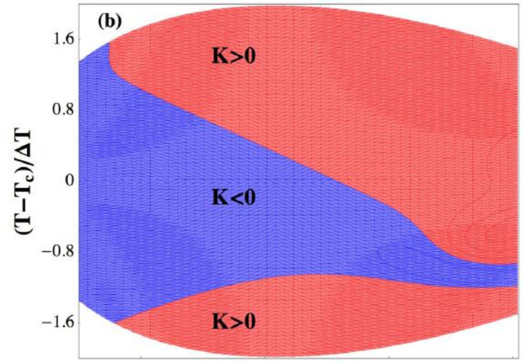
- Evolution of correlation length

Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)



- Higher orders (spatially uniform “σ” mode)

Mukherjee, Venugopalan, Yin (2015)



- Dynamical evolution in chiral fluid model

Nahrgang, Herold, … (2014~)

- Correlation functions

Kapusta, Torres-Rincon (2012)

Cumulants and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

correlation function

1-to-1 correspondence

1-dim case

$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Aim of This Study

- Describe **conserved nature** of critical fluctuation.
- We want to study **experimental observables**.
  - focus on a **conserved charge** **(baryon number)**
  - study evolution of **conserved-charge** fluctuation
- Concentrate on **2nd order cumulant**. (not higher)
- We study
  - **rapidity window denepdence** of the cumulant
  - 2-particle **correlation function**

Our Main Conclusion

Non-monotonicity in cumulants or correlation func.	=	Signal of QCD-CP

Stochastic Diffusion Equation (SDE)

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

- Describe a relaxation of a conserved density *n* toward uniform state without fluctuation

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

$$\langle \xi(\eta_1)\xi(\eta_2)\rangle \sim \chi\delta(\eta_1-\eta_2)$$

- Describe a relaxation toward fluctuating uniform state
- χ: susceptibility (fluctuation in equil.)

Soft Mode of QCD Critical Point

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

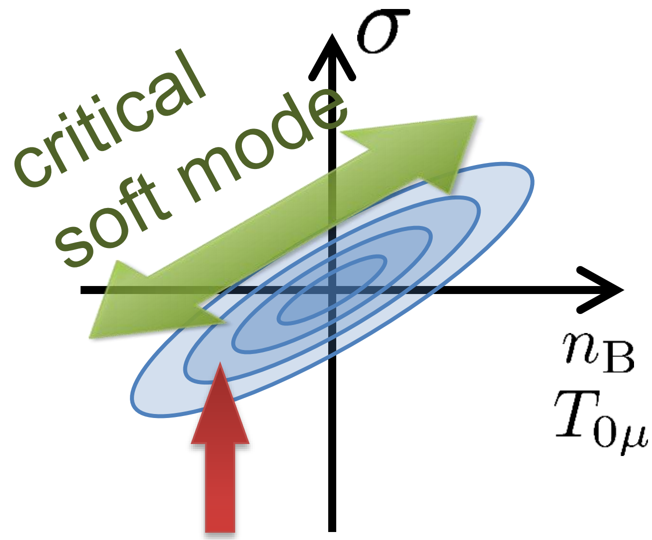
- Effective potential

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

- Time dependent Ginzburg-Landau

$$\begin{pmatrix} \dot{\sigma} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix}$$

$$\sim k^2$$



σ: fast damping

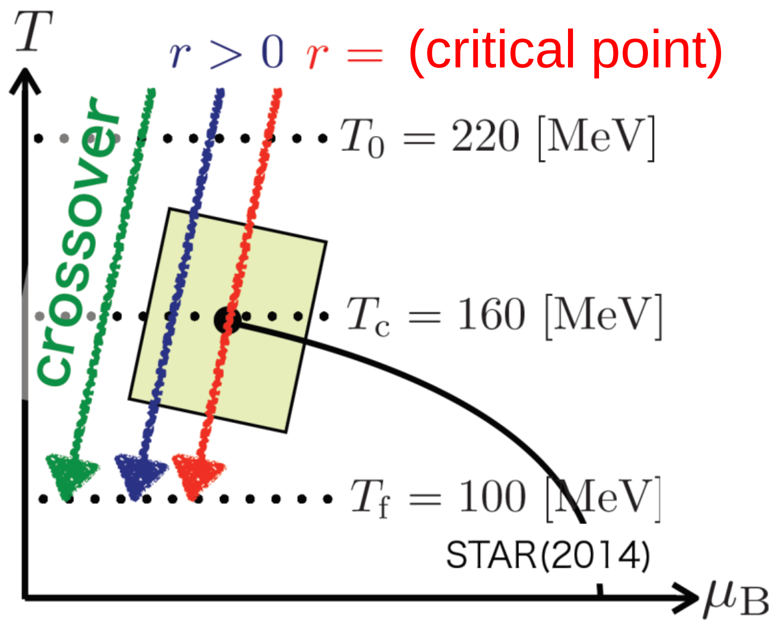
For slow and
long wavelength,

SDE $\quad \partial_\tau n = D(\tau)\partial_\eta^2 n + \partial_\eta \xi$

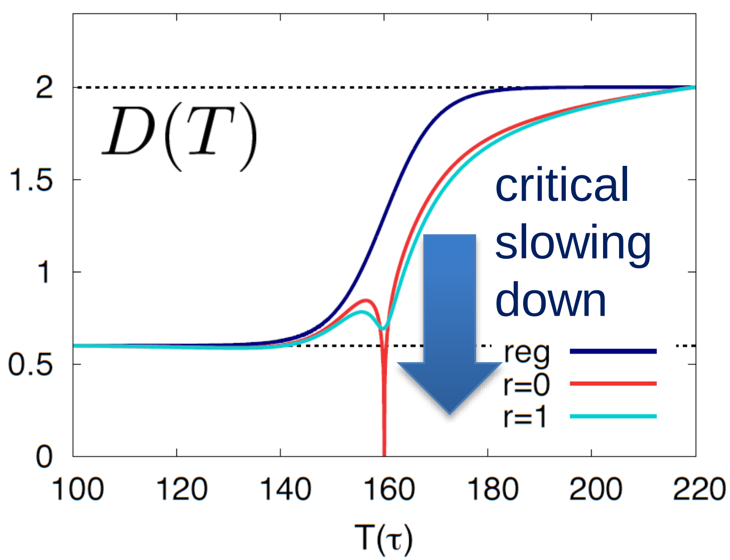
singularities in $D(\tau)$ and $\chi(\tau)$

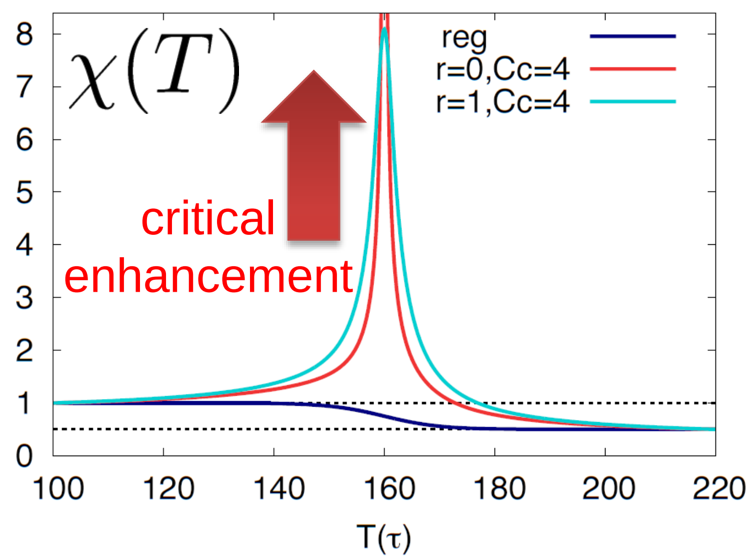
Parametrizing D(τ) and χ(τ)

- **Critical behavior**
  - 3D Ising (r,H)
  - model H



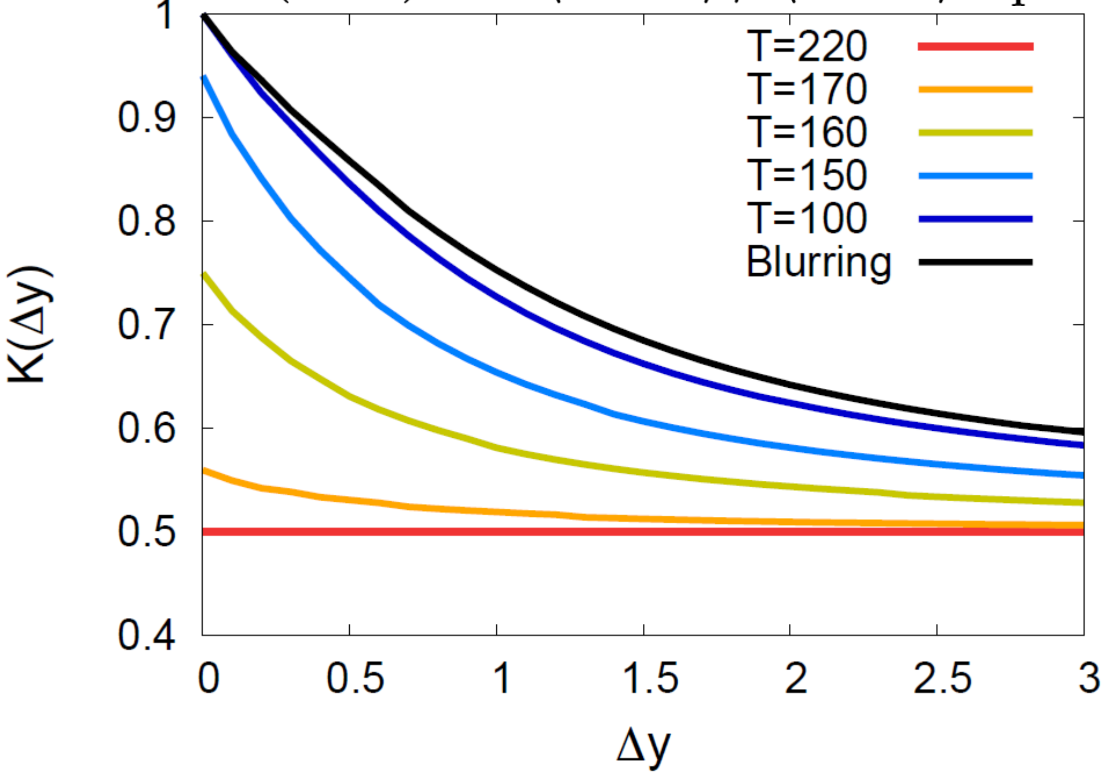
- **Temperature dep.**

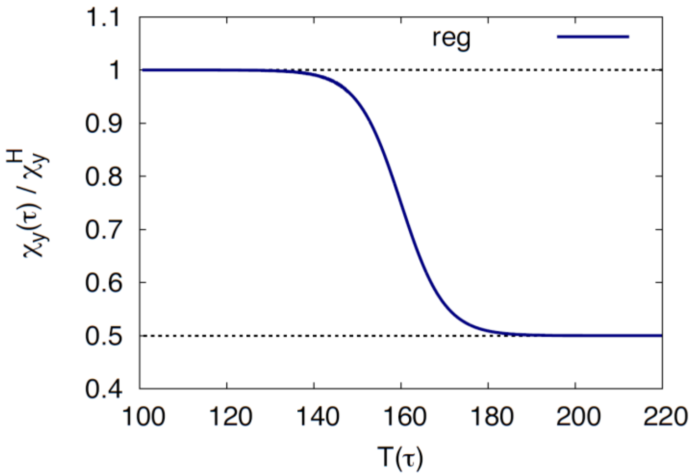


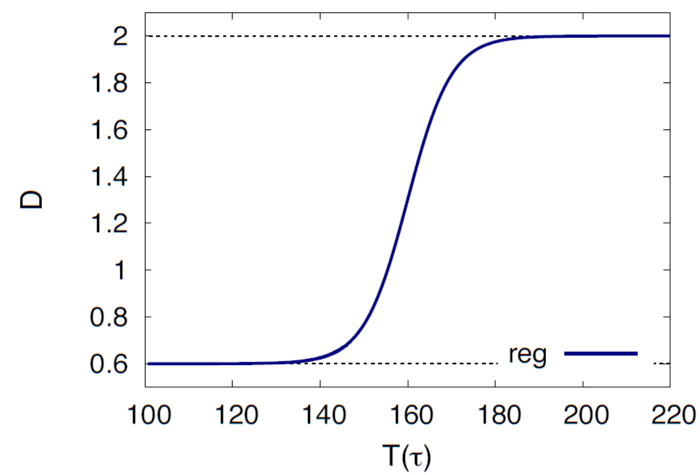


Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$





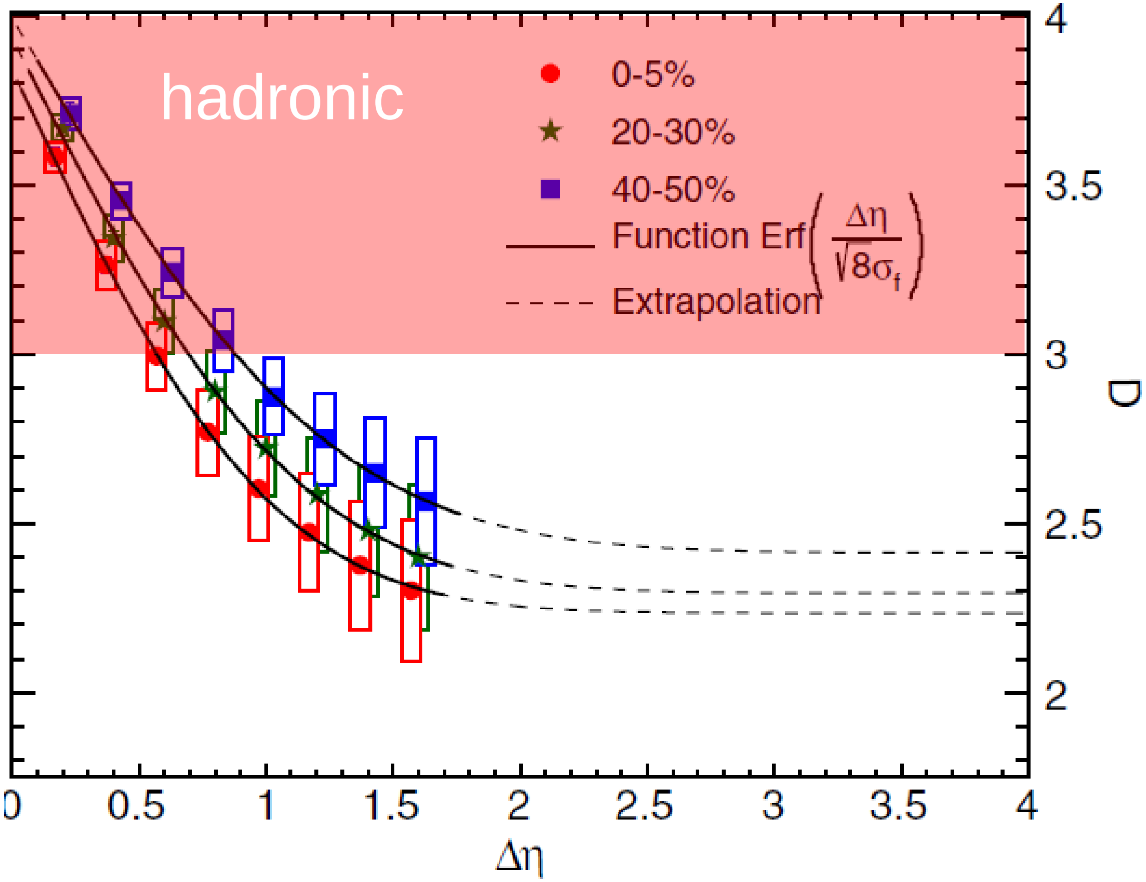


- monotonically decresing

Analytic result: $\chi(\tau)$ monotonically increasing ⇒ $K(\Delta y)$ monotonically decreasing

Δη Dependence @ ALICE

ALICE
PRL 2013

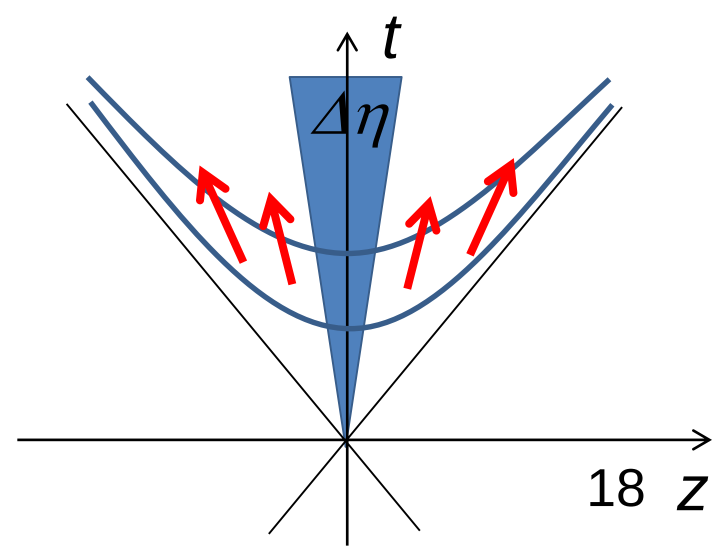


rapidity window

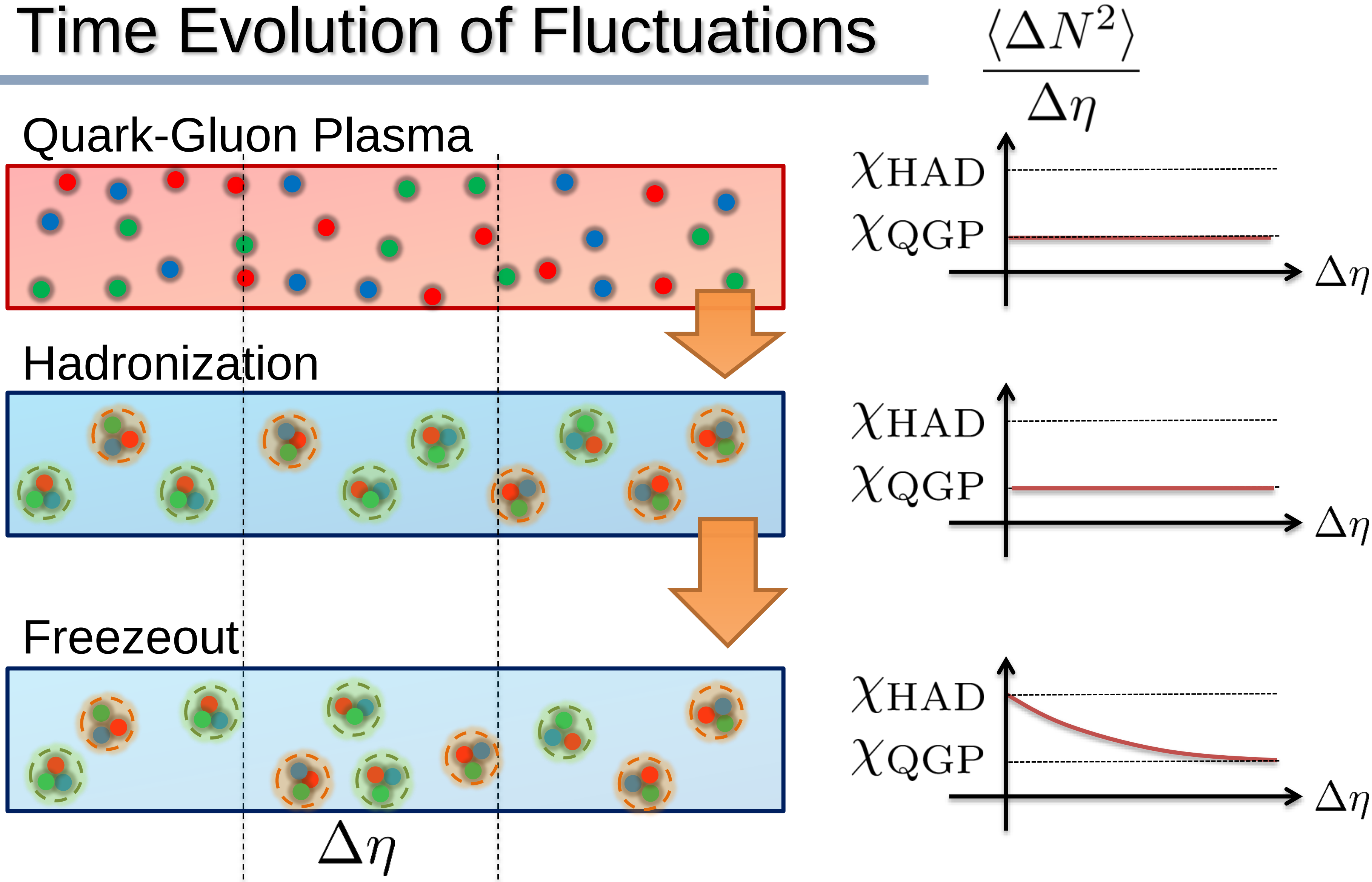
D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark



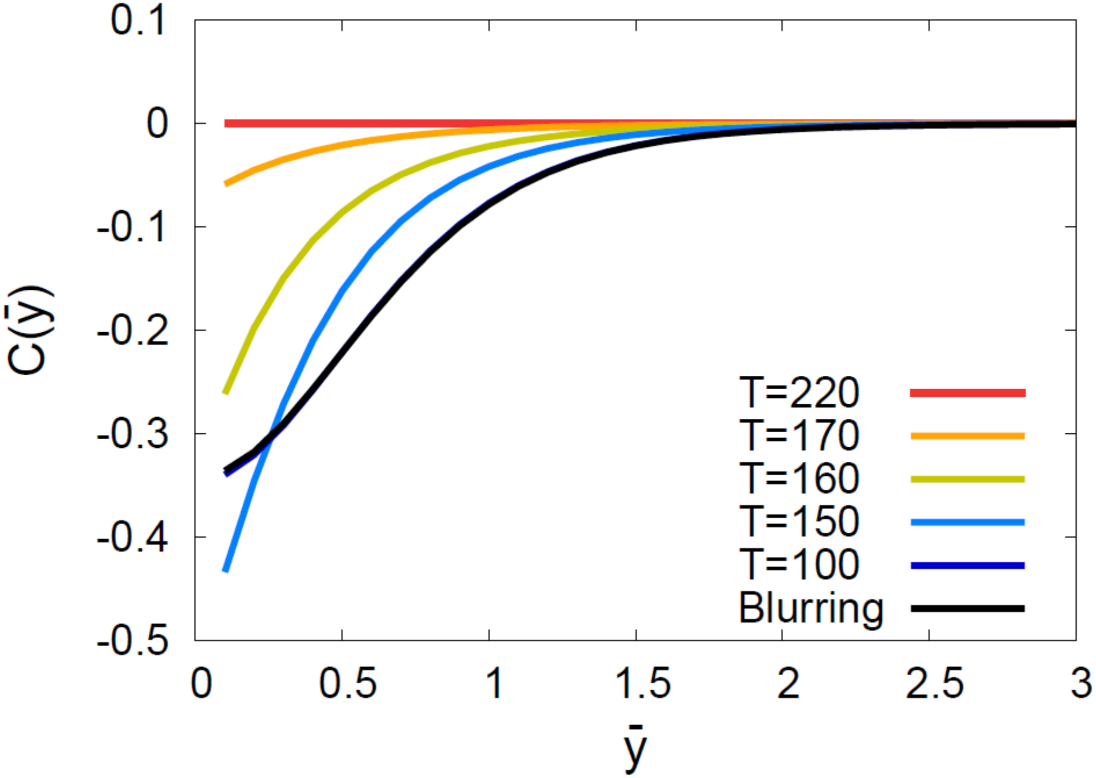
Time Evolution of Fluctuations

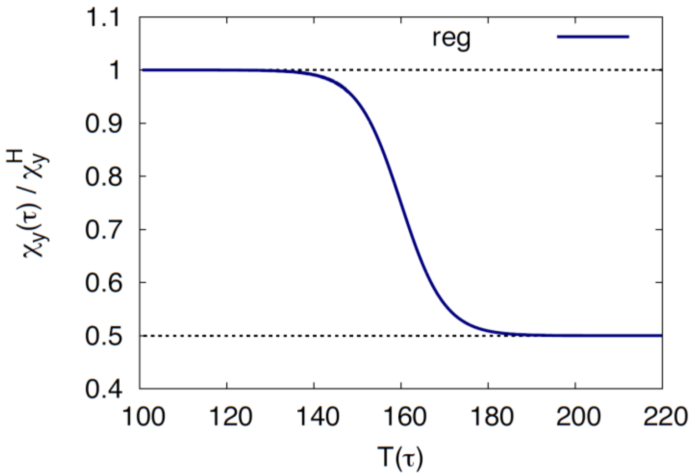


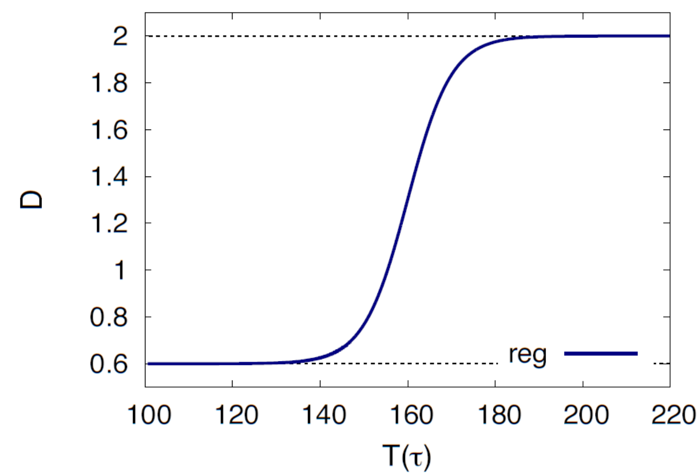
Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Crossover / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$





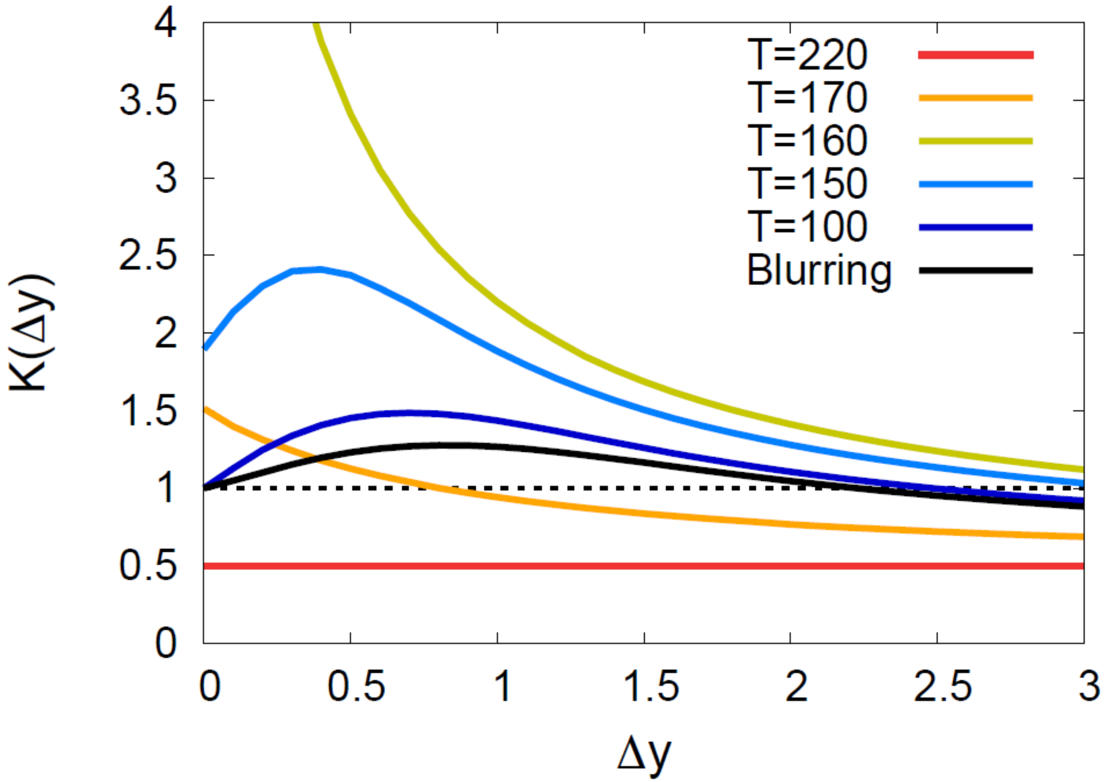


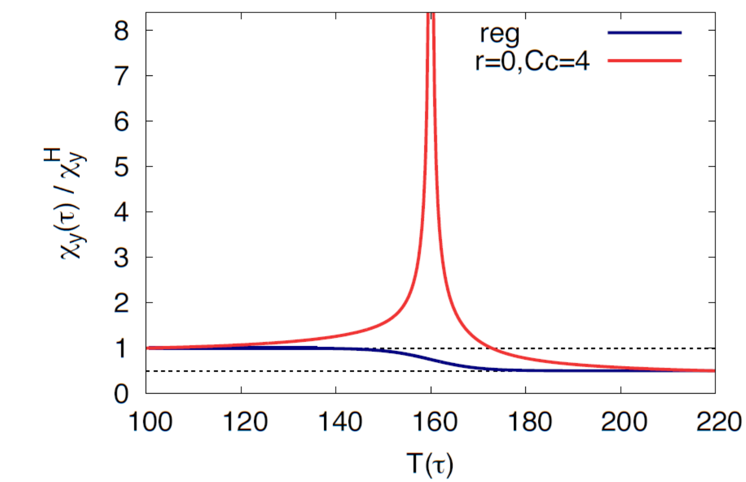
- monotonically decresing

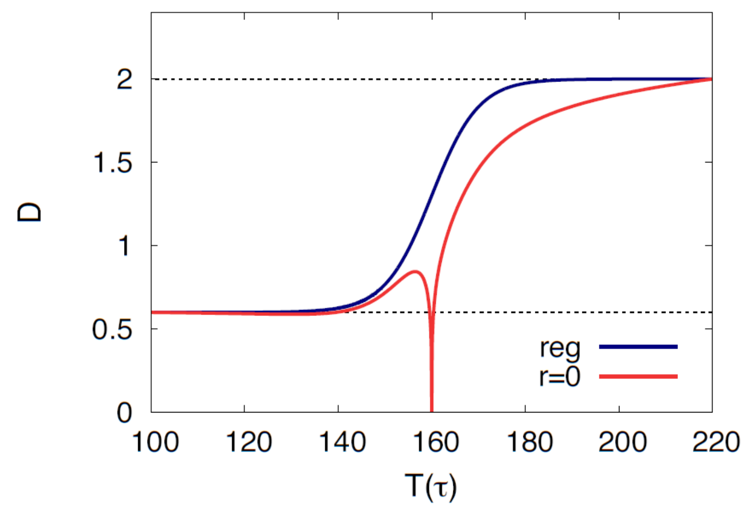
Analytic result: $\chi(\tau)$ monotonically increasing ⇒ $C(\bar{y})$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$







□ non-monotonic Δy dep.

Analytic result

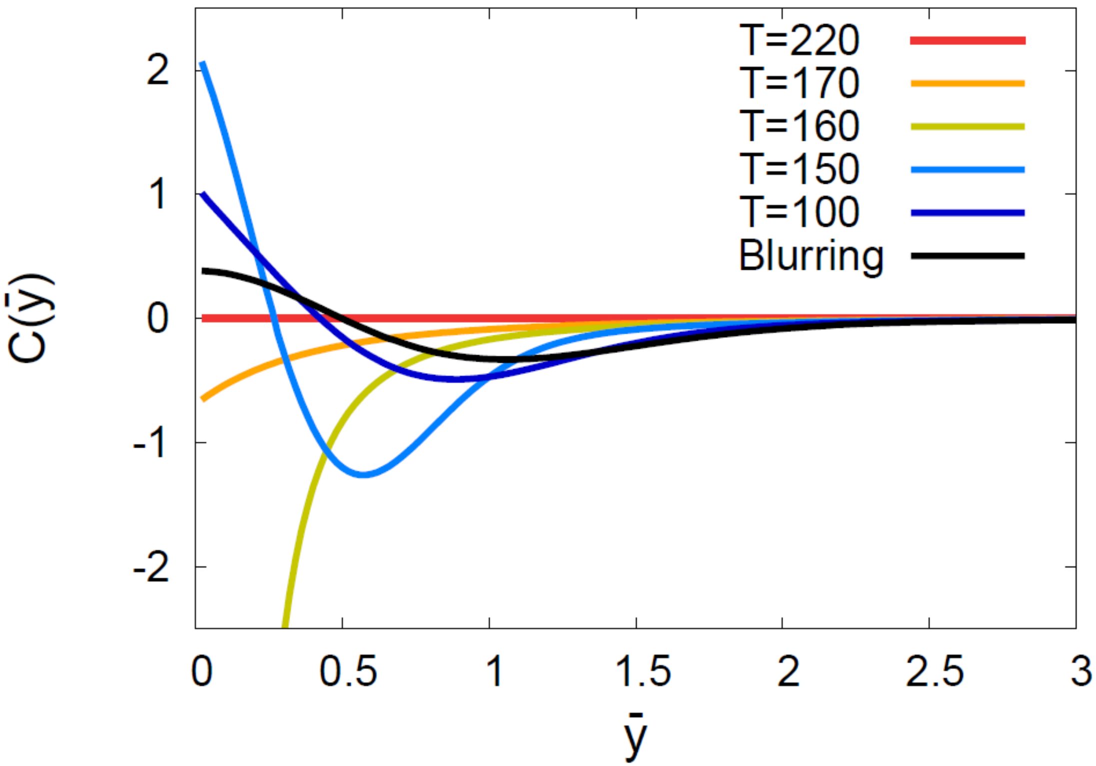
$K(\Delta y)$
non-monotonic

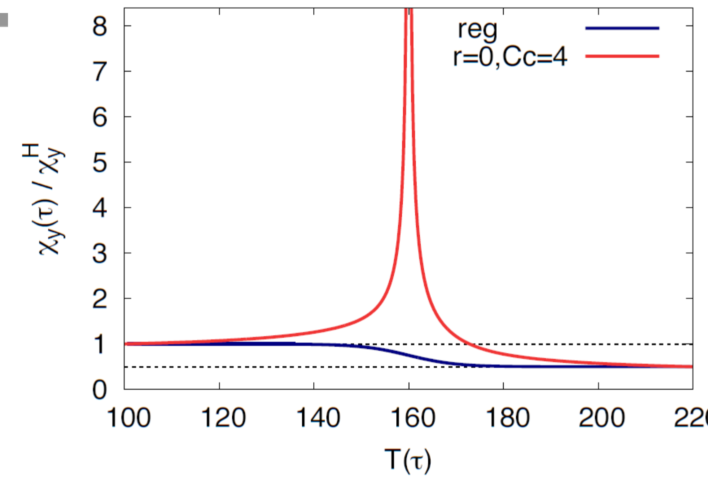
→

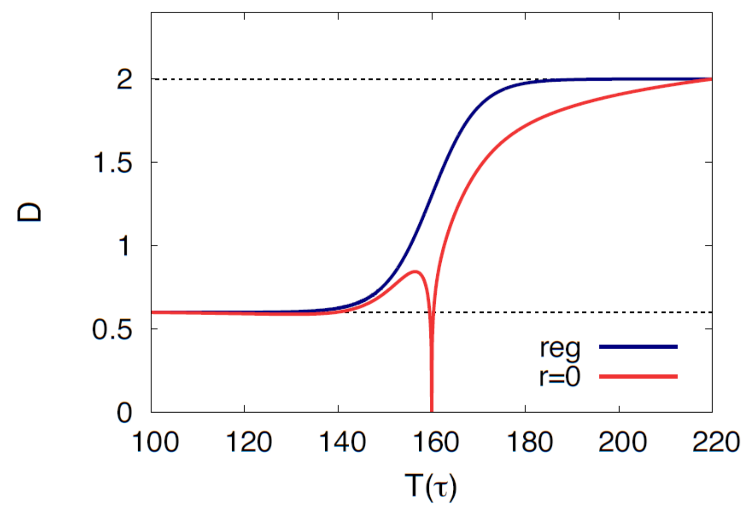
$\chi(\tau)$
non-monotonic

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$





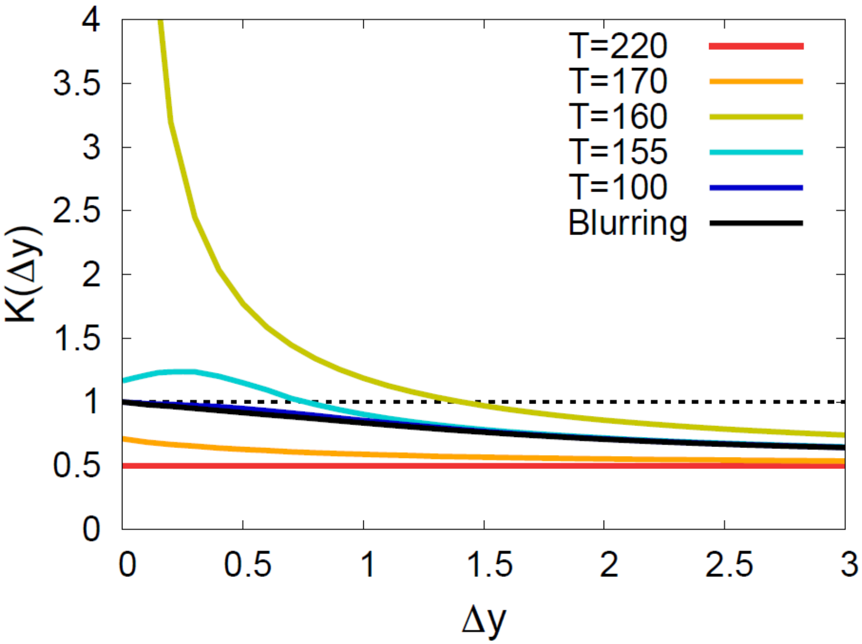


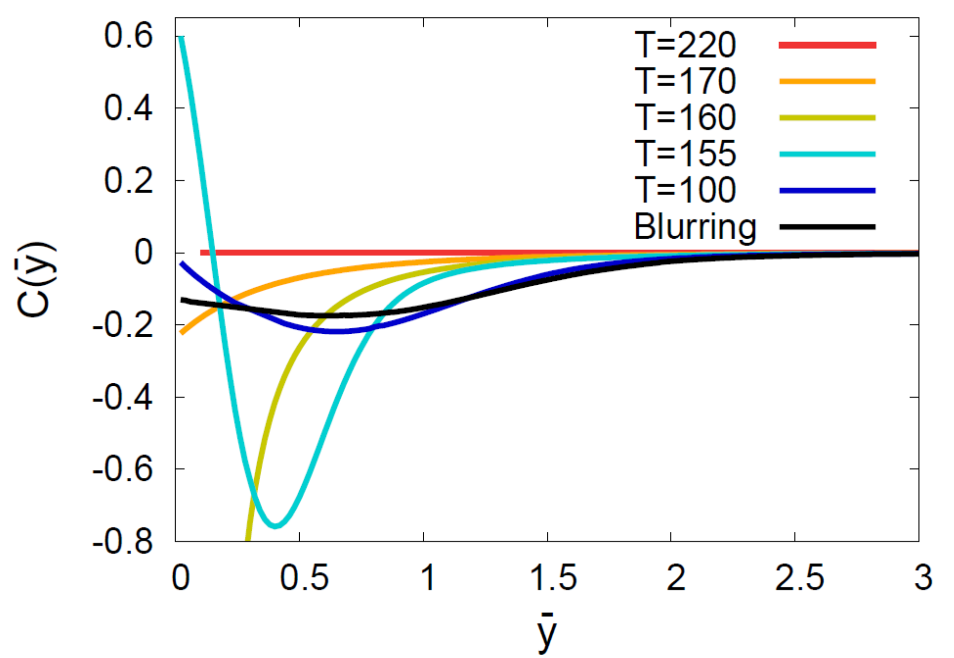
- non-monotonic Δy dep.

Analytic result: $C(\Delta y)$ non-monotonic → $\chi(\tau)$ non-monotonic

Weaker Critical Enhancement

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.} \qquad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$





- Non-monotonicity in K(Dy) disappears.
- But C(y) is still non-monotonic.

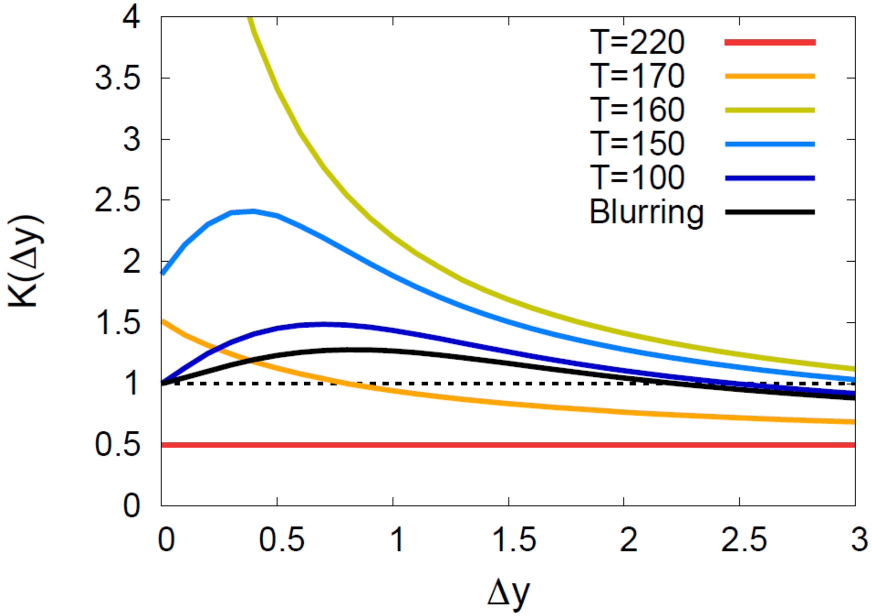
Analytic result

$K(\Delta y),\ C(\bar{y})$ monotonic ⇒ no information on $\chi(\tau)$

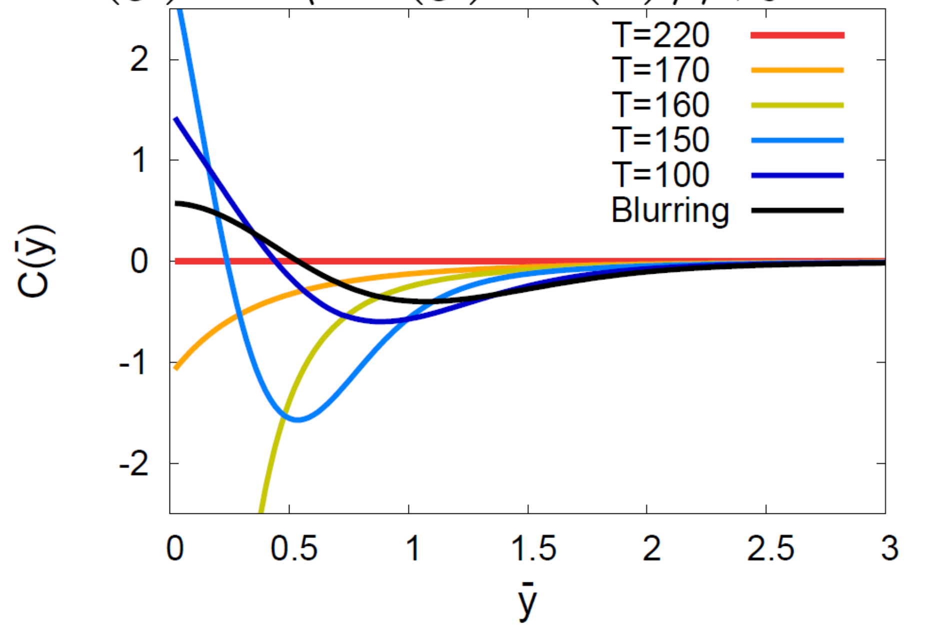
- C(y) is better to see non-monotonicity.

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$



$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$



- Signal of the critical enhancement can be clearer on a path away from the CP.

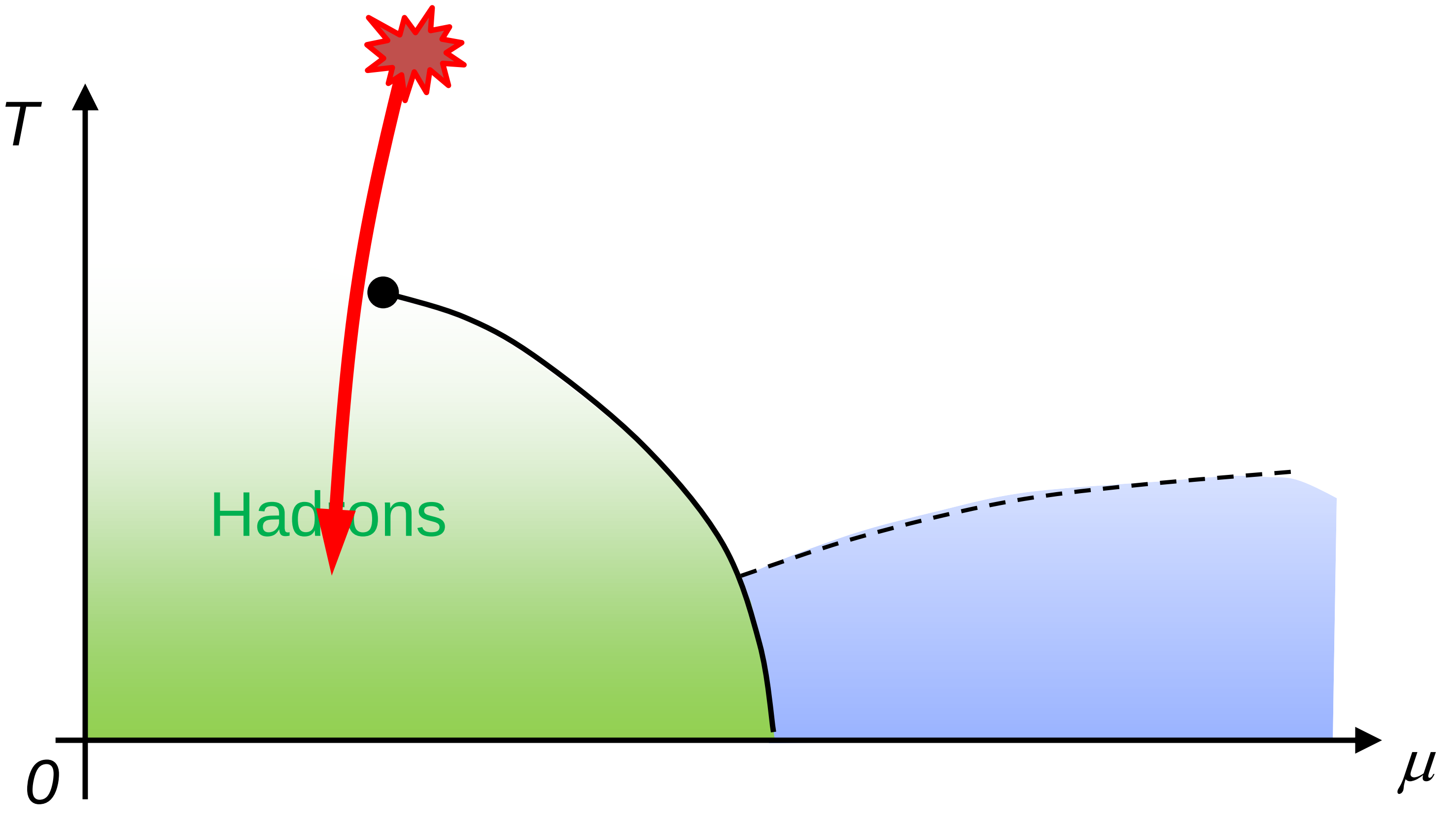
Away from the CP → Weaker critical slowing down

Summary

- ☐ Soft mode of the QCD critical point is a conserved mode. Its time evolution depends on the size defining the charge.
- ☐ Time evolution of conserved charges (especially baryon number) is well described by the stochastic diffusion equation.
- ☐ A non-monotonic behavior of cumulant or correlation function is the signal of the critical enhancement!

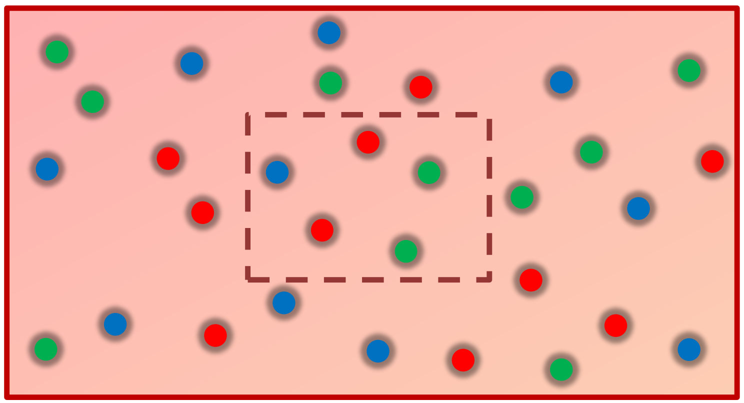
Suggestion to experimentalists

- ☐ To find the CP, measure
  - Δy dep. of 2nd order cumulant
  - y dep. of correlation function
- ☐ Study lower-order fluctuation in more detail


T
Hadrons
0
μ

Fluctuations and Elemental Charge

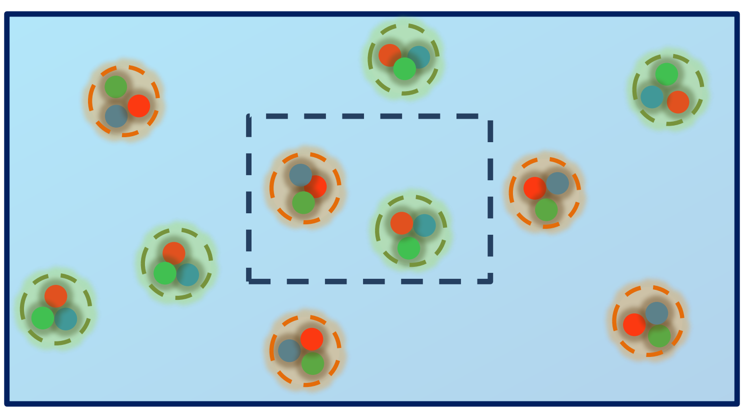
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005

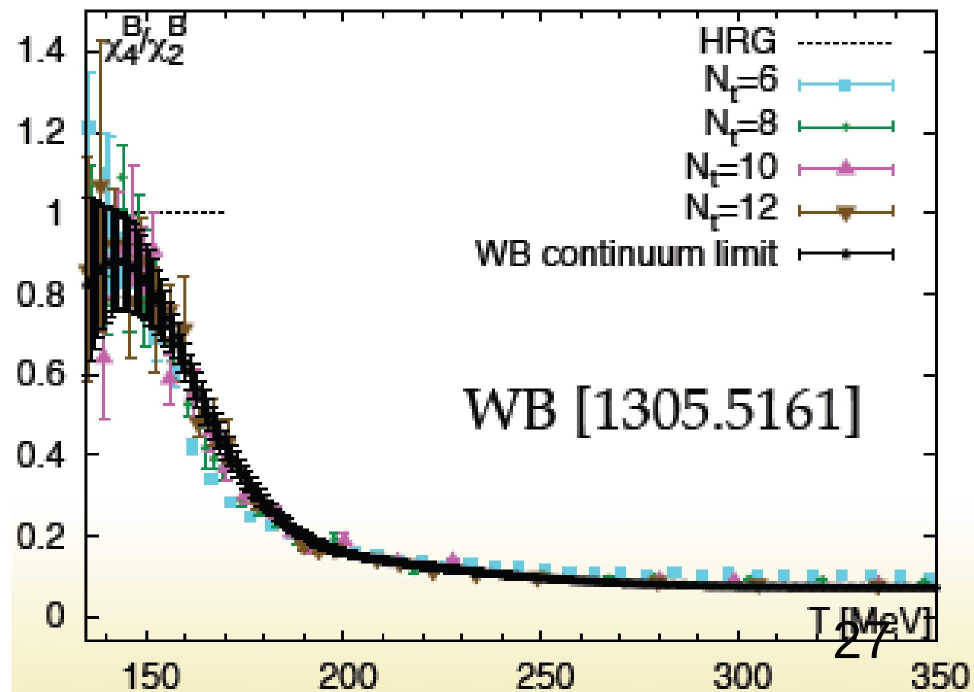


$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

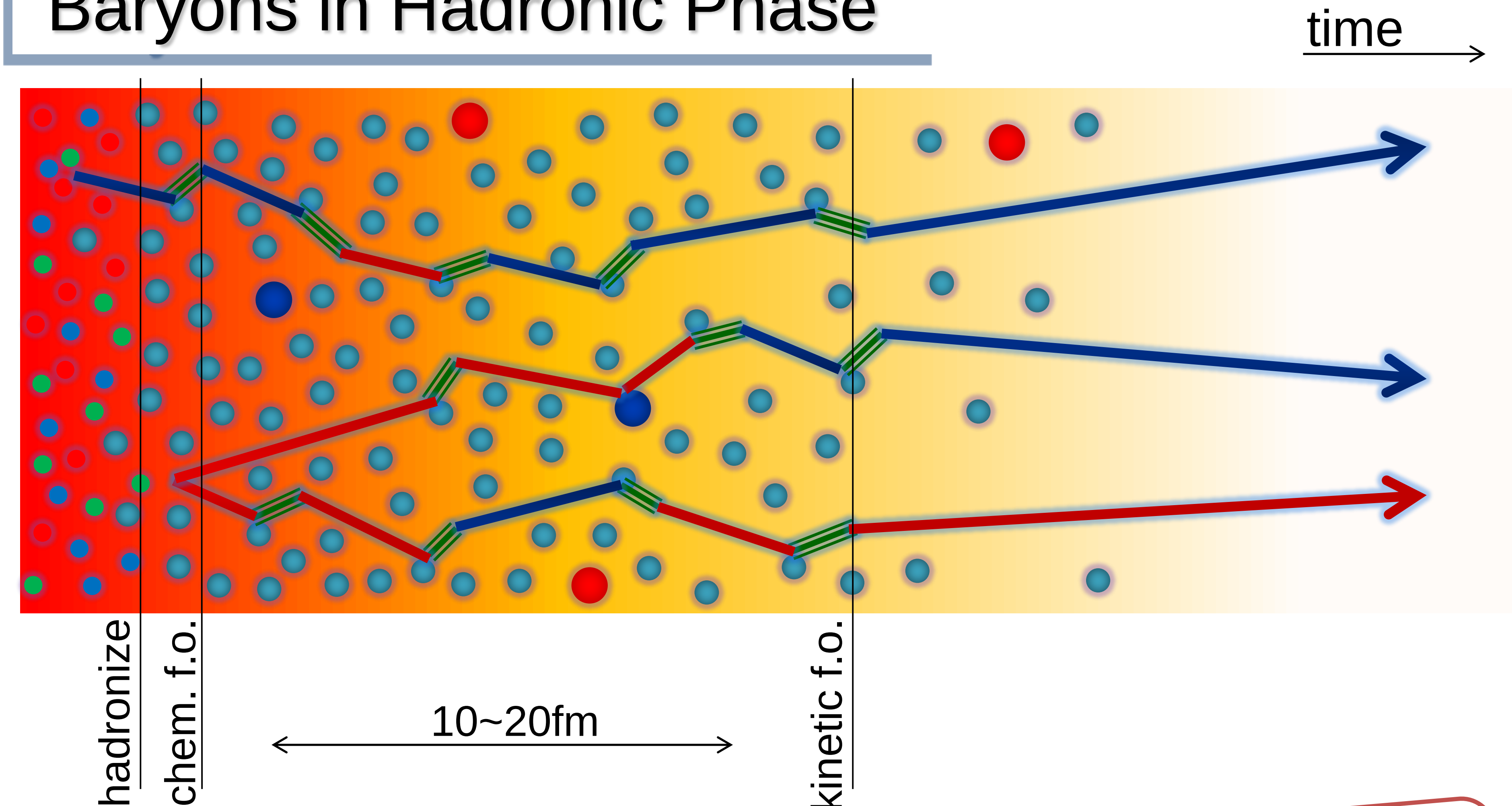
$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$





Baryons in Hadronic Phase



$p, \bar{p}$ — mesons

$n, \bar{n}$ — baryons

$\Delta(1232)$

Baryons behave like Brownian pollens in water