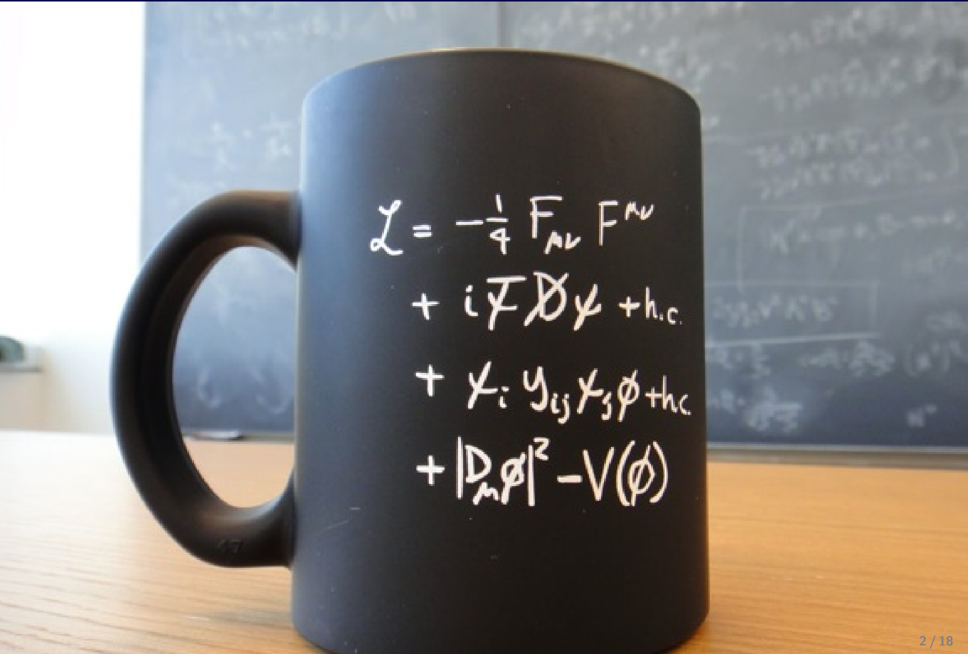
CERN, July 2017

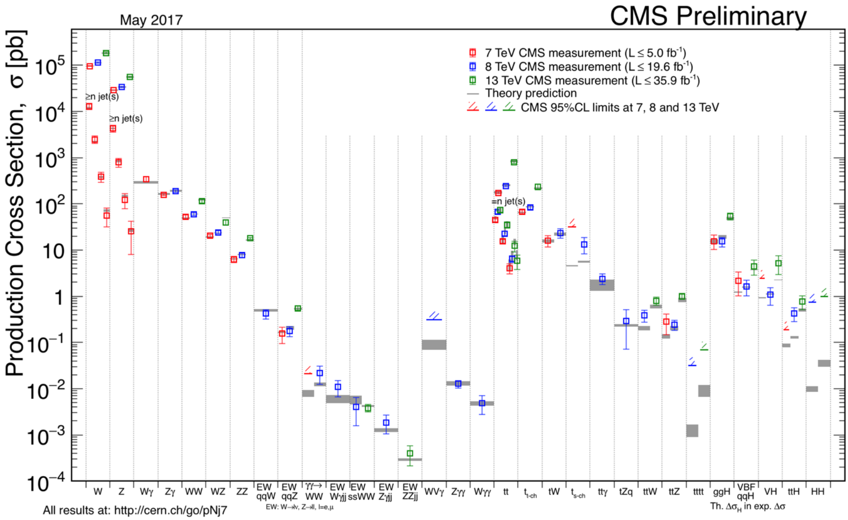
The Standard Model of particle physics

Michael Krämer (RWTH Aachen University)

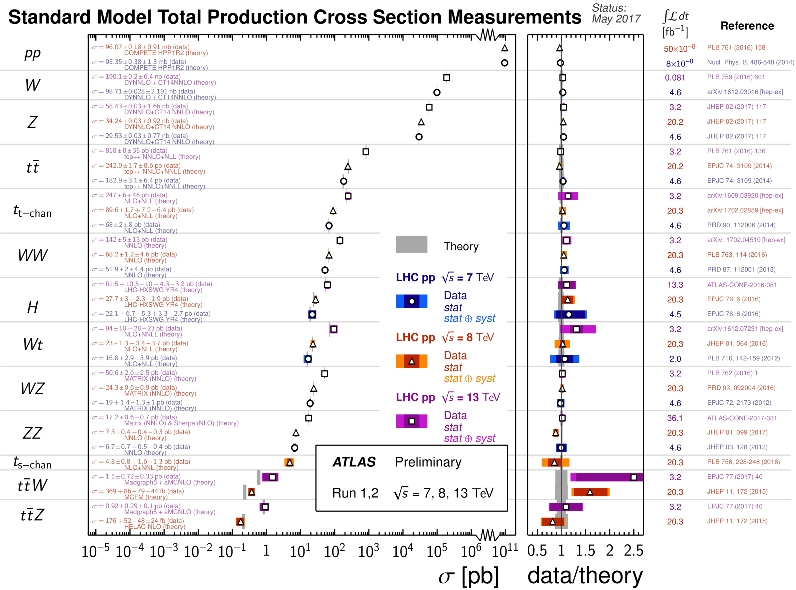
The Standard Model of particle physics



The Standard Model of particle physics



The Standard Model of particle physics



The Standard Model of particle physics

The Standard Model Lagrangian is determined by symmetries

- space-time symmetry: global Poincaré-symmetry
- internal symmetries: local $SU(n)$ gauge symmetries

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi}\not{D}\psi \qquad \text{gauge sector}$$

$$+ |D_\mu H|^2 - V(H) \qquad \text{EWSB sector}$$

$$+ \psi_i \lambda_{ij} \psi_j H + \mathrm{h.c.} \qquad \text{flavour sector}$$

...including only the operators of lowest dimension
and ignoring the strong CP-problem (see lectures by Andrew Cohen).

Outline

- QED as a gauge theory
- Quantum Chromodynamics
- Breaking gauge symmetries:

  the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism
- Exploring electroweak symmetry breaking at the LHC

Bibliography

- I.J.R. Aitchison and A.J.G. Hey, "Gauge Theories in Particle Physics", IoP Publishing.
- R.K. Ellis, W.J. Stirling and B.R. Webber, "QCD And Collider Physics," Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 8 (1996) 1.
- D. E. Soper, Basics of QCD perturbation theory, arXiv:hep-ph/0011256.
- Lectures by Keith Ellis, Douglas Ross, Adrian Signer, Robert Thorne and Bryan Webber (thanks!).

The Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi(x) = 0$$

"A great deal more was hidden in the Dirac equation than the author had expected when he wrote it down in 1928. Dirac himself remarked in one of his talks that his equation was more intelligent than its author." (Weisskopf on Dirac)

[We use natural units: $c = \hbar = 1$, so that [mass] $=$ [length]$^{-1}$ $=$ [time]$^{-1}$ $=$ (Giga) electron volt]

The Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

"A great deal more was hidden in the Dirac equation than the author had expected when he wrote it down in 1928. Dirac himself remarked in one of his talks that his equation was more intelligent than its author." (Weisskopf on Dirac)

[We use natural units: $c = \hbar = 1$, so that [mass] = [length]$^{-1}$ = [time]$^{-1}$ = (Giga) electron volt]

What would you do to reconcile quantum theory and special relativity:

$$i\frac{\partial}{\partial t}\phi = H\phi \quad \text{and} \quad E^2 = \vec{p}^2 + m^2\,?$$

The Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

"A great deal more was hidden in the Dirac equation than the author had expected when he wrote it down in 1928. Dirac himself remarked in one of his talks that his equation was more intelligent than its author." (Weisskopf on Dirac)

[We use natural units: $c = \hbar = 1$, so that [mass] = [length]$^{-1}$ = [time]$^{-1}$ = (Giga) electron volt]

What would you do to reconcile quantum theory and special relativity:

$$i\frac{\partial}{\partial t}\phi = H\phi \quad \text{and} \quad E^2 = \vec{p}^2 + m^2\,?$$

Iterate the Schrödinger equation to arrive at

$$\left(i\frac{\partial}{\partial t}\right)^2 \phi = H^2\phi = (-\vec{\nabla}^2 + m^2)\phi$$

or

$$(\Box + m^2)\phi = 0$$

where $\Box = \partial^2/\partial t^2 - \vec{\nabla}^2$.

Dirac was not satisfied with the Klein-Gordon equation $(\Box + m^2)\phi = 0$ since it contains

- solutions with negative energy $E < 0$;
- a second order derivative in time, and can thus lead to negative probability densities $|\phi|^2 < 0$.

Dirac was not satisfied with the Klein-Gordon equation $(\Box + m^2)\phi = 0$ since it contains

- solutions with negative energy $E < 0$;
- a second order derivative in time, and can thus lead to negative probability densities $|\phi|^2 < 0$.

Bohr: "What are you working on Mr. Dirac?"
Dirac: "I am trying to take the square root of something."

Dirac wanted an equation that is Lorentz covariant and first order in the time derivative:

$$i\frac{\partial\psi}{\partial t} = H_{\mathrm{Dirac}}\psi = (\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \beta m)\psi = (-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)\psi$$

Dirac was not satisfied with the Klein-Gordon equation $(\Box + m^2)\phi = 0$ since it contains

- solutions with negative energy $E < 0$;
- a second order derivative in time, and can thus lead to negative probability densities $|\phi|^2 < 0$.

Bohr: " What are you working on Mr. Dirac?"
Dirac: " I am trying to take the square root of something."

Dirac wanted an equation that is Lorentz covariant and first order in the time derivative:

$$i\frac{\partial \psi}{\partial t} = H_{\rm Dirac}\psi = (\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \beta m)\psi = (-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)\psi$$

Iterating the equation on both sides yields

$$\begin{aligned} E^2\psi &= \left(i\frac{\partial}{\partial t}\right)^2\psi = (-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)\psi \\ &= \left(-\alpha^i\alpha^j\nabla^i\nabla^j - i(\beta\alpha^i + \alpha^i\beta)m\nabla^i + \beta^2 m^2\right)\psi \\ &= (p^2 + m^2)\psi = (-\nabla^i\nabla^i + m^2)\psi\,. \end{aligned}$$

The α_i and β must satisfy

$$
\begin{aligned}
\alpha_i \alpha_j + \alpha_j \alpha_i &= 2\delta_{ij} \\
\beta \alpha_i + \alpha_i \beta &= 0 \\
\beta^2 = 1
\end{aligned}
$$

so they cannot be numbers.

Dirac proposed that the α_i and β are 4×4 matrices, and that ψ is a 4-component column vector, known as Dirac spinor.

The α_i and β must satisfy

$$\begin{aligned} \alpha_i\alpha_j + \alpha_j\alpha_i &= 2\delta_{ij} \\ \beta\alpha_i + \alpha_i\beta &= 0 \\ \beta^2 = 1 \end{aligned}$$

so they cannot be numbers.

Dirac proposed that the α_i and β are 4×4 matrices, and that ψ is a 4-component column vector, known as Dirac spinor.

One choice of matrices is

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\vec{\sigma}$ are the Pauli matrices.

The α_i and β must satisfy

$$\begin{aligned} \alpha_i\alpha_j + \alpha_j\alpha_i &= 2\delta_{ij} \\ \beta\alpha_i + \alpha_i\beta &= 0 \\ \beta^2 = 1 & \end{aligned}$$

so they cannot be numbers.

Dirac proposed that the α_i and β are 4×4 matrices, and that ψ is a 4-component column vector, known as Dirac spinor.

One choice of matrices is

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\vec{\sigma}$ are the Pauli matrices.

The Dirac spinor describes particles and antiparticles with spin $1/2$:

$$\psi = \begin{pmatrix} \psi^{\uparrow} \\ \psi^{\downarrow} \\ \psi^{\uparrow} \\ \psi^{\downarrow} \end{pmatrix}$$

There is a more compact way to write the Dirac equation.

Define the γ-matrices

$$\gamma^0 \equiv \beta \quad \text{and} \quad \vec{\gamma} \equiv \beta\vec{\alpha}$$

so that

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \,.$$

There is a more compact way to write the Dirac equation.

Define the γ-matrices

$$\gamma^0 \equiv \beta \quad \text{and} \quad \vec{\gamma} \equiv \beta\vec{\alpha}$$

so that

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \,.$$

With the previous choice of α_i and β one has

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

where $\vec{\sigma}$ are the Pauli matrices.

There is a more compact way to write the Dirac equation.

Define the γ-matrices

$$\gamma^0 \equiv \beta \quad \text{and} \quad \vec{\gamma} \equiv \beta\vec{\alpha}$$

so that

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \, .$$

With the previous choice of α_i and β one has

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

where $\vec{\sigma}$ are the Pauli matrices.

Using the γ-matrices, the Dirac equation becomes:

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0 \quad \text{or} \quad (i\not{\partial} - m)\psi(x) = 0$$

where we have introduced $\partial_\mu \equiv \partial/\partial x^\mu = (\partial/\partial t, -\vec{\nabla})$ and $\not{\partial} \equiv \gamma^\mu\partial_\mu$.

The beauty and magic of the Dirac equation

The Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

- is form-invariant (covariant) under Lorentz transformations;
- describes particles with spin $1/2$;
- predicts the correct magnetic moment $g = 2$;
- predicts the existence of anti-particles!

The Dirac Lagrangian

One can construct Lorentz scalars and vectors from Dirac spinors and the γ-matrices, e.g.

$$\begin{aligned} \overline{\psi}\psi &\xrightarrow{LT} \overline{\psi}\psi \\ \overline{\psi}\gamma^\mu\psi &\xrightarrow{LT} \Lambda^\mu_{\ \nu}\overline{\psi}\gamma^\nu\psi \end{aligned}$$

where we have defined $\overline{\psi} \equiv \psi^\dagger\gamma^0$.

The Dirac Lagrangian

One can construct Lorentz scalars and vectors from Dirac spinors and the γ-matrices, e.g.

$$\begin{aligned} \overline{\psi}\psi &\xrightarrow{LT} \overline{\psi}\psi \\ \overline{\psi}\gamma^{\mu}\psi &\xrightarrow{LT} \Lambda^{\mu}_{\ \nu}\overline{\psi}\gamma^{\nu}\psi \end{aligned}$$

where we have defined $\overline{\psi} \equiv \psi^{\dagger}\gamma^{0}$.

Using $\overline{\psi}, \psi$ and γ^{μ} one can thus construct a Lorentz covariant Lagrangian

$$\mathcal{L} = \overline{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi\,,$$

which leads to the Dirac equation through the usual Euler-Lagrange equations,

$$\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial \mathcal{L}}{\partial(\partial\overline{\psi}/\partial x_{\mu})}\right) - \frac{\partial \mathcal{L}}{\partial \overline{\psi}} = 0 \quad \text{and} \quad \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial \mathcal{L}}{\partial(\partial\psi/\partial x_{\mu})}\right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0\,.$$

Gauge transformations: QED

Consider the Lagrangian for a free Dirac field ψ:

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi\,.$$

The Lagrangian is invariant under a phase transformation of the fermion field:

$$\psi \to e^{-i\omega}\psi, \quad \overline{\psi} \to e^{i\omega}\overline{\psi},$$

where ω is a constant (i.e. independent of x).

Gauge transformations: QED

Consider the Lagrangian for a free Dirac field ψ:

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi\,.$$

The Lagrangian is invariant under a phase transformation of the fermion field:

$$\psi \to e^{-i\omega}\psi, \quad \overline{\psi} \to e^{i\omega}\overline{\psi},$$

where ω is a constant (i.e. independent of x).

The set of numbers $e^{-i\omega}$ form a group. This particular group is "abelian" which is to say that any two elements of the group commute:

$$e^{-i\omega_1}e^{-i\omega_2} = e^{-i\omega_2}e^{-i\omega_1}\,.$$

This particular group is called $U(1)$, i.e. the group of all unitary 1×1 matrices. (A unitary matrix satisfies $U^\dagger = U^{-1}$.)

Gauge transformations: QED

Consider the Lagrangian for a free Dirac field ψ:

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi\,.$$

The Lagrangian is invariant under a phase transformation of the fermion field:

$$\psi \to e^{-i\omega}\psi, \quad \overline{\psi} \to e^{i\omega}\overline{\psi},$$

where ω is a constant (i.e. independent of x).

The set of numbers $e^{-i\omega}$ form a group. This particular group is "abelian" which is to say that any two elements of the group commute:

$$e^{-i\omega_1}e^{-i\omega_2} = e^{-i\omega_2}e^{-i\omega_1}\,.$$

This particular group is called $U(1)$, i.e. the group of all unitary 1×1 matrices. (A unitary matrix satisfies $U^\dagger = U^{-1}$.)

Thus the Dirac Lagrangian is invariant under global $U(1)$ transformations.

We now require invariance under local $U(1)$ transformations, i.e.

$$\psi \to e^{-i\omega(x)}\psi, \quad \overline{\psi} \to e^{i\omega(x)}\overline{\psi},$$

where $\omega(x)$ now depends on the space-time point.

We now require invariance under local $U(1)$ transformations, i.e.

$$\psi \to e^{-i\omega(x)}\psi, \quad \overline{\psi} \to e^{i\omega(x)}\overline{\psi},$$

where $\omega(x)$ now depends on the space-time point.

Note that $\mathcal{L} = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$ is not invariant under local $U(1)$ transformations:

$$\mathcal{L} \to \mathcal{L} + \delta\mathcal{L} = \mathcal{L} \; + \overline{\psi}\gamma^\mu[\partial_\mu\omega(x)]\psi\,,$$

where we consider infinitesimal transformations

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad \overline{\psi} \to \overline{\psi} + \delta\overline{\psi} = \overline{\psi} + i\omega(x)\overline{\psi}\,.$$

We now require invariance under local $U(1)$ transformations, i.e.

$$\psi \to e^{-i\omega(x)}\psi, \quad \overline{\psi} \to e^{i\omega(x)}\overline{\psi},$$

where $\omega(x)$ now depends on the space-time point.

Note that $\mathcal{L} = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$ is not invariant under local $U(1)$ transformations:

$$\mathcal{L} \to \mathcal{L} + \delta\mathcal{L} = \mathcal{L} \; + \overline{\psi}\gamma^\mu[\partial_\mu\omega(x)]\psi\,,$$

where we consider infinitesimal transformations

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad \overline{\psi} \to \overline{\psi} + \delta\overline{\psi} = \overline{\psi} + i\omega(x)\overline{\psi}\,.$$

We can restore invariance under local $U(1)$ transformations if we introduce a vector field $A_\mu(x)$ with the interaction

$$-e\overline{\psi}\gamma^\mu A_\mu\psi\,,$$

so that the Lagrangian density becomes

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi\,.$$

The new Lagrangian

$$\mathcal{L} = \overline{\psi}\,(i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu}) - m)\,\psi$$

is invariant under local $U(1)$ transformations if we require that

$$A_{\mu} \to A_{\mu} + \delta A_{\mu} = A_{\mu} + \frac{1}{e}[\partial_{\mu}\omega(x)]\,.$$

The new Lagrangian

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi$$

is invariant under local $U(1)$ transformations if we require that

$$A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]\,.$$

We need to add a Lorentz- and gauge invariant kinetic term for the field A_μ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi\,,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,.$$

[We have fixed the coefficient of the term $\propto F_{\mu\nu}F^{\mu\nu}$ so that we recover the standard form of Maxwell's equations.]

The new Lagrangian

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi$$

is invariant under local $U(1)$ transformations if we require that

$$A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]\,.$$

We need to add a Lorentz- and gauge invariant kinetic term for the field A_μ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\,\psi\,,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,.$$

[We have fixed the coefficient of the term $\propto F_{\mu\nu}F^{\mu\nu}$ so that we recover the standard form of Maxwell's equations.]

A mass term for the new field $\propto m_A^2 A_\mu A^\mu$ is not invariant under gauge transformations,

$$\delta\mathcal{L} = \frac{2m_A^2}{e}A^\mu \partial_\mu \omega(x) \neq 0\,,$$

and thus not allowed.

It is useful to introduce the concept of a "covariant derivative" D_μ as

$$D_\mu = \partial_\mu + ieA_\mu \, .$$

With

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]$$

one finds

$$D_\mu\psi \to D_\mu\psi + \delta(D_\mu\psi) = D_\mu\psi - i\omega(x)D_\mu\psi$$

so that

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$

is gauge invariant.

It is useful to introduce the concept of a "covariant derivative" D_μ as

$$D_\mu = \partial_\mu + ieA_\mu \,.$$

With

$$\psi \to \psi + \delta\psi = \psi - i\omega(x)\psi \quad \text{and} \quad A_\mu \to A_\mu + \delta A_\mu = A_\mu + \frac{1}{e}[\partial_\mu \omega(x)]$$

one finds

$$D_\mu \psi \to D_\mu \psi + \delta(D_\mu \psi) = D_\mu \psi - i\omega(x) D_\mu \psi$$

so that

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$

is gauge invariant.

One can express $F_{\mu\nu}$ in terms of the covariant derivative:

$$F_{\mu\nu} = -\frac{i}{e}[D_\mu, D_\nu] = \ldots = \partial_\mu A_\nu - \partial_\nu A_\mu \,.$$

Gauge transformations: Summary

- The Dirac Lagrangian is invariant under local $U(1)$ transformations if we add a vector field A_μ and an interaction $-e\overline{\psi}\gamma^\mu A_\mu \psi$.
- The interaction is obtained by replacing the derivative ∂_μ with the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$.
- The gauge-invariant kinetic term for the vector field is $\propto F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} \propto [D_\mu, D_\nu]$.
- The new vector (gauge) field is massless, since a term $\propto A_\mu A^\mu$ is not gauge-invariant.
- The Lagrangian resulting from local $U(1)$ gauge-invariance is identical to that of QED.