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The Standard Model of particle physics

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Outline

- QED as a gauge theory
- Quantum Chromodynamics
- Breaking gauge symmetries:

  the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism
- Exploring electroweak symmetry breaking at the LHC

Spontaneous symmetry breaking

A $SU(n)$ gauge theory

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \overline{\psi}^i \left(i\gamma^\mu (\partial_\mu + igT^a A^a_\mu) - m \right)^j_i \psi_j$$

has massless gauge bosons A^a_μ:

To preserve gauge invariance of the Lagrangian, the A^a_μ transform under gauge transformations as

$$A^a_\mu \rightarrow A^a_\mu - f^{abc} A^b_\mu(x) \omega^c(x) + \frac{1}{g} \left[\partial_\mu \omega^a(x) \right] ,$$

and thus a mass term

$$\mathcal{L} \supset M_A^2 \, A^a_\mu A^{a\,\mu}$$

is not gauge invariant.

This is what we want for QED (massless photon) and QCD (massless gluons), but not for a gauge theory of the weak interactions.

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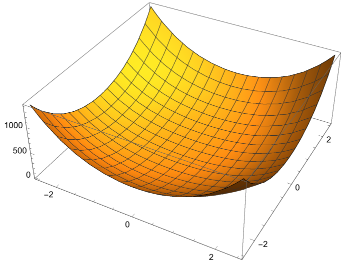

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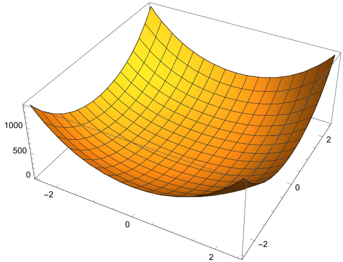

The potential and the ground state at $\vec{r} = 0$ are symmetric under rotations.

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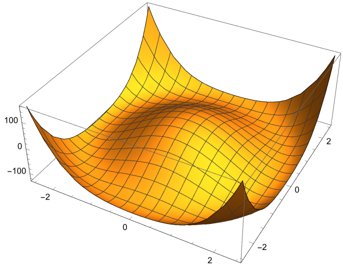

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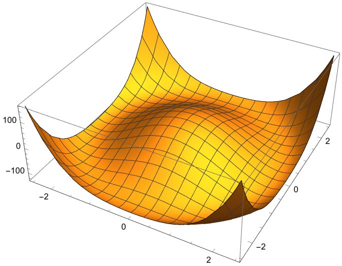

The potential is symmetric under rotations, but the ground state (any point along the circle $|\vec{r}| = \sqrt{-\mu^2/2\lambda}$) is not.

Let us consider a gauge theory with a complex scalar field Φ:

$$\mathcal{L} = (D_\mu \Phi)^* D^\mu \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi)$$

and

$$V(\Phi) = -\mu^2 \Phi^* \Phi + \lambda |\Phi^* \Phi|^2$$

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The symmetry breaking occurs in the choice made for the value of Θ. For any specific choice of Θ we have

$$\Phi \to e^{-i\omega} \Phi = e^{-i\omega} e^{i\Theta} \frac{v}{\sqrt{2}} = e^{i(\Theta - \omega)} \frac{v}{\sqrt{2}} = e^{i\Theta'} \frac{v}{\sqrt{2}} ,$$

i.e. the ground state is not invariant under gauge transformations.

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Let us expand Φ around the vacuum expectation value,

$$\Phi(x) = \frac{\rho(x)}{\sqrt{2}} e^{i\phi(x)/v} = \frac{1}{\sqrt{2}}(v + H(x))e^{i\phi(x)/v} \approx \frac{1}{\sqrt{2}}(v + H(x) + i\phi(x)),$$

and express the Lagrangian in terms of the fields H and ϕ.

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The potential becomes

$$V = \mu^2 H^2 + \mu\sqrt{\lambda}(H^3 + \phi^2 H) + \frac{\lambda}{4}(H^4 + \phi^4 + 2H^2\phi^2) + \frac{\mu^4}{4\lambda}\,.$$

There is a mass term for the field H:

$$V \supset \mu^2 H^2 \equiv \frac{M_H}{2}H^2 \quad \text{with} \quad M_H = \sqrt{2}\mu\,,$$

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but no mass term for the field ϕ.

Thus ϕ represents a massless particle, called “Goldstone boson”.

For the kinetic term we find

$$(D_\mu \Phi)^* D^\mu \Phi \supset \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{1}{2} g^2 v^2 A_\mu A^\mu + g^2 v A_\mu A^\mu H \,.$$

The gauge boson has acquired a mass term:

$$(D_\mu \Phi)^* D^\mu \Phi \supset \frac{1}{2} g^2 v^2 A_\mu A^\mu \equiv \frac{1}{2} M_A^2 A_\mu A^\mu \quad \text{with} \quad M_A = gv$$

and there is an interaction between the gauge field and the field H:

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Let us count the number of degrees of freedom:

- A complex scalar field Φ (2) + a massless gauge boson A_μ (2) = 4
- A real scalar field H (1) + a massive gauge boson A_μ (3) = 4

The 2 d.o.f. of the complex field Φ correspond to the field H and the longitudinal component of the massive gauge boson.

The Standard Model with one family

Empirically we know that the weak interactions violate parity and that the couplings are of the form vector minus axial-vector ($V - A$):

$$\overline{\psi}\gamma_\mu\psi - \overline{\psi}\gamma_\mu\gamma_5\psi\,,$$

where $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

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We define left- and right-chiral components of spinor fields as

$$\psi = \psi_L + \psi_R \quad \text{where} \quad \psi_{L/R} = \frac{1}{2}(1 \mp \gamma^5)\psi\,.$$

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The $(V - A)$ structure implies that only left-chiral fermions participate in the weak interactions:

$$\overline{\psi}\gamma_\mu\psi - \overline{\psi}\gamma_\mu\gamma_5\psi = \overline{\psi}\gamma_\mu(1-\gamma_5)\psi = \overline{\psi}_L\gamma_\mu\psi_L\,.$$

To write down a gauge invariant Lagrangian for the (electro-)weak interactions, we have to choose the gauge group. Let us try

$$SU(2)_L \times U(1)_Y \, .$$

The $SU(2)_L$ group has 3 generators, $T^a = \sigma_a/2$, a gauge coupling denoted by g and three gauge bosons W^a_μ. It is called weak isospin.

The $U(1)$ group is not the gauge group of QED, but that of hypercharge Y. The corresponding coupling and gauge boson are denoted by g' and B^μ.

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As matter content (for the first family), we have

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}; u_R; d_R; l_L \equiv \begin{pmatrix} \nu \\ e_L \end{pmatrix}; e_R; \nu_R \,.$$

The model is constructed such that $SU(2)_L$ gauge transformations only act on q_L and l_L,

$$q_L \to q_L' = e^{-i\omega^a T^a} q_L \quad \text{and} \quad l_L \to l_L' = e^{-i\omega^a T^a} l_L \,,$$

while u_R, d_R, ν_R, and e_R are $SU(2)_L$ singlets and do not couple to the corresponding gauge bosons W^a_μ.

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Under $U(1)_Y$, the matter fields transform as $\psi \to \psi' = e^{-i\omega Y_\psi}\psi$.

The Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism

We introduce a scalar field which transforms as a doublet under $SU(2)_L$, and which has a potential of the form

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$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

so that

$$D_\mu \Phi = \frac{1}{\sqrt{2}} \left(\partial_\mu + i \frac{g}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^- \\ \sqrt{2} W_\mu^+ & -W_\mu^3 \end{pmatrix} + i \frac{g'}{2} B_\mu \right) \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

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and thus

$$|D_\mu \Phi|^2 \supset \frac{1}{2} (\partial_\mu H)^2 + \frac{g^2 v^2}{4} W^{+\mu} W_\mu^- + \frac{v^2}{8} \left(g W_\mu^3 - g' B_\mu \right)^2$$

where $W_\mu^\pm = (W_\mu^1 \pm W_\mu^2)/\sqrt{2}$.

Thus the gauge bosons W_μ^3 and B_μ mix, and the physical mass eigenstates are the linear combinations

$$\begin{aligned} Z_\mu &\equiv \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu \\ A_\mu &\equiv \cos\theta_w B_\mu + \sin\theta_w W_\mu^3 \end{aligned}$$

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We can read off the masses of the gauge bosons,

$$M_W = \frac{1}{2} g v, \quad M_Z = \frac{1}{2}\frac{g v}{\cos\theta_w} \quad \text{and} \quad M_A = 0\,.$$

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One can show that the quantum numbers of the $SU(2)_L$, $U(1)_Y$ and $U(1)_{\rm em}$ gauge groups are connected through $Q = Y + T^3$.

Fermion masses

In our free Dirac Lagrangian, we included a mass term for the fermions

$$\mathcal{L} \supset m\overline{\psi}\psi = m\overline{\psi}_L\psi_R + m\overline{\psi}_R\psi_L \,.$$

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is gauge invariant. Thus, we obtain a mass term and an interaction

$$-\frac{Y_e}{\sqrt{2}}(v+H)\left(\overline{e}_L e_R + \overline{e}_R e_L\right) = -\frac{Y_e}{\sqrt{2}}(v+H)\,\overline{e}e = -m_e\overline{e}e - \frac{m_e}{v}H\overline{e}e \,,$$

where

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The strength of the interaction between the Higgs particle and the fermions is proportional to the fermion mass.

The strength of the interaction between the Higgs particle and other particles is proportional to the particle mass: