

Precision calculations for CLIC Physics

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Context

- ✓ (My viewpoint) In all likelihood, few years down the road a decision will have to be made

Which future collider to build?

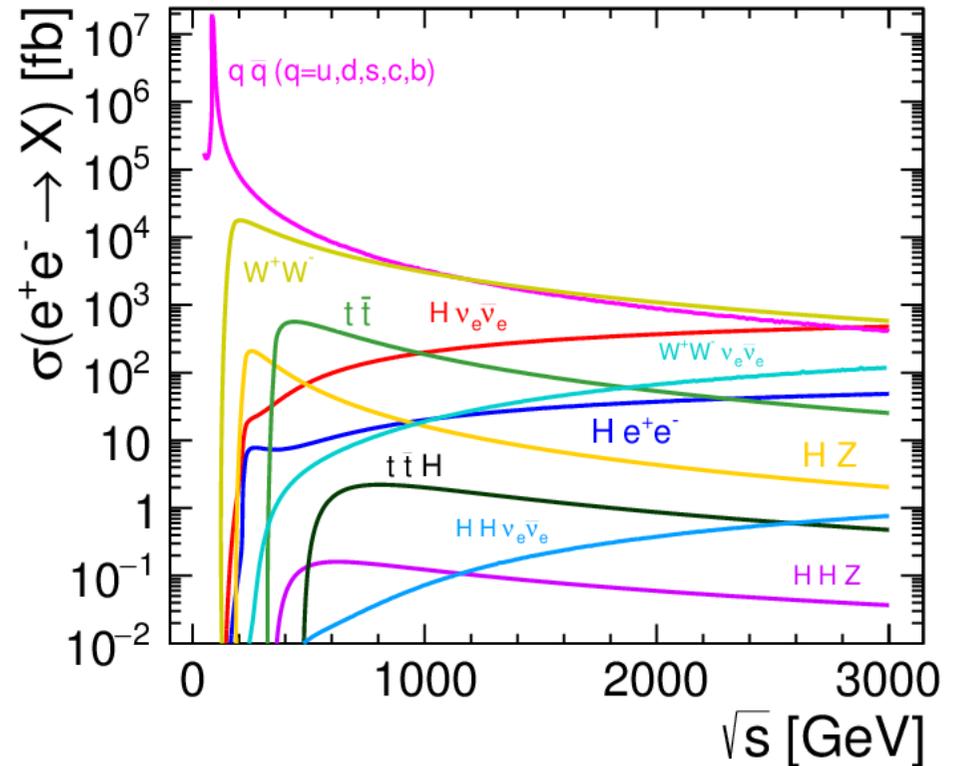
- ✓ A higher-energy hadron collider? (100 TeV too expensive? Scaled-down version ~30 TeV?)
- ✓ e^+e^- (CLIC?)
- ✓ The physics case will need to be made and it may involve direct comparison between the two
- ✓ How to view precision calculations from BSM perspective:
 1. Compute things that *can* be computed
 2. Compute things that *need* to be computed
- ✓ In this talk:
 - ✓ An overview of 1., together with some predictions/extrapolations for the far future
 - ✓ Emphasize the importance of 2. as an input from BSM.

“No problem is too big when it is important enough!”

- ✓ Precision calculations are critical – all past colliders (including LHC) are a testament for this

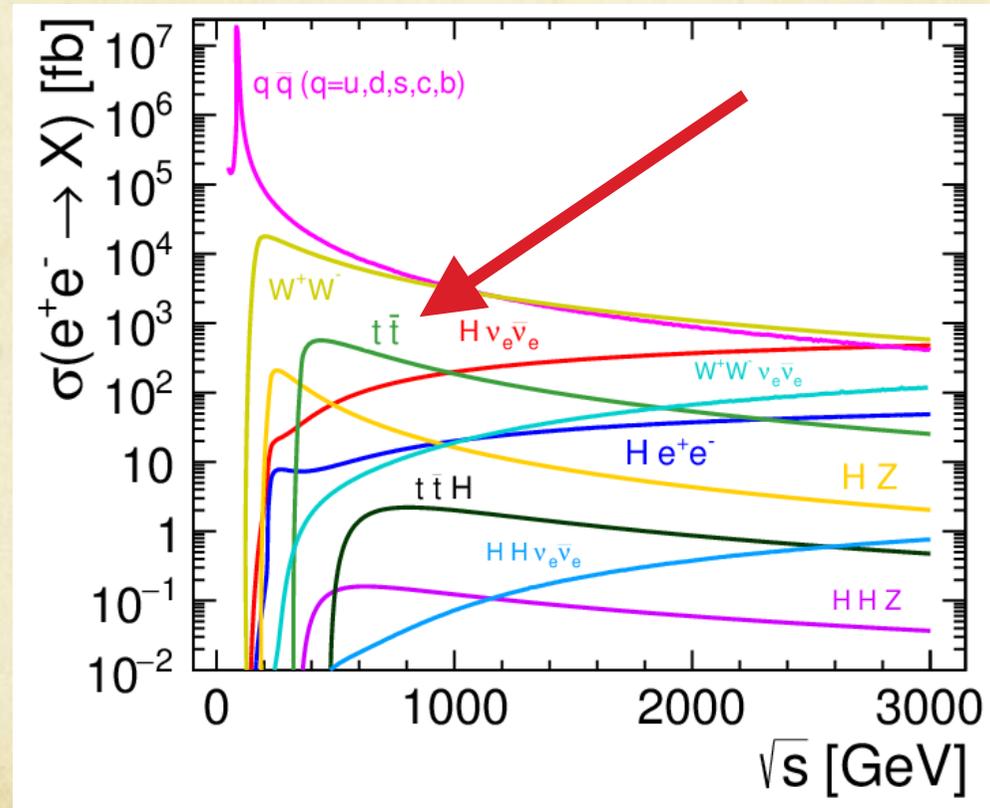
Contents

- ✓ Top physics
 - ✓ N³LO result
 - ✓ Extraction of top mass and width
- ✓ Top and Higgs: extraction of the top-Yukawa
- ✓ Higgs
- ✓ Jet production: taming QCD and measuring α_S
- ✓ W mass and width from WW threshold scan
- ✓ EW precision observables
- ✓ Event generators
 - ✓ GigaZ-like physics: re-measuring much better well-known SM stuff might be needed.



Top production

See also talk by Christoph Englert



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Stable tops

- ✓ Total inclusive cross-section known at NNLO for a long time (expansion in m/s or numerically):
Chetyrkin, Kühn, Steinhauser '96
Harlander, Steinhauser '98
- ✓ NLO EW also known for a long time
[Beenakker, Hollik, Denner, Dittmaier: since '91]
- ✓ *Fully differential* top-pair production in full NNLO QCD (with stable tops) in the continuum
 - ✓ Computed, respectively, within:
 - ✓ Phase-space slicing method
 - ✓ Antenna-subtraction method
 - ✓ No EW corrections included
 - ✓ No $t\bar{t}t$ final states included
 - ✓ (The well known) lack of convergence close to threshold is evident

Gao, Zhu '14

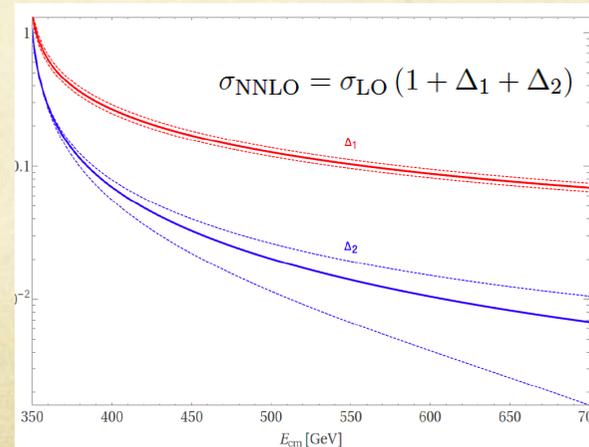
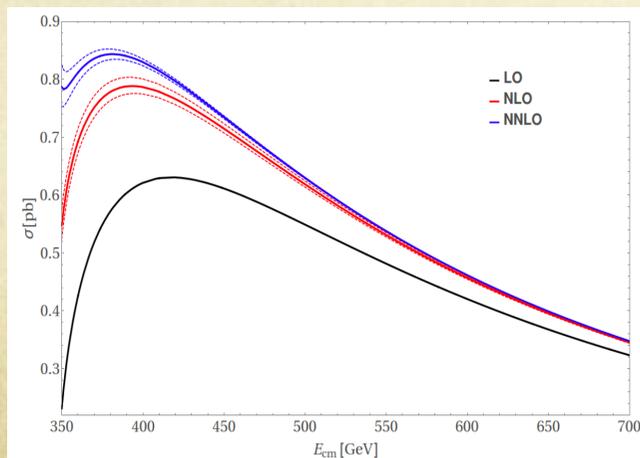
Chen, Dekkers, Heisler, Bernreuther, Si '16

Fadin, Khoze '87

Strassler, Peskin '91

Jezabek, Kühn, Teubner '92

From: Chen, Dekkers, Heisler, Bernreuther, Si '16

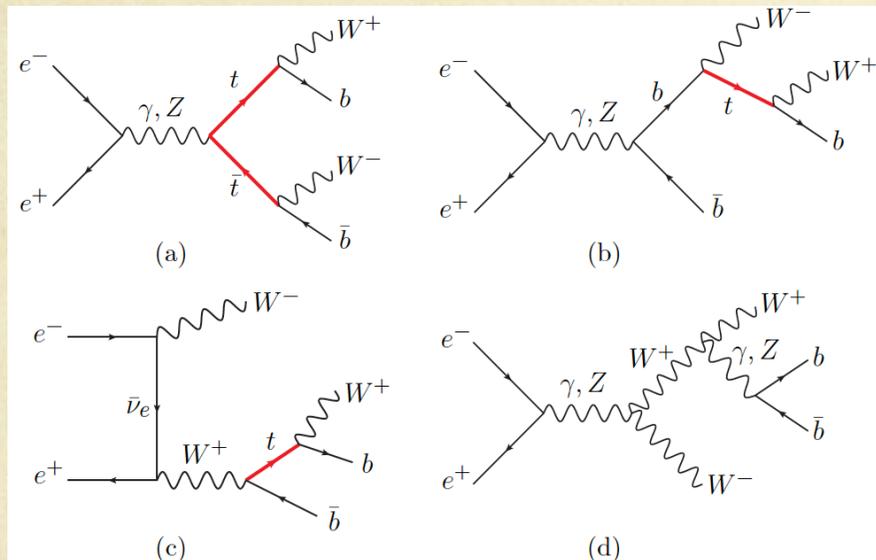


Adding top decay

- ✓ No NNLO QCD result available yet (but it is really easy to do in NWA)
- ✓ Top production and decay in NLO QCD known for a long time

Carl Schmidt '95

- ✓ Thanks to recent advances, full off-shell production $e^+e^- \rightarrow WbWb$ available in NLO QCD



Guo, Ma, Zhang, Wang '08

O. Mattelaer, [MadGraph5 AMC@NLO] '14

Weiss, Nejad, Kilian, Reuter [WHIZARD] '15

- ✓ Included are all intermediate states, including single top and no-top

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Top width

✓ NLO QCD Jezabek, Kuhn '89

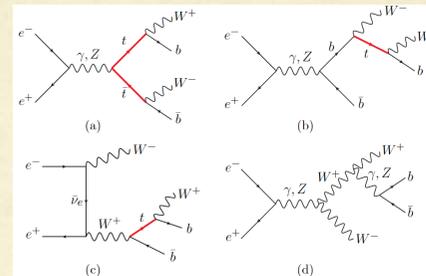
✓ NNLO QCD Czarnecki et al '10

✓ Measure it with ratios of resonance contributions of $e^+e^- \rightarrow W^+W^-bb$

Liebler, Moortgat-Pick, Papanastasiou '15

✓ Idea:

$$\frac{\sigma_{\text{single-resonant}}}{\sigma_{\text{double-resonant}}} \propto \Gamma_t$$



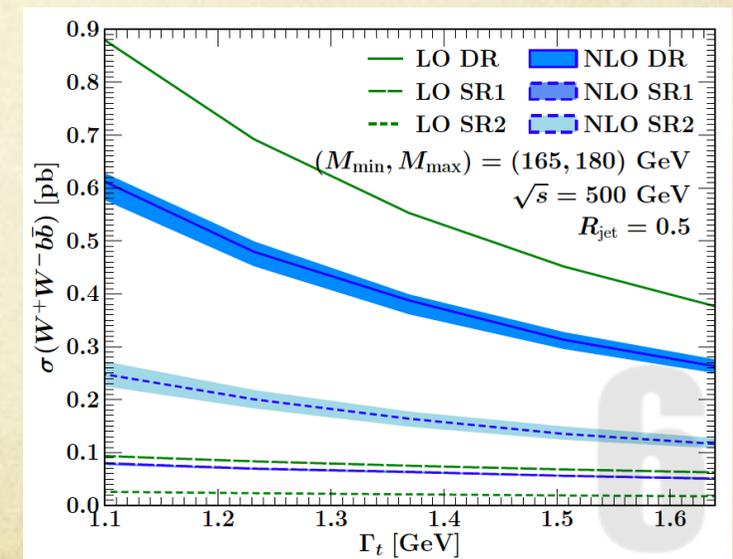
✓ We cannot divide diagrams!

✓ They can be correlated with kinematic regions for the distribution:

An example:

$$\frac{d^2\sigma_{e^+e^- \rightarrow W^+W^-bb}}{dM(W^+, J_b) dM(W^-, J_{\bar{b}})}$$

double resonant: $M(W^+, J_b) \sim m_t^2$ and $M(W^-, J_{\bar{b}}) \sim m_t^2$
 single resonant: $M(W^+, J_b) \sim m_t^2$ and $\{M(W^-, J_{\bar{b}}) \ll m_t^2 \text{ or } M(W^-, J_{\bar{b}}) \gg m_t^2\}$
 single resonant: $\{M(W^+, J_b) \ll m_t^2 \text{ or } M(W^+, J_b) \gg m_t^2\}$ and $M(W^-, J_{\bar{b}}) \sim m_t^2$
 non resonant: $\{M(W^+, J_b) \ll m_t^2 \text{ or } M(W^+, J_b) \gg m_t^2\}$ and $\{M(W^-, J_{\bar{b}}) \ll m_t^2 \text{ or } M(W^-, J_{\bar{b}}) \gg m_t^2\}$. (3.1)



✓ Calculation at NLO with aMC@NLO

B-fragmentation and top decay

✓ Detailed study of B-production in top decay requires precision beyond current showers

✓ Dedicated study in NLO QCD

Corcella, Mitov '02

✓ Could nowadays be extended to full NNLO + NNLL (soft) + NNLL (collinear) due to:

✓ Top decay at NNLO

Gao, Li, Zhu '12

Brucherseifer, Caola, Melnikov '13

✓ Soft-gluon resummation at NNLL

Cacciari, Catani '01

Andersen, Gardi '04

✓ Perturbative fragmentation function at NNLO

Mele, Nason '91

Melnikov, Mitov '04

✓ Time-like splitting functions at NNLO

Dokshitzer, Marchesini, Salam '05

Mitov, Moch, Vogt '06

✓ Dedicated ongoing work for adding fragmentation functions a-la LHAPDF by NNPDF collaboration



High-energies: top jets

- ✓ Top jets with NNLL resummation have been investigated within SCET in

Fleming, Hoang, Mantry, Stewart '16

- ✓ Of interest is the distribution

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2}$$

Where:

$$M_t^2 = \left(\sum_{i \in X_t} p_i^\mu \right)^2$$

$$M_{\bar{t}}^2 = \left(\sum_{i \in X_{\bar{t}}} p_i^\mu \right)^2$$

and for $Q \gg m \gg \Gamma$

- ✓ Of primary interest is m_{top} determination but also jet shapes etc.
- ✓ In the future it should be possible to match this to a full NNLO prediction

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Nearly massless identified tops

- ✓ At large c.m. energy $Q \gg m$, top production will be dominated by large $\log(Q/m)$
- ✓ Large means that: $\alpha_s(Q) \log(Q/m) \sim 1$
- ✓ For observables with identified tops, this will lead to large corrections that need to be resummed.
- ✓ This is routinely done for b-production: FONLL
Cacciari, Greco, Nason '98-
- ✓ But never before for top
- ✓ First partial applications at the LHC have appeared only recently
Ferroglia, Pecjak, Li Lin Yang '15-
- ✓ As mentioned before, all ingredients for a consistent study at NNLL are available now.

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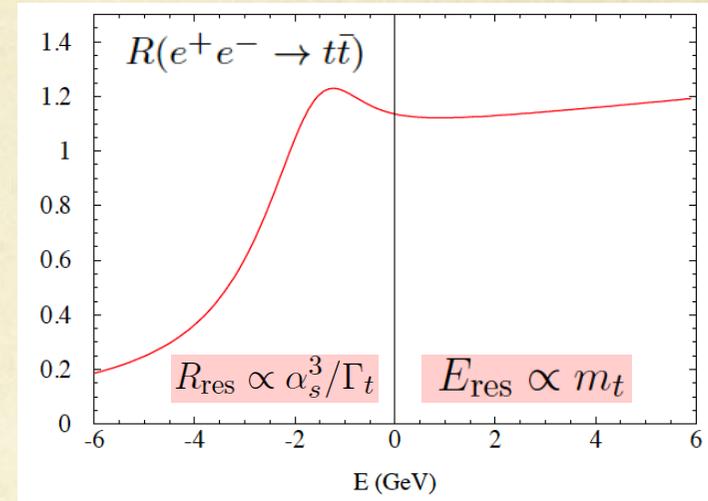
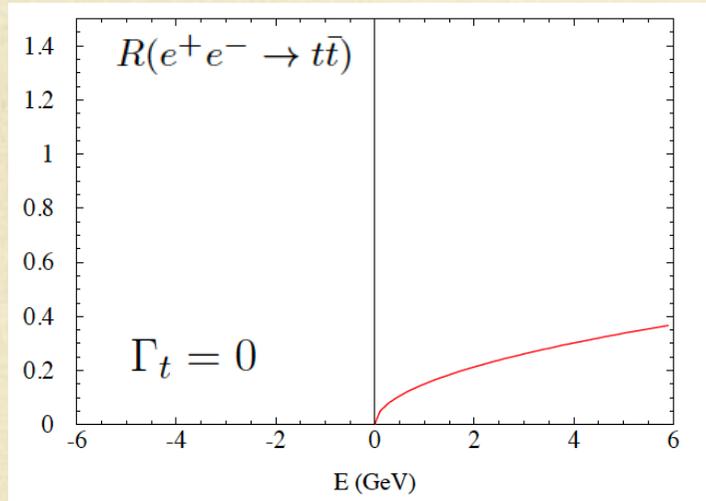
Top at threshold

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Life is better when sitting on top of a singularity!

- ✓ A threshold scan gives very clean direct access to:
 - ✓ Top mass
 - ✓ Top width

Plots courtesy of A. Penin



- ✓ Long history of calculations in NRQCD [Pineda, Soto '97] at NNLO:

Hoang, Teubner '98
Melnikov, Yelkhovsky '98
Penin, Pivovarov '98
Beneke, Signer, Smirnov '99
Nagano, Ota, Sumino '99

- ✓ The long list of papers resulted in unprecedented “concordance” paper

Hoang et al hep-ph/0001286

- ✓ Calculations were done with pole mass. Poor convergence. Improved by using running masses

Beneke '98

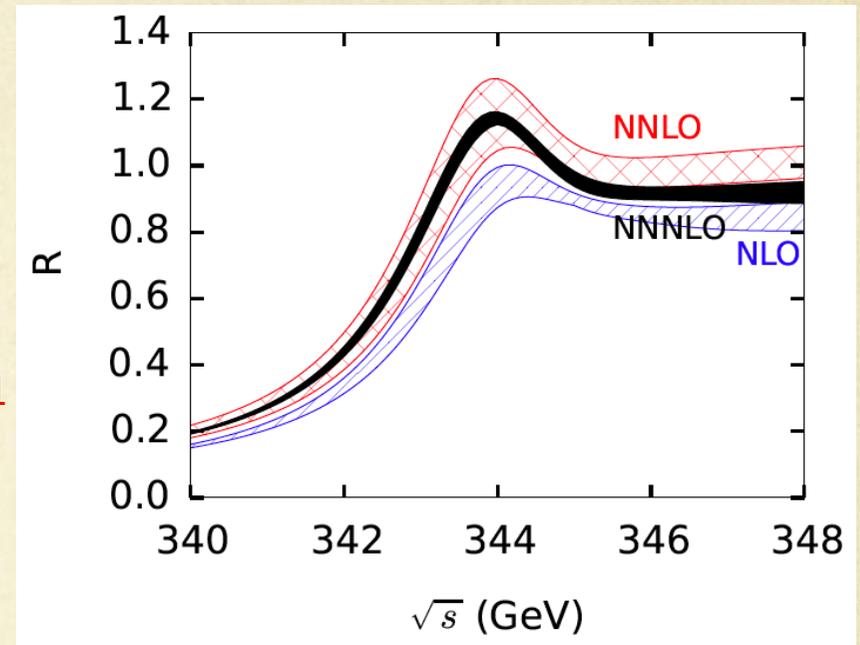
Total x-section at N³LO

- ✓ A major progress in the last few years: inclusive cross-section now computed in N³LO

Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15

- ✓ Results incorporates a number of partial results
- ✓ Since at threshold $\alpha_s/v \sim 1$,
- ✓ also included is NNLL resummation of $\log(v)$ terms

Hoang, Manohar, Stewart, Teubner '01
 Pineda, Signer '06
 Hoang, Stahlhofen '13



- ✓ Corrections to:
 - ✓ The peak position:

$$E_{res} = E_{res}^{LO} \left[1 + 3.201\alpha_s + 16.434\alpha_s^2 + (15.300 \ln \alpha_s + 84.858) \alpha_s^3 \right]$$

- ✓ The peak cross-section:

$$R_{res} = R_{res}^{LO} \left[1 - 2.131\alpha_s + (-7.256 \ln \alpha_s + 8.375)\alpha_s^2 - (16.352 \ln^2 \alpha_s + 0.682 \ln \alpha_s + 66.793) \alpha_s^3 \right]$$

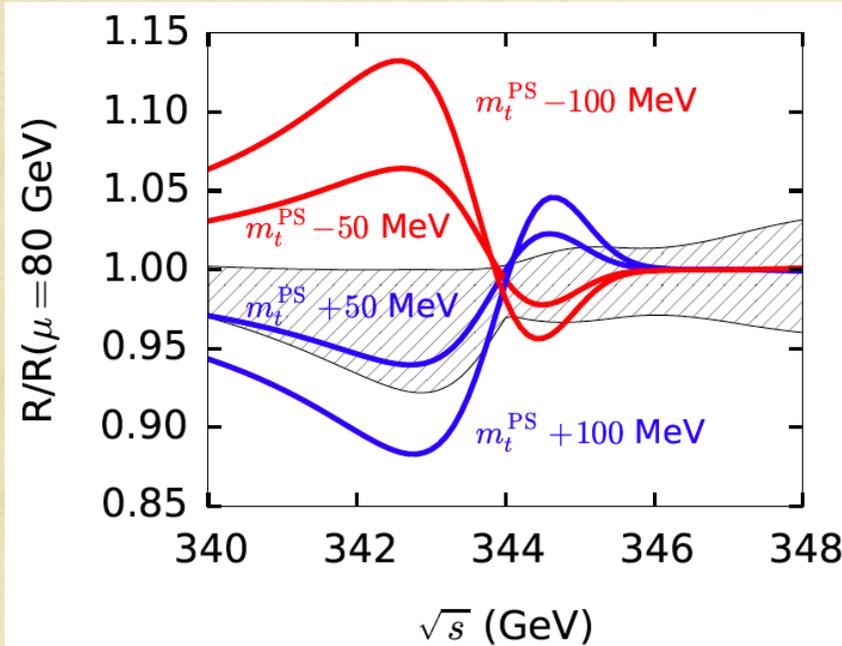
Numbers from:
 Penin, Zerf arXiv:1401.7035
 (With thanks to A. Penin)

Total x-section at N³LO

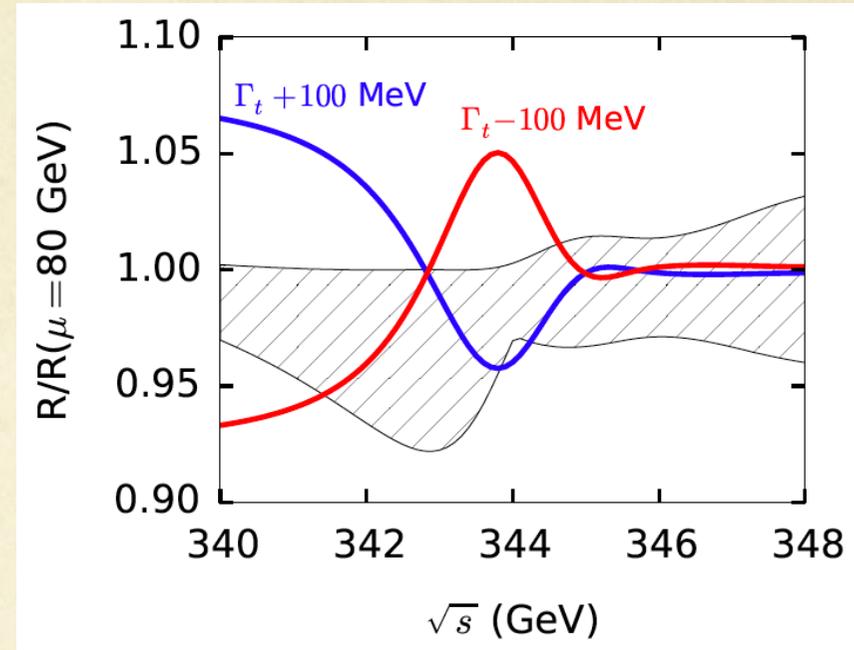
- ✓ Improved sensitivity to top mass/width at N³LO:

Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15

Top mass



Top width



- ✓ Results in terms of PS mass. Can be related to MSbar mass with ~ 20 MeV error.

Numeric inputs:

$$m_t^{\text{PS}}(\mu_f = 20 \text{ GeV}) = 171.5 \text{ GeV}, \Gamma_t = 1.33 \text{ GeV}, \\ \alpha_s(M_Z) = 0.1185 \pm 0.0006.$$

μ – renormalization scale varied in [50GeV – 350GeV]

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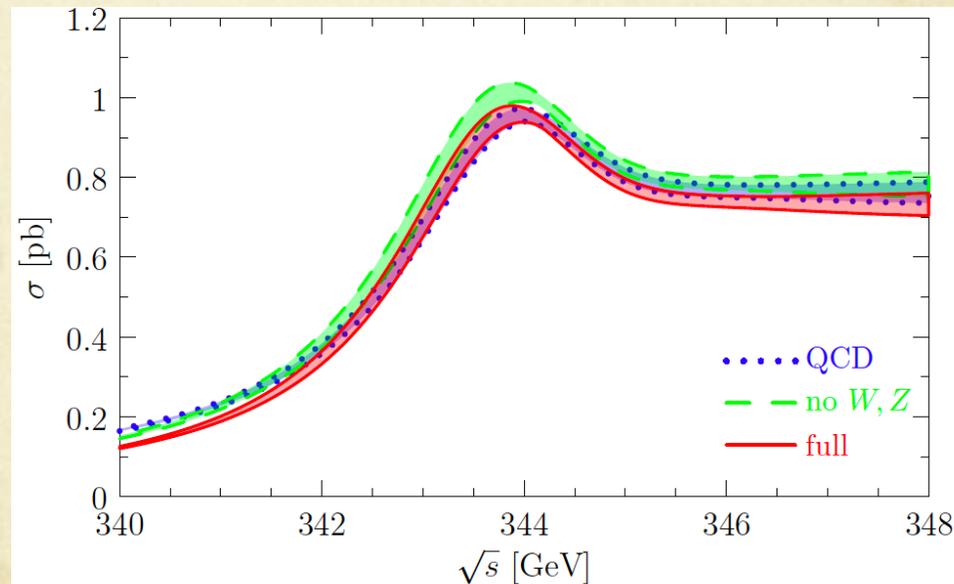
Total x-section at N³LO

✓ Public code at N³LO: **QQbar_threshold**

Beneke, Kiyo, Maier, Piclum '16

- ✓ N³LO QCD and partial electroweak corrections
- ✓ Non-resonant contributions at NLO
- ✓ Works for top and bottom quarks
- ✓ Various mass schemes: PS, 1S, MS, pole
- ✓ Loose invariant Wb mass cut: $(m_t - m_{Wb})^2 \gg \Gamma_t m_t$
- ✓ ...
- ✓ Still missing: initial state radiation (structure function approach)

Example:
effect of EW corrections



$$m_t^{\text{PS}}(20 \text{ GeV}) = 171.5 \text{ GeV}, \quad \Gamma_t = 1.33 \text{ GeV}, \quad m_H = 125 \text{ GeV}$$
$$\alpha_s(m_Z) = 0.1177, \quad \alpha(m_Z) = 1/128.944, \quad m_W, m_Z$$

Top mass: precision and scheme dependence

- ✓ Computing in terms of the pole mass is easy and natural.
- ✓ However, that particular mass has non-perturbative corrections that restrict its ultimate precision
- ✓ Recent estimate based on the 4-loop relation: pole mass \leftrightarrow \overline{m}_s mass

Marquard, Smirnov, Smirnov, Steinhauser '15

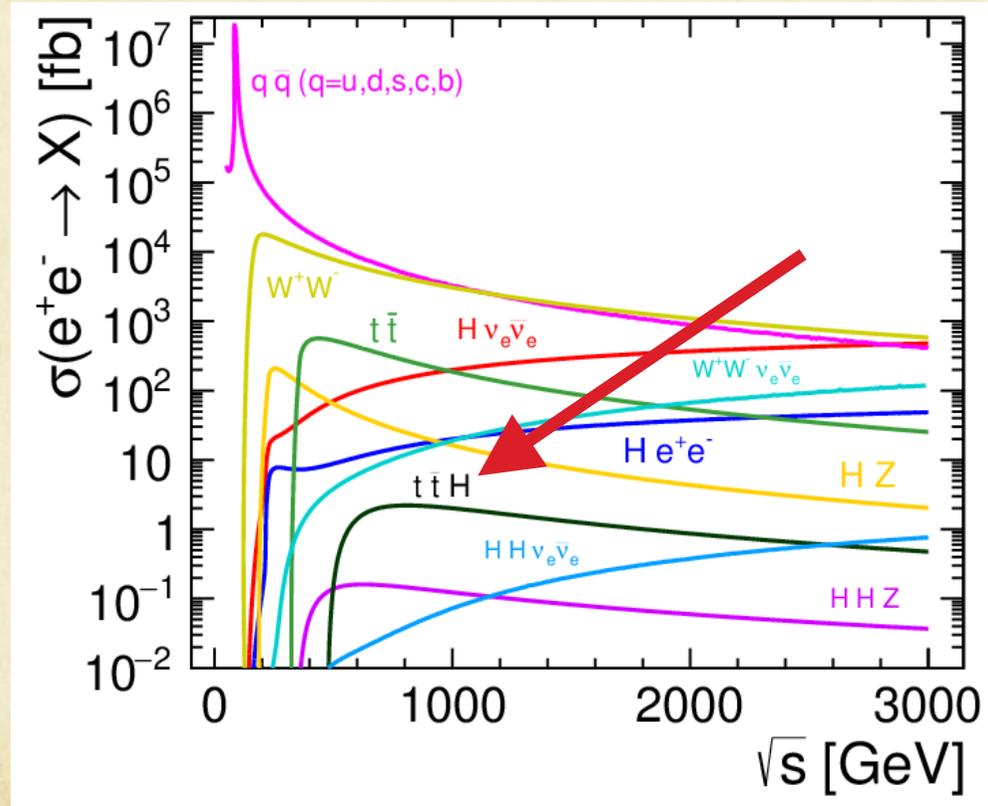
$$m_p = 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005) \text{ GeV}$$

Assuming: $\overline{m}_t = m_t(\overline{m}_t) = 163.643 \text{ GeV}$ and $\alpha_s^{(6)}(\overline{m}_t) = 0.1088$

- ✓ Exploring the leading asymptotic behavior of the above relation
Beneke '94
 - ✓ One can derive an improved relation which predicts (approximately) higher terms in the above expansion.
Beneke, Marquard, Nason, Steinhauser '16
 - ✓ The ultimate precision is taken for the term where the term-to-term difference is smallest
 - ✓ Error from the terms beyond 4 loops: $\sim 250 \text{ MeV}$
 - ✓ Ultimate intrinsic error in the above relation: $\sim 70 \text{ MeV}$
- However see new work by A. Hoang et al. '17
- ✓ Very important at e^+e^- colliders

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Top and Higgs



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Top and Higgs

- ✓ One of the main goals of a future e^+e^- machine is to run at around 500 GeV and measure the Higgs coupling to top
 - ✓ This requires good understanding of top production in the continuum
 - ✓ Expectations for top-Yukawa precision:
 - ✓ 3-4% (compared to 10% at HL-LHC) Azzi et al arXiv:1703.01626
 - ✓ 1% at Fcc-hh (from ttH/ttZ) Mangano, Plehn, Reimitz, Schell, Shao '15
- ✓ Somewhat surprising comparison; this may not be the last word on this (one is absolute measurement; the other a ratio)

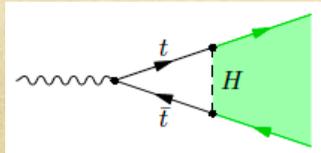
- ✓ Lots of studies in NLO QCD + EW already. Dedicated event generators:
 - ✓ aMC@NLO
 - ✓ WHIZARD

$$\frac{\Delta y_t}{y_t} = \kappa \frac{\Delta \sigma}{\sigma}$$

In SM: $\kappa \sim 0.5$

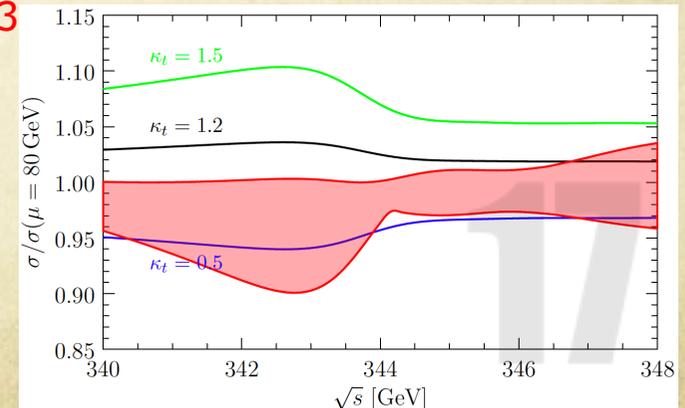
- ✓ Many dedicated calculations:
 - ✓ NLO QCD Dawson, Reina '98
 - ✓ EW corrections Denner et al.; Belanger et al., You et al. '03
 - ✓ NLL threshold logs Farrell, Hoang '05

- ✓ There is some sensitivity to the value of the top-Yukawa already from $e^+e^- \rightarrow tt$ at threshold:



From diagrams like:

Figures courtesy of A. Maier (2016)



Top and Higgs

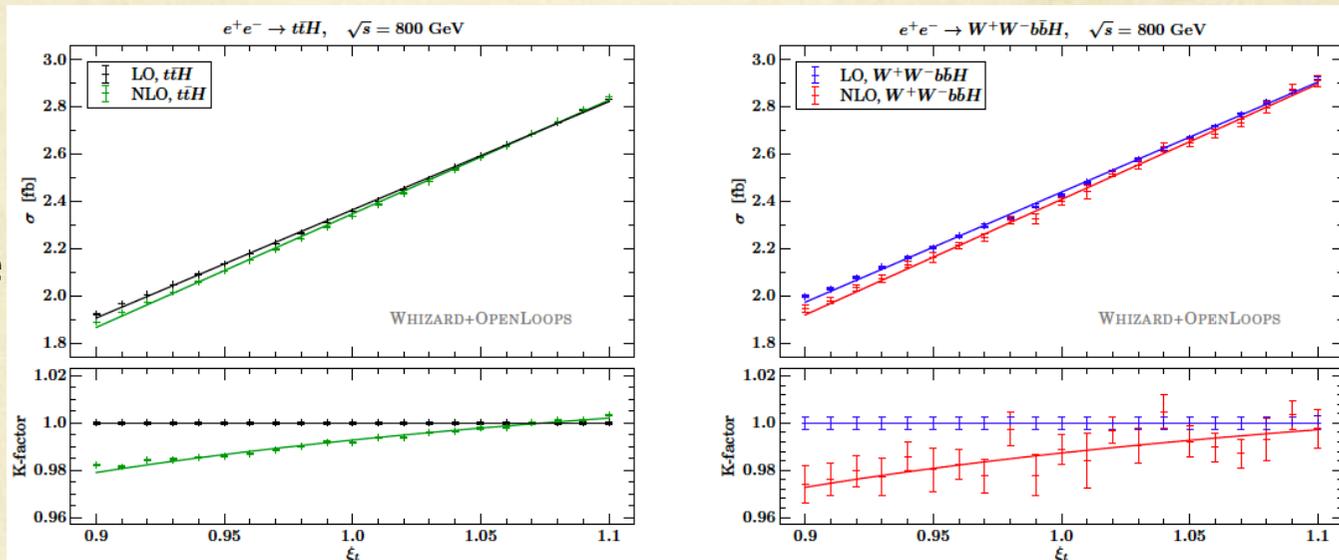
- ✓ Extensive recent study (WHIZARD MC event generator):

Chokouf Nejad, Kilian, Lindert, Pozzorini, Reuter, Weiss '16

- ✓ Few % sensitivity to y_t possible from $t\bar{t}H$:

Define: $\frac{\Delta y_t}{y_t} = \kappa \frac{\Delta \sigma}{\sigma}$ with $y_t = \xi_t y_t^{\text{SM}}$ where: $y_t^{\text{SM}} = \sqrt{2} m_t / v$

Current precision benchmark for stable and off-shell tops:

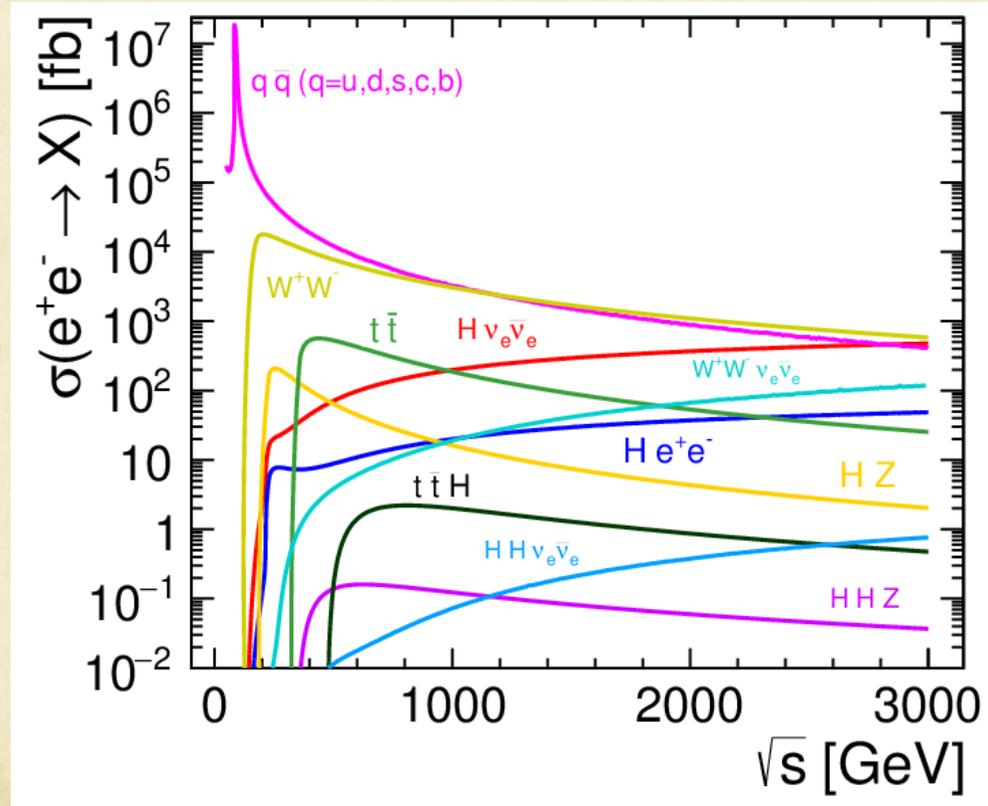


$e^+e^- \rightarrow$	κ^{LO}	κ^{NLO}	$\kappa^{\text{NLO}}/\kappa^{\text{LO}}$
$t\bar{t}H$	0.514	0.485	0.943
$W^+W^-b\bar{b}H$	0.520	0.497	0.956

- ✓ Theory improvement possible: NNLO QCD within reach in the near future!

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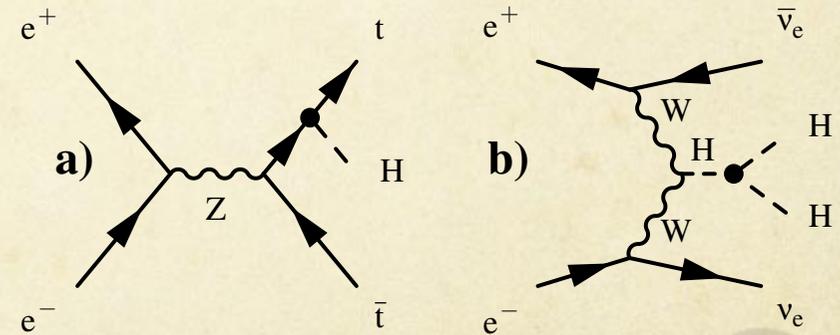
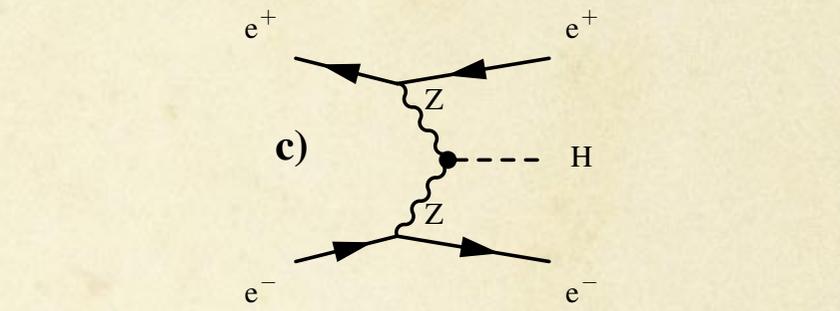
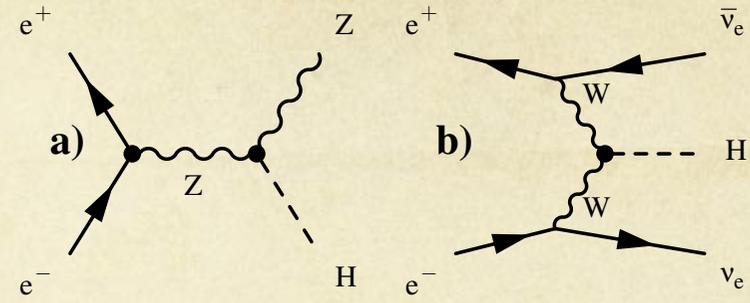
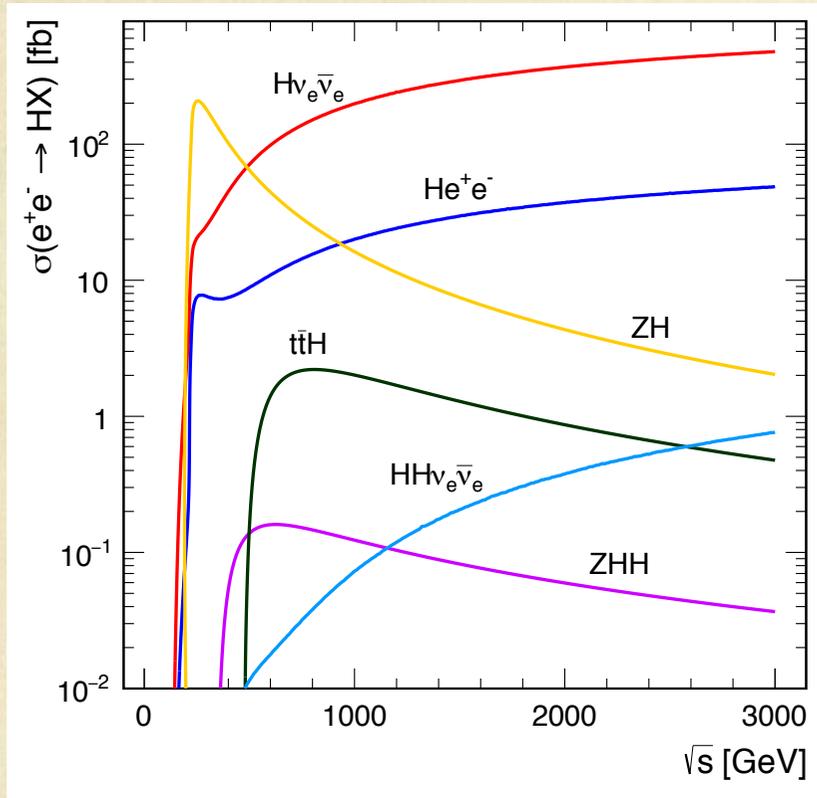
Higgs production



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Higgs

✓ Main production channels:



From: [arXiv:1608.07538](https://arxiv.org/abs/1608.07538)

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Higgs

- ✓ These are EW processes, so mostly NLO EW corrections enter
- ✓ These are available in NLO MC programs like WHIZARD and aMC@NLO
- ✓ Some higher order effects are needed and can be included
 - ✓ Photon radiation, photon initiated processes, etc.
- ✓ QCD enters through Higgs decay which is known through NNLO
- ✓ QCD corrections may become important in hadronic backgrounds to

- ✓ HH, with: $H \rightarrow qq$
- ✓ $H+X$

- ✓ 4 jet production is known at NLO only (more later) but feasible at NNLO for the future

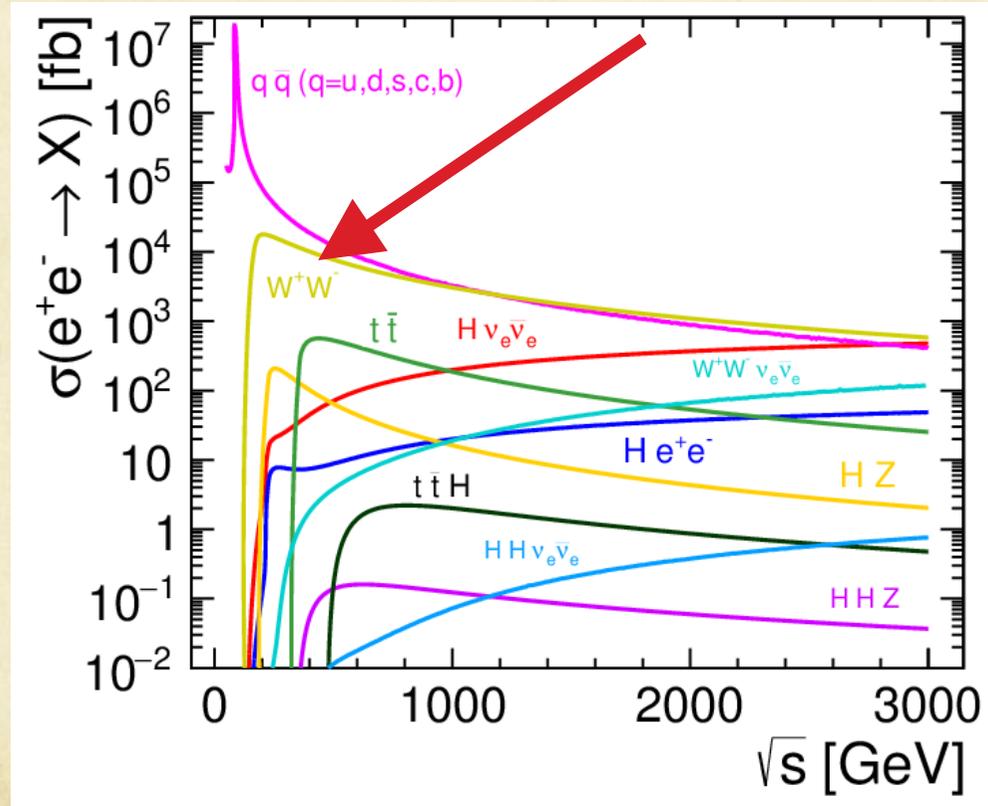
Process	σ/fb
$e^+e^- \rightarrow Hv_e\bar{\nu}_e; H \rightarrow b\bar{b}$	137
$e^+e^- \rightarrow Hv_e\bar{\nu}_e; H \rightarrow c\bar{c}$	6.9
$e^+e^- \rightarrow Hv_e\bar{\nu}_e; H \rightarrow gg$	20.7
$e^+e^- \rightarrow q\bar{q}\nu\bar{\nu}$	788
$e^+e^- \rightarrow q\bar{q}l\nu$	4310
$e^\pm\gamma \rightarrow q\bar{q}e$	16600
$e^\pm\gamma \rightarrow q\bar{q}\nu$	29300
$\gamma\gamma \rightarrow q\bar{q}$	76600

1.5 ab^{-1} at $\sqrt{s} = 1.4 \text{ TeV}$

[arXiv:1608.07538](https://arxiv.org/abs/1608.07538)

WW production

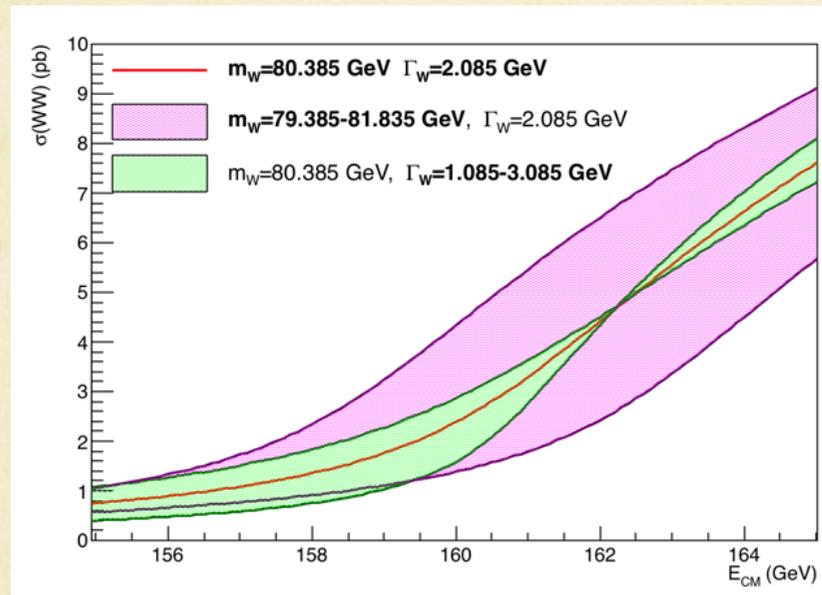
See also talk by Jiayin Gu



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W mass and width

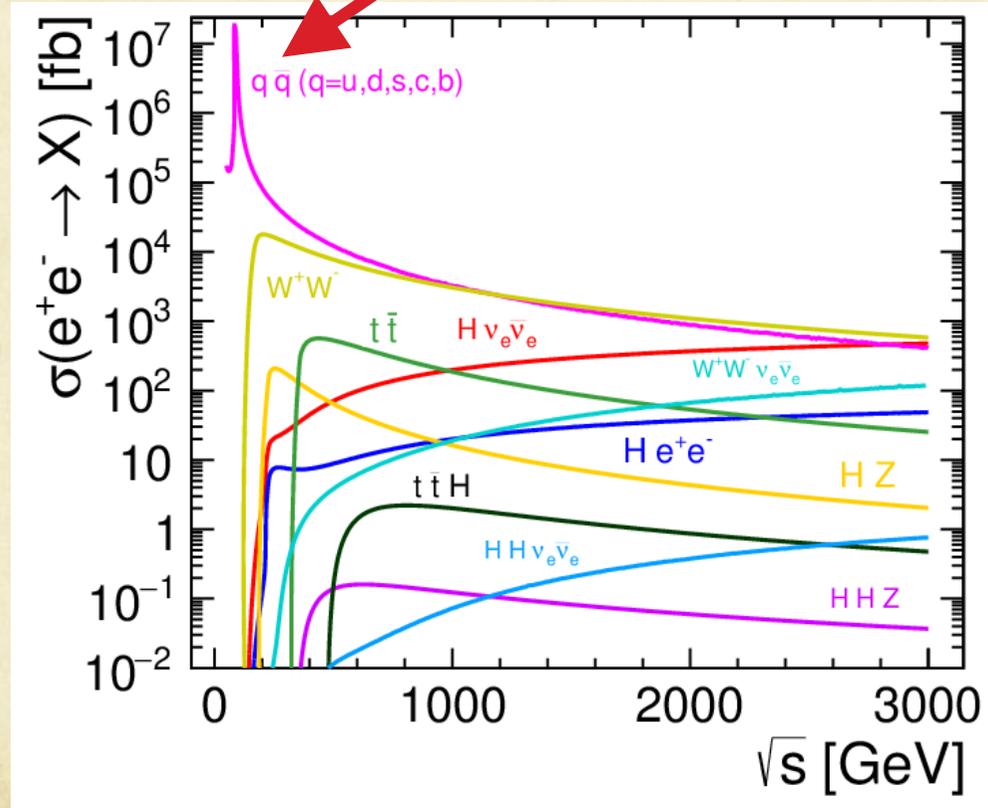
- ✓ Scan the WW threshold to explore its sensitivity to W mass and width
- ✓ State of the art calculation for WW x-section:
 - ✓ YFSWW3 [Jadach, Placzek, Skrzypek, Ward, Was '01](#)
 - ✓ RacoonWW [Denner, Dittmaier, Roth, Wackerath '02](#)
- ✓ Generally these are NLO EW with some higher order QED and some QCD corrections
 - ✓ Some 2-loop contributions [Actis, Beneke, Falgari, Schwinn '08](#)



From Paolo Azzurri '17

- ✓ Measuring the WW cross-section in two points can give W mass and width with very good precision ($\Delta m_W < 1 \text{ MeV}$). Theory is likely the limiting factor. Should be possible to improve.

Jet production



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Jet production

- ✓ Known in NNLO QCD for 2 and 3 jets

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07
S. Weinzierl '08

Del Duca, Duhr, Kardos, Somogyi, Szor, Trocsanyi, Tulipant '16

- ✓ NLO EW for 2,3 jets

Denner, Dittmaier, Gehrmann, Kurz '09

- ✓ Automatic resummation through NNLO+NNLL with programs like CAESAR and ARES

Banfi, Salam, Zanderighi '03-

Banfi, McAslan, Monni, Zanderighi '14-

- ✓ Known in NLO QCD through 5 jets

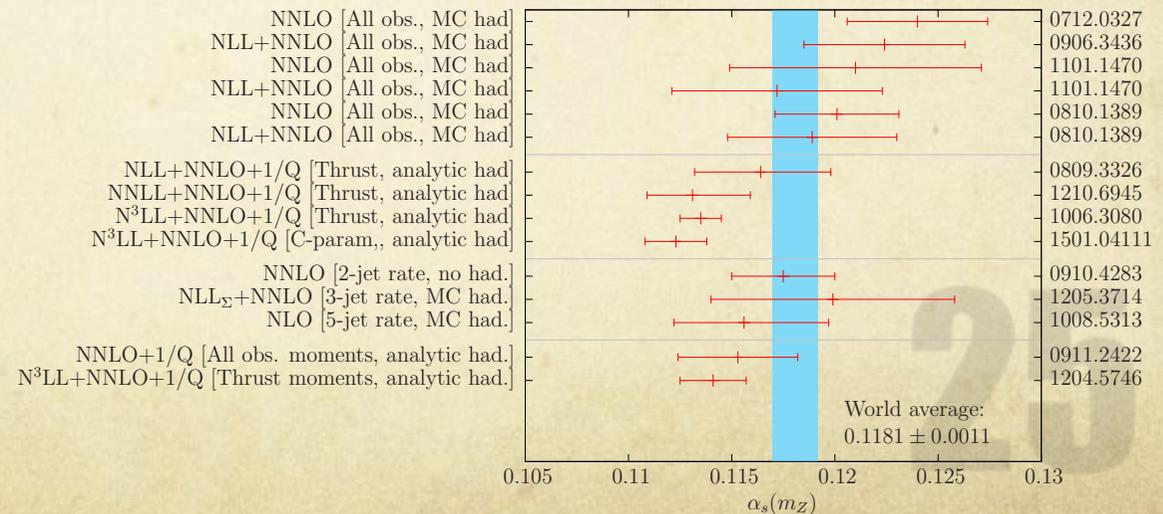
Frederix, Frixione, Melnikov, Zanderighi '10

- ✓ Jets allow studies of perturbative and non-perturbative QCD
- ✓ Backgrounds
- ✓ BSM context?
- ✓ Extraction of α_s

From: [arXiv:1008.5313v2](https://arxiv.org/abs/1008.5313v2)

Observable	$\alpha_s(M_Z)$	Ref.
τ decays	0.1197 ± 0.0016	[67]
Υ decays	0.119 ± 0.0055	[70]
3 jet observables	0.1224 ± 0.0039	[44]
jets in DIS	0.1198 ± 0.0032	[71]
DIS	0.1142 ± 0.0021	[72]
thrust	0.1135 ± 0.0011	[73]
lattice	0.1183 ± 0.0008	[74]
EW fits	0.1193 ± 0.0028	[75]
world average	0.1184 ± 0.0007	[67]
$e^+e^- \rightarrow$ five jets	0.1156 ± 0.0038	this paper

From: Gehrmann, Luisoni, Monni '15



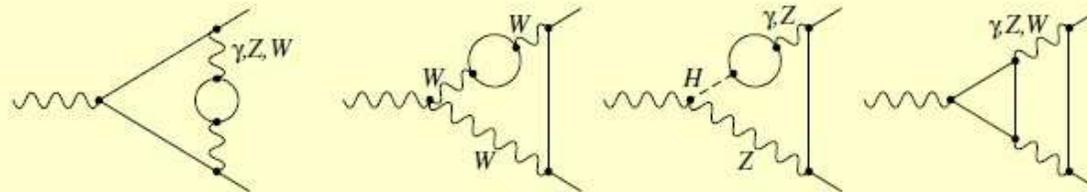
EW Precision Observables

See also talk by [Giuliano Panico](#)

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Z-decay, etc

- ✓ Full 2-loop EW corrections to $\sin^2 \theta_{\text{eff}}^b$ now available
- ✓ 2-loop bosonic EW corrections computed; work on dominant 3-loop corrections ongoing
Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16
- ✓ Prior results for EW precision observables:



From: A. Freitas '16

- Complete NNLO corrections (Δr , $\sin^2 \theta_{\text{eff}}^l$) Freitas, Hollik, Walter, Weiglein '00
Awramik, Czakon '02; Onishchenko, Veretin '02
Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
Hollik, Meier, Uccirati '05,07; Degrossi, Gambino, Giardino '14
- “Fermionic” NNLO corrections (g_{Vf} , g_{Af}) Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98
Freitas '13,14
- Partial 3/4-loop corrections to ρ/T -parameter
 $\mathcal{O}(\alpha_t \alpha_s^2)$, $\mathcal{O}(\alpha_t^2 \alpha_s)$, $\mathcal{O}(\alpha_t \alpha_s^3)$
Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker, Veretin '03
Boughezal, Tausk, v. d. Bij '05
Schröder, Steinhauser '05; Chetyrkin et al. '06
Boughezal, Czakon '06

$$(\alpha_t \equiv \frac{y_t^2}{4\pi})$$

Z-decay, etc

- ✓ Status of intrinsic uncertainties for EW precision observables

From: S. Heinemeyer '17

Quantity	FCC-ee	Current intrinsic unc.	Projected unc.
M_W [MeV]	1	4 ($\alpha^3, \alpha^2\alpha_s$)	1
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	0.6	4.5 ($\alpha^3, \alpha^2\alpha_s$)	1.5
Γ_Z [MeV]	0.1	0.5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$)	0.2
R_b [10^{-5}]	6	15 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)	7
R_l [10^{-3}]	1	5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)	1.5

These calculations are required for the projection:

- complete $\mathcal{O}(\alpha\alpha_s^2)$ corrections
- fermionic $\mathcal{O}(\alpha^2\alpha_s)$ corrections
- double-fermionic $\mathcal{O}(\alpha^3)$ corrections
- leading four-loop corrections enhanced by the top Yukawa coupling
- the $\mathcal{O}(\alpha_{\text{bos}}^2)$ corrections are not the leading uncertainties now

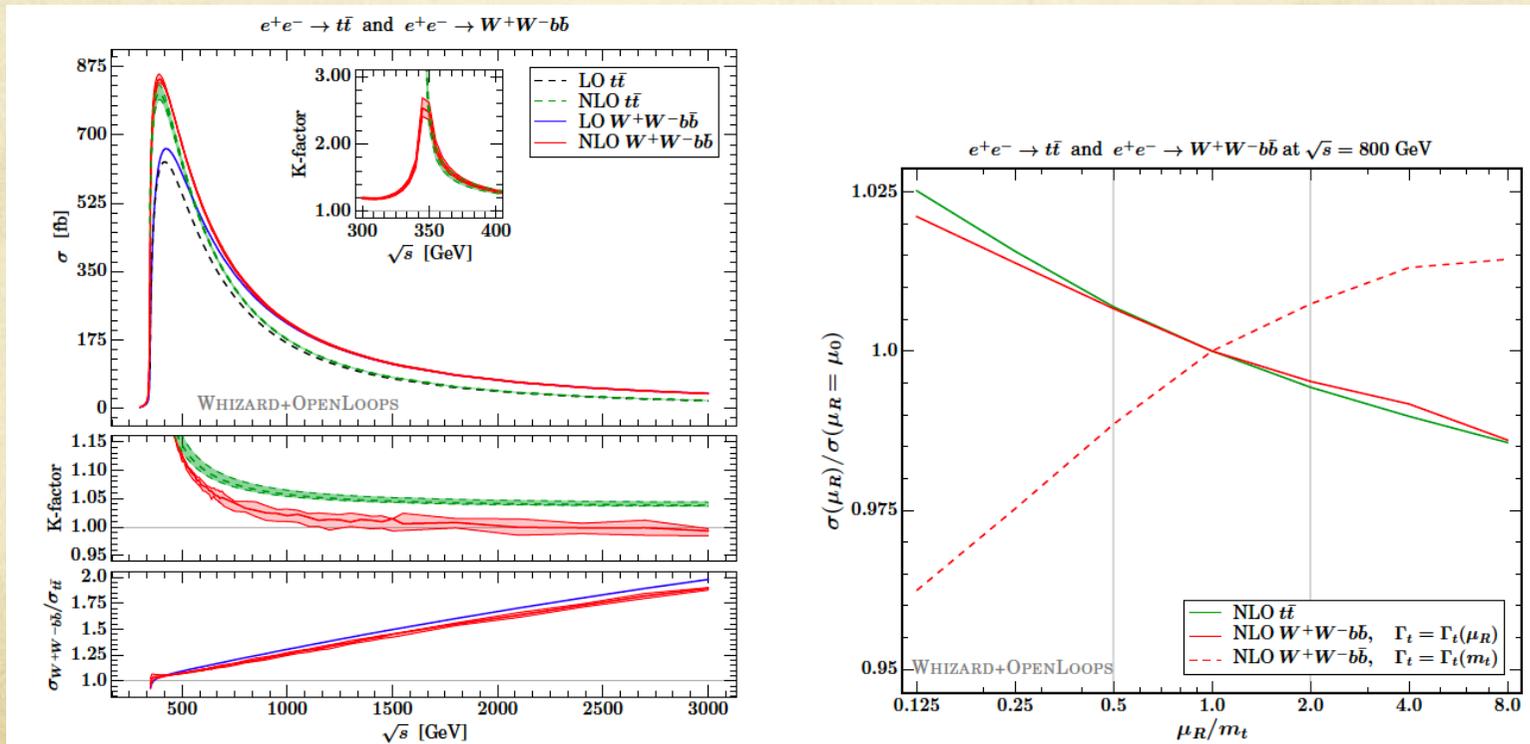
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Event Generators

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NLO event generators

- ✓ LHC advances can also be put to work for e^+e^-
 - ✓ (almost) Everything today can be computed at NLO with out of the box programs!
- ✓ aMC@NLO: full NLO QCD + EW interfaced to parton showers. Actively developed for e^+e^-
See talk by Shao-Feng Ge
- ✓ WHIZARD
 - Nejad, Kilian, Lindert, Pozzorini, Reuter, Weiss '16
 - See talk by Jürgen Reuter
- ✓ Matching to showers
- ✓ Matching continuum and threshold production



More on event generators: the future is bright

- ✓ Ongoing are very significant developments regarding event generators at the LHC
- ✓ The long reign of LL parton showers is slowly coming to an end
- ✓ Very intense work underway on extending showers to full NLL accuracy
 - Hoeche, Prestel, Krauss '16
 - Others...
- ✓ Basically, the singularities inside the shower would match the ones of a generic NNLO QCD calculation
- ✓ Such a change will have profound implication for showers since
 - ✓ They will have to be re-tuned from scratch from LEP data.
GigaZ-like physics option may be needed!
- ✓ Basically, the Event generators then will be much more sophisticated than today.
- ✓ Matching of showers to any then-existing NNLO calculation should be assumed
- ✓ In other words, we can assume MC@NNLO with improved tuning from, possibly, future data
- ✓ Showers may become more accurate also because in e^+e^- collisions they do not need to deal with "dirty" hadron physics present at the LHC.

Conclusions

- ✓ There are extensive theory developments that cover both regimes:
 - ✓ Threshold production
 - ✓ Continuum production
- ✓ The currently available results and tools are adequate for
 - ✓ Computing (almost) everything at NLO is now possible with public tool like aMC@NLO, WHIZARD etc.
- ✓ In the future, in time for data taking at CLIC, significant progress compared to today at the LHC can be expected
 - ✓ MC@NNLO style generators with NLL showers. GigaZ-like option will be beneficial.
 - ✓ NNLO calculations for small multiplicities (in fact, $e^+e^- \rightarrow t\bar{t}H$ at NNLO is not unthinkable for today)
- ✓ These should allow for 1MeV for W mass; %-level for top width; 50MeV precision for m_{top}
- ✓ 3-4 %-level determination of the top-Yukawa coupling
 - ✓ compare to 10% at HL-LHC but 1% at Fcc-hh. Perhaps there is an opportunity for further study here?
- ✓ Precision EW observables and fits benefit from very precise theory and will likely play prominent role in the future.