Implications of tau data for CP violation in K decays

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In collaboration with:

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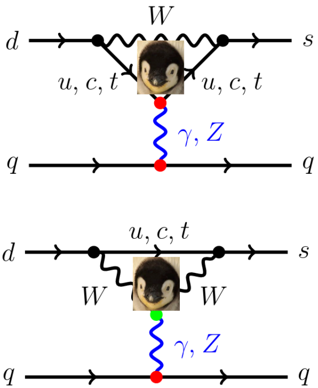
Electroweak penguins in $K \to \pi\pi$



Headless versions from Gisbert, Pich 18

$\rightarrow$

$$Q_7 = \frac{3}{2}\,(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q\,(\bar{q}q)_{\mathrm{V+A}}$$

$$Q_8 = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}$$

$$Q_9 = \frac{3}{2}\,(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q\,(\bar{q}q)_{\mathrm{V-A}}$$

$$Q_{10} = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

- Q_8 plays a crucial role in $\frac{\epsilon'}{\epsilon}$
- Q_7 and Q_8 (in $K \to \pi\pi$) do not vanish in the chiral limit ($\mathcal{O}(p^0)$)

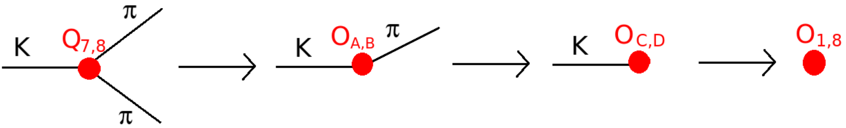
Soft-meson theorem

$$\lim_{q\to 0}\langle \phi^k(q)\beta|\mathcal{O}(0)|\alpha\rangle = \frac{i}{F}\langle\beta|[Q_5^k,\mathcal{O}(0)]|\alpha\rangle$$

GOLDSTONES (~ PIONS, KAONS)

GENERAL OPERATOR

GENERAL STATES



$$\lim_{k,p,q\to 0}\langle(\pi\pi)_{I=2}|\mathcal{Q}_7|K^0\rangle = -\frac{2}{F^3}\langle\mathcal{O}_1\rangle_\mu$$

Donoghue, Golowich '99

$$\lim_{k,p,q\to 0}\langle(\pi\pi)_{I=2}|\mathcal{Q}_8|K^0\rangle = -\frac{2}{F^3}\left(\frac{1}{2}\langle\mathcal{O}_8\rangle_\mu + \frac{1}{N_c}\langle\mathcal{O}_1\rangle_\mu\right)$$

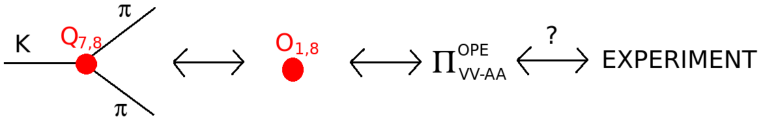
$$\langle\mathcal{O}_1\rangle_\mu \equiv \langle 0|\tfrac{1}{2}\bar{d}\,\Gamma^L_\mu u\;\bar{u}\Gamma^\mu_R d|0\rangle_\mu\,, \qquad \langle\mathcal{O}_8\rangle_\mu \equiv \langle 0|\tfrac{1}{2}\bar{d}\,\Gamma^L_\mu\lambda_i u\;\bar{u}\Gamma^\mu_R\lambda_i d|0\rangle_\mu$$

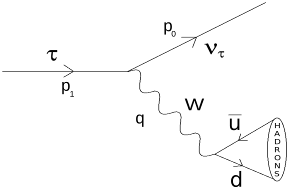
$$\Pi_{VV-AA}(q) \equiv i \int d^4x\, e^{-iqx} \langle 0 | J_V(x) J_V^\dagger(0) - J_A(x) J_A^\dagger(0) | 0 \rangle \,, \; J_{V(A)} = \bar{d}\gamma_\mu(\gamma_5) u$$

Operator Product Expansion $\quad \Pi(q) = \frac{\mathcal{O}_D}{Q^D}$ SVZ '79

- $\mathcal{O}_{D=0} = 0$. It vanishes at all orders in massless perturbative QCD
- $\mathcal{O}_{D=2,4} \approx 0$. Owing to the values of m_u and m_d
- First contribution coming from $D = 6$

  $$\mathcal{O}_{D=6} \sim 4\pi\alpha_s \langle \mathcal{O}_8 \rangle_\mu + \alpha_s^2 \left(\widetilde{C}_1 \langle \mathcal{O}_1 \rangle_\mu + \widetilde{C}_8 \langle \mathcal{O}_8 \rangle_\mu \right), \quad \widetilde{C}_i = A_i + B_i \ln(-q^2/\mu^2)$$





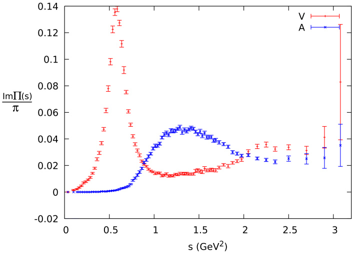
$\tau \to \nu_\tau n$ with n an arbitrary non-strange hadronic state (e.g. π or $\pi\pi$)

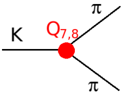
$$M \sim G_F L^\mu \langle n | J_{W\mu} | 0 \rangle \to \frac{d\Gamma_n}{ds} \sim G_F^2 L^{\mu\nu} \langle n | J_{W\mu} | 0 \rangle \langle 0 | J^\dagger_{W\nu} | n \rangle$$

Summing over all possible n that can be reached through V (A) current:

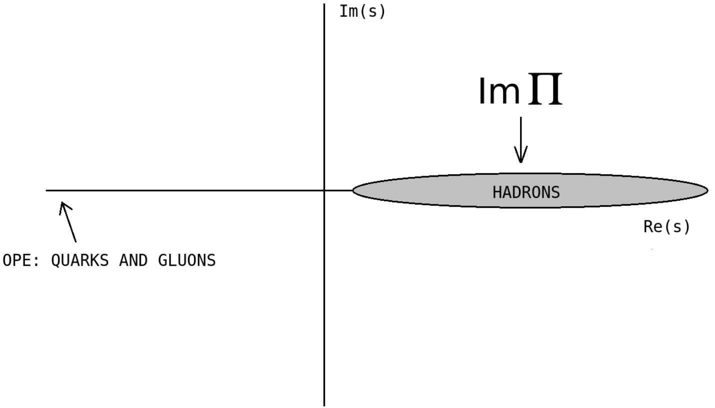
$$\frac{d\Gamma_{V(A)}}{ds} \sim \mathrm{Im}\,\Pi_{VV(AA)}(s)$$

ALEPH '14

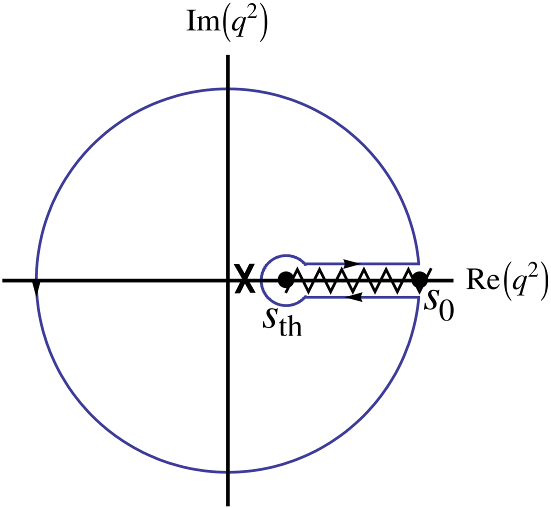




$\longleftrightarrow$ $O_{1,8}$ $\longleftrightarrow$ $\Pi^{OPE}_{VV\text{-}AA}$ $\overset{?}{\longleftrightarrow}$ $\mathrm{Im}\,\Pi$ $\longleftrightarrow$ EXP



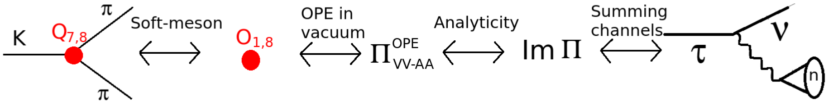
$\Pi(s)$ is analytic in all complex plane except for a cut in the positive real axis



$$\int_{s_{th}}^{s_0} \frac{ds}{s_0}\,\omega(s)\,\mathrm{Im}\,\Pi(s) - \frac{i}{2}\oint_{|s|=s_0} \frac{ds}{s_0}\omega(s)\Pi(s) = 2\pi\frac{f_\pi^2}{s_0}\omega(m_\pi^2), \quad \omega(s) \text{ analytic inside}$$

$$\underbrace{\int_{s_{th}}^{s_0} \frac{ds}{s_0}\, \omega(s)\, \mathrm{Im}\,\Pi(s)}_{\text{Experimental data}} - \underbrace{\frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi(s)}_{\text{OPE}} = 2\pi \frac{f_\pi^2}{s_0} \omega(m_\pi^2), \quad \omega(s) \text{ analytic inside}$$

$$\delta_{\rm DV}[\omega(s), s_0] \equiv \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left[\Pi - \Pi^{\rm OPE}\right](s) = \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \frac{1}{\pi}\, \mathrm{Im}(\Pi - \Pi^{\rm OPE})(s)$$



Phenomenological implications studied by DG '99, CDGM '01, CDGM '02

Motivation for phenomenological reanalysis

- Updated data sets
- Use of further techniques to assess systematic uncertainties

Operator Product Expansion $\Pi(q) = \frac{\mathcal{O}_D}{Q^D}$

$\mathcal{O}_{D=6} \sim 4\pi\alpha_s \langle \mathcal{O}_8 \rangle_\mu + \alpha_s^2 (\widetilde{C}_1 \langle \mathcal{O}_1 \rangle_\mu + \widetilde{C}_8 \langle \mathcal{O}_8 \rangle_\mu), \quad \widetilde{C}_i = A_i + B_i \ln(-q^2/\mu^2)$

Large uncertainties. We work at LO in $\alpha_s \to \mathcal{O}_{D=6} \sim 4\pi\alpha_s \langle \mathcal{O}_8 \rangle_\mu$

$$\lim_{k,p,q\to 0} \langle (\pi\pi)_{I=2} | \mathcal{Q}_7 | K^0 \rangle = -\frac{2}{F^3} \langle \mathcal{O}_1 \rangle_\mu$$

$$\lim_{k,p,q\to 0} \langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle = -\frac{2}{F^3} \left(\frac{1}{2} \langle \mathcal{O}_8 \rangle_\mu + \frac{1}{N_c} \langle \mathcal{O}_1 \rangle_\mu \right)$$

- We still need $\langle \mathcal{O}_1 \rangle_\mu$
- In the large-N_c limit, $\frac{\langle \mathcal{Q}_7 \rangle_\mu}{\langle \mathcal{Q}_8 \rangle_\mu} = 0$. Suppression confirmed by other analyses
- Taking $\frac{\langle \mathcal{Q}_7 \rangle_\mu}{\langle \mathcal{Q}_8 \rangle_\mu} \leq \frac{1}{N_c} \to \langle \mathcal{Q}_8 \rangle_\mu^{\langle \mathcal{O}_1 \rangle_\mu} \leq \frac{\langle \mathcal{Q}_8 \rangle_\mu^{\langle \mathcal{O}_8 \rangle_\mu}}{N_c^2} \frac{1}{1 - \frac{1}{N_c^2}}$
- $\lim_{p,q,k=0} \langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle_\mu \approx -\frac{\mathcal{O}_{D=6}(\mu)}{4\pi\alpha_s(\mu) F^3}$

Revisiting $\mathcal{O}_{D=6}$

Gonzalez-Alonso et al. '16

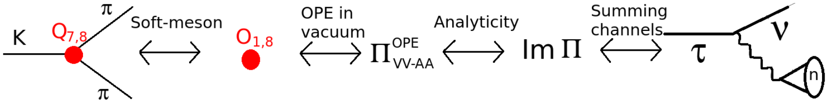
- Parametrization: $\rho(s) = \kappa e^{-\gamma s} \sin(\beta(s - s_z)), \qquad s > \hat{s}_0$
- Random tuples $(\kappa, \gamma, \beta, s_z)$. Every tuple $\rightarrow$ a possible spectral function
- Accept those spectral functions in agreement with data and sum rules
- Every spectral function $\rightarrow$ a value of $\mathcal{O}_6$. Used to estimate DV uncertainties
- $\mathcal{O}_{D=6} \approx (-3.6 \pm 1.0) \cdot 10^{-3} \mathrm{GeV}^6$

Extra tests and small changes aimed to improve it Preliminary!

- Combined fit to WSRs and $\mathcal{O}_{D=6}$ so that experimental correlation is taken into account
- Agreement with plateau observed ignoring DVs at $s_0 \sim m_\tau^2$ when pinching
- Stability by changing $\hat{s}_0$ (test of reliability) observed

$$\mathcal{O}_{D=6} = (-3.1 \pm 1.0) \cdot 10^{-3}\ \mathrm{GeV}^6 \text{ Preliminary!}$$

$\mathcal{O}_{D=6} = (-3.1 \pm 1.0) \cdot 10^{-3}\ \mathrm{GeV}^6$ Preliminary!



$\lim_{p,q,k=0} \langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle_{2\,\mathrm{GeV}} = -\frac{\mathcal{O}_{D=6}}{4\pi\alpha_s F^3} \approx (1.14 \pm 0.49)\ \mathrm{GeV}^3$ Preliminary!

- Good agreement with the large-N_c limit

$$\langle (\pi\pi)_{I=2} | \mathcal{Q}_8 | K^0 \rangle^{N_c}_{2\,\mathrm{GeV}} = 2FB_0^2 = 2\frac{M^4_{K_0} F}{(m_d+m_s)^2} = 1.1\ \mathrm{GeV}^3$$

- Good agreement with the lattice, although with larger uncertainties...

...We still can turn the table→ use the lattice to improve other tau-based results

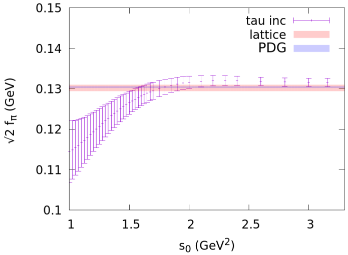
Input from the lattice

$$\langle Q_7\rangle_{3\,\mathrm{GeV}} = 0.36 \pm 0.03$$
$$\langle Q_8\rangle_{3\,\mathrm{GeV}} = 1.6 \pm 0.1$$

RBC-UKQCD '12

Excellent SD knowledge of $\Pi(s)$ $\qquad \Pi(q^2) = \frac{\mathcal{O}_D}{Q^D}$

- $\mathcal{O}_{D=0} = 0$. It vanishes at all orders in massless perturbative QCD
- $\mathcal{O}_{D=2,4} \approx 0$. Owing to the values of m_u and m_d
- $\mathcal{O}_{D=6} \sim 4\pi\alpha_s \langle\mathcal{O}_8\rangle_\mu + \alpha_s^2 \left(\widetilde{C}_1\langle\mathcal{O}_1\rangle_\mu + \widetilde{C}_8\langle\mathcal{O}_8\rangle_\mu\right)$, $\quad \widetilde{C}_i = A_i + B_i \ln(-q^2/\mu^2)$
  $\langle\mathcal{O}_1\rangle_\mu$ and $\langle\mathcal{O}_8\rangle_\mu$ known from lattice input + DG relations

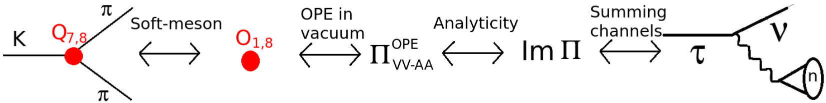


Beyond the percent level!

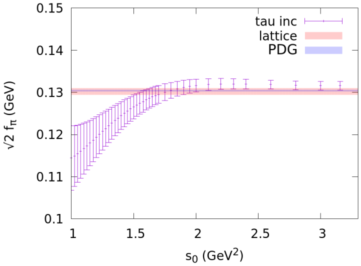
Other applications

- $\mathcal{O}_{D=8} = -(0.7 \pm 0.6) \cdot 10^{-2}\,\mathrm{GeV}^8$ Prel.
- Bound to New Physics (see G-A's talk)

Conclusions Preliminary results



$$\langle(\pi\pi)_{I=2}|\mathcal{Q}_8|K^0\rangle_{2\mathrm{GeV}} = (1.14 \pm 0.49)\,\mathrm{GeV}^3$$



Dispersive relations → very precise predictions where p-QCD and χPT do not work!

BACK UP

Dispersion relations with polynomial kernels at NLO in α_s

$$\Pi^{(1+0)}(Q^2=-q^2)=\sum_{p=D/2}\frac{a_p(\mu)+b_p(\mu)\ln\frac{Q^2}{\mu^2}}{Q^{2p}}$$

$$\int_{s_{th}}^{s_0}\frac{ds}{s_0}\omega(s)\,\mathrm{Im}\,\Pi(s)=-\int_{s_0}^{\infty}\frac{ds}{s_0}\omega(s)\,\mathrm{Im}\,\Delta_{DV}(s)-\frac{\pi}{(-s_0)^p}\sum_{n=0}^{\infty}\sum_{p=1}^{\infty}c_n d_p^{(n)}$$
$$+\,2\pi\frac{f_\pi^2}{s_0}\omega(m_\pi^2)\,,$$

with $\omega(s)=\sum_n c_n x^n$,

$$d_p^{(n)}=\begin{cases}a_p^M & \text{if}\quad p=n+1\\ \frac{b_p}{n-p+1} & \text{if}\quad p\neq n+1\end{cases}\,,$$

$$a_p^M(\mu,s_0)=a_p(\mu)+b_p(\mu)\ln\left(\frac{s_0}{\mu^2}\right)\,.$$

Sum rules in the chiral limit

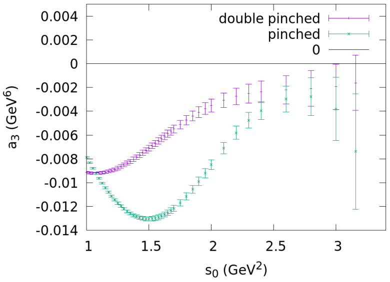
$$\omega^{(2,0)}(s) \equiv \left(1 - \tfrac{s}{s_0}\right)^2$$

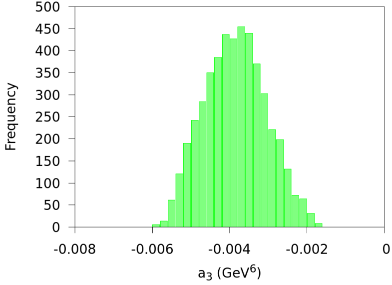
$$\int_{s_{th}}^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2 \mathrm{Im}\,\Pi(s) = 2\pi\frac{f_\pi^2}{s_0} - 4\pi\frac{f_\pi^2 m_\pi^2}{s_0^2} + 2\pi\frac{f_\pi^2 m_\pi^4}{s_0} + \pi\frac{a_3}{s_0^3} + \delta_{DV}^{\omega^{(2,0)}}(s_0)\,,$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2 \mathrm{Im}\,\Pi_{m_q=0}(s) = 2\pi\frac{F^2}{s_0} + \pi\frac{a_3^0}{s_0^3} + \delta_{DV}^{\omega^{(2,0)},\,0}(s_0)\,,$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2 \left(\mathrm{Im}\,\Pi(s) - \mathrm{Im}\,\Pi_{m_q=0}(s)\right) \approx 2\pi\frac{f_\pi^2 - F^2}{s_0}\;.$$

$$2\pi\tfrac{f_\pi^2 - F^2}{s_0} \sim 0.0028,\; \pi\tfrac{|a_3|}{s_0^3} \sim 0.0015$$





$\hat{s}_0(\mathrm{GeV}^2)$	1.25	1.4	1.55	1.7	1.9
$a_3(10^{-3}\mathrm{GeV}^6)$	$-5.3^{+0.7}_{-0.5}$	$-5.1^{+0.7}_{-0.5}$	$-5.3^{+0.5}_{-0.3}$	$-3.7^{+1.3}_{-0.9}$	$-3.8^{+1.8}_{-1.0}$