INCLUSIVE τ DETERMINATION(S) OF $|V_{us}|$

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PLB781 (2018) 206 and arXiv:1803.07228 [hep-lat]

with

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OUTLINE

- *Systematic issues in the conventional BF-only FB FESR implementation (and the related inclusive τ V_{us} puzzle)*

- *An alternate FB FESR implementation solution*

- *A new, lattice+inclusive us V+A τ approach (advantages, results, and future prospects)*

CONTEXT

- Non-τ vs inclusive τ determinations

$\lvert V_{us}\rvert$	Source
0.2257(9)(?)	3-family unitarity, HT16 $\lvert V_{ud}\rvert$
$0.2233(4)_{exp}(4)_{latt}$	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
$0.2254(3)_{exp}(8)_{latt}$	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π

c.f. "conventional" (BFs-only) inclusive τ FB FESR analysis result: 0.2195(19) (Lusiani, CKM 2018)

- Interesting (if true) given hints of lepton universality breakdown in $R[D^{(*)}]$ [e.g. Nature 546 (2017) 227]

HADRONIC τ DECAY, FB FESR BASICS

- $R_{ij;V/A} \equiv \Gamma[\tau \to \nu_\tau \,\mathrm{hadrons}_{ij;V/A}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$

- With $y_\tau \equiv s/m_\tau^2$, flavor ij decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2 \, |V_{ij}|^2 S_{EW}}{m_\tau^2} \, [1 - y_\tau]^2 \, \tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1 + 2y_\tau) \, \rho^{(J=1)}_{ij;V+A}(s) + \rho^{(J=0)}_{ij;V+A}(s) \right]$$

kinematic weight : $w_\tau(y) = (1 - y)^2 (1 + 2y)$

- FESRs for $\Pi = \Pi^{(J=0+1)}_{ud-us;V+A}(Q^2)$, $\rho = \rho^{(J=0+1)}_{ud-us;V+A}(s)$

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$

with $|V_{ij}|^2\rho^{(0+1)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$

- $R^w_{ij;V/A}(s_0)$: re-weighted $R^{(0+1)}_{ij;V/A}$ analogue

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR^{(0+1)}_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

- FB differences $\delta R^w(s_0) \equiv \frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - \frac{R^w_{us;V+A}(s_0)}{|V_{us}|^2}$

- **FESR, OPE for $\delta R^w(s_0)$, input $|V_{ud}|$** $\Rightarrow$

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

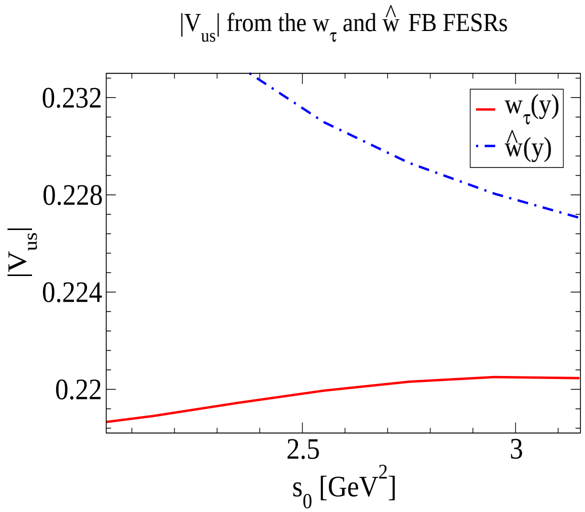
Self-consistency: $|V_{us}|$ independent of s_0, w

- **The conventional implementation** [Gamiz et al. JHEP03(2003)060]
  - $s_0 = m_\tau^2$, $w = w_\tau$ only [spectral integrals from inclusive ud, us BFs, **but no self-consistency tests**]
  - w_τ degree 3 $\Rightarrow$ OPE to $D = 8$
  - Assumptions needed for $D = 6$ (VSA), $D = 8$ (~ 0)

- Variable-s_0, -w FB FESR tests with same $D = 6, 8$ assumptions show [KM et al. PLB781 (2018) 206]
  - strong unphysical s_0 dependence
  - strong unphysical w dependence even at $s_0 = m_\tau^2$
  - apparent convergence for all w beyond $s_0 = m_\tau^2$

- Results ⇒
  - Sizeable theoretical systematic in conventional (BF-only) implementation (more below)
  - Use of conventional implementation (unfortunately) must be abandoned going forward

Conventional implementation $|V_{us}|$ from w_τ, $\hat{w}$ FESRs

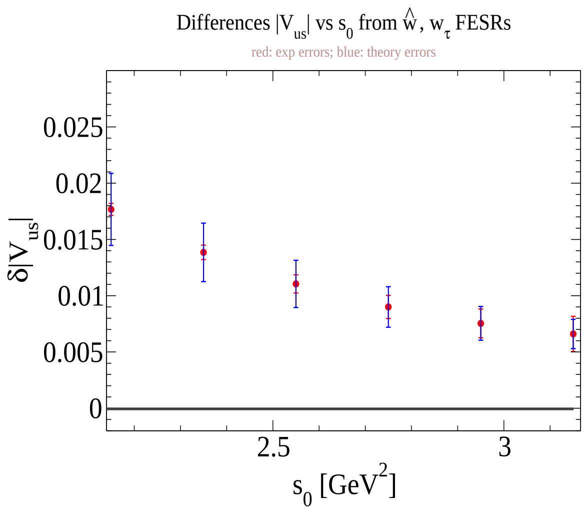


D=6 contributions identical (up to sign)
D=8 contributions differ by a factor of -1/2

$$w_\tau(y) = 1 - 3y^2 + 2y^3,\ \hat{w}(y) = 1 - 3y + 3y^2 - y^3$$

Correlated differences, $|V_{us}|$ from conventional implementation w_τ, $\hat{w}$ FESRs

Differences should be zero within errors if D=6,8 assumptions are reliable



$$w_\tau(y) = 1 - 3y^2 + 2y^3, \ \hat{w}(y) = 1 - 3y + 3y^2 - y^3$$

AN ALTERNATE FB FESR IMPLEMENTATION

- Theory side
  - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data **(N.B. requires variable s_0)**
  - 3-loop-truncated FOPT $D = 2$, standard $D = 2 + 4$ error estimates [from comparison to lattice data]
  - C_{2N+2} ($D = 2N + 2$ condensate), $|V_{us}|$ both from $w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ FESR ($y = s/s_0$)
  - $|V_{us}|$ from different w_N as self-consistency check

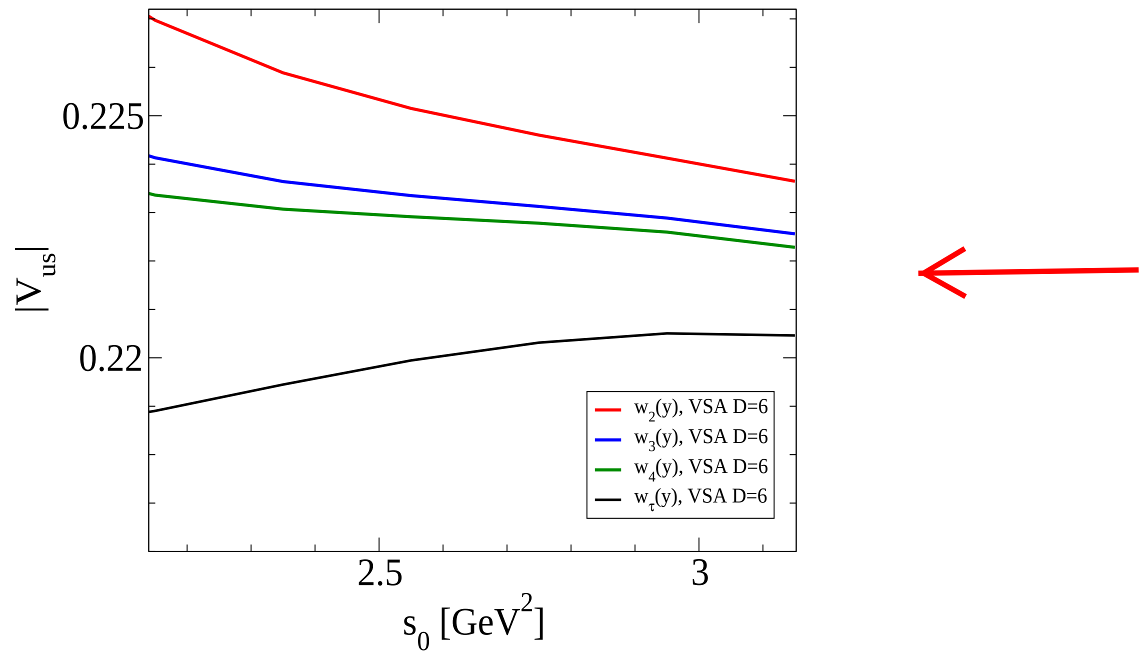
- Experimental input (also for new lattice approach)
  - Updated/corrected 2013 ALEPH for ud V+A
  - us V+A from sum over exclusive modes
    - K from $K_{\mu 2}$ or $B[\tau \to K\nu_\tau]$
    - $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$: BaBar, Belle unit-normalized distributions

      [Note: Normalized using BFs from HFLAV 2017 combined with near-completed BaBar Adametz thesis $B[K^- n\,\pi^0]$ results (Lusiani CKM 2018)]
    - Remaining ("residual modes"): 1999 ALEPH, with exclusive-mode, Lusiani CKM 2018 BF rescalings (∼ 25% errors, some MC)

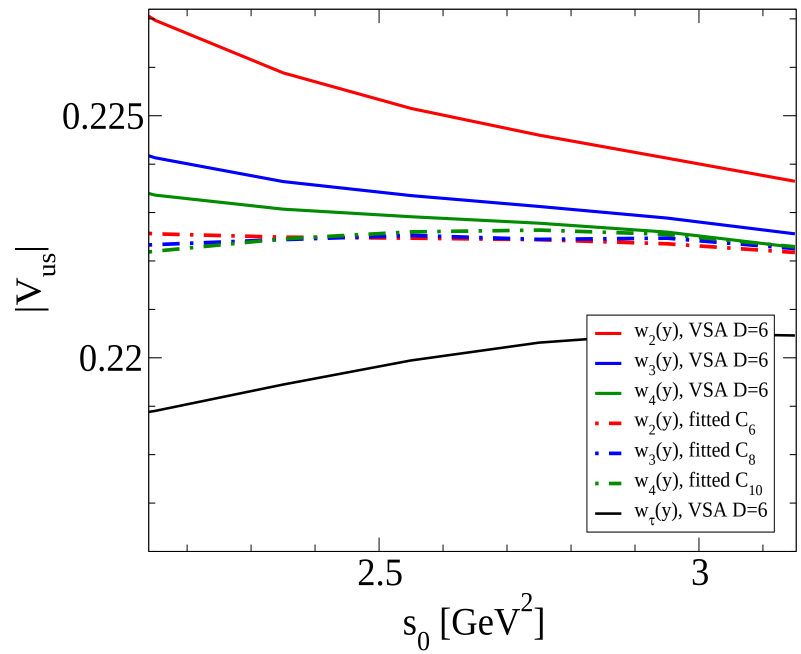
NEW FB FESR RESULTS

- With fitted $D > 4$ condensates
  - unphysical s_0-, $w(y)$-dependence problems resolved
  - $|V_{us}|$ increased by ~ 0.0020: 0.2195(19)(?) $\rightarrow$ 0.2212(23) (0.2219(22) if use $K_{\mu 2}$+SM for K)
- Favorable (~ 0.0005) theory error situation
- us spectral integral uncertainty strongly dominant
- Sizeable 25% residual mode error $\Rightarrow$ significant near-term us error improvement highly unlikely

Conventional implementation w_τ, w_2, w_3, w_4 results



Alternate vs conventional implementation results



A LATTICE τ-BASED ALTERNATIVE

arXiv:1803.07228, with R. Lewis, J. Hudspith (York), P. Boyle, T. Izubuchi, A. Jüttner, C. Lehner, H. Ohki, A. Portelli, M. Spraggs (RBC/UKQCD)

- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us V+A polarizations with weights having poles at Euclidean Q^2
  - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$)
  - Theory: Lattice us 2-point function data (no OPE)
  - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- $|V_{us}|^2\,\tilde{\rho}_{us;V+A}(s)$ from experimental $dR_{us;V+A}/ds$

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1+2\frac{s}{m_\tau^2}\right)\rho^{(J=1)}_{us;V+A}(s) + \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J=0$ subtraction required)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1-2\frac{Q^2}{m_\tau^2}\right)\Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \to \infty$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

$$\int_{th}^{\infty} ds\, w_N(s)\tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)} \equiv \tilde{F}_{w_N}$$

  - Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS
  - LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$
  - $w_N(s)$: rapid fall-off if all $Q_k^2 < 1\ GeV^2$

    $\Rightarrow$ K, $K\pi$ **dominate LHS, near-endpoint multi-particle,** $s > m_\tau^2$ **contributions strongly suppressed**
  - Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error

- A few details:

  - Near-physical-point RBC/UKQCD $n_f = 2+1$ DWF ensembles

    * $48^3 \times 96$, $1/a = 1.73\ GeV$, $m_\pi = 0.139\ GeV$, $m_K = 0.499\ GeV$

    * $64^3 \times 128$, $1/a = 2.36\ GeV$, $m_\pi = 0.139\ GeV$, $m_K = 0.508\ GeV$

    * (Small) retuning to physical valence masses [PQ]

  - Time-momentum representation for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ entering sum in $\tilde{F}_{w_N}$

- Weighted spectral integrals (include $|V_{us}|^2$ factor)

$$\tilde{R}_{w_N} \equiv \int_0^{m_\tau^2} \frac{m_\tau^2}{12\pi^2 S_{EW}(1-y_\tau)^2} \frac{dR_{us;V+A}(s)}{ds} w_N(s) ds$$

- $\tilde{F}_{w_N}^{pQCD} \equiv \int_{m_\tau^2}^\infty [\tilde{\rho}_{us}(s)]^{\rm pQCD}\, w_N(s) ds$

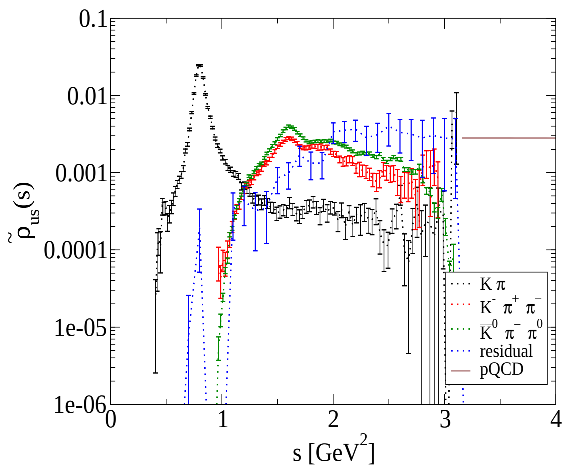
- $|V_{us}| = \sqrt{\tilde{R}_{us;w_N} / \left[\tilde{F}_{\omega_N} - \tilde{F}_{w_N}^{pQCD}\right]}$

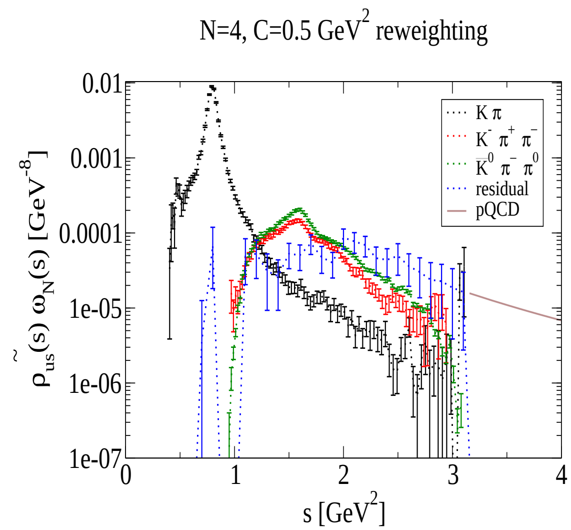
- Expect continuum spectral A, $J=0$ channel contributions negligible for w_N employed (confirmed by lattice data), hence exclusive A, $J=0$ analysis also possible (using only K spectral contribution)

- w_N below: uniform pole spacing Δ, centroid C

Reweighting of exclusive mode distributions

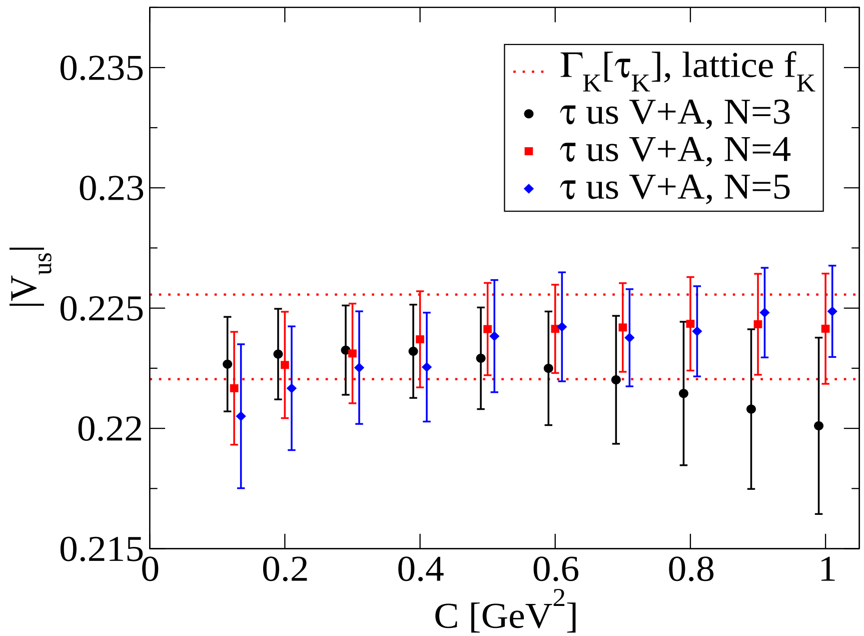
Left panel: un-re-weighted experimental data; right panel: $N = 4$, $C = 0.5\ GeV^2$ re-weighted version





Inclusive determinations

Inclusive $|V_{us}|$ (Lusiani CKM2018 BFs, including B_K)



INCLUSIVE ANALYSIS RESULTS SUMMARY

- Error optimized for $N = 4$, $C \simeq 0.7\ GeV^2$

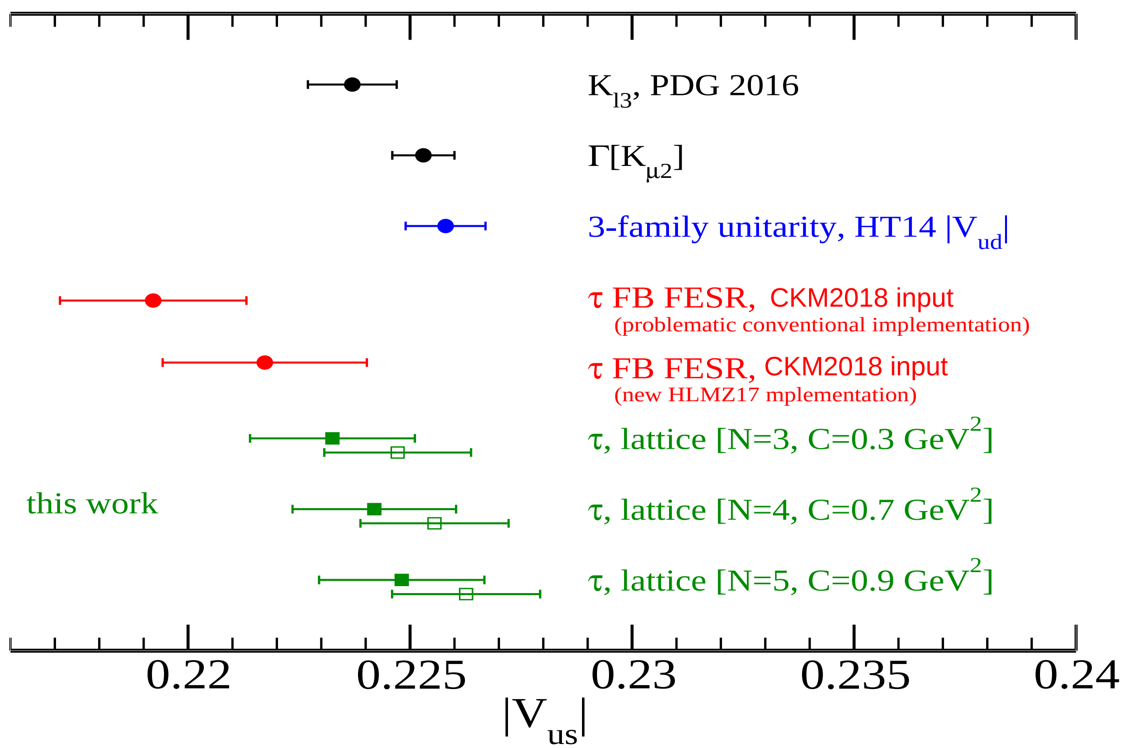
- Optimized $|V_{us}|$ results:

  - With $B[\tau \to K\nu_\tau]$ for K: $0.2240\,(13)_{exp}\,(13)_{th}$

  - With $\Gamma[K_{\mu 2}]$+SM for K: $0.2254\,(10)_{exp}\,(13)_{th}$

- Residual mode sum error becoming relevant ($K^-\pi^0\pi^0$ distribution desirable)

Comparison to results from other methods



Comments/Prospects

- Lattice analysis confirms larger $|V_{us}|$ from τ decay data [as per alternate HMLZ FB FESR implementation]

- Significant error reduction from lattice approach c.f. FB FESR determination employing same data

- Theory uncertainty under better control for lattice than for OPE

- Improved suppression of high-s, higher-error spectral contributions in lattice approach *without blowing up theory errors*

- **Lattice thus superior to FB FESR approach and should replace it going forward**

- Trend to $|V_{us}|$ fall-off for $N = 3$, larger C compatible with missing high-s, higher-multiplicity spectral strength (larger impact on FB FESR than lattice results)

- Theory (lattice) errors straightforwardly reducible through improved statistics

- Signifiant experimental error reduction from improved K, $K\pi$ τ BFs even without unit-normalized distribution improvements (e.g., Belle II)

SUMMARY

- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
  - **Conventional FB FESR implementation using only inclusive BFs no longer tenable**
  - Alternate, no-assumptions implementation: $|V_{us}|$ higher by ~ 0.0020, compatible with other determinations
  - Near-term improvements via us exclusive BFs
  - Highly favorable theoretical error situation
  - However, competitive $|V_{us}|$ needs improvements to old ALEPH higher-multiplicity, low-statistics data

- Advantage of new lattice-inclusive us $V+A$ τ approach
  - Theory:
    * Lattice in place of OPE; no us $J=0$ subtraction; improvement through increased statistics
    * *Parasitic on lattice a_μ effort (a major effort in the lattice community)*
  - Spectral integrals:
    * Theory errors still small for weights strongly suppressing higher multiplicity contributions
    * Strong K, $K\pi$ dominance of spectral integral
    * Experimental improvements currently possible through improved K BF, $K\pi$ BFs, distributions

BACKUP SLIDES

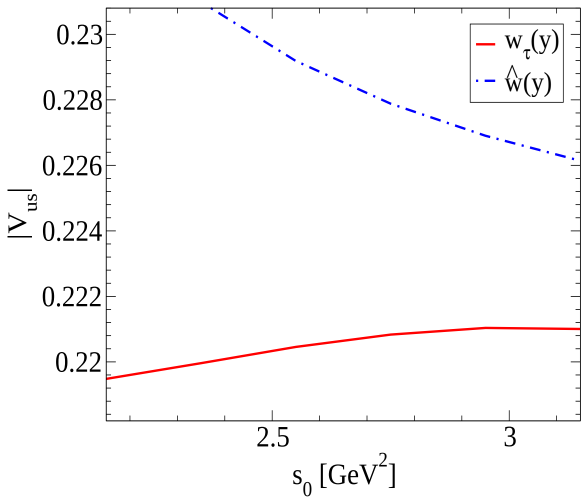
- Relative exclusive mode $R^w_{us:V+A}$ contributions

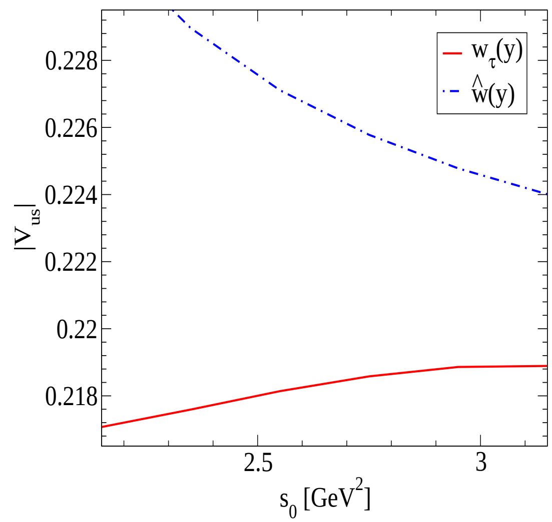
Wt	s_0 [GeV^2]	K	$K\pi$	$K\pi\pi$ (B-factory)	Other
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

- **"Other":** 1999 ALEPH data/MC, $\sim$ 25% error

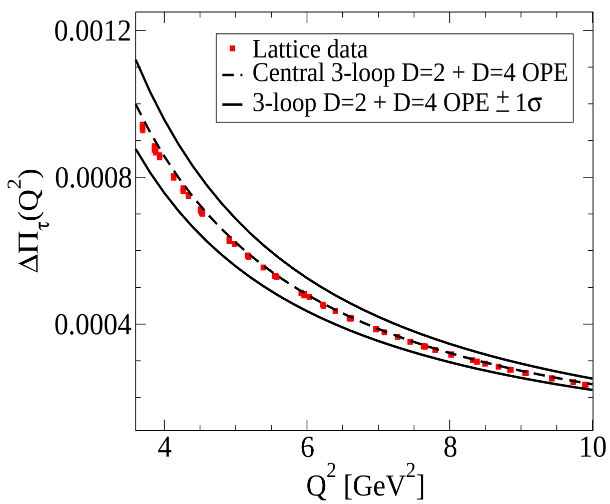
- **$\Rightarrow$ sub-0.5% $|V_{us}|$ (sub-% $R^w_{us;V+A}$ error) requires experimentally (much) more challenging higher-multiplicity mode improvements**

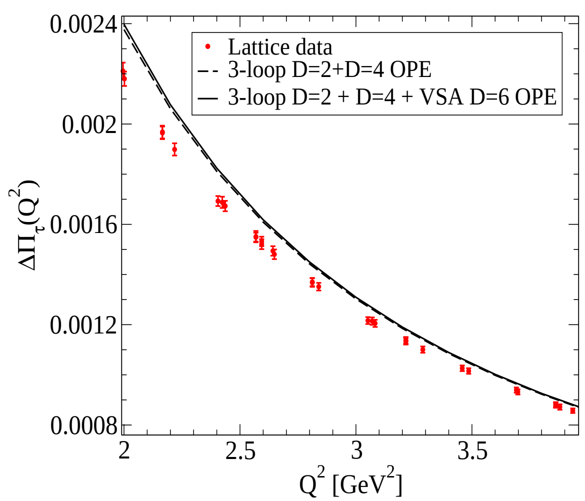
LHS: ACLP2013 $K\pi$ normalization; RHS: HFAG2016 $K\pi$ normalization





Lattice vs OPE results for $\Delta\Pi_\tau(Q^2)$





- Finite t behavior

  - Current-current two-point function

$$C^{\mu\nu}_{V/A}(t) = \sum_{\vec{x}} \langle J^{\nu}_{V/A}(\vec{x}, t)(J^{\mu}_{V/A}(0,0))^{\dagger} \rangle$$

  - $J = 0,\, 1$ components: $C^{(1)}_{V/A}(t) = \frac{1}{3}\sum_{k=x,y,z} C^{kk}_{V/A}(t)$, $C^{(0)}_{V/A}(t) = C^{tt}_{V/A}(t)$

  - $Q^2 = 0$-subtracted $J = 0,\, 1$ polarizations

    [Bernecker, Meyer Eur. Phys. J. A47 (2011) 47]

$$\Pi^{(J)}_{V/A}(Q^2) - \Pi^{(J)}_{V/A}(0) = \sum_{t} K(Q,t)\, C^{(J)}_{V/A}(t)$$

$$K(Q,t) = \frac{\cos \hat{Q}t - 1}{\hat{Q}^2} + \frac{1}{2}t^2$$

- Resulting $J = 0$, 1, V, A contributions to $\tilde{F}_{w_N}$

$$\tilde{F}^{(J)}_{V/A;w_N} = \lim_{t\to\infty} L^{(J)}_{V/A;w_N}(t)$$

where

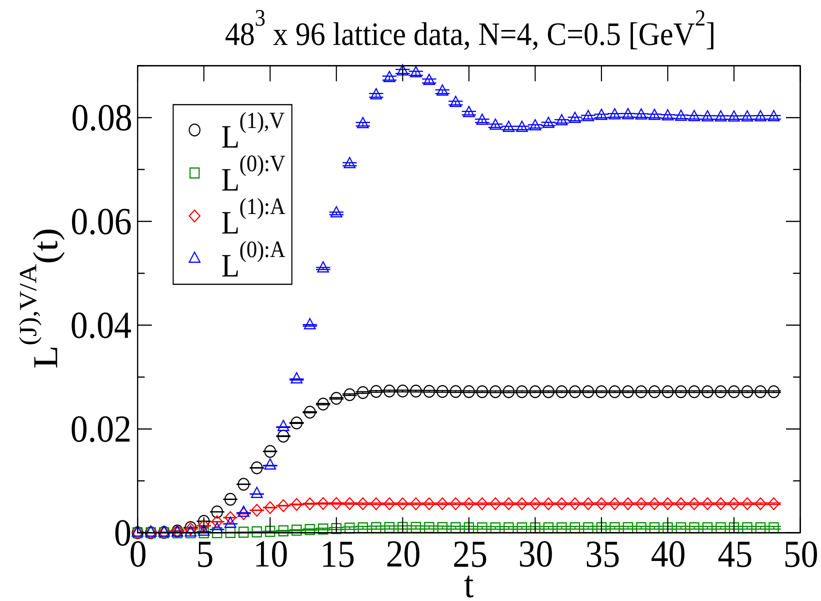
$$L^{(J)}_{V/A;\omega_N}(t) = \sum_{l=-t}^{t} w^{(J)}_N(l) C^{(J)}_{V/A}(l)$$

with

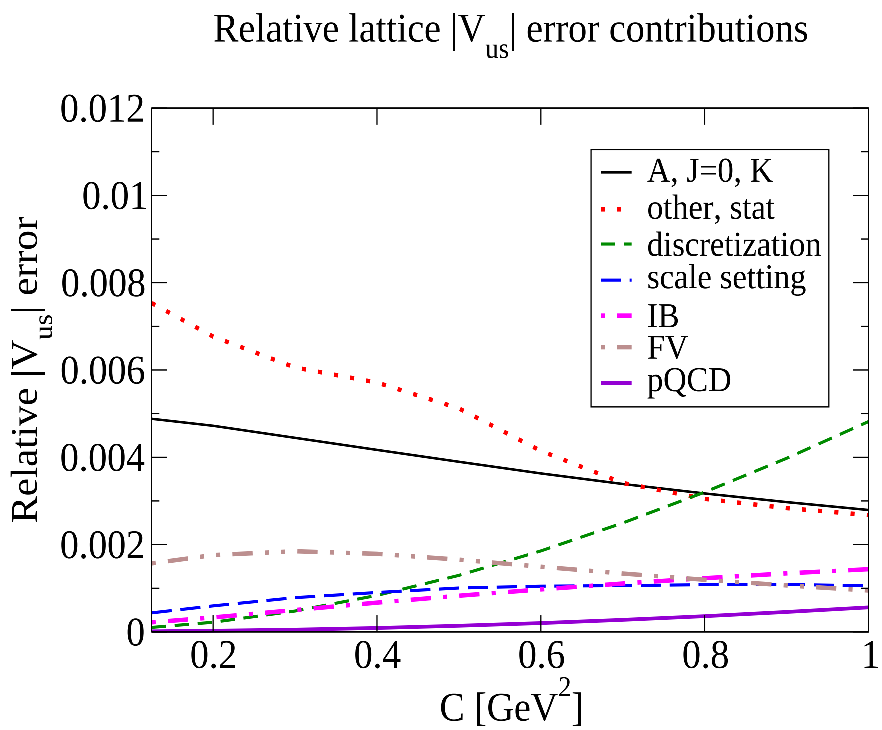
$$w^{(1)}_N = \sum_{k=1}^{N} K\left(\sqrt{Q_k^2}, t\right)\left(1 - \frac{2Q_k^2}{m_\tau^2}\right) \mathrm{Res}\left[w_N(s)\right]_{s=-Q_k^2}$$

$$w^{(0)}_N = \sum_{k=1}^{N} K\left(\sqrt{Q_k^2}, t\right) \mathrm{Res}\left[w_N(s)\right]_{s=-Q_k^2}$$

◦ Large t convergence behavior



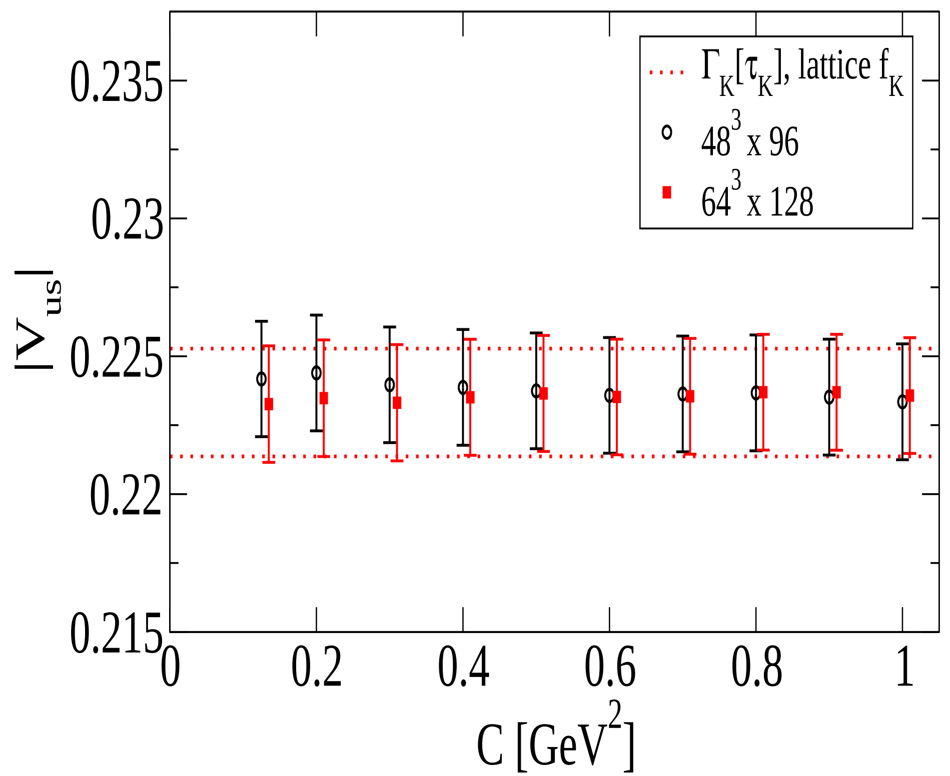
Lattice error breakdown vs. C, $N = 4$



Error budget (updated BF at Charm 2018 by A. Lusiani)

contribution		relative error (%)			
	[N, C[GeV^2]]	$[3, 0.3]$	$[3, 1]$	$[4, 0.7]$	[5, 0.9]
theory	f_K	0.37	0.20	0.34	0.36
	others, stat.	0.41	0.19	0.34	0.41
	discretization	0.10	0.80	0.25	0.27
	scale setting	0.09	0.08	0.11	0.11
	IB	0.10	0.21	0.11	0.10
	FV	0.10	0.04	0.13	0.18
	pQCD	0.05	0.26	0.03	0.01
	total	0.59	0.91	0.58	0.65
experiment	K	0.39	0.22	0.36	0.39
	$K\pi$	0.16	0.27	0.19	0.18
	$K^-\pi^+\pi^-$	0.06	0.16	0.06	0.05
	$\bar{K}^0\pi^-\pi^0$	0.03	0.09	0.03	0.03
	residual	0.40	1.34	0.41	0.27
	total	0.59	1.39	0.58	0.51
Combined	total	0.83	1.67	0.82	0.83

- Exp error reduces about 10%. Now theory and experimental errors are same for the preferred $N = 4$ value



Exclusive A, $J = 0$ channel determination