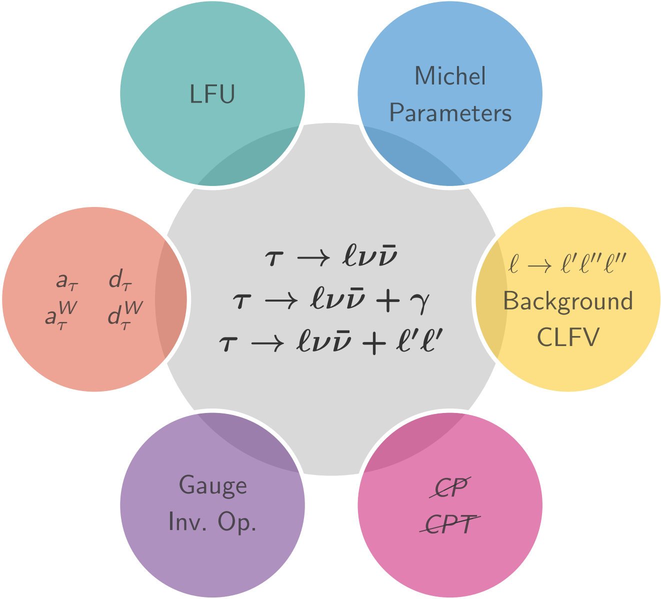
NLO prediction for the decays $\tau \to \ell\ell'\ell'\nu\bar{\nu}$ and $\mu \to eee\nu\bar{\nu}$

Matteo Fael

24 Sept. 2018 – Tau 2018 – Amsterdam

in collaboration with C. Greub arXiv:1611.03726
see also M. Pruna, A. Signer, Y. Ulrich arXiv:1611.03617

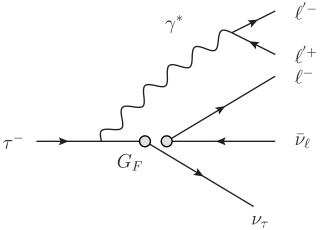
UNIVERSITÄT SIEGEN


LFU
Michel Parameters
a_τ d_τ
a_τ^W d_τ^W
$\tau \to \ell \nu \bar{\nu}$
$\tau \to \ell \nu \bar{\nu} + \gamma$
$\tau \to \ell \nu \bar{\nu} + \ell' \ell'$
$\ell \to \ell' \ell'' \ell''$
Background CLFV
Gauge Inv. Op.
$\not{CP}$
$\not{CPT}$

- $\tau \to \ell\nu\bar{\nu} + \ell'\ell'$ @ LO
  - D. Yu. Bardin, T. G. Istatkov, and G. Mitselmakher, Sov.J.Nucl.Phys. 15 (1972) 161
  - P. M. Fishbane and K. J. F. Gaemers, PRD 33 (1986) 159
  - R. M. Djilkibaev and R. V. Konoplich, PRD 79 (2009) 073004
  - Flores-Tlalpa, Lopez Castro, Roig JHEP 1604 (2016) 185
  - Arroyo-Urena, Diaz, Meza-Aldama, Tavares-Velasco, Int.J.Mod.Phys. A32 (2017) 1750195
- Analytic LO Branching Ratio in the $m_e \to 0$ limit:
  - van Ritbergen, Stuart, NPB 564 (2000) 343
- $\tau \to \ell\nu\bar{\nu} + \ell'\ell'$ @ NLO:
  Original papers focus only on muon's rare decay:
  - C. Greub & MF, JHEP 1701 (2017) 084.
  - M. Pruna, A. Signer, Y. Ulrich, Phys.Lett. B765 (2017) 280.

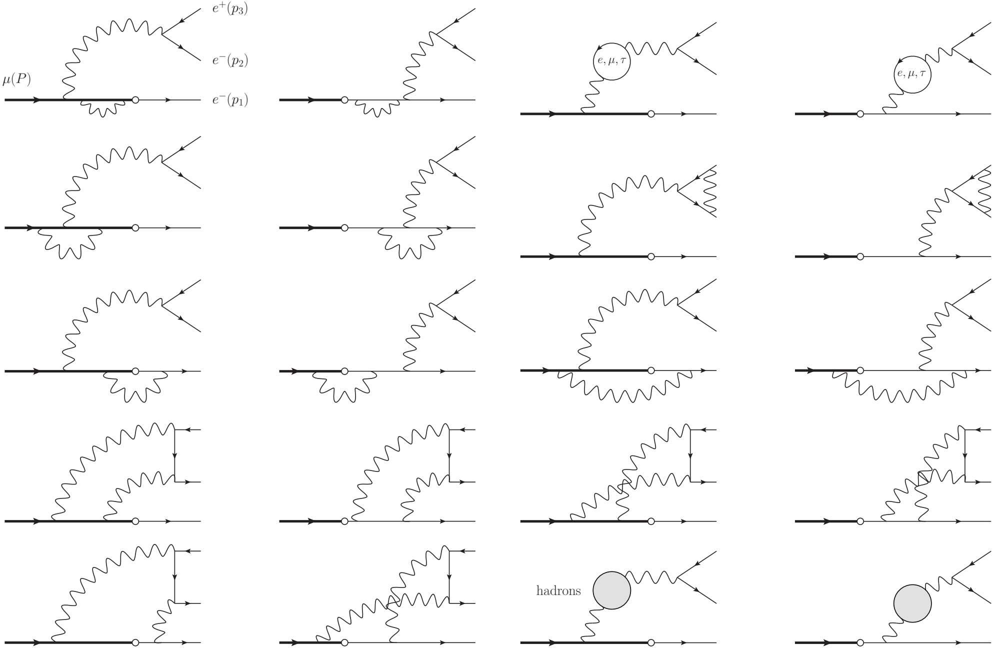
Technical Ingredients

$$\mathcal{L} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{QCD}} - \frac{4G_F}{\sqrt{2}} \left(\bar{\psi}_{\nu_\mu} \gamma^\mu P_L \psi_\mu\right) \left(\bar{\psi}_e \gamma_\mu P_L \psi_{\nu_e}\right) + \mathrm{h.c.}$$



- virtual: 22 diagrams + 2 had.
- real: 10 diagrams

(everything $\times 2$ if $\ell = \ell'$).



$e^+(p_3)$
$e^-(p_2)$
$\mu(P)$
$e^-(p_1)$
e, μ, τ
e, μ, τ
hadrons

The Montecarlo code:

- Full dependence on m_e, m_μ.
- Algebraic manipulation of tree-level and one-loop diagrams with `Form`
- `LoopTools` and `Collier` evaluates one-loop tensor coefficients.
  T. Hahn, M. Perez-Victoria, Comput.Phys.Commun. 118 (1999) 153;
  A. Denner, S. Dittmaier,L. Hofer, Comput.Phys.Commun. 212 (2017) 220.
- Very good numerical stability with `Collier` for $\tau \to ee e\nu\bar{\nu}$.
- $\Pi^{\mathrm{had}}(t)$ and $R^{\mathrm{had}}(z)$ provided by Jegerlehner's package `alphaQED`:
  `www-com.physik.hu-berlin.de/~fjeger/alphaQEDc17.tar.gz`

Real emission

- Phase space slicing:

$$\int d\phi_{n+1} |\mathcal{M}_{\text{real}}|^2 = \int_{k_\gamma^0 < \omega_0} d^3k \, d\phi_n \sum_{ij} \frac{p_i \cdot p_j}{k \cdot p_i k \cdot p_j} |\mathcal{M}_{\text{Born}}|^2 + \int_{k_\gamma^0 > \omega_0} d\phi_{n+1} |\mathcal{M}_{\text{real}}|^2$$

Real emission

- Phase space slicing:

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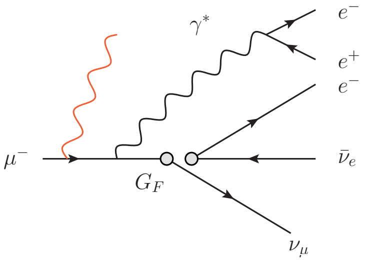
Numerically very challenging for $\tau \to eee\nu\bar{\nu}$ and $\tau \to \mu ee\nu\bar{\nu}$.

Real emission



- QED dipole subtraction:
  Catani, Seymour, Phys.Lett. B378 (1996) 287;
  S. Dittmaier, Nucl.Phys. B565 (2000) 69.

$$\int d\phi_{n+1} |\mathcal{M}_{\mathrm{real}}|^2 = \int d\phi_{n+1} \left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2 \right) + \int d\phi_n d^3k |\mathcal{M}_{\mathrm{sub}}|^2$$

where

$$|\mathcal{M}_{\mathrm{sub}}|^2 = \sum_{i \neq j} g_{ij}(p_i, p_j, k) |\mathcal{M}_{\mathrm{Born}}|^2$$

$$\tau \to \ell \nu \bar{\nu} \ell' \ell'$$

	$\mathcal{B}_{\mathrm{LO}}$	$\delta\mathcal{B}_{\mathrm{NLO,QED}}$	$\delta\mathcal{B}_{\mathrm{NLO,had}}$	$\delta\mathcal{B}/\mathcal{B}$
$\tau \to ee e\nu\bar{\nu}$	$4.2488\,(4) \times 10^{-5}$	$-4.2\,(1) \times 10^{-8}$	-1.0×10^{-9}	-0.1%
$\tau \to \mu e e\nu\bar{\nu}$	$1.989\,(1) \times 10^{-5}$	$4.4\,(1) \times 10^{-8}$	-6.6×10^{-10}	0.2%
$\tau \to e\mu\mu\nu\bar{\nu}$	$1.2513\,(6) \times 10^{-7}$	$2.70\,(1) \times 10^{-9}$	-3.6×10^{-10}	1.8%
$\tau \to \mu\mu\mu\nu\bar{\nu}$	$1.1837\,(1) \times 10^{-7}$	$2.276\,(2) \times 10^{-9}$	-3.5×10^{-10}	1.6%
$\mu \to eee\nu\bar{\nu}$	$3.6054\,(1) \times 10^{-5}$	$-6.69\,(5) \times 10^{-8}$	-1.8×10^{-11}	0.2%

Tau lifetime uncertainty:

$$\delta\tau_\tau/\tau_\tau = 1.7 \times 10^{-3}$$

Shift of the fine structure constant:

$$\Delta\alpha(4m_\mu^2) = 6 \times 10^{-3}$$

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Shift of the fine structure constant:

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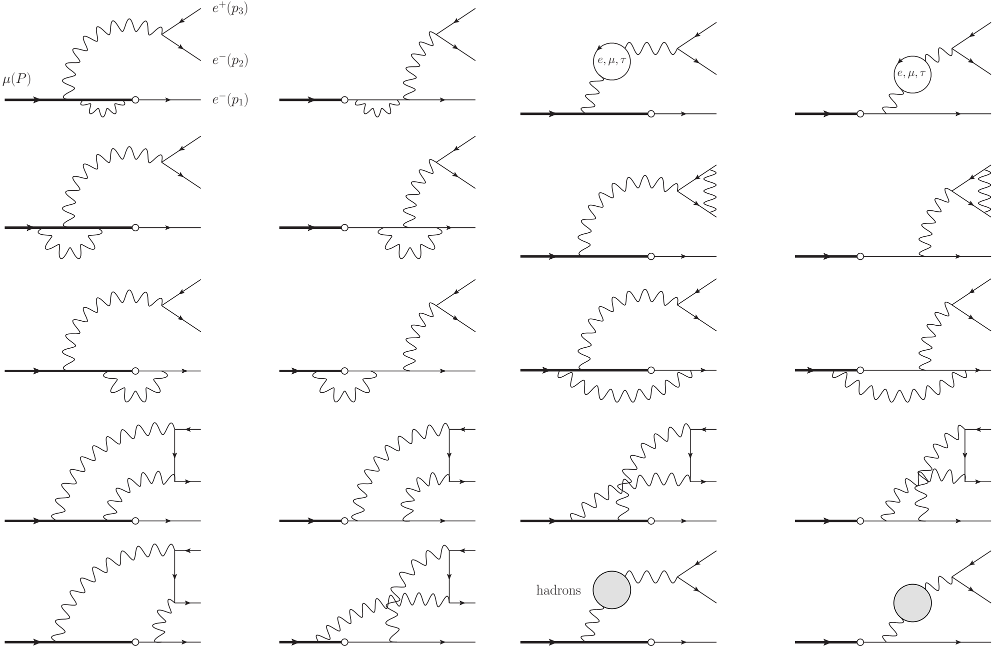
$\tau \to \ell\ell'\ell'\nu\bar{\nu}$ @ Belle

from Yifan Jin's talk:

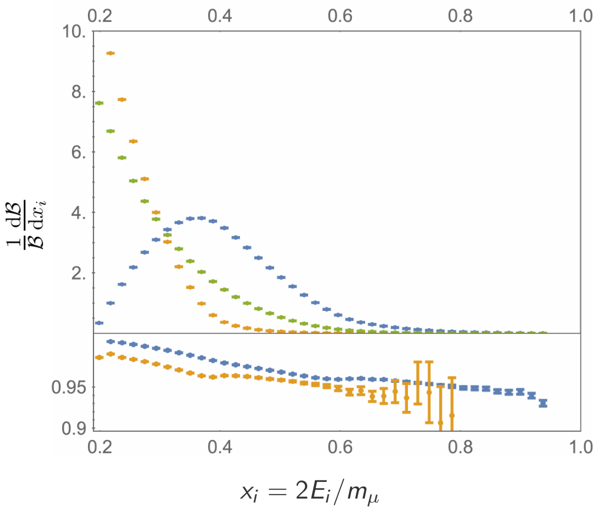
Modes:	$\tau \to e\, \nu_e\, \nu_\tau\, e^+ e^-$	$\tau \to \mu\, \nu_\mu \nu_\tau\, e^+ e^-$	$\tau \to e\, \nu_e\, \nu_\tau\, \mu^+ \mu^-$	$\tau \to \mu\, \nu_\mu\, \nu_\tau\, \mu^+ \mu^-$
Detection efficiency	(1.790 ± 0.001) %	(1.090 ± 0.003) %	(3.561 ± 0.006) %	(1.674 ± 0.004) %
Main BKG	$e\, \nu_e \nu_\tau \gamma(\to e\, e)$; $\pi\, \pi^0\, \nu_\tau$	$\mu\, \nu_\mu\, \nu_\tau\, \gamma(\to e\, e)$; $\pi\, \pi^0\, \nu_\tau$	$\pi\, \pi^0\, \nu_\tau$	$\pi\, \pi^+ \pi^- \nu_\tau$
Expected number of signal	1300	430	8	4
Purity of signal region	47%	50%	37%	16%

We expect to observe the first two modes with Belle data and to set upper limits on the rest two modes.

see also J. Sasaki (Belle) J.Phys.Conf.Ser. 912 (2017) 012002


$e^+(p_3)$
$e^-(p_2)$
$\mu(P)$
$e^-(p_1)$
e, μ, τ
e, μ, τ
hadrons

$\mu^+ \to e^+ e^+ e^- \nu \bar{\nu}$



Mu3e setup

$E_i > 10\text{MeV}$

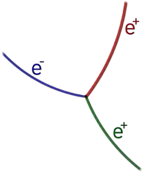
$|\cos\theta_i| < 0.8$

- hard e^+
- soft e^+
- e^-

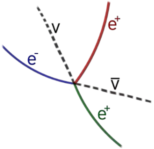
M. Pruna, A. Signer, Y. Ulrich, Phys.Lett. B772 (2017) 452

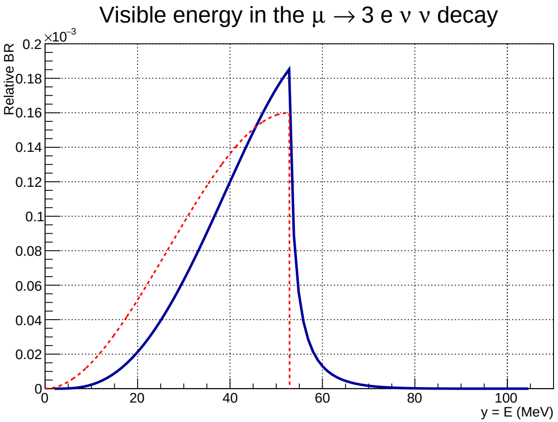
Searching for CLFV with Mu3e

Signal: $\mu \to eee$



Background: $\mu \to eee\nu\bar{\nu}$

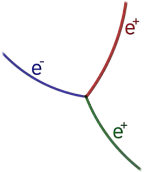




Calibbi, Signorelli, Riv.Nuovo Cim. 41 (2018) 1

Searching for CLFV with Mu3e

Signal: $\mu \to eee$



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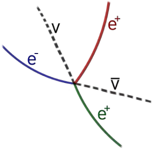
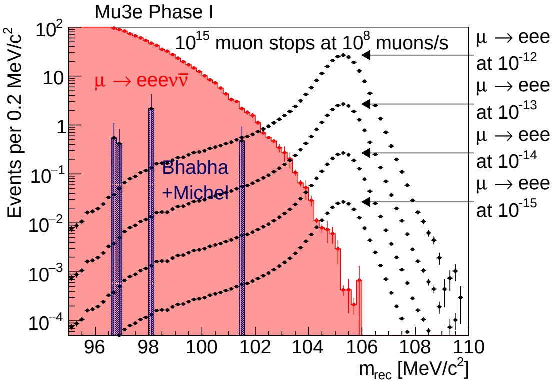
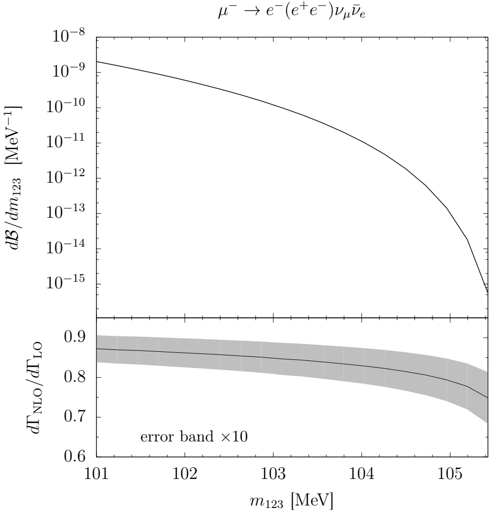
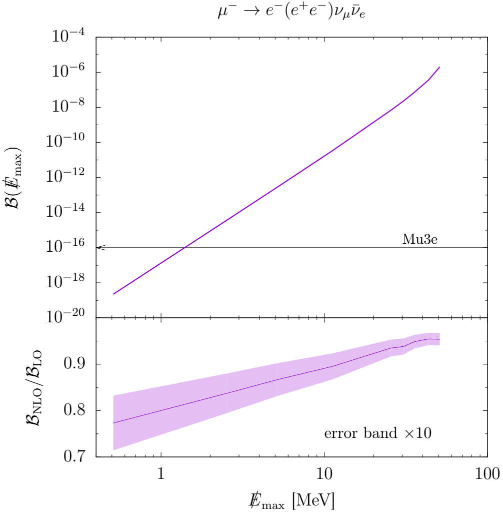




A. Perrevoort (Mu3e), 1802.09851 [physics.ins-det]

see also talk by A. Bravar and poster by A. K. Perrevoort



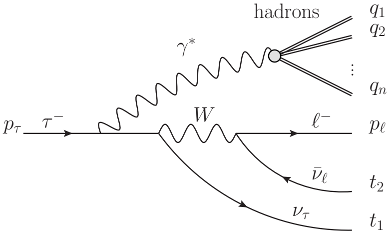


C. Greub & MF, JHEP 1701 (2017) 084.

$\gamma^* \rightarrow$ **hadrons?**

$\tau \to \ell\nu\bar{\nu}+$ had.

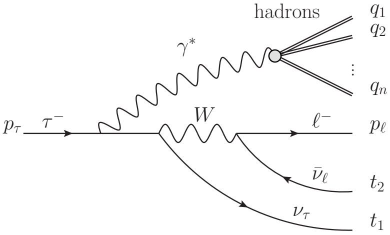
- Phase space factorization
- Optical theorem



$$\frac{d\Gamma}{dq^2} = \frac{\alpha}{3\pi}\frac{R_{\mathrm{had}}(q^2)}{q^2}\Gamma_{\tau\to\ell\nu\bar{\nu}\gamma^*}(q^2)$$

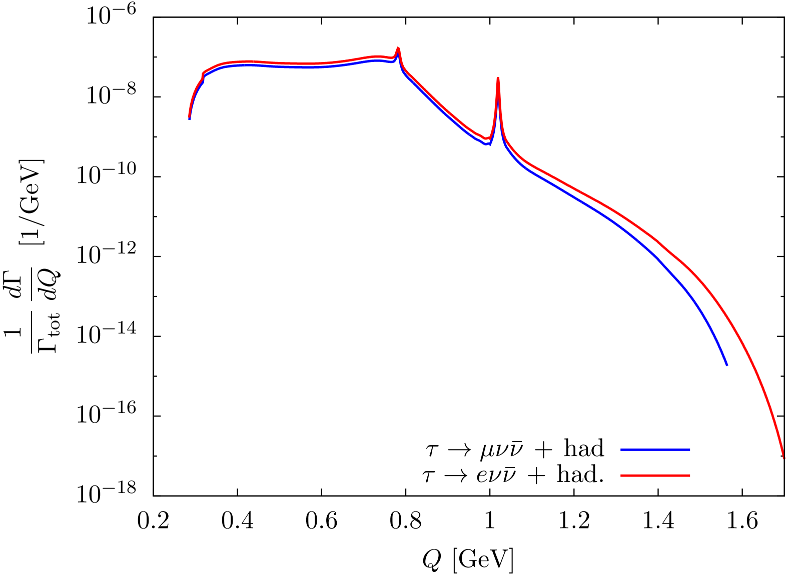
$\tau \to \ell\nu\bar{\nu}+$ had.

- Phase space factorization
- Optical theorem



$$\frac{d\Gamma}{dq^2} = \frac{\alpha}{3\pi}\frac{R_{\mathrm{had}}(q^2)}{q^2}\Gamma_{\tau\to\ell\nu\bar{\nu}\gamma^*}(q^2)$$

$$\mathcal{B}(\tau \to e\nu\bar{\nu} + \text{had}) = 2.25 \times 10^{-8},$$
$$\mathcal{B}(\tau \to \mu\nu\bar{\nu} + \text{had}) = 1.80 \times 10^{-8}.$$



10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Q [GeV]
$\frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dQ}$ [1/GeV]
$\tau \to \mu\nu\bar{\nu}$ + had
$\tau \to e\nu\bar{\nu}$ + had.

Conclusions

- Two independent calculations at NLO are available for $\tau \to \ell\ell'\ell'\nu\bar{\nu}$ and $\mu \to ee e\nu\bar{\nu}$.
- Corrections to $\mathcal{B}(\tau \to \ell\nu\bar{\nu}\ell'\ell')$ are of order 0.1%, for $\ell' = e$, and 1%, for $\ell' = \mu$.
- Locally in the phase space corrections can be $O(1-5\%)$.
- Detector acceptance or stringent cuts can enhance rad. corrections at the 10 % level.
- The decays $\tau \to \ell\nu\bar{\nu}+$ had. are within the reach of Belle-II.

Backup

$\mathcal{B}$	Dicus&Vega 1994	Volobouev (CLEO) 1996	Flores-Tlalpal 2016	Diaz 2017	Fael 2018		PSU		
					LO	corr	LO	corr	rel.
$\tau \to e\ e^+e^-$	$4.15(6) \cdot 10^{-5}$	$4.457(6) \cdot 10^{-5}$	$4.21(1) \cdot 10^{-5}$	$4.22(2) \cdot 10^{-5}$	$4.2488(4) \cdot 10^{-5}$	$-4.2(1) \cdot 10^{-8}$	$4.2489(1) \cdot 10^{-5}$	$-4.0(2) \cdot 10^{-8}$	-0.000944281
$\tau \to \mu\ e^+e^-$	$1.97(2) \cdot 10^{-5}$	$2.089(3) \cdot 10^{-5}$	$1.984(4) \cdot 10^{-5}$	$1.987(3) \cdot 10^{-5}$	$1.989(1) \cdot 10^{-5}$	$4.4(1) \cdot 10^{-8}$	$1.9879(2) \cdot 10^{-5}$	$4.43(5) \cdot 10^{-8}$	0.00222725
$\tau \to e\ \mu^+\mu^-$	$1.257(3) \cdot 10^{-7}$	$1.347(2) \cdot 10^{-7}$	$1.247(1) \cdot 10^{-7}$	$1.246(2) \cdot 10^{-7}$	$1.2513(6) \cdot 10^{-7}$	$2.70(1) \cdot 10^{-9}$	$1.2513(2) \cdot 10^{-7}$	$2.708(2) \cdot 10^{-9}$	0.0216386
$\tau \to \mu\ \mu^+\mu^-$	$1.190(2) \cdot 10^{-7}$	$1.276(5) \cdot 10^{-7}$	$1.183(1) \cdot 10^{-7}$	$1.184(1) \cdot 10^{-7}$	$1.1837(1) \cdot 10^{-7}$	$2.276(2) \cdot 10^{-9}$	$1.1838(1) \cdot 10^{-7}$	$2.276(1) \cdot 10^{-9}$	0.0192223

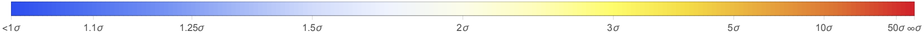
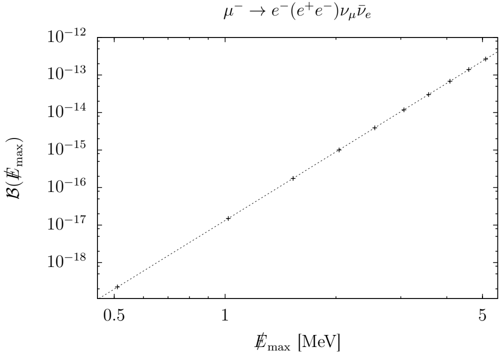


table by Y. Ulrich

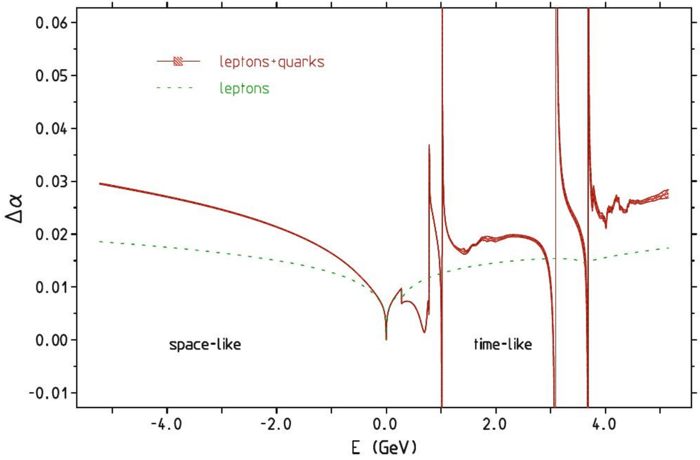
	MF,Greub	Pruna, Signer, Ulrich
Full mass dependence	✓	✓
Decaying μ	unpolarized	polarized
One-loop	LoopTools, Collier	GoSam
IR	PS slicing, dipoles	FKS
Phase space	analytic integration νs PS	fully differential
Had. corrections	✓	$\times$



<u>Fit:</u>

$$\mathcal{B}(\not{E}_{\rm max}) = \kappa \left(\frac{\not{E}_{\rm max}}{m_e}\right)^{\gamma}$$

- $\kappa_{\rm NLO} = 2.217\,(2) \times 10^{-19}$
- $\gamma_{\rm NLO} = 6.0768\,(4)$



F. Jegerlehner, The anomalous magnetic moment of the muon (2nd Ed.), Springer.