Neutrino-less double beta decay — theory

Vincenzo Cirigliano
Los Alamos National Laboratory
Outline

• Neutrino mass and Lepton Number Violation
  • LNV probes: $0\nu\beta\beta$, meson & lepton decays, LHC

• EFT framework for LNV and $0\nu\beta\beta$ physics reach:
  • $0\nu\beta\beta$ from light Majorana $\nu$ exchange
  • $0\nu\beta\beta$ from (multi)TeV-scale dynamics & collider connections
Neutrino mass and new physics

- Neutrino mass requires introducing new degrees of freedom

Dirac mass:

\[ m_D \bar{\psi}_L \psi_R + h.c. \]

Majorana mass:

\[ m_M \psi_L^T C \psi_L + h.c. \]

- Violates \( L_e, \mu, \tau \), conserves \( L \)

- Violates \( L_e, \mu, \tau \) and \( L \) (\( \Delta L = 2 \))
Neutrino mass and new physics

- Neutrino mass requires introducing **new degrees of freedom**

\[ \mathcal{L}_\nu \text{SM} = \mathcal{L}_\text{SM} + \mathcal{L}_\nu - \text{mass} + \ldots \]

- Key question:
  - Are neutrino Majorana particles? Or equivalently:
  - Is **Lepton Number** a good symmetry of the **new dynamics**?

- Implications:
  - Model building
  - Generation of baryon asymmetry via “Leptogenesis” (need LNV

Fukujita-Yanagida 1987
Thought experiment (B. Kayser): generate ν beam from π⁺ → μ⁺ν and check whether it produces μ⁺ on a target downstream:

- A Dirac neutrino in either helicity state won’t do that
- A Majorana neutrino with helicity=+1 (R-handed) will do that

\[ \pi^+ \rightarrow \mu^+ \nu \]
• **Thought experiment** (B. Kayser): generate $\nu$ beam from $\pi^+ \rightarrow \mu^+ \nu$ and check whether it produces $\mu^+$ on a target downstream:
  - A Dirac neutrino in either helicity state won’t do that
  - A Majorana neutrino with helicity=$+1$ (R-handed) will do that

![Diagram showing the reaction $\pi^+ \rightarrow \mu^+ \nu$]

• But fraction of R-helicity $\nu$’s produced in $\pi^+ \rightarrow \mu^+ \nu$ is $\sim (m_\nu/E_\nu)^2 < 10^{-16}!!$

---

**Neutrinoless processes are our best bet!**
Neutrinoless probes of $\Delta L=2$ dynamics

Minimal setup: Majorana mass insertion

Heavier neutrinos or other particles can also mediate LNV. Six-fermion (dim9) operator $\sim 1/\Lambda^5$ at low energy
Neutrinoless probes of $\Delta L=2$ dynamics

Minimal setup: Majorana mass insertion

Observation of any of these processes (regardless of the mechanism) would demonstrate that neutrinos are Majorana fermions

Shechter-Valle 1982
Neutrinoless probes of $\Delta L=2$ dynamics

- Neutrinoless double beta decay

$$(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$$

$T_{1/2} > \# 10^{25\text{yr}}$

See talk by L. Cardani

Observable in certain even-even nuclei for which single beta decay is energetically forbidden
Neutrinoless probes of $\Delta L=2$ dynamics

- Neutrinoless double beta decay
  \[(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-\]
  \[T_{1/2} > \#10^{25}\text{yr}\]

- Decays of mesons or leptons
  See talk by A. Teixeira
  \[K^+ \rightarrow \pi^- \ell_1^+ \ell_2^+\]
  \[l_{1,2} = e, \mu\]
  \[\text{BRs} < \#10^{-10}\]
  \[B^+ \rightarrow h^- \ell_1^+ \ell_2^+\]
  \[h = \pi, K\]
  \[l_{1,2} = e, \mu\]
  \[\text{BR} (\pi^- \mu^+ \mu^+) < \#10^{-9}\]
  \[\tau^- \rightarrow l^+ h_1^- h_2^-\]
  \[l = e, \mu\]
  \[h_{1,2} = \pi, K\]
  \[\text{BRs} < \#10^{-8}\]
  \[\mu^- \rightarrow e^+\text{ conversion BR at }10^{-12}\text{ level}\]
Neutrinoless probes of $\Delta L=2$ dynamics

- Neutrinoless double beta decay
  \[(N, Z) \to (N - 2, Z + 2) + e^- + e^-\]
  \[T_{1/2} > \# 10^{25}\text{yr}\]

- Decays of mesons or leptons
  \[K^+ \to \pi^- \ell_1^+ \ell_2^+\]
  \[\ell_{1,2} = e, \mu\]
  \[BRs < \# 10^{-10}\]
  \[B^+ \to h^- \ell_1^+ \ell_2^+\]
  \[h = \pi, K\]
  \[\ell_{1,2} = e, \mu\]
  \[BR (\pi^- \mu^+ \mu^+) < \# 10^{-9}\]
  \[\tau^- \to \ell^+ h_1^- h_2^-\]
  \[\ell = e, \mu\]
  \[h_{1,2} = \pi, K\]
  \[BRs < \# 10^{-8}\]

- Same sign di-lepton production at LHC
  \[pp \to \ell\ell + 2\text{ jets}\]
  \[\ell = e, \mu, \tau\]
Neutrinoless probes of $\Delta L=2$ dynamics

- Neutrinoless double beta decay
  - Currently probing $m_{ee}=m_{\beta\beta} \sim 0.1$ eV and $\Lambda_{6\text{-fermion}} \sim$ few TeV

- Decays of mesons or leptons
  - Not competitive in high-scale LNV models: probing $m_{ab} \sim$ eV and $\Lambda_{6\text{-fermion}} \sim$ TeV requires BRs $\sim 10^{-30}$. (Avogadro number wins!)
  - Competitive for low-scale LNV models

- Same sign di-lepton production at LHC
  - Competitive probe of LNV mechanism at the TeV scale

A. Teixeira et al. 1712.03984
• Ton-scale $0\nu\beta\beta$ searches ($T_{1/2} > 10^{27-28}$ yr) probe at unprecedented levels LNV from a variety of mechanisms

Impact of $0\nu\beta\beta$ searches most efficiently analyzed in EFT framework
EFT framework

BSM dynamics

Example:
Left-Right Symmetric Model

SM-EFT

- $\Delta L=2$ operators appear only at odd dimension
- Insertions of small dimensionless (Yukawa) coupling can make $\text{dim}=5,7,9$ operators equally important for $\Lambda \sim \text{TeV}$
- $\nu_R$ with mass $< M_W$ can be included in the framework

$\nu_{ew}, M_W$

$\Lambda_x$ ($\sim \text{GeV}$)

$k_F, m_\pi$

$E$

$\Lambda (> \text{TeV})$

$\nu_{ew}, M_W$

$\Lambda_x$ ($\sim \text{GeV}$)

$k_F, m_\pi$
The EFT framework allows for the study of BSM dynamics in various energy scales. The figure illustrates the hierarchy of scales:

- BSM dynamics (E > TeV)
- SM-EFT
- SM-EFT' (dim5, dim7, dim9)
- Left-Right Symmetric Model (Example)

The EFT framework is particularly useful for understanding hadronic matrix elements for processes like $n p \to n p$, $e^- e^-$, and $d u \to d u$, where $L=2$ and the interactions are described in terms of currents and potentials at different energy scales.
EFT framework

\[ \Lambda (> \text{TeV}) \]

\[ v_{\text{ew}}, M_W \]

\[ \Lambda_x (~\text{GeV}) \]

\[ k_F, m_\pi \]

\[ \mathbf{BSM \ dynamics} \]

\[ \text{SM-EFT} \]

\[ \text{SM-EFT}' \]

\[ \text{Chiral EFT (N, \pi, \ldots)} \]

\[ \text{Many body QM} \]

\[ \text{Nuclear potential} \]

Example:
Left-Right Symmetric Model
EFT framework

$E$ 

$\Lambda$ (> TeV) 

$\Lambda_x$ (~GeV) 

$k_F, m_\pi$ 

BSM dynamics

Example: Left-Right Symmetric Model

$v_{ew}, M_W$

Nuclear potential

$76 Ge$ $V_{I=2}$ $76 Se$

SM-EFT

SM-EFT'

Chiral EFT ($N, \pi, \ldots$)

Many body QM
EFT framework

$E > \text{TeV}$

$\Lambda_{\nu, M_W}$

$\Lambda_{\chi} \approx \text{GeV}$

$k_F, m_\pi$

Chain of EFTs + lattice QCD & many-body methods

$T_{1/2} \sim (m_W/\Lambda)^A (\Lambda_{\chi}/m_W)^B (k_F/\Lambda_{\chi})^C$
$0\nu\beta\beta$ from light Majorana neutrino exchange (dim-5 operator)
$0\nu\beta\beta$ from light $\nu_M$ exchange

Decay amplitude

$$A \propto m_{\beta\beta} \langle f | \sum_{a,b} V^{(a,b)}_\nu | i \rangle$$

$$m_{\beta\beta} = \sum U_{ei}^2 m_i$$

$$V^{(a,b)}_\nu = \tau^{+,a} \tau^{+,b} \frac{1}{q^2} \left( J_V^{(a)}(q) J_V^{(b)}(-q) + J_A^{(a)}(q) J_A^{(b)}(-q) \right)$$

$$J_V \sim 1$$

$$J_A \sim g_A \sigma$$

In this case $0\nu\beta\beta$ is a direct probe of $\nu$ mass and mixing: $\Gamma \propto |M_{0\nu}|^2 (m_{\beta\beta})^2$
**m_{\beta\beta} phenomenology**

- Strong correlation of 0νββ with oscillation parameters: \( \Gamma \propto (m_{\beta\beta})^2 \)

\[
\langle m_{\beta\beta} \rangle^2 = | \sum U_{ei}^2 m_{\nu i} |^2
\]

See talk by L. Cardani

- Assuming current range for matrix elements, discovery **possible** for inverted spectrum or \( m_{\text{lightest}} > 50 \text{ meV} \) (regardless of mass ordering)
**$m_{\beta\beta}$ phenomenology**

- Strong correlation of $0\nu\beta\beta$ with oscillation parameters: $\Gamma \propto (m_{\beta\beta})^2$

$$\langle m_{\beta\beta} \rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$

See talk by L. Cardani

Dark bands: unknown phases

Light bands: uncertainty from oscillation parameters (90% CL)

Assume range for nuclear matrix elements from different many-body methods

$$(T_{1/2}^{0\nu})^{-1} = g_A^4 G_{01} |M_{0\nu}|^2 \frac{m_{\beta\beta}}{m_e^2}$$
Room for theory improvement?

• Steps towards controllable uncertainties in matrix elements:

  • Use chiral EFT as guiding principle

  • Use first-principles results in light nuclei (A=12) as a benchmark

    S. Pastore et al., 1710.05026

  • First principles nuclear structure calculations in sight for $^{48}$Ca and $^{76}$Ca, with QCD-rooted chiral potentials

    G. Hagen, T. Papenbrock et al., to appear
Light $\nu_M$ exchange in chiral EFT

- Leading order (LO) term in $Q/\Lambda_X$ ($Q \sim k_F \sim m_{\pi}$): tree-level $\nu_M$ exchange

- Renormalization of $nn \rightarrow ppee$ amplitude in presence of LO strong potential requires a LO short-range coupling $g_{\nu} \sim 1/F_{\pi}^2 \sim 1/k_F^2$!
Estimating finite part of $g_\nu$

1) Match $\chi$EFT & lattice QCD calculation of hadronic amplitude $nn \to pp$

$$S_{\Delta L=2}^{\text{eff}} = \frac{i8G_F^2V_{ud}^2m_{\beta\beta}}{2!} \int d^4x \bar{e}_L(x)e^c_L(x) \int d^4y \, S(x-y) \, T\left(\bar{u}_L\gamma_\mu d_L(x) \bar{u}_L\gamma_\mu d_L(y)\right)g^{\mu\nu}$$

Scalar massless propagator (remnant of $\nu$ propagator)
Estimating finite part of $g_\nu$

1) Match $\chi$EFT & lattice QCD calculation of hadronic amplitude $nn \rightarrow pp$

$$S^{\Delta L=2}_{\text{eff}} = \frac{i8G_F^2V_{ud}^2m_{\beta\beta}}{2!} \int d^4x \bar{e}_L(x)e^c_L(x) \int d^4y S(x-y) T\left(\bar{u}_L\gamma_\mu d_L(x) \bar{u}_L\gamma_\mu d_L(y)\right)g^{\mu\nu}$$

Scalar massless propagator (remnant of $\nu$ propagator)

$$ (J^+ \times J^+) \ \text{vs} \ \ (J_{\text{EM}} \times J_{\text{EM}}) \ I=2 $$

2) Chiral symmetry relates $g_\nu$ to one of two $I=2$ EM LECs
1) Match χEFT & lattice QCD calculation of hadronic amplitude \( nn \rightarrow pp \)

\[
S_{\Delta L=2}^{\text{eff}} = \frac{i8G_F^2V_{ud}^2m_{\beta\beta}}{2!} \int d^4 x \bar{e}_L(x) e_L^c(x) \int d^4 y S(x-y) T \left( \bar{u}_L \gamma_\mu d_L(x) \bar{u}_L \gamma_\mu d_L(y) \right) g^{\mu\nu}
\]

Scalar massless propagator (remnant of \( \nu \) propagator)

\((J^+ \times J^+)\) vs \((J^{\text{EM}} \times J^{\text{EM}})\) \( I=2 \)

2) Chiral symmetry relates \( g_\nu \) to one of two \( I=2 \) EM LECs

- Only one combination of the EM constants fixed by NN scattering
- Using this as rough estimate of \( g_\nu \), we get \( O(50\%) \) shift for the matrix element in light nuclei \((A=12)\)
- Strong motivation to pursue lattice QCD calculation
$0\nu\beta\beta$ from higher-dimensional operators
$0\nu\beta\beta$ from $\mathcal{L}_{\Delta L=2}^{(7)}$

Long range effect:
\(\nu\) exchange \textit{without} mass insertion

LNV vertex

- Hadronic input in good shape: isovector nucleon charges \((V,A,S,P,T)\)
- Nuclear m.e. related to the one needed for light \(\nu_M\) exchange

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Horoi and Neacsu, 1706.05391
Pass, Hirsch, Klapdor-Kleingrothaus, Kovalenko 1999
Doi, Kotani, Takasugi 1985
$0\nu\beta\beta$ from $\mathcal{L}^{(9)}_{\Delta L=2}$

Pion-range effects

Short-range effects

$V_{I=2} \supset (c_{\pi\pi} V_{\pi\pi} + c_{\pi N} V_{\pi N} + c_{NN} V_{NN})$

$c_\alpha \sim$ short-distance coupling (model-dep.) $\times$ hadronic matrix element
\(0\nu\beta\beta\) from \(\mathcal{L}_{\Delta L=2}^{(9)}\)

- Pion-range effects
- Short-range effects

- Relative importance of \(V_{\pi\pi, \pi N, NN}\) depends on \(O_i\)’s chiral properties
- Non-perturbative renormalization \(\rightarrow\) \(V_{\pi\pi}\) and \(V_{NN}\) are both leading
- \(<\pi^+|O_i|\pi^->\) known from lattice QCD
- \(<pp|O_i|nn>\) not yet known from LQCD (only factorization model)
- Nuclear matrix elements related to the ones for light \(V_M\) exchange
What scales are being probed?

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1806.02780

**Dim 7 in SM-EFT**

\[(v/\Lambda)^3\]

**Dim 9 in SM-EFT**

\[(v/\Lambda)^5\]

Bounds reflect dependence on $\Lambda_X/\Lambda$ and $Q/\Lambda_X$
Dim-7 phenomenology (I)

- Dim-5 ($m_{\beta\beta}$) + Dim-7 operator (leptoquark-induced)

- Same leptonic structure as in $\nu_M$ exchange: can cancel $m_{\beta\beta}$ !!

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Dim-7 phenomenology (I)

- Dim-5 ($m_{\beta\beta}$) + Dim-7 operator (leptoquark-induced)

$$m_{\beta\beta}^{(\text{eff})} = \frac{m_e}{g_{A,M}^2} \left( \frac{T_{1/2}^{0,\nu}}{G_{01}} \right)^{-1/2}$$

- Same leptonic structure as in $\nu_M$ exchange: can cancel $m_{\beta\beta}$!!

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Dim-7 phenomenology (2)

• Dim-5 ($m_{\beta\beta}$) + Dim-7 operator (LRSM-induced)

• Different leptonic structure from light $v_M$ exchange: no cancellation

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Dim-7 phenomenology (2)

- Dim-5 ($m_{\beta\beta}$) + Dim-7 operator (LRSM-induced)

$$m_{\beta\beta}^{(\text{eff})} = \frac{m_e}{g_A^2 M_\nu} \left( \frac{T_{1/2}^{0\nu}}{G_{01}} \right)^{-1/2}$$

**Inverted Hierarchy**

**Normal Hierarchy**

$$\Lambda = 40 \, \text{TeV}$$

- Different leptonic structure from light $\nu_M$ exchange: no cancellation

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Dim-7 phenomenology (2)

- Dim-5 \( (m_{\beta\beta}) \) + Dim-7 operator (LRSM-induced)

- In this case, electron \( \theta \) and \( E \) distributions distinguish dim5 and dim7

\[
\Lambda = 40 \text{ TeV}
\]

\[
|m_{\beta\beta}| = 0.05 \text{ eV}, \quad C_{\text{LeudH}} = e^{i\alpha} / \Lambda^3
\]

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos(\theta)}
\]

\[
\frac{m_e}{\Gamma} \frac{d\Gamma}{dE_1}
\]

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390
Simplified RPV-SUSY-like model

• Sensitivity study: $0\nu\beta\beta$ vs LHC

Peng, Ramsey-Musolf, Winslow, 1508.0444

Plot assumes 30% uncertainty in the nuclear & hadronic matrix elements and VSA for $\pi\pi$ matrix element ($\sim 2 \times$ lattice result)

Dim-9 $\Delta L=2$ six-fermion operator at low energy
Left-Right symmetric model

- Generates ops. at dim-5 ($m_{\beta\beta}$) + dim-7 & dim-9

![Diagram of Left-Right symmetric model with chiral enhancement for ($\xi \neq 0$)]

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1806.02780
Left-Right symmetric model

- Generates ops. at dim-5 ($m_{\beta\beta}$) + dim-7 & dim-9
- LHC bounds exist
Left-Right symmetric model

- Generates ops. at dim-5 ($m_{\beta\beta}$) + dim-7 & dim-9
- Dim-9 contribution sizable in NH

Illustrative LHC-safe parameters

$m_{WR} = 4.5$ TeV  $m_{\Delta R} = 10$ TeV  $m_{\nu R} = O(10$ TeV)  $U_R = U_{PMNS}$

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1806.02780
Conclusions

• Ton-scale $0\nu\beta\beta$ searches ($T_{1/2} > 10^{27-28}$ yr) will probe LNV from a variety of mechanisms — please don’t get stuck on “Ah, but if it’s normal hierarchy we’ll never see anything…”

• EFT approach provides a general framework to:
  1. Relate $0\nu\beta\beta$ to underlying LNV dynamics (and collider processes)
     • Illustrated by studying dim 7 and dim 9 operators in the SM-EFT and simple models (~LRSM, RPV-SUSY)
  2. Organize contributions to hadronic and nuclear matrix elements
     • Leading potentials from dim5 & dim9 LNV involve new “contacts”

Improving the theory uncertainty is challenging, but there are exciting prospects thanks to advances in EFT, lattice QCD, and nuclear structure
Backup
Low scale seesaw: intriguing example with one light sterile $\nu_R$ with mass ($\sim$eV) and mixing ($\sim0.1$) to fit short baseline anomalies

- Extra contribution to effective mass

\[ m_{\beta\beta} = m_{\beta\beta}\big|_{\text{active}} + |U_{e4}|^2 e^{2i\Phi} m_4 \]

Usual phenomenology turned around!
• Strong correlation of $0\nu\beta\beta$ with oscillation parameters: $\Gamma \propto (m_{\beta\beta})^2$

$$\langle m_{\beta\beta} \rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$

Unitary mixing in CC vertex:
3 angles (known), 1 + 2 phases (unknown)
\( m_{\beta\beta} \) phenomenology

- Strong correlation of \( 0\nu\beta\beta \) with oscillation parameters: \( \Gamma \propto (m_{\beta\beta})^2 \)

\[
\langle m_{\beta\beta} \rangle^2 = |\sum U^2_{ei} m_{\nu_i}|^2
\]

Mass ordering still not fixed by oscillation data
Anatomy of $0\nu\beta\beta$ amplitude from light $\nu$ exchange

Expansion parameter $Q/\Lambda_x$ with $Q \sim k_F \sim m_{\pi}$ and $\Lambda_x \sim M_n$

**Leading order**

$$g_\nu$$

$V_{l=2}$

**Related to matrix elements and excitation energies needed to predict $2\nu\beta\beta$ decay**

**N2LO**

VC, W. Dekens, M. Graesser, E. Mereghetti, S. Pastore, J. de Vries, U. van Kolck 1802.10097

VC, W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

Figure adapted from Primakoff-Rosen 1969
Scaling of contact term in $0\nu\beta\beta$

- $nn \rightarrow ppee$ amplitude with LO strong potential

\[ \tilde{C} \sim \frac{1}{F_{\pi}^2} \]

from fit to $a_{NN}$

\[ \sim \frac{1}{2} (1 + 2g_A^2) \left( \frac{m_N \tilde{C}}{4\pi} \right)^2 \left( \frac{1}{4 - d + \log \mu^2} \right) \]
Scaling of contact term in $0\nu\beta\beta$

- $nn \rightarrow ppee$ amplitude with LO strong potential

This effect is not included in current nuclear m.e. calculations

Finite part of the “low-energy coupling” $g_\nu$ is currently unknown

\[ \tilde{C} \sim 1/F_{\pi}^2 \text{ from fit to } a_{NN} \]

\[ \sim \frac{1}{2} (1 + 2g_A^2) \left( \frac{m_N \tilde{C}}{4\pi} \right)^2 \left( \frac{1}{4 - d} + \log \mu^2 \right) \]

\[ \sim 1/F_{\pi}^2 \]
Estimating finite part of $g_{\nu}$

1) Match $\chi$EFT & lattice QCD calculation of hadronic amplitude $nn \rightarrow pp$

\[
S_{\text{eff}}^{\Delta L=2} = \frac{i8G_F^2V_{ud}^2m_{\beta\beta}}{2!} \int d^4x \, \bar{e}_{L}(x)e_{L}^{c}(x) \int d^4y \, S(x-y) \quad T \left( \bar{u}_{L}\gamma_{\mu}d_{L}(x) \, \bar{u}_{L}\gamma_{\mu}d_{L}(y) \right) g^{\mu\nu}
\]

Scalar massless propagator

$(J^+ \times J^+) \quad \text{vs} \quad (J_{\text{EM}}^+ \times J_{\text{EM}}) \quad I=2$

2) Chiral symmetry relates $g_{\nu}$ to $I=2$ electromagnetic LECs (hard $\nu$ vs $\gamma$)

\[
e^2 C_1 \left( \bar{N}Q_L N \bar{N}Q_L N - \frac{\text{Tr}[Q_L^2]}{6} \bar{N} \tau N \cdot \bar{N} \tau N + L \rightarrow R \right)
\]

\[
e^2 C_2 \left( \bar{N}Q_L N \bar{N}Q_R N - \frac{\text{Tr}[Q_L Q_R]}{6} \bar{N} \tau N \cdot \bar{N} \tau N + L \rightarrow R \right)
\]

\[
Q_L = u^\dagger Q_L u \\
Q_R = u Q_R u^\dagger
\]

\[
u = 1 + \frac{i\pi \cdot \tau}{2F_\pi} + \ldots
\]

Two $I=2$ NN non-derivative operators: chiral symmetry $\Rightarrow g_{\nu} = C_1$
0νββ vs EM isospin breaking

- NN observables cannot disentangle $C_1$ from $C_2$ (need pions), but provide data-based estimate of $C_1 + C_2$

- $C_1 + C_2$ controls ClB combination of $^1S_0$ scattering lengths $a_{nn} + a_{pp} - 2 a_{np}$

- Fit to data, including Coulomb potential, pion EM mass splitting, and contact terms confirms that $C_1 + C_2 \sim 1/F_\pi^2 >> 1/(4\pi F_\pi)^2$
Estimating numerical impact

- Assume $C_1 = C_2$ and hence $g_\nu = (C_1 + C_2)/2$ at some scale $R_S$
- Compute effect in light nuclei: use wave-functions obtained via Variational Monte Carlo from AV18 (NN) + U9 (NNN) potentials — short range correlations included
- Hybrid calculation at this stage: can’t expect $R_S$-independence

Plot matrix element densities $\rho(r)$

$$A = \int dr \; \rho(r)$$
Estimating numerical impact of $g_\nu$

Transitions of experimental interest ($^{76}\text{Ge} \rightarrow ^{76}\text{Se}, \ldots$) have $\Delta l=2$ (and node) $\Rightarrow$ expect significant effect
\( 0\nu\beta\beta \) from \( \mathcal{L}_{\Delta L=2}^{(9)} \)

- Example: scalar operators

\[ \mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \bar{u}_L \gamma_\mu d_L \]
\[ \mathcal{O}_2 = \bar{u}_L d_R \bar{u}_L d_R, \quad \mathcal{O}_3 = \bar{u}_L^\alpha d_R^\beta \bar{u}_L^\beta d_R^\alpha \]
\[ \mathcal{O}_4 = \bar{u}_L \gamma^\mu d_L \bar{u}_R \gamma_\mu d_R, \quad \mathcal{O}_5 = \bar{u}_L^\alpha \gamma^\mu d_L^\beta \bar{u}_R^\beta \gamma_\mu d_R^\alpha \]

- Hadronic realization depends on \( \mathcal{O}_i \)'s chiral properties

\[ \mathcal{L}_{NN} = \left( g_{1L}^{NN} C_{1L}^{(9)} + g_{2L}^{NN} C_{2L}^{(9)} + g_{3L}^{NN} C_{3L}^{(9)} + g_{4L}^{NN} C_{4L}^{(9)} + g_{5L}^{NN} C_{5L}^{(9)} \right) (\bar{p}n) (\bar{p}n) \frac{\bar{e}_L C e_T}{v^5} \]
\[ \mathcal{L}_\pi = \frac{F_0^2}{2} \left[ \frac{5}{3} g_{1L}^{\pi\pi} C_{1L}^{(9)} \partial_\mu \pi^- \partial^\mu \pi^- + \left( g_{4L}^{\pi\pi} C_{4L}^{(9)} + g_{5L}^{\pi\pi} C_{5L}^{(9)} - g_{2L}^{\pi\pi} C_{2L}^{(9)} - g_{3L}^{\pi\pi} C_{3L}^{(9)} \right) \pi^- \pi^- \right] \times \frac{\bar{e}_L C e_T}{v^5} + (L \leftrightarrow R) + \ldots \]

\[ g_{1\pi\pi}^{\pi\pi} \sim \mathcal{O}(1), \quad g_{2,3,4,5}^{\pi\pi} \sim \mathcal{O}(\Lambda^2_\chi) \]

\[ g_{1NN}^{NN} \sim \mathcal{O}(1), \quad g_{2,3,4,5}^{NN} \sim \mathcal{O}\left( \frac{\Lambda^2_\chi}{F^2_\pi} \right) \]
Pion matrix elements from LQCD

- Quite consistent with result obtained from kaon m.e. via chiral SU(3)

Nicholson et al., 1805/02634

MS-bar at $\mu=2\text{GeV}$

$g_1^{\pi\pi} = +0.4$

$g_2^{\pi\pi} = -(1.8 \text{ GeV})^2$

$g_3^{\pi\pi} = +(1.0 \text{ GeV})^2$

$g_4^{\pi\pi} = -(1.7 \text{ GeV})^2$

$g_5^{\pi\pi} = -(3.6 \text{ GeV})^2$

VC , W. Dekens, M. Graesser, E. Mereghetti 1701.01443
Pion matrix elements from LQCD

• Result is < 1/2 × “vacuum insertion approximation”, commonly used in literature!
• \( <pp|O_i|nn> \) (\( <p\pi^+|O_i|n> \)) not yet known from LQCD (only factorization model)
• In some instances, using “wrong hadronization” (e.g. no pion range) leads to factor >10 change in sensitivity to short-distance couplings