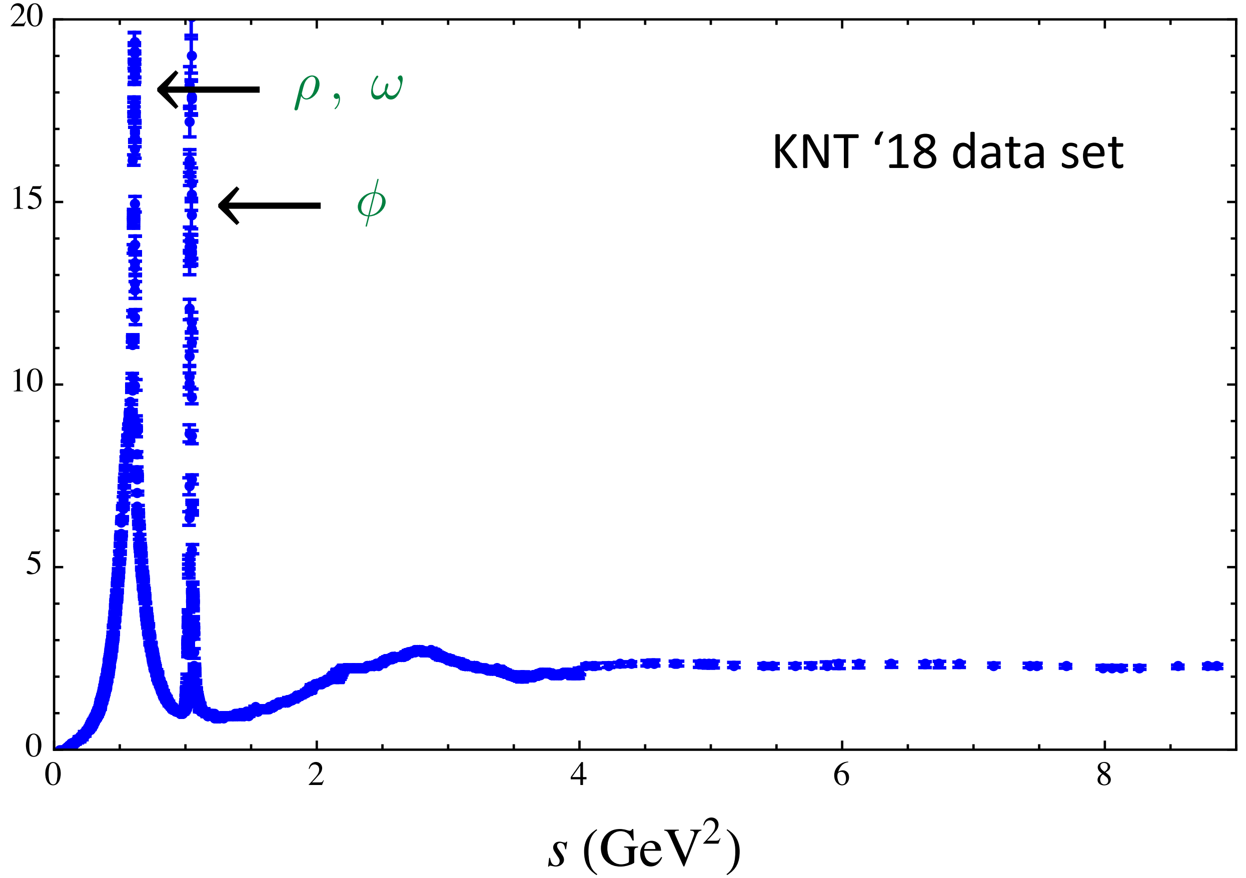
The strong coupling from $e^+e^- \to$ hadrons

Diogo Boito, Maarten Golterman, Alex Keshavarzi,
Kim Maltman, Daisuke Nomura, Santi Peris, Thomas Teubner

Tau 2018, Amsterdam, Sept. 24-28, 2018

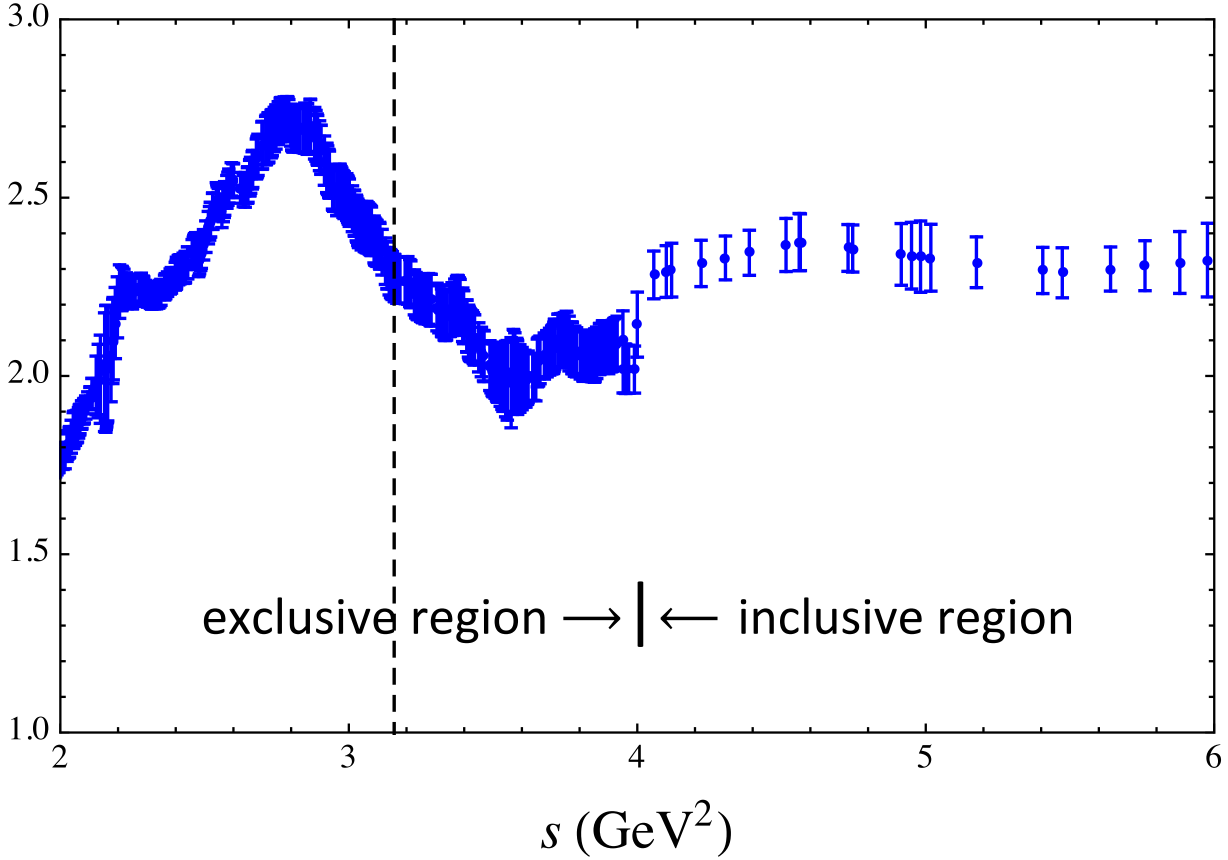
New compilation of R-ratio data

(Keshavarzi, Nomura, Teubner, '18
see also Davier *et al.*, '17,
Jegerlehner, 16)



Large s: $R(s) \approx 2$ parton-model value + perturbative corrections

Blow up of KNT data

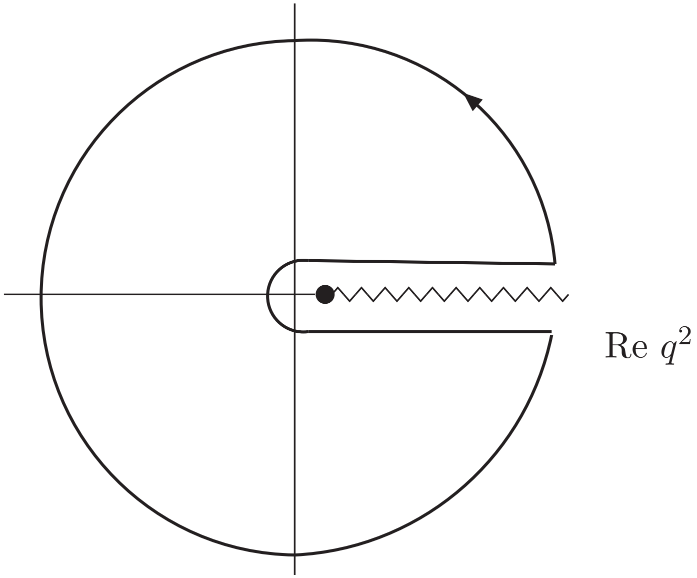


A lot more data in the exclusive region –
they will determine the precision with which α_s can be extracted!
Finite energy sum rules as tool to make use of low-s data

Finite energy sum rules (FESRs)

Vacuum polarization $\Pi(q^2)$ is analytic in q^2 plane, hence, choosing

$$w(y) \to 1$$
$$1 - y^2$$
$$(1-y)^2(1+2y)$$
$$(1-y^2)^2$$



complex $z = q^2$ plane

$$I^{(w)}(s_0) = \frac{1}{s_0}\int_0^{s_0} ds\, w\left(\frac{s}{s_0}\right) \underbrace{\frac{1}{12\pi^2}R(s)}_{\rho(s)} = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w\left(\frac{z}{s_0}\right)\Pi(z)$$

Advantage: weighted integrals over ***all*** spectral data up to $s = s_0$ reduces errors

Finite energy sum rule

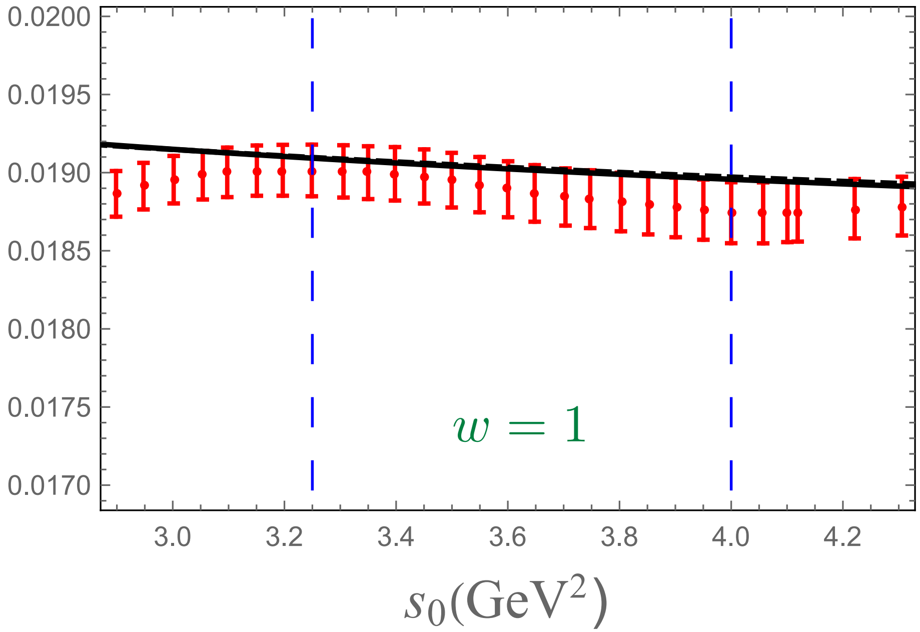
$$I^{(w)}(s_0) = \frac{1}{s_0}\int_0^{s_0} ds\, w\left(\frac{s}{s_0}\right)\underbrace{\frac{1}{12\pi^2}R(s)}_{\rho(s)} = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w\left(\frac{z}{s_0}\right)\Pi(z)$$

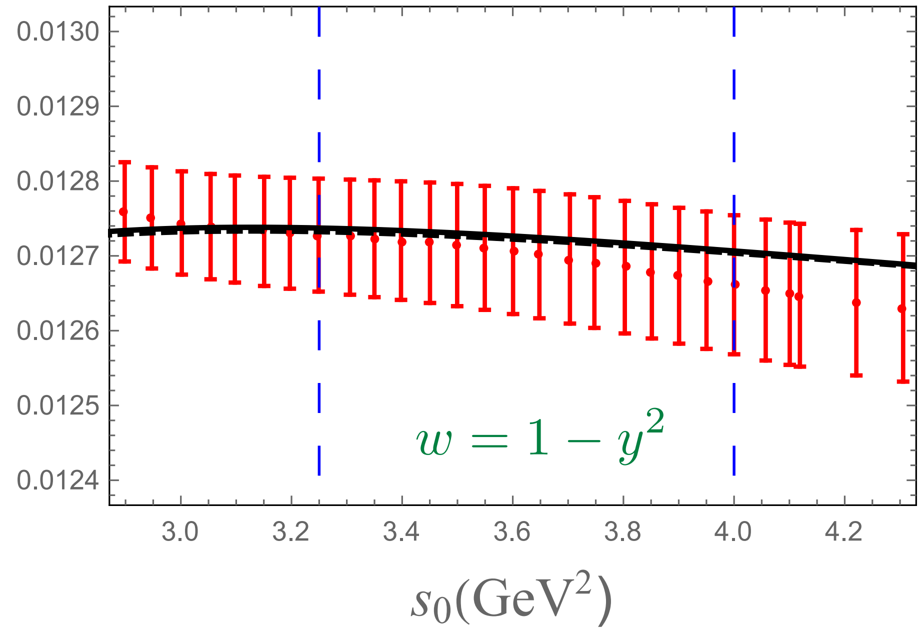
Split $\Pi(q^2) = \underbrace{\Pi_{\text{pert}}(q^2)}_{\to\alpha_s} + \underbrace{\Pi_{\text{OPE}}(q^2)}_{\sum_{k=1}^{\infty}\frac{C_{2k}}{(-q^2)^k}} + \underbrace{\Pi_{\text{DV}}(q^2)}_{\text{resonances}}$ (pert.th. to 5 loops: Baikov, Chetyrkin, Kühn, '08)

quantity	OPE coeff: $D = 2k$	error at $s_0 = 4$ GeV^2
$R(s_0)$	–	4.3%
$I^{(w=1)}(s_0)$	$D = 2$	1.04%
$I^{(w=1-y^2)}(s_0)$	$D = 2, 6$	0.73%
$I^{(w=(1-y)^2(1+2y))}(s_0)$	$D = 2, 6, 8$	0.56%
$I^{(w=(1-y^2)^2)}(s_0)$	$D = 2, 6, 10$	0.59%

Note: D = 2 OPE contribution known from perturbation theory

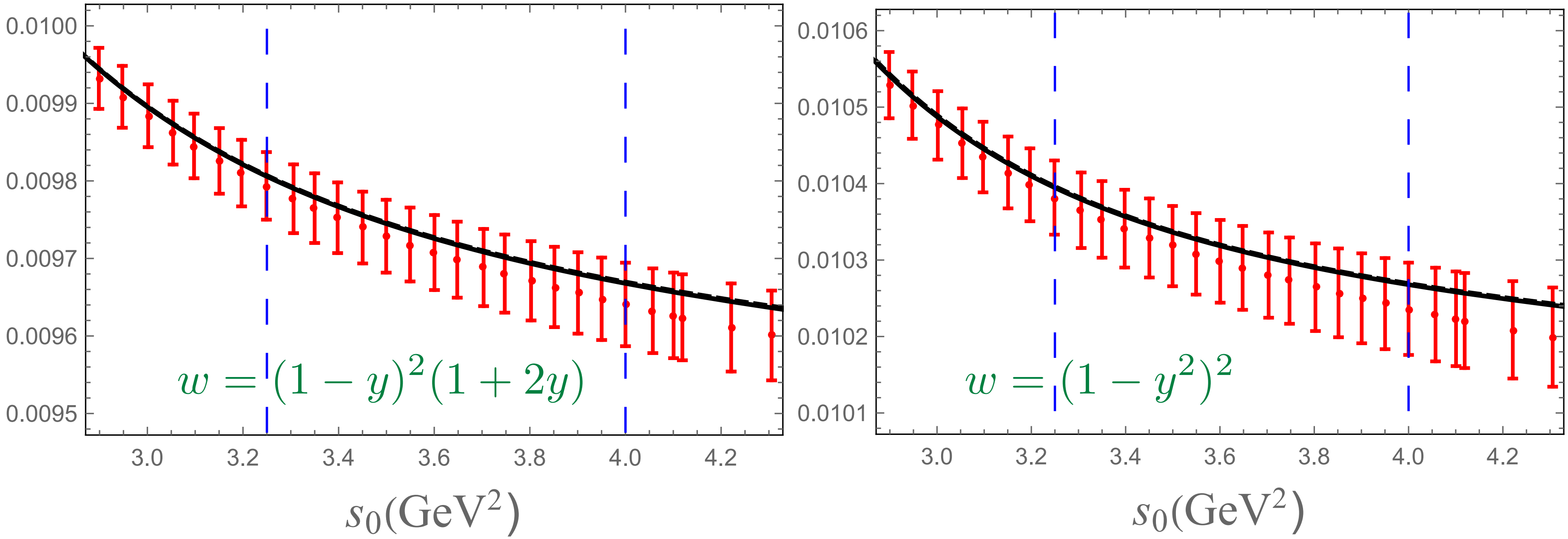
Results: fits





- No integrated duality-violating (resonance) effects (test below)
- Fit windows: $3.25\ \mathrm{GeV}^2 \leq s_0^{\mathrm{min}} \leq 3.80\ \mathrm{GeV}^2,\ s_0^{\mathrm{max}} = 4.0\ \mathrm{GeV}^2$; correlated fits
- Note good quality of fits *without* duality violations! p-values 0.12 to 0.42

Results: fits



- Again, no integrated duality-violating (resonance) effects
- Fit windows: $3.25\ \mathrm{GeV}^2 \le s_0^{\mathrm{min}} \le 3.80\ \mathrm{GeV}^2,\ s_0^{\mathrm{max}} = 4.0\ \mathrm{GeV}^2$; correlated fits
- Note good quality of fits *without* duality violations! p-values 0.09 to 0.32

Results

weight	$\alpha_s(m_\tau^2)$ (FOPT)	$\alpha_s(m_\tau^2)$ (CIPT)
1	$0.299(16)$	$0.308(19)$
$1-y^2$	$0.298(17)$	$0.305(19)$
$(1-y)^2(1+2y)$	$0.298(18)$	$0.303(20)$
$(1-y^2)^2$	$0.297(18)$	$0.303(20)$

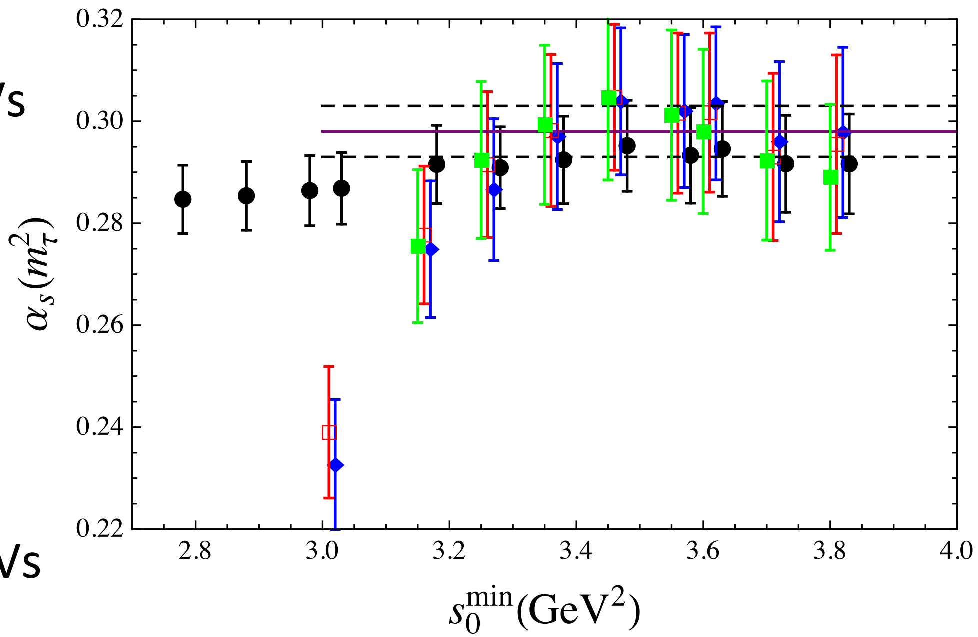
- Two resummation methods in pert. th.: Fixed-Order and Contour-Improved (*cf*. poster by Boito at this workshop)
- Note excellent consistency between different weights
- OPE coeff. C_6 obtained from all weights except $w=1$, also consistent
- Error combination of fit error and variation of fit window (all fit windows have $3.25\ \mathrm{GeV}^2 \leq s_0 \leq 4\ \mathrm{GeV}^2$)

Results – tests

- inclusion of $s > 4\ \mathrm{GeV}^2$ data leads to consistent results, but no overall error reduction

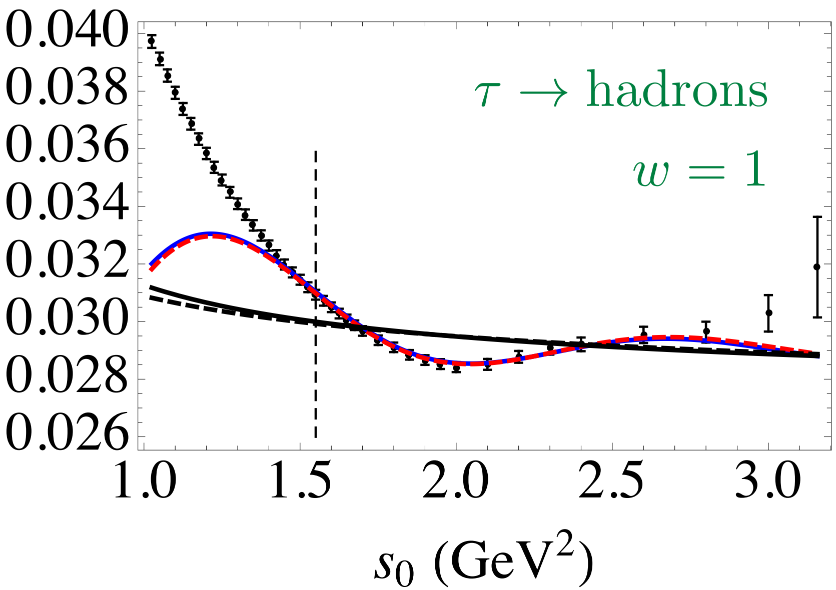
- consistency test for effect of possible duality violations (resonance effects) using information for $I=1$ from hadronic τ decays

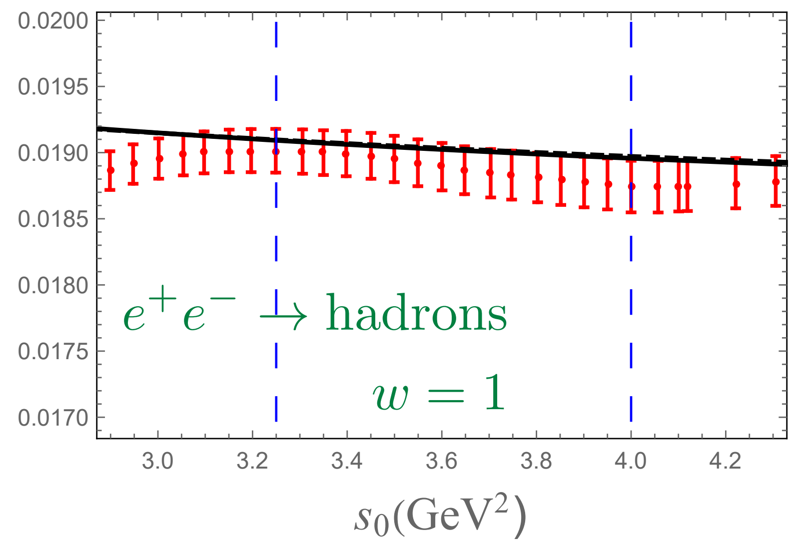
  - colored points: fits with no DVs (3 weights; FOPT)
  - horizontal line: central value
  - dashed lines: error bar
  - black points: include DVs



See S. Peris poster for more on DVs

Difference with determination from hadronic τ decays

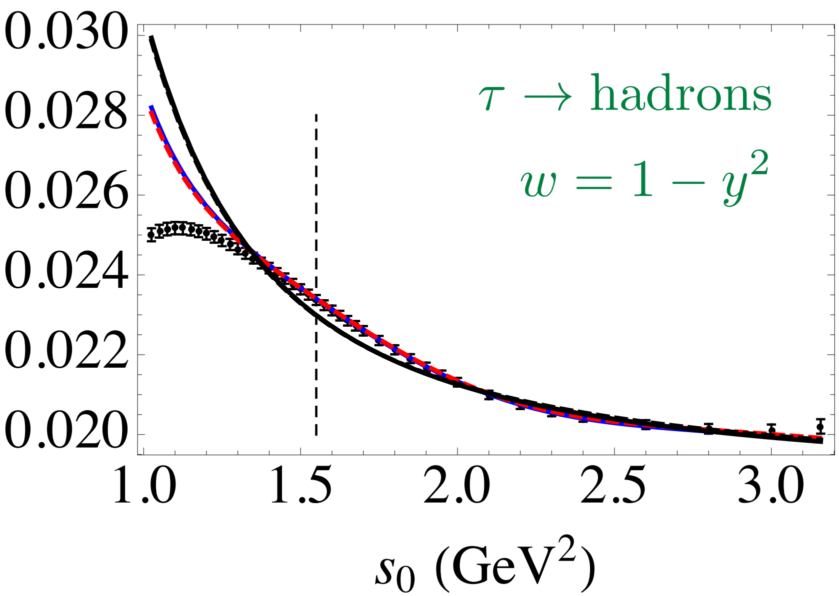


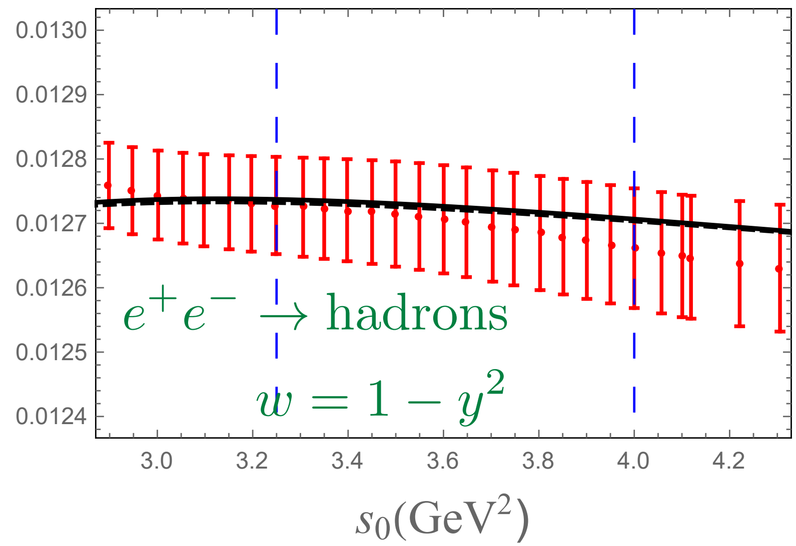


- fit interval
  $1.55 \text{ GeV}^2 \le s_0 \le 3.16 \text{ GeV}^2$
- duality violations clearly visible!
  black curves: pert.th. + OPE
  colored curves: + DVs
- serious "pinching" needed!
  leads to issues with OPE

- fit interval
  $3.25 \text{ GeV}^2 \le s_0 \le 4 \text{ GeV}^2$
- duality violations barely visible!
  black curves: pert.th. + OPE

Difference with determination from hadronic τ decays





- fit interval
  $1.55\ \mathrm{GeV}^2 \leq s_0 \leq 3.16\ \mathrm{GeV}^2$
- duality violations clearly visible!
  black curves: pert.th. + OPE
  colored curves: + DVs
- serious "pinching" needed!
  leads to issues with OPE

- fit interval
  $3.25\ \mathrm{GeV}^2 \leq s_0 \leq 4\ \mathrm{GeV}^2$
- duality violations not visible!
  black curves: pert.th. + OPE

➢ see S. Peris poster for more

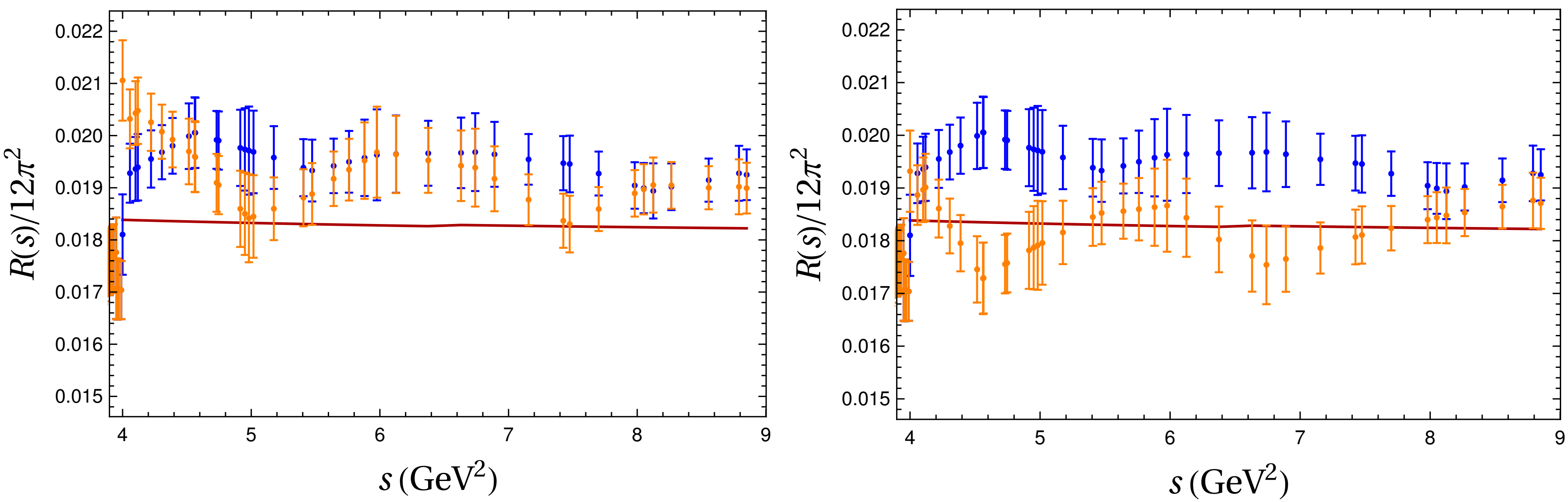
Final results

- from $e^+e^- \to \text{hadrons}$: $\alpha_s(m_\tau^2) = 0.298(17)$ (FOPT)
  $= 0.304(19)$ (CIPT)
- from $\tau \to \text{hadrons}$: $\alpha_s(m_\tau^2) = 0.303(9)$ (FOPT)
  (Boito *et al.* '15) $= 0.319(12)$ (CIPT)
  excellent agreement! Note much reduced FOPT-CIPT difference!
- At the Z mass from e^+e^-: $\alpha_s(m_Z^2) = 0.1158(22)$ (FOPT)
  $= 0.1166(25)$ (CIPT)

  consistent with 4- and 5-loop running
- error dominated by experimental errors
- Also of interest: e^+e^--based tests of τ analysis "truncated OPE approach" (*e.g.*, Davier *et al*. '13, Pich *et al*. '16) show serious systematic problems
  [see S. Peris poster for details]

BACK-UP SLIDES

Perturbation theory and the R-ratio

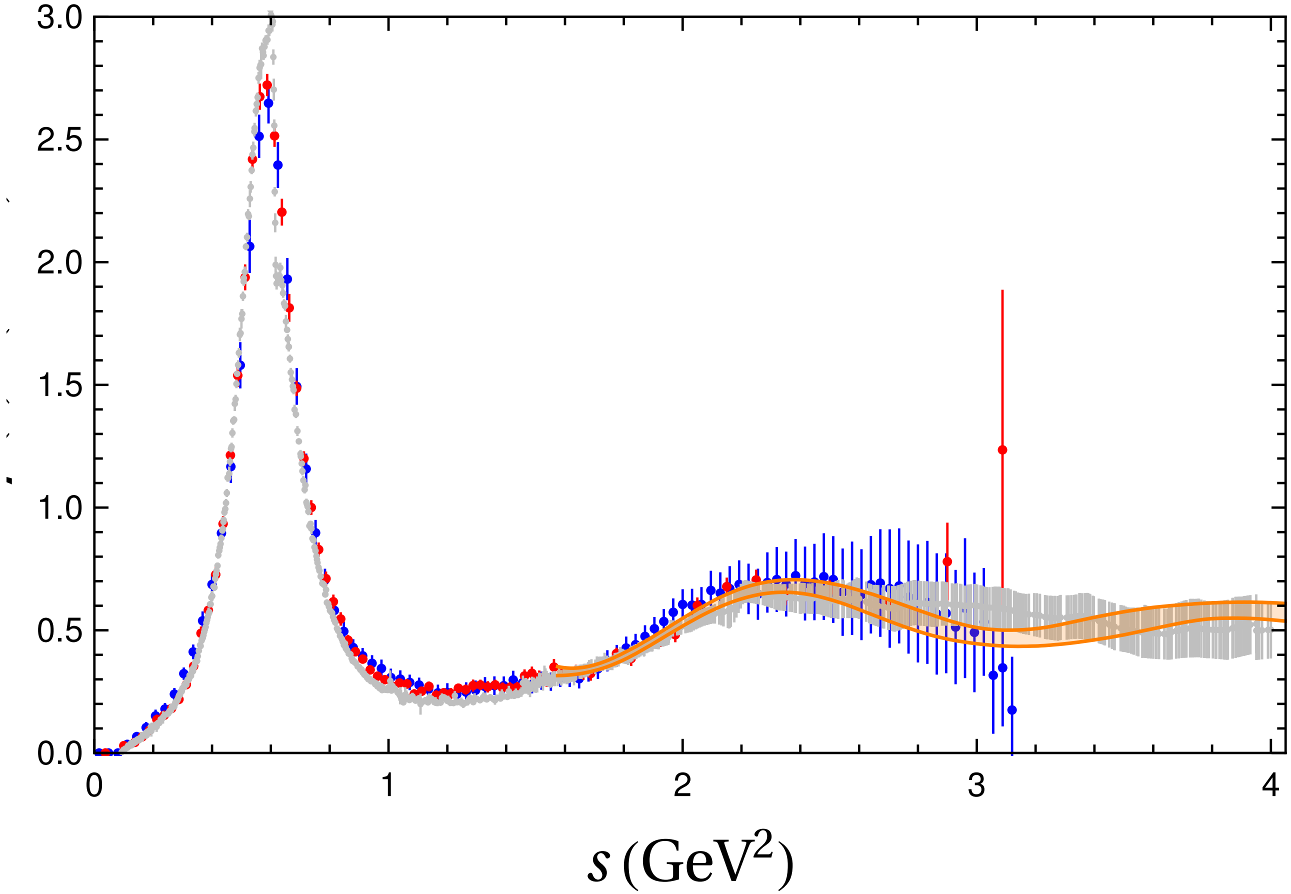


real data (blue) and two different mock data sets (orange) generated from pert.th. with $\alpha_s(m_\tau^2) = 0.3$ and real-data covariances

try fit from (inclusive) data above $s = 4\ \mathrm{GeV}^2$, leads to $\alpha_s(m_\tau^2) \approx 0.4 \pm 0.1$

conclusion: difference compatible with statistical fluctuation

Comparison of $I = 1$ spectral functions



grey KNT data

blue OPAL data

red ALEPH data

orange
$I = 1$ spectral function fitted to OPAL/ALEPH data

Test of the "truncated OPE" approach (2 slides from poster)

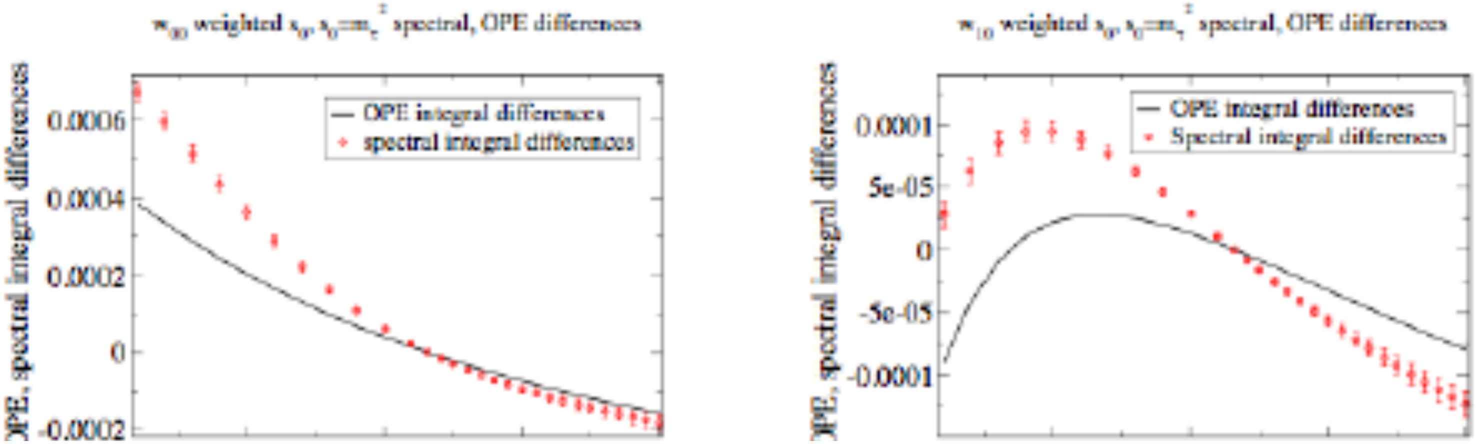
$$I^{(w)}_{\mathrm{exp/th}}(s_0) - I^{(w)}_{\mathrm{exp/th}}(m_\tau^2)$$

with

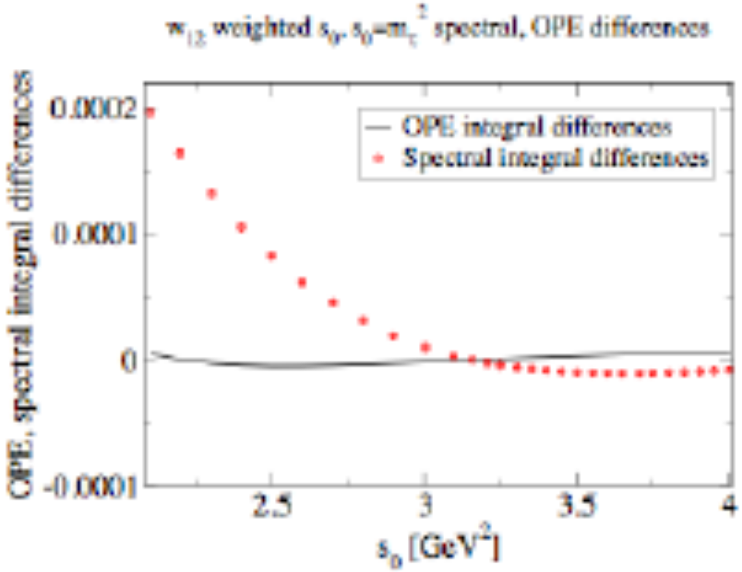
$$I^{(w)}_{\mathrm{exp}}(s_0) = \int_0^{s_0} dt\, w(t)\, \rho(t)$$

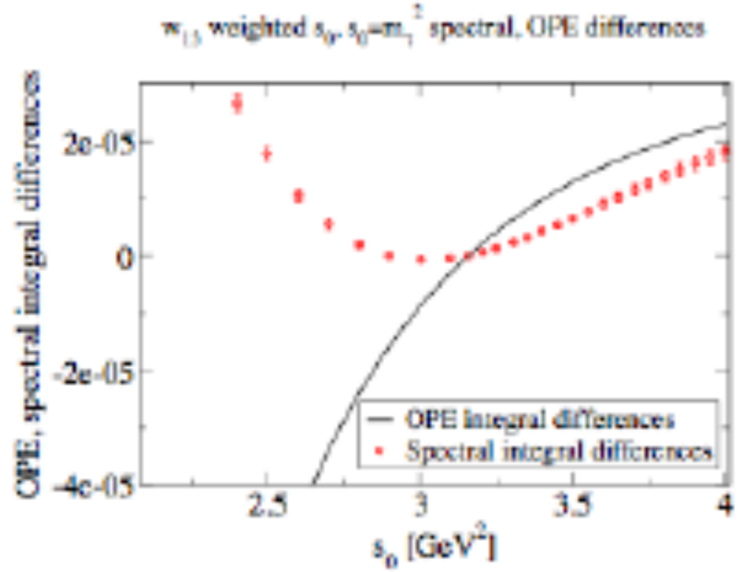
with different weight functions w(t) suggested in the literature

- Test of truncated OPE strategy with moments $w_{kl}(x) = (1-x)^{2+k}(1+2x)x^l$ (frequently used in the literature) Davier et al, '13
Pich and Rodríguez-Sánchez, '16



R(s) data from the recent compilation of Keshavarzi, Nomura, and Teubner, '18





- Similar problems for alternate weight choices, e.g. “optimal weights” defined in Pich and Rodríguez-Sánchez, '16

Failure of $s_0 = m_\tau^2$-only, neglected higher D fits $\Rightarrow$ use of s_0-dependence required, hence lower s_0 and larger DVs. *Need for best possible theoretical representation of DVs.*