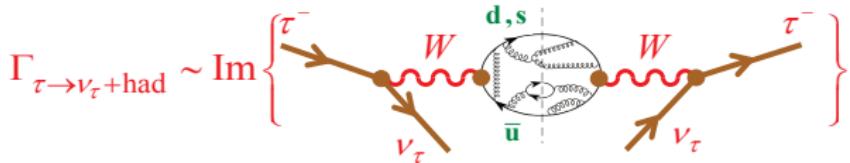


α_s from τ decay data

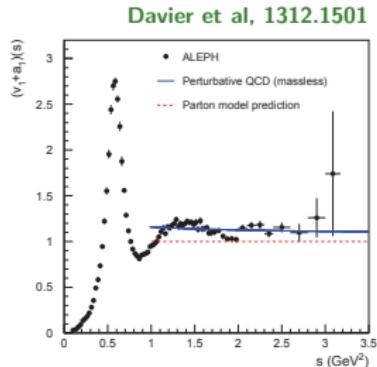
Antonio Pich
IFIC, Univ. Valencia – CSIC

The 15th International Workshop on Tau Lepton Physics (TAU 2018)
Vondelkerk, Amsterdam, The Netherlands, 24–28 September 2018

τ Hadronic Width: R_τ



$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right) + |V_{us}|^2 \Pi_{us,V+A}^{(J)}(s)$$



$$R_\tau = \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

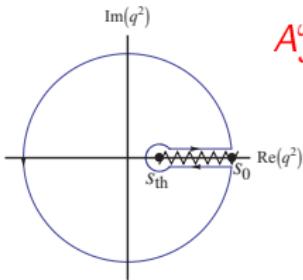
$$= 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(0+1)}(s) - 2\frac{s}{m_\tau^2} \text{Im} \Pi^{(0)}(s) \right]$$

$$i \int d^4x \ e^{iqx} \langle 0 | T \left[\mathcal{J}_{ij}^\mu(x) \mathcal{J}_{ij}^{\nu\dagger}(0) \right] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2)$$

$$A_\mathcal{J}^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_\mathcal{J}^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_\mathcal{J}^{(J)}(s)$$

$$\Pi_\mathcal{J}^{(J)}(s) \approx \Pi_\mathcal{J}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

$$i \int d^4x \, e^{iqx} \langle 0 | T \left[\mathcal{J}_{ij}^\mu(x) \mathcal{J}_{ij}^{\nu\dagger}(0) \right] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2)$$



$$A_{\mathcal{J}}^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_{\mathcal{J}}^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{\mathcal{J}}^{(J)}(s)$$

$$\Pi_{\mathcal{J}}^{(J)}(s) \approx \Pi_{\mathcal{J}}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

$$R_\tau = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

Braaten-Narison-Pich '92

$$= 6\pi i \oint_{|x|=1} (1-x)^2 \left[(1+2x) \Pi^{(0+1)}(m_\tau^2 x) - 2x \Pi^{(0)}(m_\tau^2 x) \right]$$

$$\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$$

Baikov-Chetyrkin-Kühn '08

$$a_\tau \equiv \alpha_s(m_\tau^2)/\pi , \quad S_{EW} = 1.0201(3)$$

Marciano-Sirlin, Braaten-Li, Erler

$$\delta_{NP} = -0.0064 \pm 0.0013 \quad (\text{Fitted from data})$$

Davier et al '14

Perturbative Contribution ($m_q = 0$)

$$a_\tau \equiv \frac{\alpha_s(m_\tau^2)}{\pi}$$

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} \textcolor{red}{K}_n a_s (-s)^n$$



$$\delta_P = \underbrace{\sum_{n=1} \textcolor{red}{K}_n A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=1} \textcolor{red}{r}_n a_\tau^n}_{\text{FOPT}}$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} \left(1 - 2x + 2x^3 - x^4\right) \left(\frac{\alpha_s(-m_\tau^2 x)}{\pi}\right)^n = a_\tau^n + \dots$$

Perturbative Contribution ($m_q = 0$)

$$a_\tau \equiv \frac{\alpha_s(m_\tau^2)}{\pi}$$

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n a_s (-s)^n$$



$$\delta_P = \underbrace{\sum_{n=1} K_n A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=1} r_n a_\tau^n}_{\text{FOPT}}$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} \left(1 - 2x + 2x^3 - x^4\right) \left(\frac{\alpha_s(-m_\tau^2 x)}{\pi}\right)^n = a_\tau^n + \dots$$

1) The dominant corrections come from the contour integration

Large running of α_s along the circle $s = m_\tau^2 e^{i\phi}$, $\phi \in [-\pi, \pi]$

n	1	2	3	4	5
K_n	1	1.6398	6.37101	49.0757	?
r_n	1	5.2023	26.3659	127.079	$307.78 + K_5$
$r_n - K_n$	0	3.5625	19.9949	78.0029	307.78

Baikov-Chetyrkin-Kühn '08

Le Diberder-Pich '92

Perturbative Contribution ($m_q = 0$)

$$a_\tau \equiv \frac{\alpha_s(m_\tau^2)}{\pi}$$

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n a_s (-s)^n$$



$$\delta_P = \underbrace{\sum_{n=1} K_n A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=1} r_n a_\tau^n}_{\text{FOPT}}$$

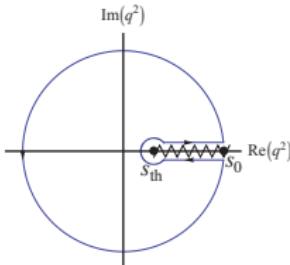
$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} \left(1 - 2x + 2x^3 - x^4\right) \left(\frac{\alpha_s(-m_\tau^2 x)}{\pi}\right)^n = a_\tau^n + \dots$$

2) CIPT gives rise to a well-behaved perturbative series

$a_\tau = 0.11$	$A^{(1)}(\alpha_s)$	$A^{(2)}(\alpha_s)$	$A^{(3)}(\alpha_s)$	$A^{(4)}(\alpha_s)$	δ_P
$\beta_{n>1} = 0$	0.14828	0.01925	0.00225	0.00024	0.20578
$\beta_{n>2} = 0$	0.15103	0.01905	0.00209	0.00020	0.20537
$\beta_{n>3} = 0$	0.15093	0.01882	0.00202	0.00019	0.20389
$\beta_{n>4} = 0$	0.15058	0.01865	0.00198	0.00018	0.20273
$\beta_{n>5} = 0$	0.15041	0.01859	0.00197	0.00018	0.20232
$\mathcal{O}(a_\tau^4) \text{ FOPT}$	0.16115	0.02431	0.00290	0.00015	0.22665

FOPT overestimates δ_P by 11%

Non-Perturbative Contribution

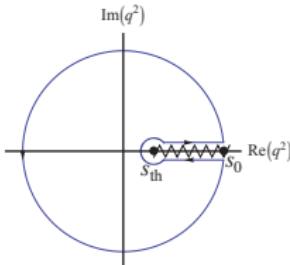


$$A_J^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_J^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_J^{(J)}(s)$$

$$\Pi_J^{(J)}(s) \approx \Pi_J^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,J}^{(J)}}{(-s)^{D/2}}$$

$$A_J^{\omega,\text{NP}}(s_0) = \pi \sum_D a_{-1,D} \frac{\mathcal{O}_{D,J}^{(J)}}{s_0^{D/2}} \quad , \quad \omega(-s_0 x) = \sum_n a_{n,D} x^{n+D/2}$$

Non-Perturbative Contribution



$$A_{\mathcal{J}}^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_{\mathcal{J}}^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{\mathcal{J}}^{(J)}(s)$$

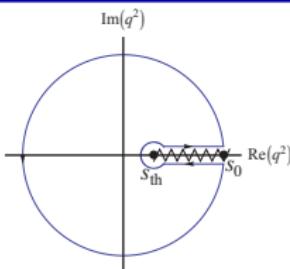
$$\Pi_{\mathcal{J}}^{(J)}(s) \approx \Pi_{\mathcal{J}}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

$$A_{\mathcal{J}}^{\omega, \text{NP}}(s_0) = \pi \sum_D a_{-1,D} \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{s_0^{D/2}} \quad , \quad \omega(-s_0 x) = \sum_n a_{n,D} x^{n+D/2}$$

- Strong power suppression at $s_0 = m_\tau^2$: $\sim (\Lambda_{\text{QCD}}/m_\tau)^D$, $D \geq 4$

$$\mathcal{O}_{4,V/A} \approx 4\pi^2 \left\{ \frac{1}{12\pi} \langle \alpha_s GG \rangle + (m_u + m_d) \langle \bar{q}q \rangle \right\} \approx [(6.7 \pm 3.2) - 0.6] \cdot 10^{-3} \times m_\tau^4$$

Non-Perturbative Contribution



$$A_{\mathcal{J}}^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_{\mathcal{J}}^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{\mathcal{J}}^{(J)}(s)$$

$$\Pi_{\mathcal{J}}^{(J)}(s) \approx \Pi_{\mathcal{J}}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

$$A_{\mathcal{J}}^{\omega, \text{NP}}(s_0) = \pi \sum_D a_{-1,D} \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{s_0^{D/2}} \quad , \quad \omega(-s_0 x) = \sum_n a_{n,D} x^{n+D/2}$$

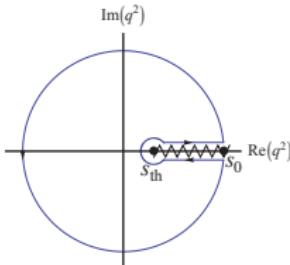
- Strong power suppression at $s_0 = m_\tau^2$: $\sim (\Lambda_{\text{QCD}}/m_\tau)^D$, $D \geq 4$

$$\mathcal{O}_{4,V/A} \approx 4\pi^2 \left\{ \frac{1}{12\pi} \langle \alpha_s GG \rangle + (m_u + m_d) \langle \bar{q}q \rangle \right\} \approx [(6.7 \pm 3.2) - 0.6] \cdot 10^{-3} \times m_\tau^4$$

- R_τ : $\omega(x) = 1 - 3x^2 + 2x^3 \rightarrow \delta_{\text{NP}} = -3 \frac{\mathcal{O}_{6,V+A}}{m_\tau^6} - 2 \frac{\mathcal{O}_{8,V+A}}{m_\tau^8}$

Additional chiral suppression in $|\mathcal{O}_{6,V+A}| < |\mathcal{O}_{6,V-A}| \approx (1.1 \pm 0.2) \cdot 10^{-4} \times m_\tau^6$

Non-Perturbative Contribution



$$A_J^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_J^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_J^{(J)}(s)$$

$$\Pi_J^{(J)}(s) \approx \Pi_J^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,J}^{(J)}}{(-s)^{D/2}}$$

$$A_J^{\omega,\text{NP}}(s_0) = \pi \sum_D a_{-1,D} \frac{\mathcal{O}_{D,J}^{(J)}}{s_0^{D/2}} \quad , \quad \omega(-s_0 x) = \sum_n a_{n,D} x^{n+D/2}$$

- Strong power suppression at $s_0 = m_\tau^2$: $\sim (\Lambda_{\text{QCD}}/m_\tau)^D$, $D \geq 4$

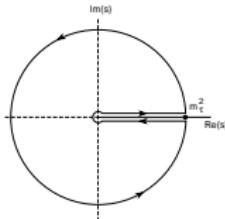
$$\mathcal{O}_{4,V/A} \approx 4\pi^2 \left\{ \frac{1}{12\pi} \langle \alpha_s GG \rangle + (m_u + m_d) \langle \bar{q}q \rangle \right\} \approx [(6.7 \pm 3.2) - 0.6] \cdot 10^{-3} \times m_\tau^4$$

- R_τ : $\omega(x) = 1 - 3x^2 + 2x^3 \rightarrow \delta_{\text{NP}} = -3 \frac{\mathcal{O}_{6,V+A}}{m_\tau^6} - 2 \frac{\mathcal{O}_{8,V+A}}{m_\tau^8}$

Additional chiral suppression in $|\mathcal{O}_{6,V+A}| < |\mathcal{O}_{6,V-A}| \approx (1.1 \pm 0.2) \cdot 10^{-4} \times m_\tau^6$

- Sensitivity to \mathcal{O}_D with different $\omega(x)$ \rightarrow Measure δ_{NP}

R_τ suitable for a precise α_s determination



$$R_\tau = 6\pi i \oint_{|x|=1} (1-x)^2 \left[(1+2x) \Pi^{(0+1)}(m_\tau^2 x) - 2x \Pi^{(0)}(m_\tau^2 x) \right]$$

$$\Pi_{\mathcal{J}}^{(J)}(s) \approx \Pi_{\mathcal{J}}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

- Known to $\mathcal{O}(\alpha_s^4)$. Sizeable $\delta_P \sim 20\%$. Strong sensitivity to α_s
- m_τ large enough to safely use the OPE. Flat **V + A** distribution
- OPE only valid away from real axis: $(1-x)^2$ pinched at $s = m_\tau^2$
- $m_{u,d} = 0 \rightarrow s \Pi^{(0)} = 0 \rightarrow R_{\tau,V+A} = 6\pi i \oint_{|x|=1} (1-3x^2+2x^3) \Pi_{ud,V+A}^{(0+1)}(m_\tau^2 x)$
 $\rightarrow \delta_{NP} \sim 1/m_\tau^6$ Strong suppression of non-perturbative effects
- $D=6$ OPE contributions have opposite sign for **V** & **A**. Cancellation
- δ_{NP} can be determined from data: $\delta_{NP} = -0.0064 \pm 0.0013$ Davier et al

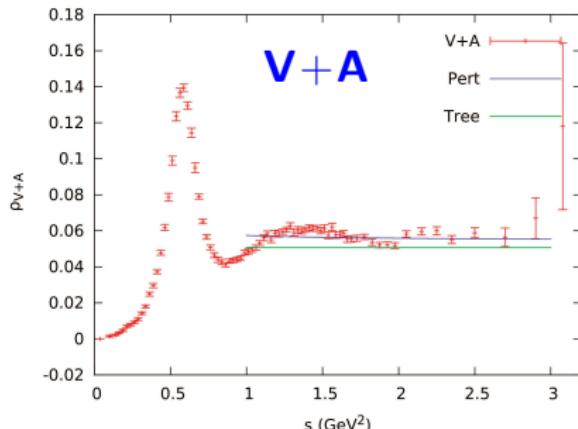
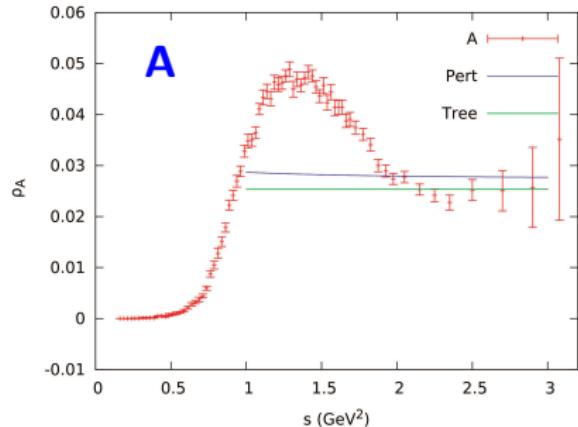
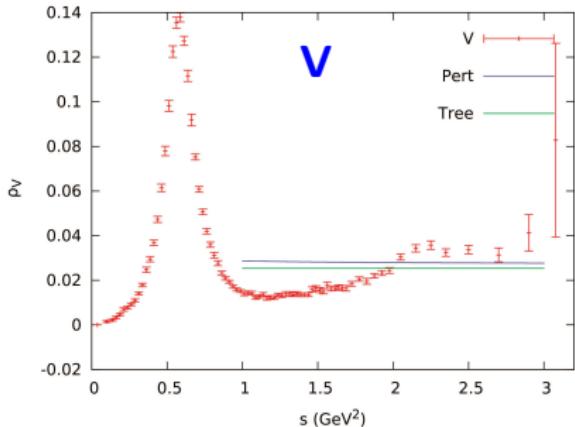
R_τ

$$\alpha_s(m_\tau^2) = 0.331 \pm 0.013$$

Pich 2014

ALEPH Spectral Functions

Davier et al. 2014



$$\alpha_s(m_\tau^2) = 0.329$$

Parton Model

New analysis of ALEPH data

Rodríguez-Sánchez, Pich, arXiv:1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments ¹	$0.339^{+0.019}_{-0.017}$	$0.319^{+0.017}_{-0.015}$	$0.329^{+0.020}_{-0.018}$
Mod. ALEPH moments ²	$0.338^{+0.014}_{-0.012}$	$0.319^{+0.013}_{-0.010}$	$0.329^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments ³	$0.336^{+0.018}_{-0.016}$	$0.317^{+0.015}_{-0.013}$	$0.326^{+0.018}_{-0.016}$
s_0 dependence ⁴	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform ⁵	$0.328^{+0.014}_{-0.013}$	$0.318^{+0.015}_{-0.012}$	$0.323^{+0.015}_{-0.013}$
Combined value	0.335 ± 0.013	0.320 ± 0.012	0.328 ± 0.013



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$1) \quad \omega_{kl}(x) = (1+2x)(1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

$$2) \quad \tilde{\omega}_{kl}(x) = (1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

$$3) \quad \omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^m (k+1)x^k = 1 - (m+2)x^{m+1} + (m+1)x^{m+2}, \quad 1 \leq m \leq 5$$

$$4) \quad \omega^{(2,m)}(x) \quad 0 \leq m \leq 2, \quad 1 \text{ single moment in each fit}$$

$$5) \quad \omega_a^{(1,m)}(x) = (1-x^{m+1})e^{-ax} \quad 0 \leq m \leq 6$$

α_s determination with ALEPH-like fit

Rodríguez-Sánchez, A.P.

$$\omega_{kl}(x) = (1-x)^{2+k} x^l (1+2x) \quad , \quad x = s/m_\tau^2 \quad , \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

Channel	$\alpha_s(m_\tau^2)$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ (10^{-3} GeV 4)	\mathcal{O}_6 (10^{-3} GeV 6)	\mathcal{O}_8 (10^{-3} GeV 8)
V (FOPT)	$0.328^{+0.013}_{-0.007}$	8^{+7}_{-14}	$-3.2^{+0.8}_{-0.5}$	$5.0^{+0.4}_{-0.7}$
V (CIPT)	$0.352^{+0.013}_{-0.011}$	-8^{+7}_{-7}	$-3.5^{+0.3}_{-0.3}$	$4.9^{+0.4}_{-0.5}$
A (FOPT)	$0.304^{+0.010}_{-0.007}$	-15^{+5}_{-8}	$4.4^{+0.5}_{-0.4}$	$-5.8^{+0.3}_{-0.4}$
A (CIPT)	$0.320^{+0.011}_{-0.010}$	-25^{+5}_{-5}	$4.3^{+0.2}_{-0.2}$	$-5.8^{+0.3}_{-0.3}$
V+A (FOPT)	$0.319^{+0.010}_{-0.006}$	-3^{+6}_{-11}	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
V+A (CIPT)	$0.339^{+0.011}_{-0.009}$	-16^{+5}_{-5}	$0.9^{+0.3}_{-0.4}$	$-1.0^{+0.5}_{-0.7}$

- High sensitivity to α_s . Bad sensitivity to power corrections
- Cancellation in $\mathcal{O}_{6,V+A}$ confirmed. V + A more reliable
- $K_5 = 275 \pm 400$, $\mu^2 = (0.5, 2) m_\tau^2$
- Best values taken from V + A. Errors increased with sensitivity to \mathcal{O}_{10}

$$\alpha_s(m_\tau^2)^{\text{CIPT}} = 0.339^{+0.019}_{-0.017}$$

$$\alpha_s(m_\tau^2)^{\text{FOPT}} = 0.319^{+0.017}_{-0.015}$$



$$\alpha_s(m_\tau^2) = 0.329^{+0.020}_{-0.018}$$

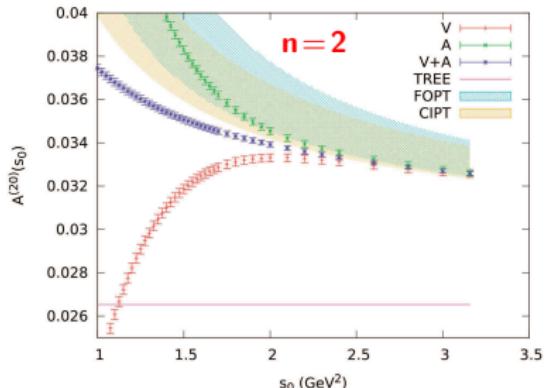
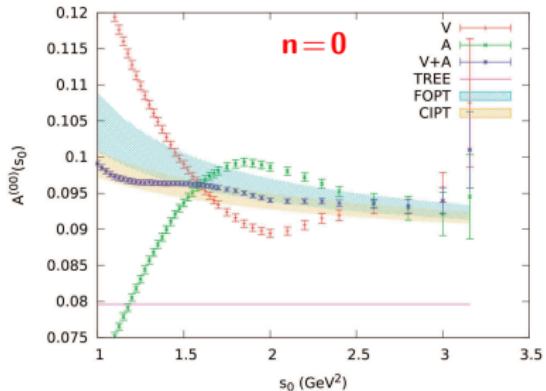
Good agreement with Davier et al.: $\alpha_s(m_\tau^2) = 0.332 \pm 0.012$

(arXiv:1312.1501)

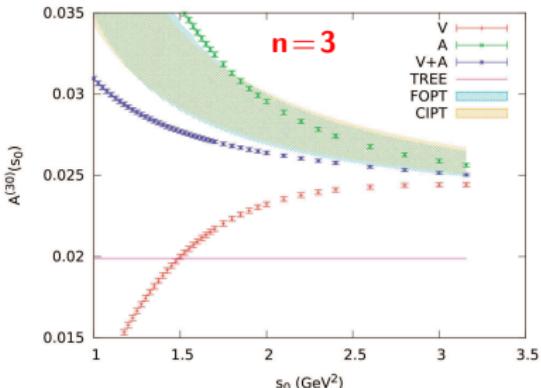
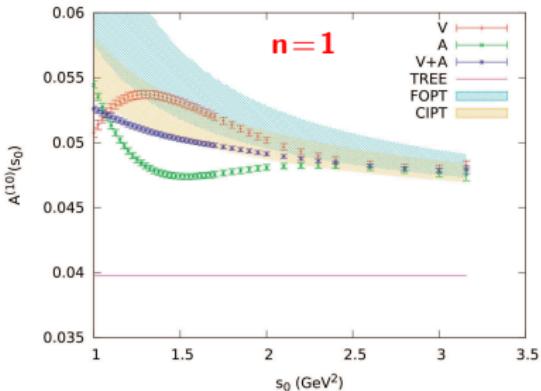
Experiment vs. (pinched) Perturbation Theory (only)

Rodríguez-Sánchez, A.P.

$$\omega^{(n,0)}(s = s_0 x) = (1 - x)^n \rightarrow \mathcal{O}_{D \leq 2}(n+1)$$



$$\alpha_s(m_\tau^2) = 0.329^{+0.020}_{-0.018}$$



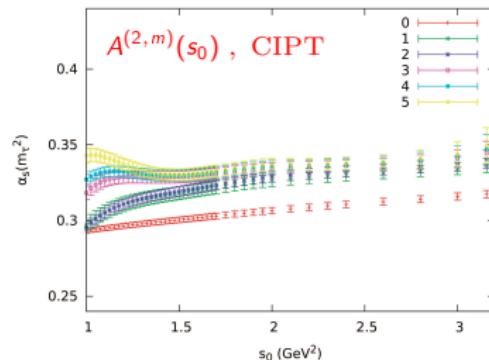
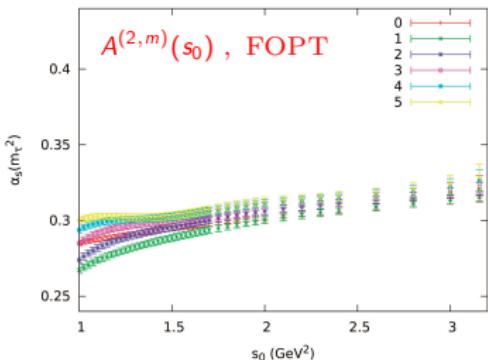
Non-Perturbative Contributions Neglected

Rodríguez-Sánchez, A.P.

$$\omega^{(1,m)}(x) = 1 - x^{m+1} \rightarrow \mathcal{O}_{2m+4}$$

$$\omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^m (k+1)x^k = 1 - (m+2)x^{m+1} + (m+1)x^{m+2} \rightarrow \mathcal{O}_{2m+4, 2m+6}$$

Moment (n, m)	$\alpha_s(m_\tau^2)$		Moment (n, m)	$\alpha_s(m_\tau^2)$	
	FOPT	CIPT		FOPT	CIPT
(1,0)	$0.315^{+0.012}_{-0.007}$	$0.327^{+0.012}_{-0.009}$	(2,0)	$0.311^{+0.015}_{-0.011}$	$0.314^{+0.013}_{-0.009}$
(1,1)	$0.319^{+0.010}_{-0.006}$	$0.340^{+0.011}_{-0.009}$	(2,1)	$0.311^{+0.011}_{-0.006}$	$0.333^{+0.009}_{-0.007}$
(1,2)	$0.322^{+0.010}_{-0.008}$	$0.343^{+0.012}_{-0.010}$	(2,2)	$0.316^{+0.010}_{-0.005}$	$0.336^{+0.011}_{-0.009}$
(1,3)	$0.324^{+0.011}_{-0.010}$	$0.345^{+0.013}_{-0.011}$	(2,3)	$0.318^{+0.010}_{-0.006}$	$0.339^{+0.011}_{-0.008}$
(1,4)	$0.326^{+0.011}_{-0.011}$	$0.347^{+0.013}_{-0.012}$	(2,4)	$0.319^{+0.009}_{-0.007}$	$0.340^{+0.011}_{-0.009}$
(1,5)	$0.327^{+0.015}_{-0.013}$	$0.348^{+0.014}_{-0.012}$	(2,5)	$0.320^{+0.010}_{-0.008}$	$0.341^{+0.011}_{-0.009}$



V+A

Exp.
errors
only

Non-Perturbative Contributions Neglected

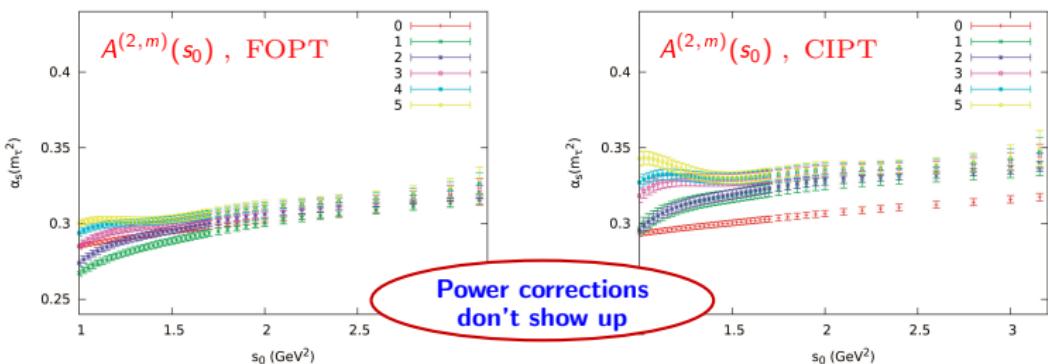
Rodríguez-Sánchez, A.P.

$$\omega^{(1,m)}(x) = 1 - x^{m+1} \rightarrow \mathcal{O}_{2m+4}$$

$$\omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^m (k+1)x^k = 1 - (m+2)x^{m+1} + (m+1)x^{m+2} \rightarrow \mathcal{O}_{2m+4, 2m+6}$$

Moment (n, m)	$\alpha_s(m_\tau^2)$		Moment (n, m)	$\alpha_s(m_\tau^2)$	
	FOPT	CIPT		FOPT	CIPT
(1,0)	$0.315^{+0.012}_{-0.007}$	$0.327^{+0.012}_{-0.009}$	(2,0)	$0.311^{+0.015}_{-0.011}$	$0.314^{+0.013}_{-0.009}$
(1,1)	$0.319^{+0.010}_{-0.006}$	$0.340^{+0.011}_{-0.009}$	(2,1)	$0.311^{+0.011}_{-0.006}$	$0.333^{+0.009}_{-0.007}$
(1,2)	$0.322^{+0.010}_{-0.008}$	$0.343^{+0.012}_{-0.010}$	(2,2)	$0.316^{+0.010}_{-0.005}$	$0.336^{+0.011}_{-0.009}$
(1,3)	$0.324^{+0.011}_{-0.010}$	$0.345^{+0.013}_{-0.011}$	(2,3)	$0.318^{+0.010}_{-0.006}$	$0.339^{+0.011}_{-0.008}$
(1,4)	$0.326^{+0.011}_{-0.011}$	$0.347^{+0.013}_{-0.012}$	(2,4)	$0.319^{+0.009}_{-0.007}$	$0.340^{+0.011}_{-0.009}$
(1,5)	$0.327^{+0.015}_{-0.013}$	$0.348^{+0.014}_{-0.012}$	(2,5)	$0.320^{+0.010}_{-0.008}$	$0.341^{+0.011}_{-0.009}$

Amazing stability



V+A

Exp. errors only

Models of Duality Violation

$$\Delta A_{V/A}^{\omega}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{\text{OPE}}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta \rho_{V/A}^{\text{DV}}(s)$$

Models of Duality Violation

$$\Delta A_{V/A}^{\omega}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{\text{OPE}}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta \rho_{V/A}^{\text{DV}}(s)$$

Ansatz: $\Delta \rho_{V/A}^{\text{DV}}(s) = s^{\lambda_{V/A}} e^{-(\delta_{V/A} + \gamma_{V/A}s)} \sin(\alpha_{V/A} + \beta_{V/A}s)$, $s > \hat{s}_0$

1) Boito et al.: $\lambda_{V/A} = 0$, $\hat{s}_0 \sim 1.55 \text{ GeV}^2$, $\omega(x) = 1$

- Fit s_0 dependence: $\rightarrow \{A^{(00)}(s_0), \rho(s_0 + \Delta s_0), \dots, \rho(s_0 + (n-1)\Delta s_0)\}$
- Direct fit of the spectral function. **OPE not valid**

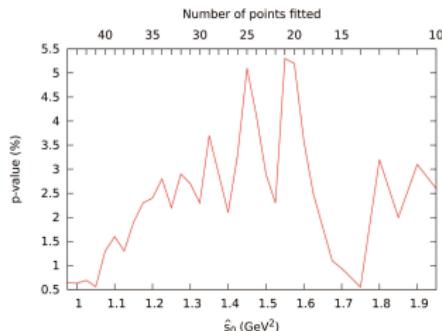
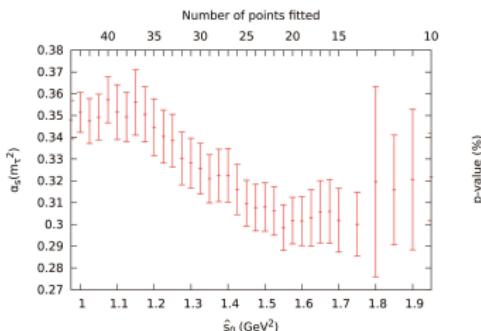
Models of Duality Violation

$$\Delta A_{V/A}^{\omega}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{\text{OPE}}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta \rho_{V/A}^{\text{DV}}(s)$$

Ansatz: $\Delta \rho_{V/A}^{\text{DV}}(s) = s^{\lambda_{V/A}} e^{-(\delta_{V/A} + \gamma_{V/A}s)} \sin(\alpha_{V/A} + \beta_{V/A}s)$, $s > \hat{s}_0$

1) Boito et al.: $\lambda_{V/A} = 0$, $\hat{s}_0 \sim 1.55 \text{ GeV}^2$, $\omega(x) = 1$

- Fit s_0 dependence: $\rightarrow \{A^{(00)}(s_0), \rho(s_0 + \Delta s_0), \dots, \rho(s_0 + (n-1)\Delta s_0)\}$
- Direct fit of the spectral function. **OPE not valid**



Rodríguez-Sánchez, A.P.

FOPT , V

(too large errors in A)

Bad quality fit (Model dependence. Instabilities. Very low p-value)

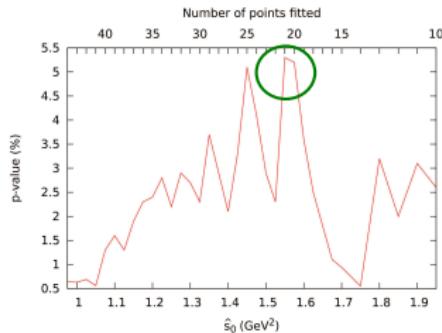
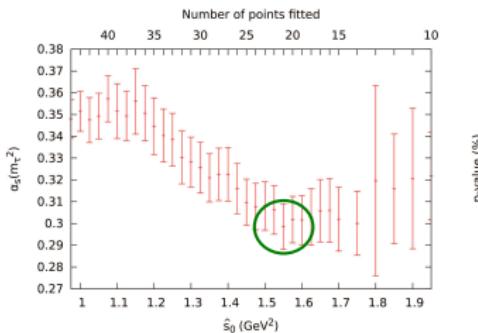
Models of Duality Violation

$$\Delta A_{V/A}^{\omega}(s_0) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{\text{OPE}}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta \rho_{V/A}^{\text{DV}}(s)$$

Ansatz: $\Delta \rho_{V/A}^{\text{DV}}(s) = s^{\lambda_{V/A}} e^{-(\delta_{V/A} + \gamma_{V/A} s)} \sin(\alpha_{V/A} + \beta_{V/A} s)$, $s > \hat{s}_0$

1) Boito et al.: $\lambda_{V/A} = 0$, $\hat{s}_0 \sim 1.55 \text{ GeV}^2$, $\omega(x) = 1$

- Fit s_0 dependence: $\rightarrow \{A^{(00)}(s_0), \rho(s_0 + \Delta s_0), \dots, \rho(s_0 + (n-1)\Delta s_0)\}$
- Direct fit of the spectral function. **OPE not valid**



Rodríguez-Sánchez, A.P.

FOPT , V

(too large errors in A)

Boito et al. value

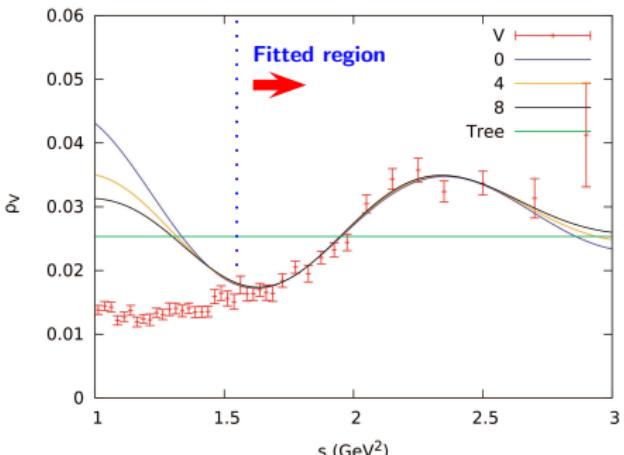
Bad quality fit (Model dependence. Instabilities. Very low p-value)

Ansatz: $\Delta\rho_{V/A}^{\text{DV}}(s) = s^{\lambda_{V/A}} e^{-(\delta_{V/A} + \gamma_{V/A}s)} \sin(\alpha_{V/A} + \beta_{V/A}s)$, $s > \hat{s}_0$

2) $\lambda_V \geq 0$: $\hat{s}_0 \sim 1.55 \text{ GeV}^2$, $\omega(x) = 1$

Rodríguez-Sánchez, A.P.

λ_V	$\alpha_s(m_\tau^2)^{\text{FOPT}}$	δ_V	γ_V	α_V	β_V	p-value
0	0.298 ± 0.010	3.6 ± 0.5	0.6 ± 0.3	-2.3 ± 0.9	4.3 ± 0.5	5.3 %
1	0.300 ± 0.012	3.3 ± 0.5	1.1 ± 0.3	-2.2 ± 1.0	4.2 ± 0.5	5.7 %
2	0.302 ± 0.011	2.9 ± 0.5	1.6 ± 0.3	-2.2 ± 0.9	4.2 ± 0.5	6.0 %
4	0.306 ± 0.013	2.3 ± 0.5	2.6 ± 0.3	-1.9 ± 0.9	4.1 ± 0.5	6.6 %
8	0.314 ± 0.015	1.0 ± 0.5	4.6 ± 0.3	-1.5 ± 1.1	3.9 ± 0.6	7.7 %



- Fitted α_s is model dependent
- $\lambda_V = 0$ (Boito) gives the worse fit
- Fit quality & α_s increase with λ_V
- closer to data at $s < \hat{s}_0$
- $\Delta\hat{s}_0 \rightarrow 3$ times larger errors

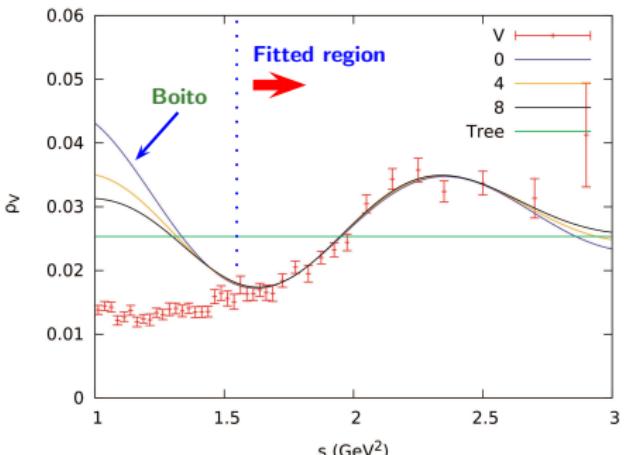
Not competitive & unreliable

Ansatz: $\Delta\rho_{V/A}^{DV}(s) = s^{\lambda_{V/A}} e^{-(\delta_{V/A} + \gamma_{V/A}s)} \sin(\alpha_{V/A} + \beta_{V/A}s)$, $s > \hat{s}_0$

2) $\lambda_V \geq 0$: $\hat{s}_0 \sim 1.55 \text{ GeV}^2$, $\omega(x) = 1$

Rodríguez-Sánchez, A.P.

	λ_V	$\alpha_s(m_\tau^2)^{\text{FOPT}}$	δ_V	γ_V	α_V	β_V	p-value
Boito	0	0.298 ± 0.010	3.6 ± 0.5	0.6 ± 0.3	-2.3 ± 0.9	4.3 ± 0.5	5.3 %
	1	0.300 ± 0.012	3.3 ± 0.5	1.1 ± 0.3	-2.2 ± 1.0	4.2 ± 0.5	5.7 %
	2	0.302 ± 0.011	2.9 ± 0.5	1.6 ± 0.3	-2.2 ± 0.9	4.2 ± 0.5	6.0 %
	4	0.306 ± 0.013	2.3 ± 0.5	2.6 ± 0.3	-1.9 ± 0.9	4.1 ± 0.5	6.6 %
	8	0.314 ± 0.015	1.0 ± 0.5	4.6 ± 0.3	-1.5 ± 1.1	3.9 ± 0.6	7.7 %



- Fitted α_s is model dependent
- $\lambda_V = 0$ (Boito) gives the worse fit
- Fit quality & α_s increase with λ_V
- closer to data at $s < \hat{s}_0$
- $\Delta\hat{s}_0 \rightarrow 3$ times larger errors

Not competitive & unreliable

New analysis of ALEPH data

Rodríguez-Sánchez, Pich, arXiv:1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments ¹	$0.339^{+0.019}_{-0.017}$	$0.319^{+0.017}_{-0.015}$	$0.329^{+0.020}_{-0.018}$
Mod. ALEPH moments ²	$0.338^{+0.014}_{-0.012}$	$0.319^{+0.013}_{-0.010}$	$0.329^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments ³	$0.336^{+0.018}_{-0.016}$	$0.317^{+0.015}_{-0.013}$	$0.326^{+0.018}_{-0.016}$
s_0 dependence ⁴	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform ⁵	$0.328^{+0.014}_{-0.013}$	$0.318^{+0.015}_{-0.012}$	$0.323^{+0.015}_{-0.013}$
Combined value	0.335 ± 0.013	0.320 ± 0.012	0.328 ± 0.013



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$1) \quad \omega_{kl}(x) = (1+2x)(1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

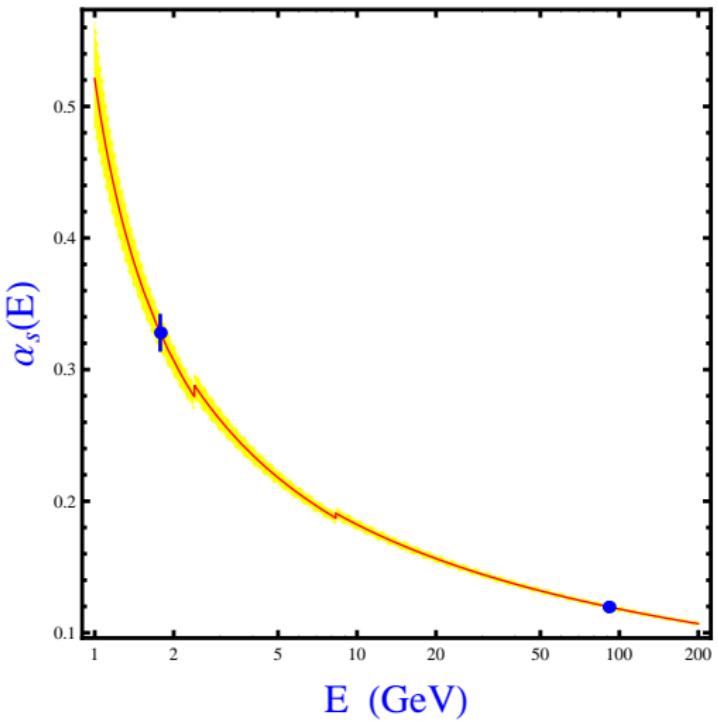
$$2) \quad \tilde{\omega}_{kl}(x) = (1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

$$3) \quad \omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^m (k+1)x^k = 1 - (m+2)x^{m+1} + (m+1)x^{m+2}, \quad 1 \leq m \leq 5$$

$$4) \quad \omega^{(2,m)}(x) \quad 0 \leq m \leq 2, \quad 1 \text{ single moment in each fit}$$

$$5) \quad \omega_a^{(1,m)}(x) = (1-x^{m+1})e^{-ax} \quad 0 \leq m \leq 6$$

α_s at N³LO from τ and Z



$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$

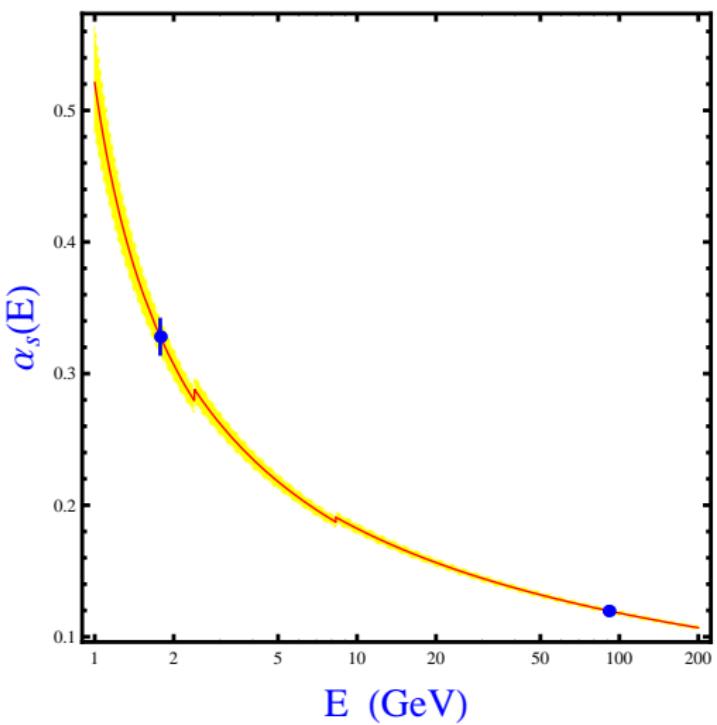
$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_Z \text{ width} = 0.1196 \pm 0.0030$$

A very precise test of
Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0001 \pm 0.0015_\tau \pm 0.0030_Z$$

α_s at N³LO from τ and Z



$$\alpha_s(M_Z^2)_{\text{Lattice}} = 0.11852 \pm 0.00084$$

A. Pich

α_s from τ data

$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{\text{Z width}} = 0.1196 \pm 0.0030$$

A very precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0001 \pm 0.0015_\tau \pm 0.0030_Z$$

(ALPHA, arXiv:1706.03821)

A scenic night view of a canal in Bruges, Belgium. The image shows a row of historic buildings with intricate facades and tall spires reflected in the calm water of the canal. The buildings are illuminated from within, casting a warm glow onto the water. A small bridge arches over the canal on the left. The sky is a deep blue, suggesting it's either dusk or dawn.

Backup

The 15th International Workshop on Tau Lepton Physics (TAU 2018)
Vondelkerk, Amsterdam, The Netherlands, 24–28 September 2018

α_s determination with ALEPH-like fit

Rodríguez-Sánchez, A.P.

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right)$$

$$(k, l) = (0, 0) \rightarrow \alpha_s, \mathcal{O}_{6V/A}, \mathcal{O}_{8V/A}$$

$$(k, l) = (1, 0) \rightarrow \alpha_s, \langle a_s GG \rangle, \mathcal{O}_{6V/A}, \mathcal{O}_{8V/A}, \mathcal{O}_{10V/A}$$

$$(k, l) = (1, 1) \rightarrow \alpha_s, \langle a_s GG \rangle, \mathcal{O}_{6V/A}, \mathcal{O}_{8V/A}, \mathcal{O}_{10V/A}, \mathcal{O}_{12V/A}$$

$$(k, l) = (1, 2) \rightarrow \alpha_s, \mathcal{O}_{6V/A}, \mathcal{O}_{8V/A}, \mathcal{O}_{10V/A}, \mathcal{O}_{12V/A}, \mathcal{O}_{14V/A}$$

$$(k, l) = (1, 3) \rightarrow \alpha_s, \mathcal{O}_{8V/A}, \mathcal{O}_{10V/A}, \mathcal{O}_{12V/A}, \mathcal{O}_{14V/A}, \mathcal{O}_{16V/A}$$

Channel	$\alpha_s(m_\tau^2)$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ (10^{-3} GeV 4)	\mathcal{O}_6 (10^{-3} GeV 6)	\mathcal{O}_8 10^{-3} GeV 8)
V (FOPT)	$0.328^{+0.013}_{-0.007}$	8^{+7}_{-14}	$-3.2^{+0.8}_{-0.5}$	$5.0^{+0.4}_{-0.7}$
V (CIPT)	$0.352^{+0.013}_{-0.011}$	-8^{+7}_{-7}	$-3.5^{+0.3}_{-0.3}$	$4.9^{+0.4}_{-0.5}$
A (FOPT)	$0.304^{+0.010}_{-0.007}$	-15^{+5}_{-8}	$4.4^{+0.5}_{-0.4}$	$-5.8^{+0.3}_{-0.4}$
A (CIPT)	$0.320^{+0.011}_{-0.010}$	-25^{+5}_{-5}	$4.3^{+0.2}_{-0.2}$	$-5.8^{+0.3}_{-0.3}$
V+A (FOPT)	$0.319^{+0.010}_{-0.006}$	-3^{+6}_{-11}	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
V+A (CIPT)	$0.339^{+0.011}_{-0.009}$	-16^{+5}_{-5}	$0.9^{+0.3}_{-0.4}$	$-1.0^{+0.5}_{-0.7}$

Good agreement with Davier et al. (arXiv:1312.1501)

① Fit one more condensate to test stability/uncertainties

Channel	$\alpha_s(m_\tau^2)$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ (10^{-3} GeV 4)	\mathcal{O}_6 (10^{-3} GeV 6)	\mathcal{O}_8 (10^{-3} GeV 8)	\mathcal{O}_{10} (10^{-3} GeV 10)
V (FOPT)	$0.320^{+0.016}_{-0.014}$	10^{+9}_{-17}	-4^{+3}_{-2}	6^{+2}_{-2}	-2^{+5}_{-5}
V (CIPT)	$0.337^{+0.020}_{-0.019}$	-1^{+10}_{-10}	-5^{+2}_{-2}	6^{+2}_{-2}	-4^{+4}_{-4}
A (FOPT)	$0.347^{+0.022}_{-0.021}$	-31^{+16}_{-33}	11^{+5}_{-4}	-12^{+4}_{-4}	15^{+9}_{-9}
A (CIPT)	$0.373^{+0.029}_{-0.029}$	-50^{+18}_{-16}	10^{+3}_{-3}	-11^{+3}_{-3}	14^{+7}_{-7}
V+A (FOPT)	$0.333^{+0.013}_{-0.012}$	-8^{+10}_{-24}	7^{+7}_{-4}	-5^{+4}_{-6}	12^{+12}_{-9}
V+A (CIPT)	$0.355^{+0.016}_{-0.015}$	-23^{+10}_{-8}	5^{+3}_{-3}	-5^{+3}_{-3}	10^{+8}_{-8}

- Good stability of α_s with respect to previous fit
- Larger variation in condensates values and increased errors

② Take central values from first fit, adding differences as errors

α_s determination with ALEPH-like fit

Rodríguez-Sánchez, A.P.

$$\omega_{kl}(x) = (1-x)^{2+k} x^l (1+2x) \quad , \quad x = s/m_\tau^2 \quad , \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

Channel	$\alpha_s(m_\tau^2)$	$\langle \frac{\alpha_s}{\pi} GG \rangle$ (10^{-3} GeV 4)	\mathcal{O}_6 (10^{-3} GeV 6)	\mathcal{O}_8 (10^{-3} GeV 8)
V (FOPT)	$0.328^{+0.013}_{-0.007}$	8^{+7}_{-14}	$-3.2^{+0.8}_{-0.5}$	$5.0^{+0.4}_{-0.7}$
V (CIPT)	$0.352^{+0.013}_{-0.011}$	-8^{+7}_{-7}	$-3.5^{+0.3}_{-0.3}$	$4.9^{+0.4}_{-0.5}$
A (FOPT)	$0.304^{+0.010}_{-0.007}$	-15^{+5}_{-8}	$4.4^{+0.5}_{-0.4}$	$-5.8^{+0.3}_{-0.4}$
A (CIPT)	$0.320^{+0.011}_{-0.010}$	-25^{+5}_{-5}	$4.3^{+0.2}_{-0.2}$	$-5.8^{+0.3}_{-0.3}$
V+A (FOPT)	$0.319^{+0.010}_{-0.006}$	-3^{+6}_{-11}	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
V+A (CIPT)	$0.339^{+0.011}_{-0.009}$	-16^{+5}_{-5}	$0.9^{+0.3}_{-0.4}$	$-1.0^{+0.5}_{-0.7}$

- High sensitivity to α_s . Bad sensitivity to power corrections
- Cancellation in $\mathcal{O}_{6,V+A}$ confirmed. V + A more reliable
- $K_5 = 275 \pm 400$, $\mu^2 = (0.5, 2) m_\tau^2$
- Best values taken from V + A. Errors increased with sensitivity to \mathcal{O}_{10}

$$\alpha_s(m_\tau^2)^{\text{CIPT}} = 0.339^{+0.019}_{-0.017}$$

$$\alpha_s(m_\tau^2)^{\text{FOPT}} = 0.319^{+0.017}_{-0.015}$$



$$\alpha_s(m_\tau^2) = 0.329^{+0.020}_{-0.018}$$

Good agreement with Davier et al.: $\alpha_s(m_\tau^2) = 0.332 \pm 0.012$

(arXiv:1312.1501)

Playing with the s_0 dependence

$$\omega^{(20)}(s = s_0 x) = (1 - x)^2 \quad \mathcal{O}_4, \mathcal{O}_6$$

$$\omega^{(21)}(s = s_0 x) = (1 - x)^2 (1 + 2x) \quad \mathcal{O}_6, \mathcal{O}_8$$

$$\omega^{(22)}(s = s_0 x) = (1 - x)^2 (1 + 2x + 3x^2) \quad \mathcal{O}_8, \mathcal{O}_{10}$$

Fit to the (9) $s_0 > 2 \text{ GeV}^2$ points. One moment only (avoid correlations)

V + A channel

Rodríguez-Sánchez, A.P.

Moment	Method	$\alpha_s(m_\tau^2)$	Lower-D Condensate (10^{-3} GeV^D)	Higher-D Condensate (10^{-3} GeV^D)
$A\omega^{(20)}(s_0)$	FOPT	$0.331^{+0.013}_{-0.018}$	-9^{+12}_{-4}	-4^{+3}_{-7}
	CIPT	$0.333^{+0.011}_{-0.009}$	-11^{+7}_{-6}	0 ± 1
$A\omega^{(21)}(s_0)$	FOPT	$0.322^{+0.010}_{-0.006}$	3^{+1}_{-2}	0 ± 2
	CIPT	$0.334^{+0.011}_{-0.009}$	0 ± 1	2 ± 2
$A\omega^{(22)}(s_0)$	FOPT	$0.319^{+0.009}_{-0.006}$	-2^{+3}_{-2}	-1^{+4}_{-5}
	CIPT	$0.334^{+0.011}_{-0.009}$	2 ± 2	-5 ± 4

$$\alpha_s(m_\tau^2)^{\text{CIPT}} = 0.335 \pm 0.014$$

$$\alpha_s(m_\tau^2)^{\text{FOPT}} = 0.323 \pm 0.012$$



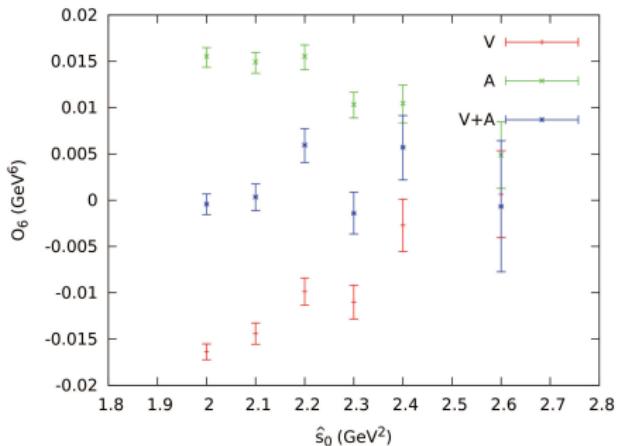
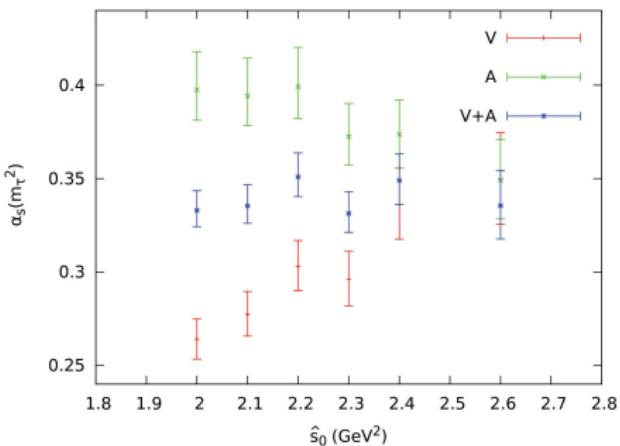
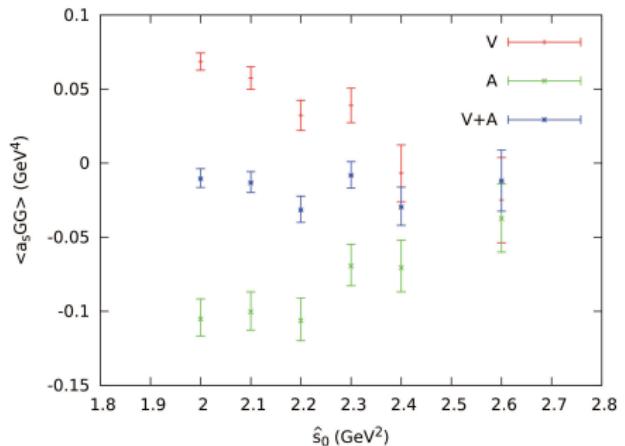
$$\alpha_s(m_\tau^2) = 0.329 \pm 0.013$$

$$A^{\omega(20)}(s_0)$$

CIPT

$$\omega^{(20)}(s = s_0 x) = (1 - x)^2$$

Fit to last $n = 4, \dots, 9$ s_0 points
vs. starting s_0 fitted value



$$\alpha_s(m_\tau^2)^{\text{CIPT}} = 0.335 \pm 0.014$$

$$\alpha_s(m_\tau^2)^{\text{FOPT}} = 0.323 \pm 0.012$$



$$\alpha_s(m_\tau^2) = 0.329 \pm 0.013$$

BUT...

- **Bad quality fit ($\chi^2_{\min}/\text{d.o.f.}$)**
- **Much worse behaviour in separate V & A channels**
- **Fitting the s_0 dependence removes pinching:**

Fitting m points of $A^{(n0)}(s_0)$ is equivalent to a fit of

$$\left\{ A^{(n0)}(s_0), A^{(n-10)}(s_0), \dots, A^{(00)}(s_0), \text{Im } \Pi(s_0), \frac{d \text{Im } \Pi(s_0)}{ds_0}, \dots, \frac{d^{m-n-1} \text{Im } \Pi(s_0)}{ds_0} \right\}$$

Local duality assumed!

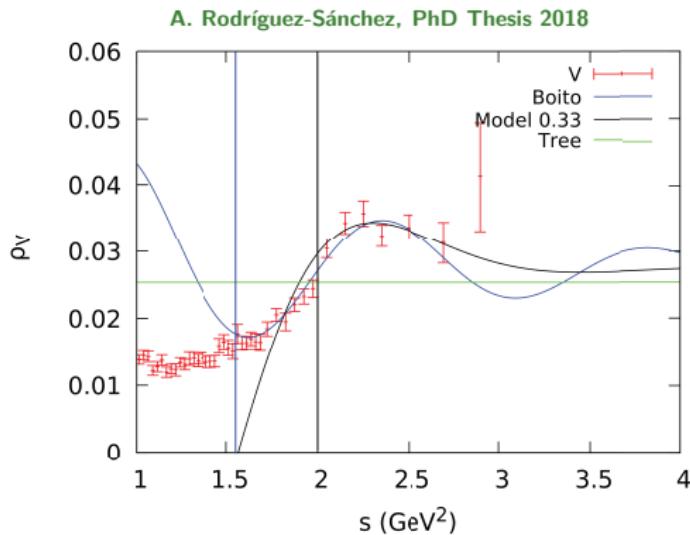


Violations of Duality

(estimated uncertainty included in quoted error)

Ansatz: $\Delta\rho_V^{\text{DV}}(s) = e^{-(\delta_V + \gamma_V s)} \sin(\alpha_V + \beta_V s)$, $s > \hat{s}_0$

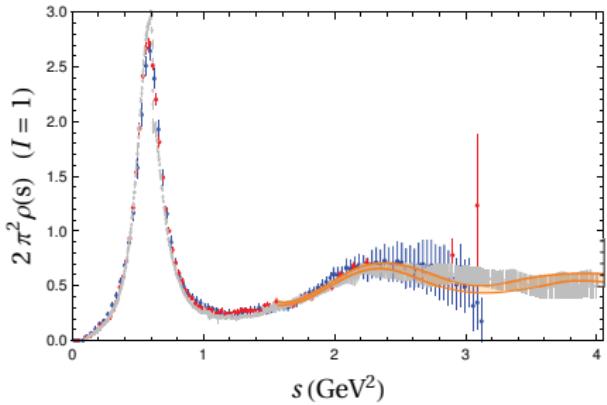
Fitted α_s is model dependent: $\{\alpha_s(m_\tau)^{\text{FOPT}}; \delta_V, \gamma_V, \alpha_V, \beta_V; \hat{s}_0\}$



- **Boito et al model:** $\{0.298 ; 3.6, 0.6, -2.3, 4.3 ; 1.55 \text{ GeV}\}$ p-value = 5%
- **Model 0.33:** $\{0.330 ; 0.51, 1.88, 0.84, 2.78 ; 2 \text{ GeV}\}$ p-value = 8%

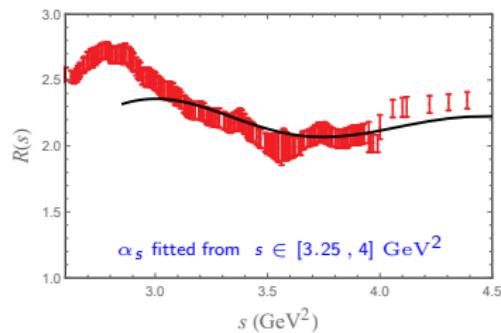
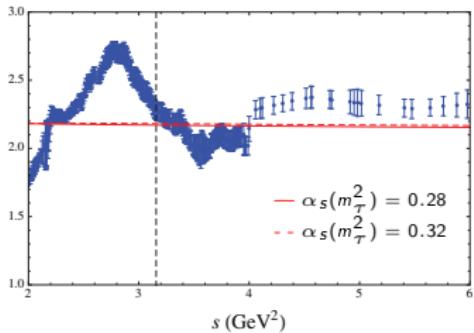
R_{ee} data analysis

Boito et al, 1805.08176

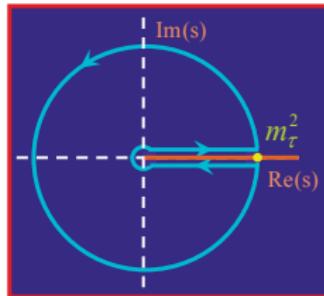


Data don't agree with
fitted (τ) DV ansatz

- ALEPH
- OPAL
- e^+e^- data
- DV ansatz (Boito et al)



$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) a_\tau (-x m_\tau^2)^n = a_\tau^n + \dots ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$



$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24} \beta_1 a_\tau^2 + \left[\beta_1^2 \left(\frac{265}{288} - \frac{\pi^2}{12} \right) - \frac{19}{24} \beta_2 \right] a_\tau^3 + \dots$$

$$a(-s) \simeq \frac{a_\tau}{1 - \frac{\beta_1}{2} a_\tau \log(-s/m_\tau^2)} = \frac{a_\tau}{1 - i \frac{\beta_1}{2} a_\tau \phi} = a_\tau \sum_n \left(i \frac{\beta_1}{2} a_\tau \phi \right)^n ; \quad \phi \in [0, 2\pi]$$

FOPT expansion only convergent if $\alpha_\tau < 0.14$ (0.11) [at 1 (3) loops]

Experimentally $\alpha_\tau \approx 0.11$ **FOPT should not be used** (divergent series)

FOPT suffers a large renormalization-scale dependence (Le Diberder- Pich , Menke)

The difference between FOPT and CIPT grows at higher orders