

Lepton Dipole Moments
Theoretical Perspective

&

(Tau Connection)

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Outline

1. Introduction

2. Anomalous Magnetic Moments: Status

i) **Electron** (5 loops!) (4 loops known functions! **Laporta**)

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{SM}} = -87(28)_{\text{exp}}(23)_{\text{th}} \times 10^{-14}$$

(**2.4 σ**) Very Large! Expect $O(6 \times 10^{-14}) \sim m_e^2/m_\mu^2 \Delta a_\mu$

$$\alpha^{-1}(a_e) = 137.035999149(33)_{\text{exp}}$$

$$\alpha^{-1}(^{133}\text{Cs}) = 137.035999046(27)$$

$$\alpha^{-1}(^{87}\text{Rb}) = 137.035998995(85)$$

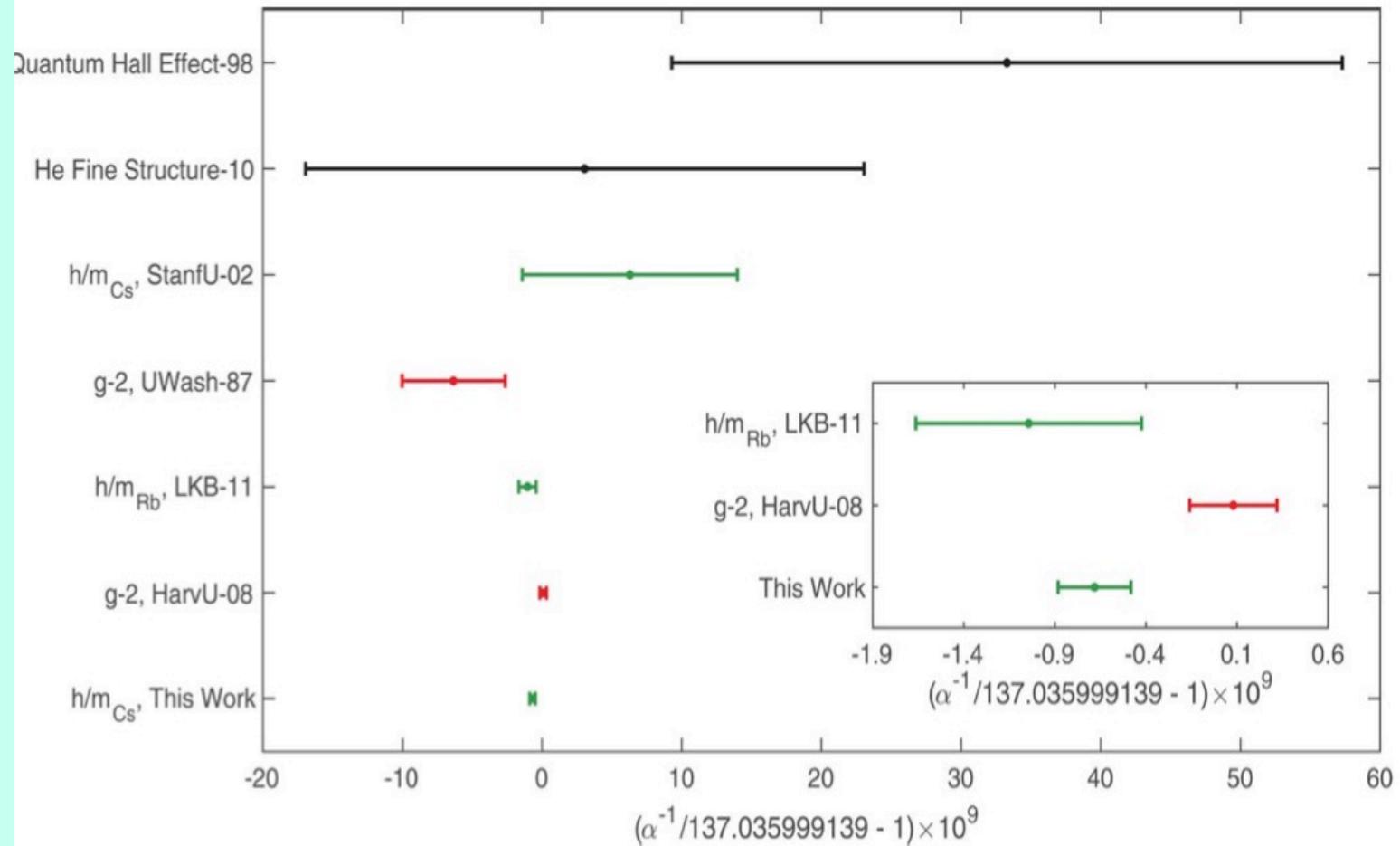
ii) **Muon** Discrepancy

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(63)_{\text{exp}}(37)_{\text{theory}} \times 10^{-11} \quad (\mathbf{3.7\sigma!})$$

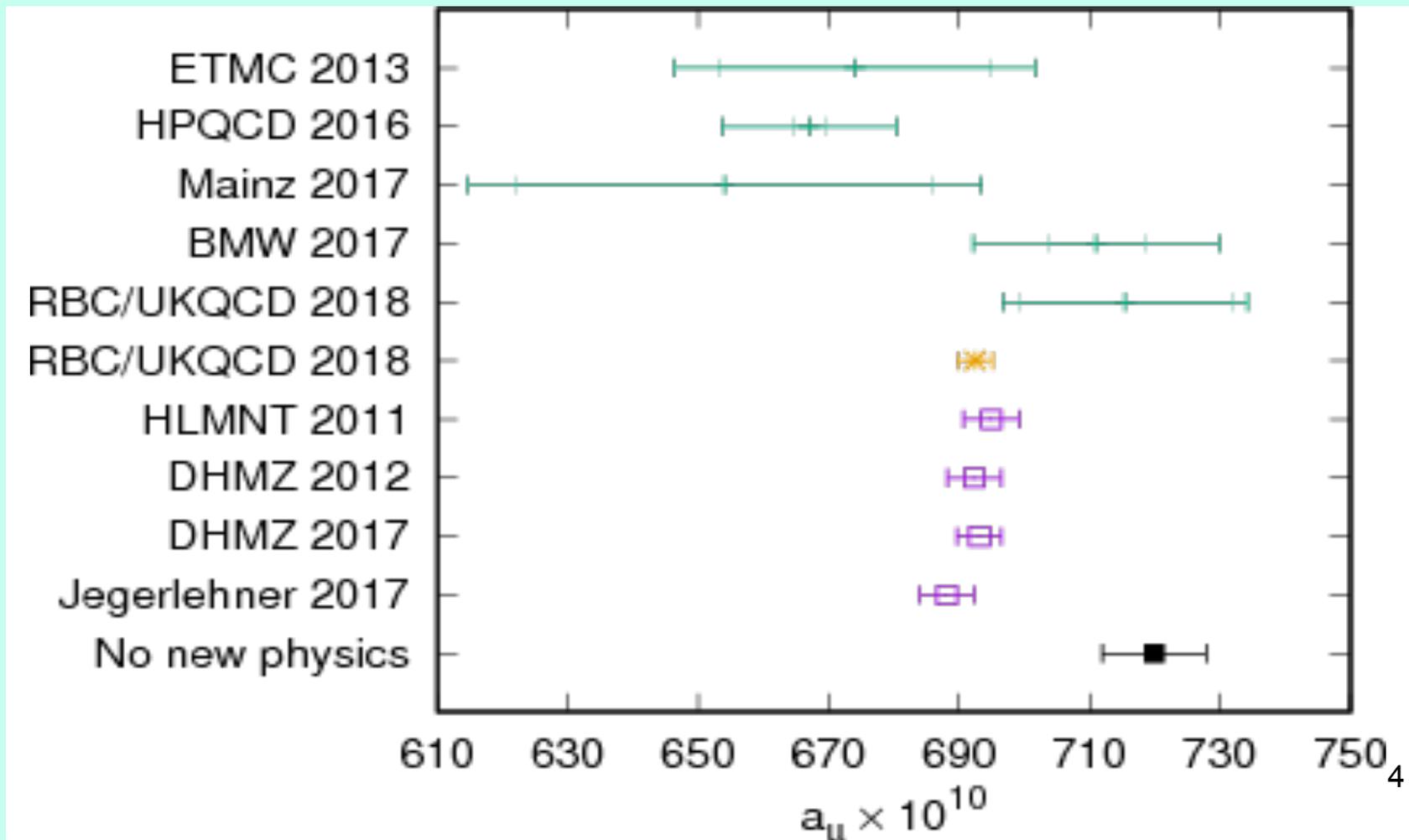
(**Hadronic effects?, Experiment? or New Physics?**)

Lattice QCD comes of Age: HVP & LBL

New Measurement of alpha: From Parker et al. Science 2018



From Blum et al. PRL 2018
Green = Lattice QCD
Purple = e^+e^- data
Gold = Mixed LQCD & e^+e^- data



Electroweak 2 Loop Corrections: Lessons Learned

3. Some g_{μ}^{-2} “New Physics” Scenarios

i) Supersymmetry - LHC Tension?

**ii) 2 Higgs Doublet Models (new favorite)*

Relatively Light (Pseudo)Scalar (2 loop g^{-2})

$$10\text{GeV} < m_A < 25\text{GeV} \quad 30 < \tan\beta = v_2/v_1 < 80$$

D. Chang et al. (2001), K. Cheung et al.(2001), ...

L. Wang et al. (2018), S. Dutta et al.(2018) ...

iii) “Light” Dark Photon

iv) Light Scalar $O(0.25\text{-few GeV})$

Giudice et al (2012), Chen et al (2016), Batell et al (2016)...

Implications For Tau Physics

e^+e^- , $\mu^+\mu^-$ Bump Hunting in tau decay or production

Many Other Scenarios

$L_\mu - L_\tau$, $B-L$ etc. gauged...

Leptoquarks & B anomalies

Dynamical Mass Generation

$$\Delta a_\mu = m_\mu^2/M^2 \quad \underline{M \leq 1\text{TeV!}}$$

etc.

Challenge: Dual solution to Δa_μ & Δa_e ?

see Davoudiasl & WJM (2018) 1 & 2 loop effects

The Genius of Dirac

The Dirac Equation (1928)

90th Anniversary

Unified

QM+Special Rel.+Spin+EM Gauge Invariance

First Order Equation

$$i(\partial_\mu - ieA_\mu(\mathbf{x}))\gamma^\mu\psi(\mathbf{x}) = m_e\psi(\mathbf{x}),$$

$$4\times 4 \gamma^\mu \text{ (Dirac) matrices: } \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}I$$

$g_e=2$ Success & Antimatter Predicted!

Post WWII Physics Boom (1947-48)

Dirac Eq. Predicted: $g_e=2$ Exactly?

1947 Small Anomalous Atomic Fine Structure Effects

1948 Schwinger Calculates: $a_e = \alpha/2\pi \approx \underline{0.00116}$

($\alpha = e^2/4\pi = 1/137$)

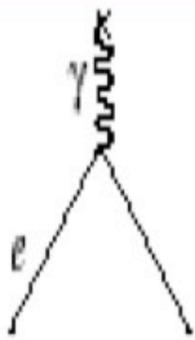
Agreed with measurement of Kusch & Foley!

$a_e^{\text{exp}} \approx \underline{0.00119(5)}$

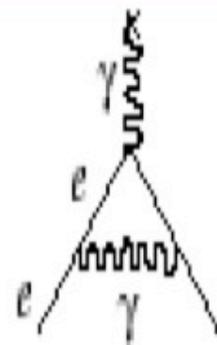
Great Success of QED-Quantum Field Theory

2018 Schwinger Centennial Year

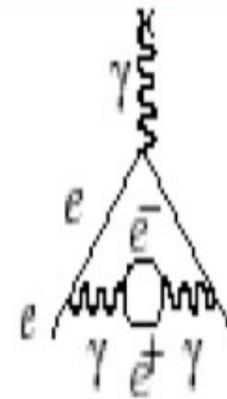
Some Anomalous Magnetic Moment Contributions



Dirac



Schwinger



2. Anomalous Magnetic Moments Today

$$a_l = (g_l - 2)/2 \quad l = e, \mu$$

$$a_e(\text{exp}) = 0.00115965218073(28) \quad \text{unc. } \pm 28 \times 10^{-14}!$$

[\(Hanneke, Fogwell, Gabrielse: PRL 2008\)](#)

$$g_e = 2.00231930436146(56)$$

Most precisely known dimensionless physical quantity!

Future: factor ≥ 4 improvement? Good Goal

$$a_e(\text{SM}) = \alpha/2\pi - 0.328478444002546(\alpha/\pi)^2$$

$$+ 1.181234016(\alpha/\pi)^3 - 1.912245764(\alpha/\pi)^4$$

$$+ \underline{6.675(192)(\alpha/\pi)^5} \dots + 1.69(1) \times 10^{-12}(\text{had}) + 0.03 \times 10^{-12}(\text{EW})$$

[Aoyama, Kinoshita, & Nio 2017 Update!](#)

[Spectacular Achievement](#)

Uncertainty $\pm 1.3 \times 10^{-14}$ (QED theory) $\pm 1.2 \times 10^{-14}$ (Hadronic)

Alpha determination using Rydberg Constant

$$R_\infty = 1.0973731568527(73) \times 10^7 \text{m}^{-1}$$

$$\frac{1}{2}m_e \alpha^2 = 13.60569253(30) \text{eV}$$

$$\alpha^{-1}(\text{Cs}) = 137.035999046(27)$$

R. Parker et al. Science (2018) major improvement!

$$\Delta a_e = a_e(\text{exp}) - a_e(\text{theory}) = -87(36) \times 10^{-14} \text{ Note Sign}$$

Error Budget: $\pm 23 \times 10^{-14}$ (alpha) $\pm 28 \times 10^{-14}$ (exp.) $\pm 2 \times 10^{-14}$ (th.)

Ongoing Cs exp. Goal - additional factor of 10 improvement!

Δa_e 3.5 sigma Sensitivity

Further Improvement? New Experiment (4 x better a_e)

Is it a harbinger of “New Physics”?

H. Davoudiasl & WJM (2018) – Light Scalar

Muon Anomalous Magnetic Moment

1957 Garwin, Lederman & Weinrich study $\pi \rightarrow \mu \nu$, $\mu \rightarrow e \nu \nu$
found parity violation & measured $g_\mu = 2.00 \pm 0.10$

Parity Violation Decay \rightarrow Self Analyzing Polarimeter
led to Three Classic CERN Exps. ending in 1977

“The Last $g_\mu - 2$ Experiment”

- Until Experimental E821 at BNL (2004 Final)

- $a_\mu^{\text{exp}} \equiv (g_\mu - 2)/2 = 116592091(54)_{\text{stat}}(33)_{\text{sys}}$

- $= \underline{116592091(63)} \times 10^{-11}$

Factor of 14 improvement over CERN results (statistics)

(Goal: Factor 4 further Improvement at FNAL) $\pm 16 \times 10^{-11}$

New JPARC low energy experiment

Standard Model Prediction

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{Hadronic}} \text{ (quark/gluon loops)}$$

QED Contributions:

- $a_{\mu}^{\text{QED}} = 0.5(\alpha/\pi) + 0.765857425(\alpha/\pi)^2 +$
 $24.05050996(\alpha/\pi)^3 +$
 $130.8796(\alpha/\pi)^4 +$
 $752.173(\alpha/\pi)^5 + \dots$

2017 Update: Aoyama, Kinoshita, & Nio

$$\alpha^{-1}(^{133}\text{Cs}) = 137.035999046(27) \text{ from } h/m_{\text{Cs}}$$

$$a_{\mu}^{\text{QED}} = \underline{116584718.92(3)} \times 10^{-11} \text{ Very Precise!}$$

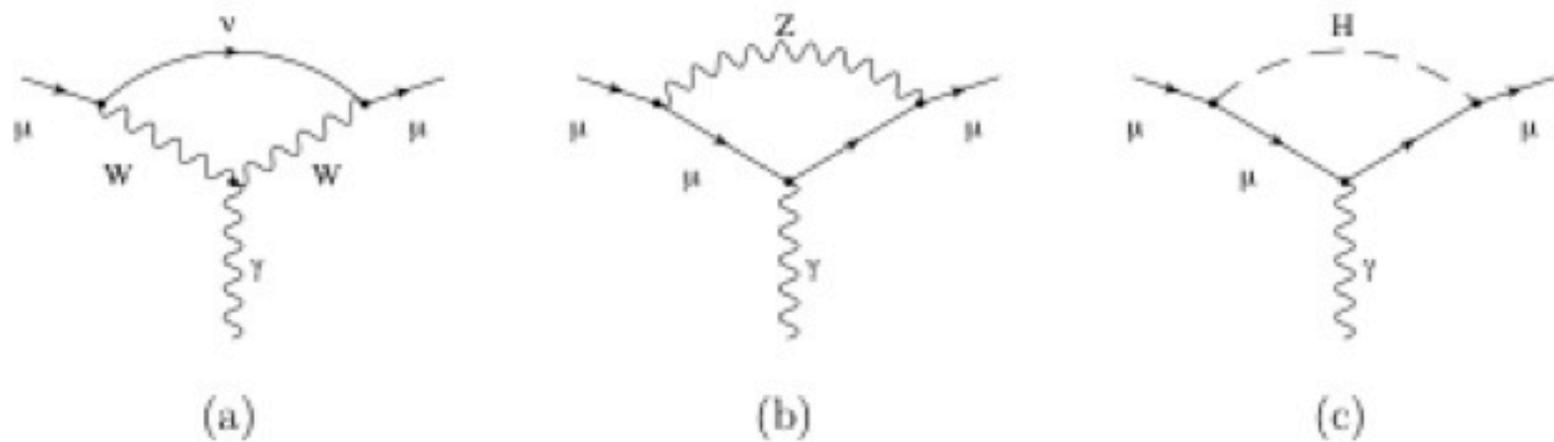


Figure 2: One-loop electroweak radiative corrections to a_μ .

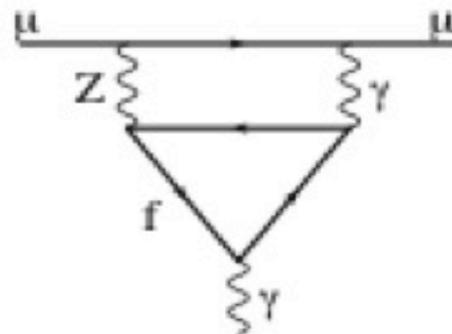


FIG. 3: Effective $Z\gamma\gamma^*$ coupling induced by a fermion triangle, contributing to a_μ^{EW} .

2.) Early EW Radiative Corrections

Muon Anomalous Magnetic Moment: $a_\mu^{\text{EW}} = \Delta(g_\mu - 2)/2$

Jackiw & Weinberg (1972) Bars & Yoshimura (1972)

Altarelli, Cabibbo & Maiani (1972) Fujikawa, Lee & Sanda (1972)

Bardeen, Gastmans & Lautrup (1972)

Finite because $g_W = 2$ three 1 loop diagrams W, Z & H

$$a_\mu^{\text{EW}}(1 \text{ loop}) = 5G_F m_\mu^2 / 24(2^{1/2})\pi^2 [1 + 1/5(1 - 4\sin^2\theta_W) + O(m_\mu^2/M^2)] = \underline{195 \times 10^{-11}}$$

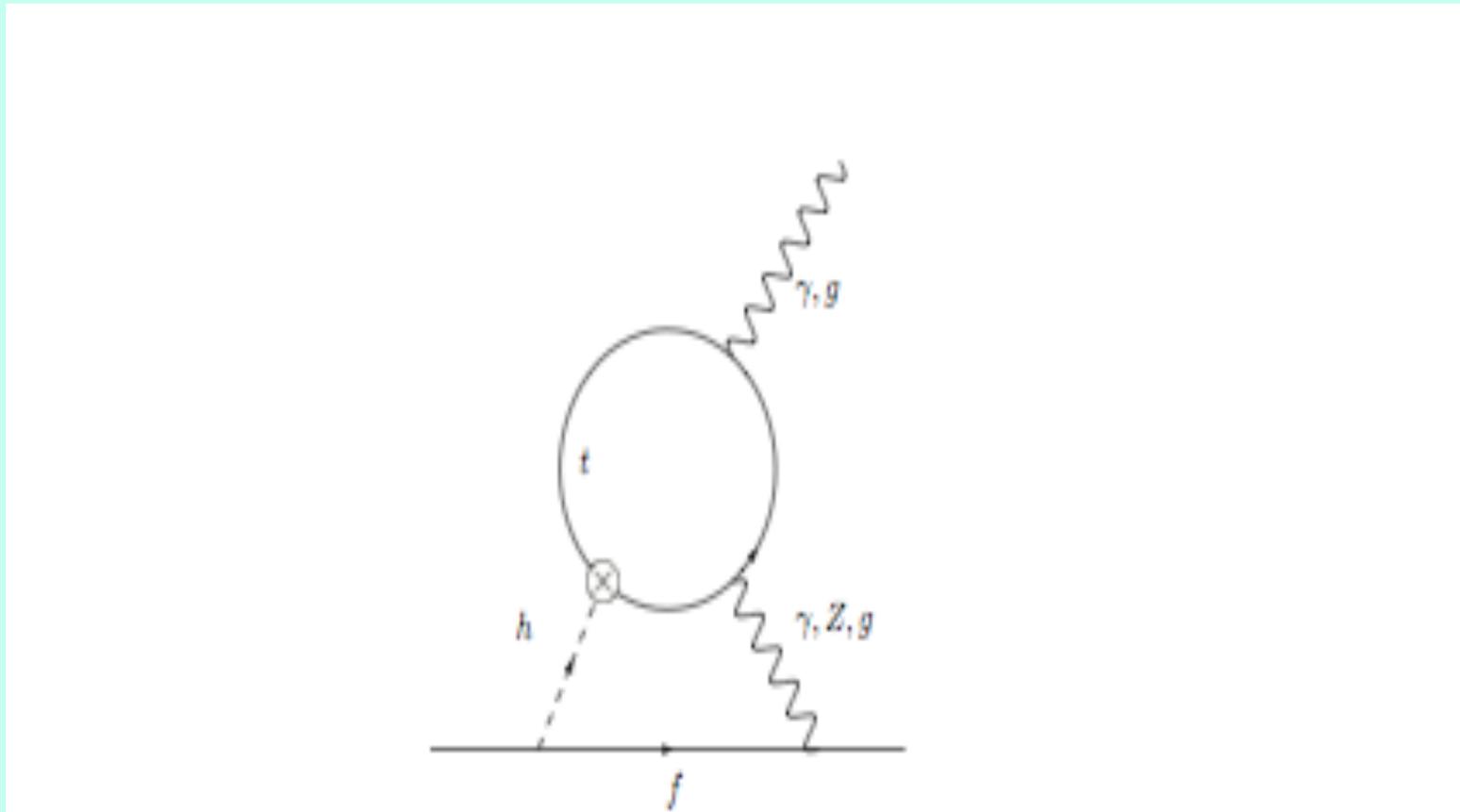
non-linear gauge, Dimensional, Regularization renormalization 1678

2 loops (Czarnecki, Krause, WJM 1995) -20% reduction to 154×10^{-11}

Currently: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 276(73) \times 10^{-11}$ (**3.7σ**) deviation

New Problem: $a_e^{\text{exp}} - a_e^{\text{SM}} = -87(36) \times 10^{-14}$ (**2.4σ**) deviation

Two Loop Higgs Contribution to fermion Dipole Moments Barr-Zee Diagrams



Electroweak Loop Effects

$a_{\mu}^{EW}(1 \text{ loop}) = \underline{194.8 \times 10^{-11}}$ original goal of E821

$a_{\mu}^{EW}(2 \text{ loop}) = \underline{-41.2(1.0) \times 10^{-11}}$ (Higgs Mass = 125 GeV)

Higgs 2 Loop $\sim 3 \times 10^{-11}$

3 loop EW leading logs very small $O(10^{-12})$

- $a_{\mu}^{EW} = \underline{154(1) \times 10^{-11}}$ *Non Controversial*

- Hadronic Contributions (HVP & HLBL)

$a_{\mu}^{\text{Had}}(\text{V.P.})^{\text{LO}} = \underline{6923(37) \times 10^{-11}}$

$a_{\mu}^{\text{Had}}(\text{V.P.})^{\text{NLO+NNLO}} = -86(1) \times 10^{-11}$

$a_{\mu}^{\text{Had}}(\text{LBL}) = 105(26) \times 10^{-11}$ (Consensus?) 3 loop $(\alpha/\pi)^3$ QCD

$a_{\mu}^{\text{SM}} = \underline{116591815(37) \times 10^{-11}}$ *(Future Improvement?)*

$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = \underline{274(63)(37) \times 10^{-11}}$ *(3.7 σ deviation!)*

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Comparison of Experiment and Theory

- $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(63)(37) \times 10^{-11}$ (3.7σ !) see C. Lehner Talk

This is a very large deviation!

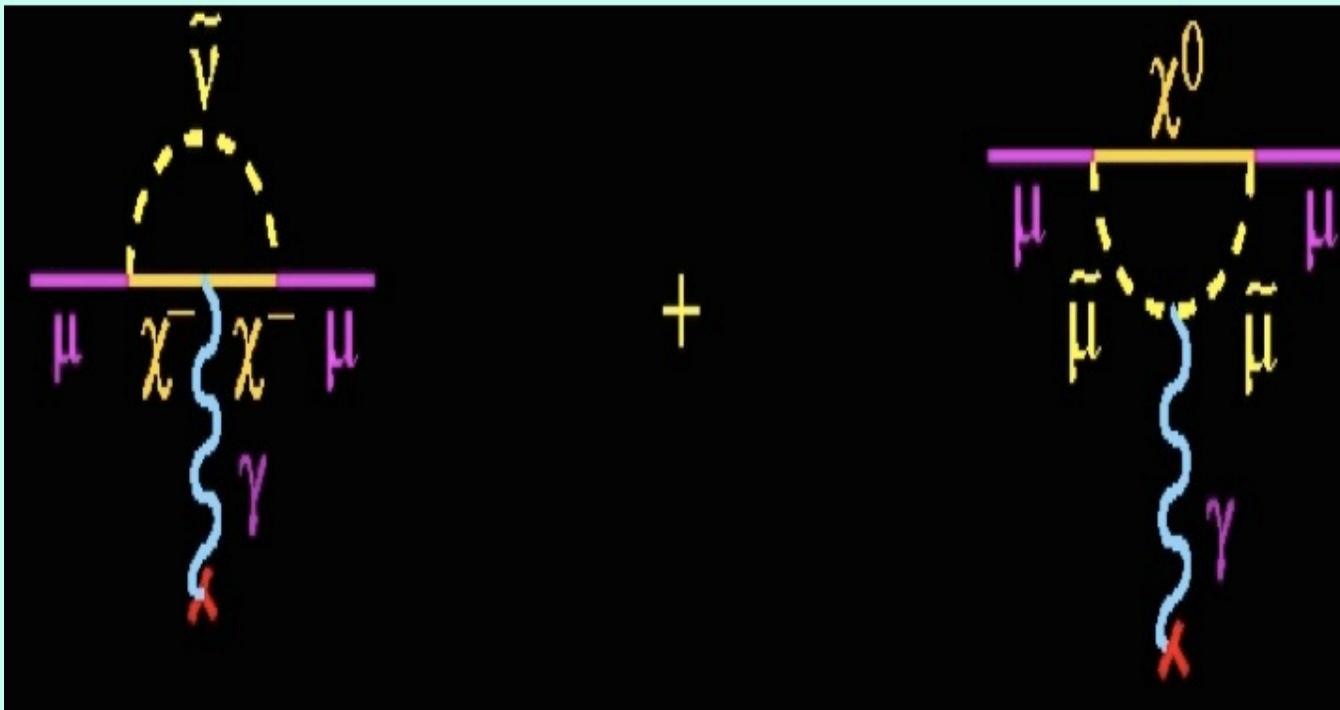
Remember, the EW contribution is only 154×10^{-11}

New Physics Nearly 2x Electroweak?

Why don't we see it in other measurements?

“New Physics” Scale Implied $< m_\mu / |\Delta a_\mu|^{1/2} < \mathbf{O}(\text{TeV})$ LHC?

3.2 “New Physics” Effects
_SUSY 1 loop a_μ Corrections
(Most Likely Scenario)



- SUSY Loops are like EW, but depend on:
- 2 spin 1/2 χ^- (charginos)
- 4 spin 1/2 χ^0 (neutralinos) including dark matter candidate!
- spin 0 sneutrinos and sleptons with *mixing!*
(light stau may dominate)
- Enhancement factor $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle \sim 3-40!$ Important

Interpretations

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(73) \times 10^{-11} \quad (3.7\sigma!)$$

Generic 1 loop MSSM SUSY Contribution:

$$a_\mu^{\text{SUSY}} = (\text{sgn}\mu) 130 \times 10^{-11} (100 \text{ GeV} / m_{\text{susy}})^2 \tan\beta$$

$$\tan\beta \approx 3-40, \quad m_{\text{susy}} \approx 100-500 \text{ GeV} \quad \text{\underline{LHC-MSSM Tension}}$$

Other Explanations: *Hadronic e^+e^- Data? HLBL(3loop)?*

Lattice Gauge Theory Efforts!

Multi-Higgs Models (2 loop effects) light A

New Dynamics < 1-2 TeV

* Dark Photons $\sim 10-200 \text{ MeV}$, $\alpha' = 10^{-8}$

Light Higgs Like Scalar $O(1 \text{ GeV})$

etc.

The Dark Boson – A Portal to Dark Matter

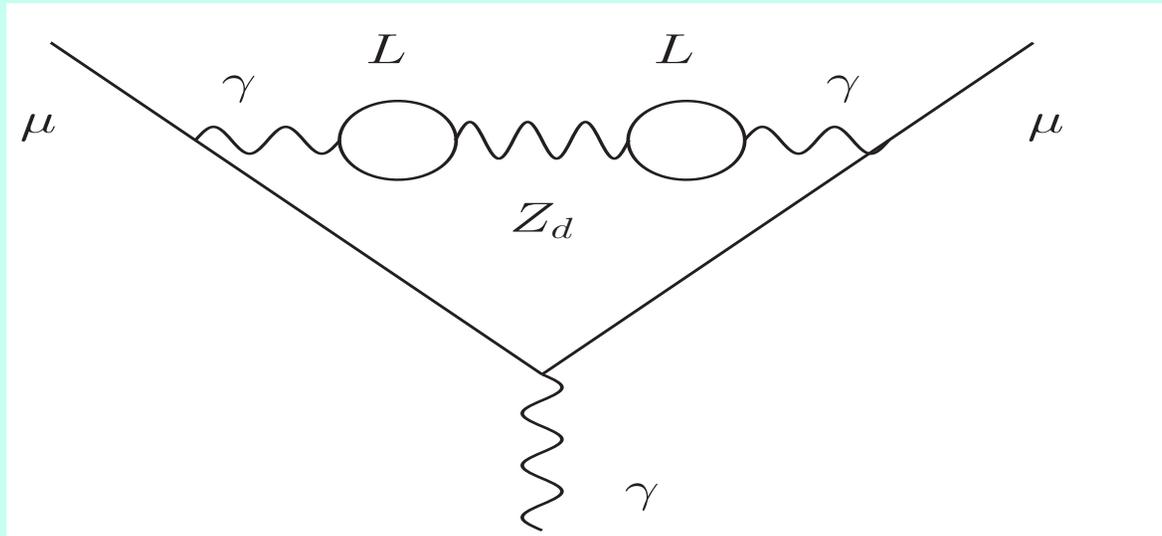
- What if some $U(1)_d$ gauge symmetry from the Dark or some Other Sector contains a “Light” *Dark Photon* (γ_d), *U Boson*, *Hidden Boson... Dark Z* (Z_d)

- Introduced for:
- 1) Sommerfeld Enhancement $D+D \rightarrow Z_d+Z_d$
 - 2) $Z_d \rightarrow e^+e^-$ (source of positrons, *γ -rays*)
 - 3) Cosmic Dark Matter Stability via global $U(1)_d$
 - *4) Light Dark Matter Abundance
 - *5) *Muon Anomalous Magnetic Moment*

Can we find direct evidence for such a light boson in the laboratory?

Effective 3 loop g_μ -2 Dark Photon Diagram

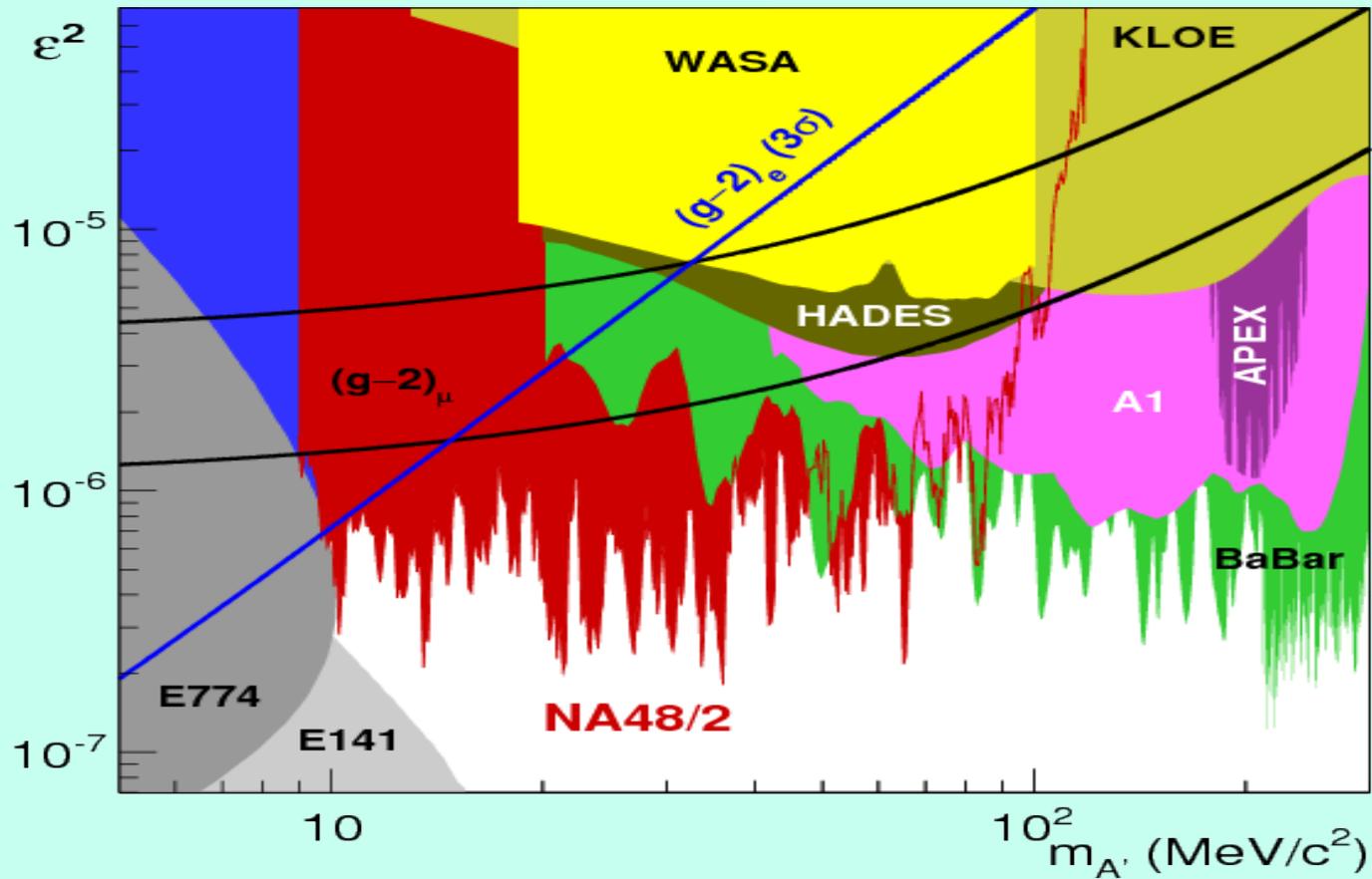
$a_\mu^{Z_d} = \alpha/2\pi\epsilon^2 F(m_{Z_d}/m_\mu)$, $F(0)=1$ solves g_μ -2 discrepancy
for $\epsilon^2 \approx 3-5 \times 10^{-6}$ & $m_{Z_d} \approx 20-200 \text{ MeV}$ (see figure)



NA48/2 Updated Bounds on Dark Photon
Simple $g_\mu-2$ discrepancy solution ruled out

Assumes $BR(\gamma_d \rightarrow e+e-) \sim 1$

Alternative $\gamma_d \rightarrow$ light dark matter



“Light” Higgs

CHEN, DAVOUDI ASL, MARCIANO, ZHANG (2016)

Batell, Lange, McKeen, Pospelov, Ritz (2016)

Light scalar $\phi \sim O(\text{GeV})$ possible remnant of $U(1)_d$ breaking

Coupling to $\mu^+\mu^- \sim 10^{-3}$ solves $g_\mu - 2$ discrepancy

Why such a relatively large coupling (Higgs size)?

Misalignment (Mass & $H\mu\mu$) due to several sources of μ mass!

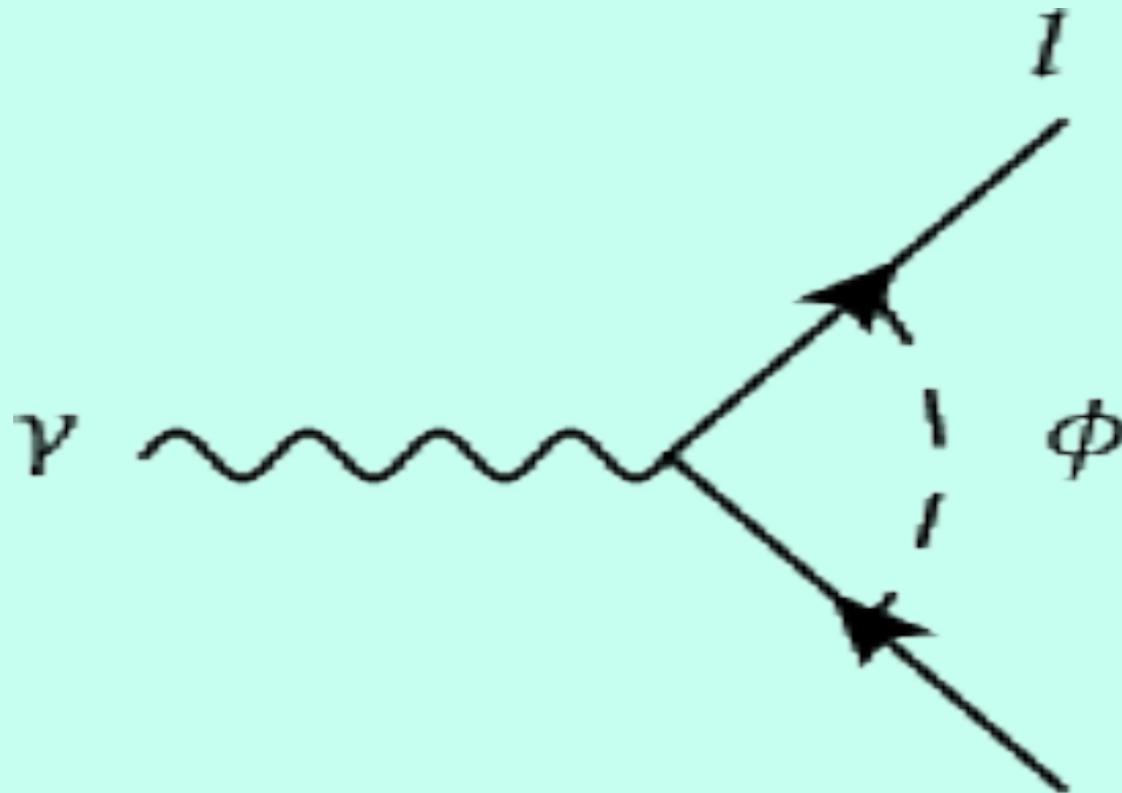
SM Higgs + Heavy charged lepton mixing effects + ...

or Multi-Higgs Mixing (2 doublets + singlet)

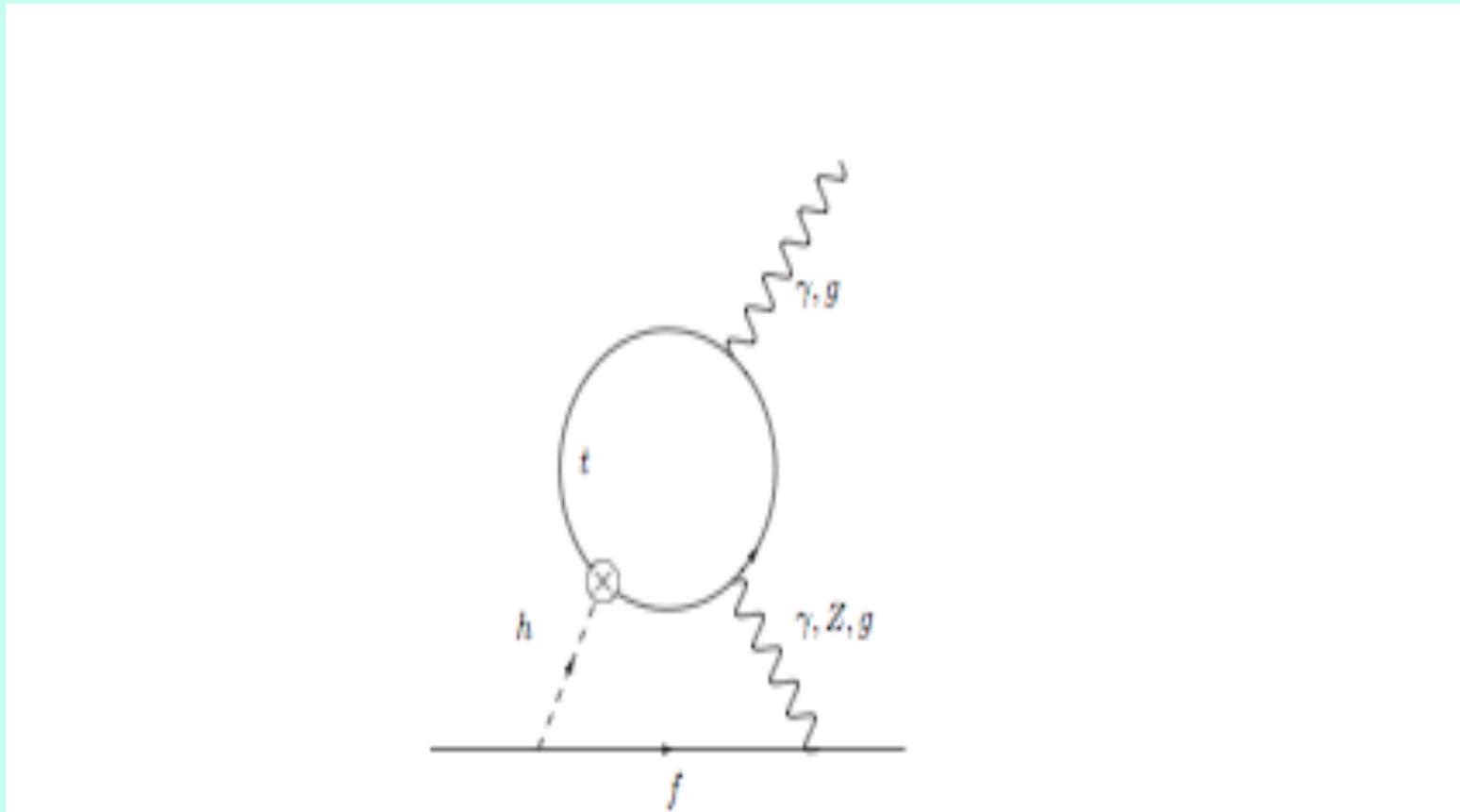
Signatures: tau decays + $\phi \rightarrow \mu^+\mu^-$ or e^+e^- BR < 10^{-6}

$e^+e^- \rightarrow \tau^+\tau^-$ + $\phi \rightarrow \mu^+\mu^-$ or e^+e^- (see Batell et al.)

One Loop Dark Higgs ($m_\phi < 1\text{ GeV}$) Contribution to Dipole Moments
Scalar Φ (0^{++}) solves Δa_μ discrepancy for $\Phi\mu\mu$ coupling $\sim 10^{-3}$



Two Loop Higgs Contribution to fermion Dipole Moments Barr-Zee Diagrams



Vector-like Heavy Leptons Dark Variant *Davoudiasl, Lee & WJM*

- Example: A Possible Dark Sector

Two Left Handed Doublets $(N_i^0, E_i^-)_L$ $i=1,2$

Two Right-Handed Doublets $(N_i^0, E_i^-)_R$

Dark Charges ± 1

Four Left Handed Singlets N_{jL}, E_{jL} $j=3,4$

Four Right Handed Singlets N_{jR}, E_{jR}

Dark Charges ± 1

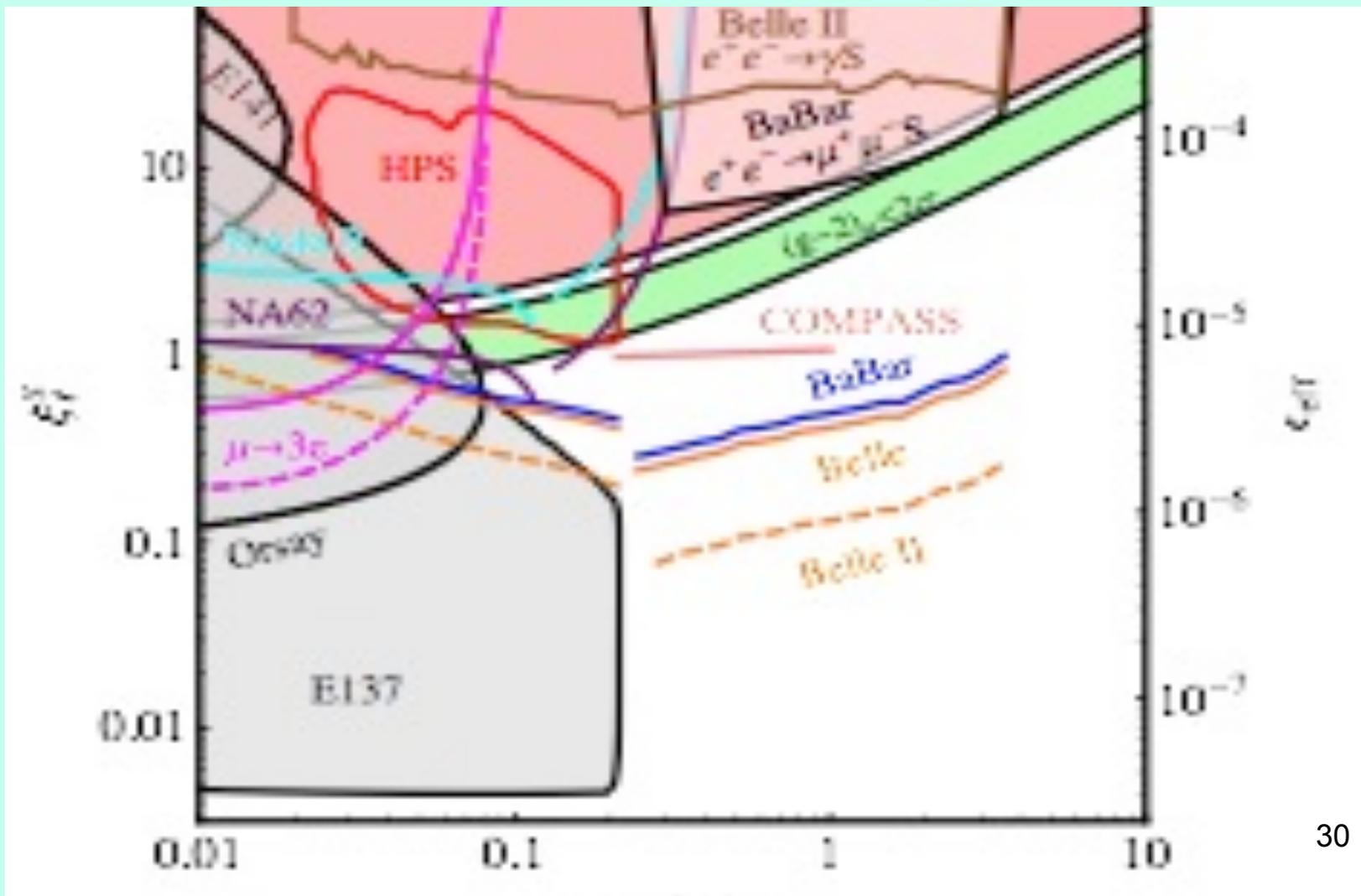
Gauge Invariant Mass Terms + Higgs Couplings \rightarrow Mixing

All interactions vector-like under $SU(2)_L \times U(1)_Y \times U(1)_d$

4 Charged & 3 Neutral Leptons Unstable

Lightest Neutral: Potential Stable Dark Matter?

From Batell et al. (2016)
Constraints on $S_{\mu\mu}$ coupling and allowed muon $g-2$ band
Possible future bounds



Cross section for $e^+e^- \rightarrow \tau^+\tau^- + S \rightarrow \mu^+\mu^-$ in fb
 from Batell et al. (2016)

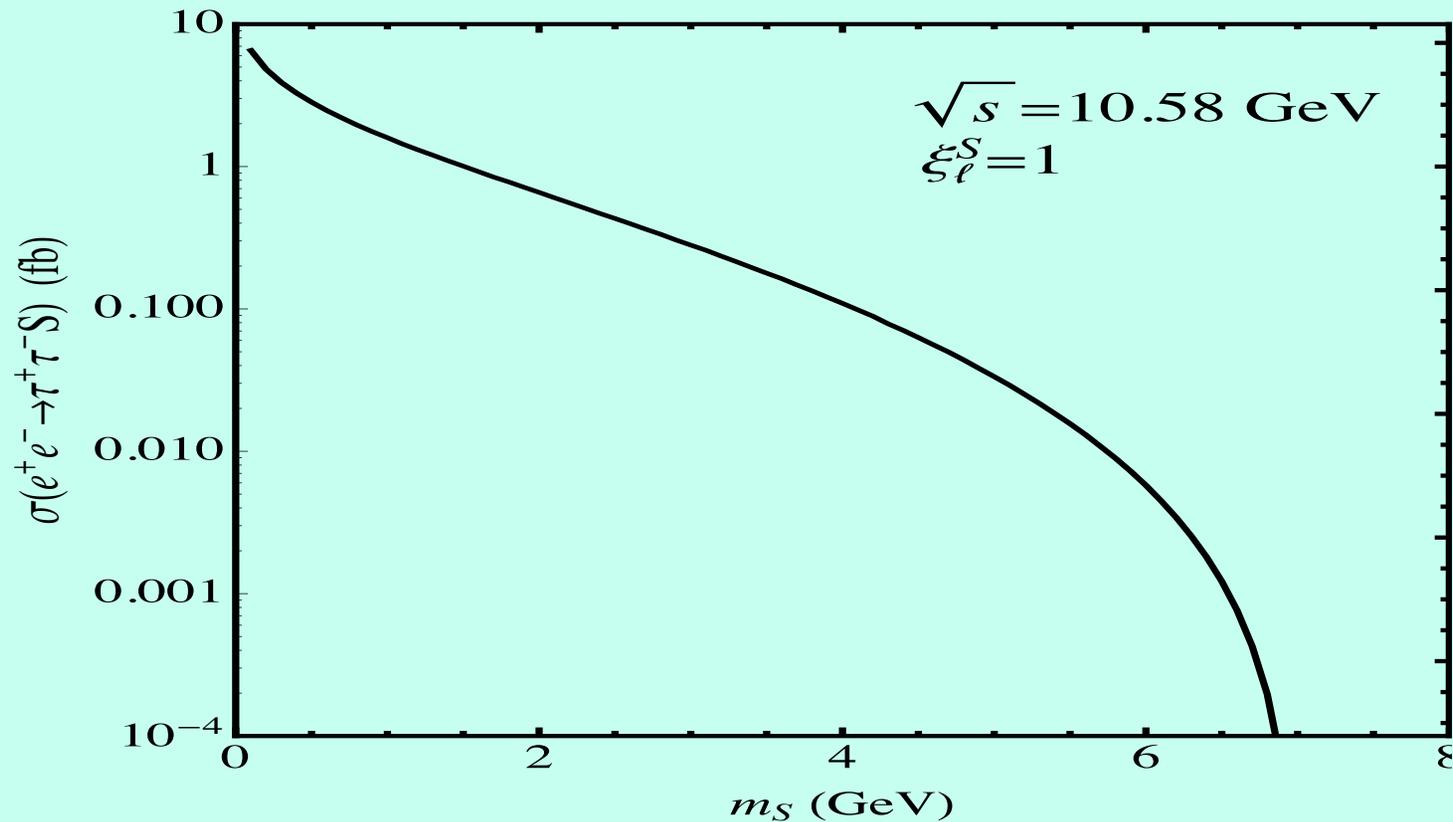


Fig. 3. Production rate for S in association with taus
 ories, as a function of m_S . The cross section is pro-
 portional to $(\xi_\ell^S)^2$, and we have set $\xi_\ell^S = 1$.

Outlook

Precision a_e and a_μ test SM and Probe “New Physics”

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 274(63)(37) \times 10^{-11} \quad (3.7\sigma)$$

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{SM}} = -87(36) \times 10^{-14} \quad (\text{Note Sign}) \quad (2.4\sigma)$$

Both a_e and a_μ should be pushed as far as possible

New Physics may be Tau Dominated

Tentative single scalar solution: 1 loop $\rightarrow \Delta a_\mu$

2 loop $\rightarrow \Delta a_e$

Requires large Yukawa & 2 photon couplings

Look for $e^+e^- \rightarrow \text{tau}^+\text{tau}^- + \phi \rightarrow \mu^+\mu^-$ or e^+e^- (Batell et al)

& bumps in $\text{tau} \rightarrow e\nu\nu + l^+l^-$

Push edm searches & rare mu & tau decays