Hadronic vacuum polarization

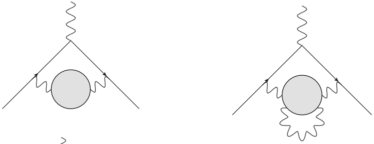
Christoph Lehner (BNL)

September 27, 2018 – Tau, Amsterdam

There is a tension of 3.7σ for the muon $a_\mu = (g_\mu - 2)/2$:

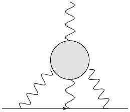
$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4 \underbrace{(2.7)}_{\mathrm{HVP}} \underbrace{(2.6)}_{\mathrm{HLbL}} \underbrace{(0.1)}_{\mathrm{other}} \underbrace{(6.3)}_{\mathrm{EXP}} \times 10^{-10}$$

HVP



this talk

HLbL



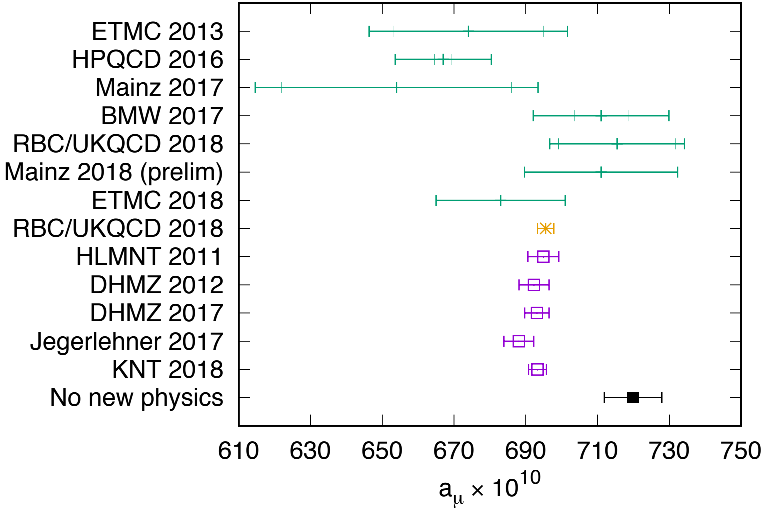
Harvey's talk

2019: $\delta a_\mu^{\mathrm{EXP}} \to 4.5 \times 10^{-10}$ (avg. of BNL/estimate of 2019 Fermilab result)

Targeted final uncertainty of Fermilab E989: $\delta a_\mu^{\mathrm{EXP}} \to 1.6 \times 10^{-10}$

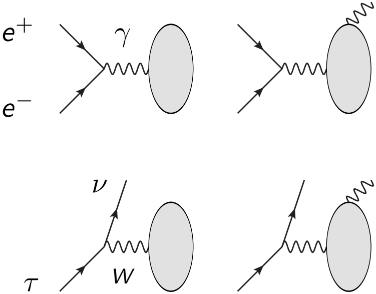
$\Rightarrow$ by 2019 consolidate HVP/HLbL, over the next years uncertainties to $O(1 \times 10^{-10})$

Status of HVP determinations



Green: LQCD, Orange: LQCD+Dispersive, Purple: Dispersive

Dispersive method - Overview



$$e^+e^- \to \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$

$$\tau \to \nu \mathrm{hadrons}(\gamma)$$

$$J_\mu = V_\mu^{I=1,I_3=\pm 1} - A_\mu^{I=1,I_3=\pm 1}$$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data. (Poster by M. Bruno)

Can have both energy-scan and ISR setup.

Dispersive method - e^+e^- status

Recent results by Keshavarzi et al. 2018, Davier et al. 2017:

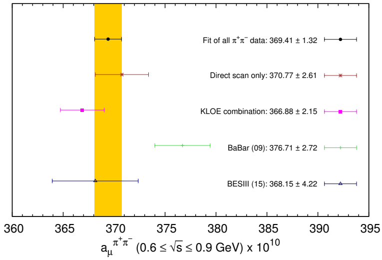
Channel	This work (KNT18)	DHMZ17 [78]	Difference
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
. . .			
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

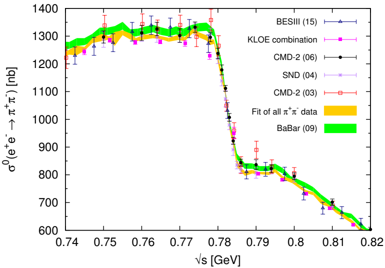
Good agreement for total, individual channels disagree to some degree. **Muon g-2 Theory Initiative workshops** recently held at Fermilab, KEK, UConn, and Mainz, intend to facilitate discussions and further understanding of these tensions.

One difference: treatment of correlations, impactful in particular in case when not all experimental data agrees

Dispersive method - e^+e^- status

Tension in 2π experimental input. BaBar and KLOE central values differ by $\delta a_\mu = 9.8(3.5) \times 10^{-10}$, compare to quoted total uncertainties of dispersive results of order $\delta a_\mu = 3 \times 10^{-10}$.





Conflicting input limits the precision and reliability of the dispersive results.

Looking for more data and insight: energy-scans update from CMD-3 in Novosibirsk and ISR updates from KLOE2, BaBar, Belle, BESIII and BelleII.

Dispersive method - τ status

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ ($2m_\pi^\pm$ – 1.8 GeV)

Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm$ – 1.8 GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm$ – 1.937 GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial. Progress towards a first-principles calculation from LQCD+QED, see poster by M. Bruno.

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Diagrams – Isospin limit

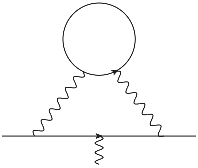
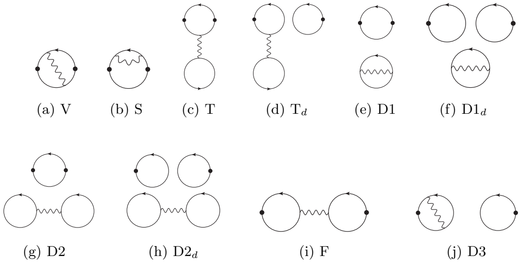


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

The quark-disconnected contribution is now calculated at physical pion mass by RBC/UKQCD 2015 and BMW 2017 and progress has been shown by the Mainz group at lattice 2018.

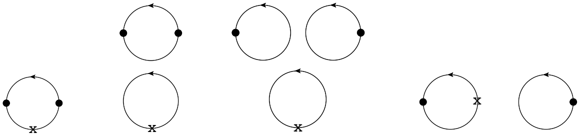
Diagrams – QED corrections



For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

BMW 2017 included phenomenological estimates of these diagrams, RBC/UKQCD 2018 V, S, and F (dominant diagrams in SU(3) and $1/N_c$) at physical pion mass, work in progress by ETMC on V and S presented at lattice 2018; RBC/UKQCD 2018 update will include values or bounds for all diagrams.

Diagrams – Strong isospin breaking



(a) M (b) R (c) R_d (d) O

For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.

M computed by HPQCD/MILC 2017, RBC/UKQCD 2018, and preliminary results shown at lattice 2018 for ETMC. BMW 2017 estimated M phenomenologically. RBC/UKQCD 2018 update will include O as well.

Regions of precision (R-ratio data here is from **Fred Jegerlehner** 2017)

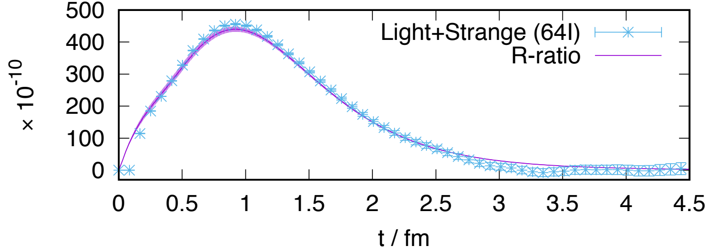


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large t, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.

Window method (implemented in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

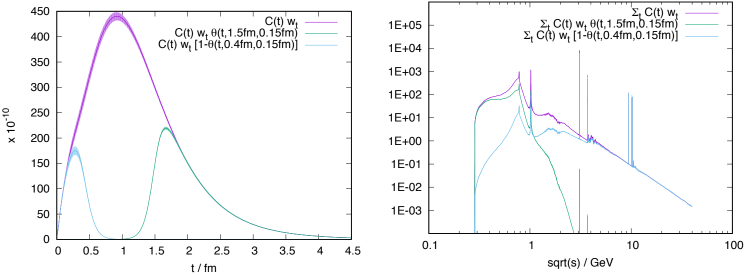
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

with

$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= \left[1 + \tanh\left[(t - t')/\Delta\right]\right]/2\,.
\end{aligned}$$

In this version of the calculation, we use $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD}.

How does this translate to the time-like region?



Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.

Example error budget from RBC/UKQCD 2018 (**Fred's alphaQED17 results used for window result**)

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.1)_{\mathrm{V}}(0.2)_{\mathrm{A}}(0.2)_{\mathrm{Z}}$	$649.7(14.2)_{\mathrm{S}}(2.8)_{\mathrm{C}}(3.7)_{\mathrm{V}}(1.5)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(0.1)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$	$53.2(0.4)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.3)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_{\mathrm{S}}(0.1)_{\mathrm{C}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$	$14.3(0.0)_{\mathrm{S}}(0.7)_{\mathrm{C}}(0.1)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$	$-11.2(3.3)_{\mathrm{S}}(0.4)_{\mathrm{V}}(2.3)_{\mathrm{L}}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}$	$5.9(5.7)_{\mathrm{S}}(0.3)_{\mathrm{C}}(1.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.1)_{\mathrm{E}}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}$	$-6.9(2.1)_{\mathrm{S}}(0.4)_{\mathrm{C}}(1.4)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.3)_{\mathrm{E}}$
a_μ^{SIB}	$0.1(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E48}}$	$10.6(4.3)_{\mathrm{S}}(0.6)_{\mathrm{C}}(6.6)_{\mathrm{V}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.3)_{\mathrm{E48}}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.1)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$	$705.9(14.6)_{\mathrm{S}}(2.9)_{\mathrm{C}}(3.7)_{\mathrm{V}}(1.8)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(2.3)_{\mathrm{L}}(0.1)_{\mathrm{E48}}$ $(0.1)_{\mathrm{E64}}(0.0)_{\mathrm{M}}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}$	$9.5(7.4)_{\mathrm{S}}(0.7)_{\mathrm{C}}(6.9)_{\mathrm{V}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(1.3)_{\mathrm{E48}}$
$a_\mu^{\text{R}-\text{ratio}}$	$460.4(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$	
a_μ	$692.5(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}$ $(0.0)_{\mathrm{b}}(0.1)_{\mathrm{c}}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$	$715.4(16.3)_{\mathrm{S}}(3.0)_{\mathrm{C}}(7.8)_{\mathrm{V}}(1.9)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(2.3)_{\mathrm{L}}$ $(1.5)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}(0.3)_{\mathrm{b}}(0.2)_{\mathrm{c}}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

For the pure lattice number the dominant errors are (S) statistics, (V) finite-volume errors, and (C) the continuum limit extrapolation uncertainty.

For the window method there are additional R-ratio systematic (RSY) and R-ratio statistical (RST) errors.

Improved systematics – compute finite-volume effects from first-principles

RBC/UKQCD study of QCD at **physical pion mass** at three different volumes:

$\boxed{L = 4.66 \text{ fm},\ L = 5.47 \text{ fm},\ L = 6.22 \text{ fm}}$

Results for light-quark isospin-symmetric connected contribution:

- $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD)
- First time this is resolved from zero in a first-principles calculation at physical pion mass (previously bound in E. Shintani et al., arXiv:1805.04250)
- Need to do better than sQED in finite-volume

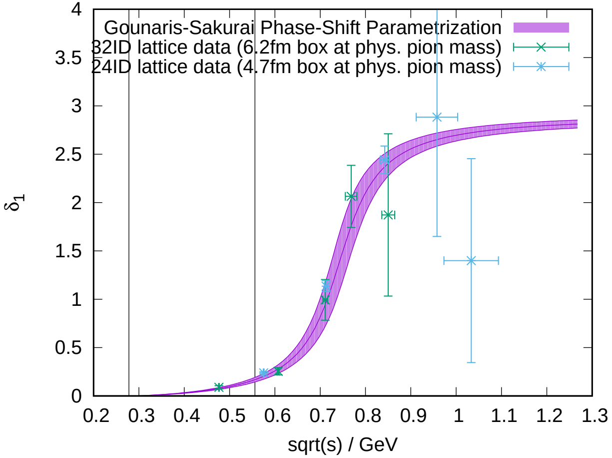
Gounaris-Sakurai-Lüscher method [H. Meyer 2012, Mainz 2017]

- Produce FV spectrum and matrix elements from phase-shift study (Lüscher method for spectrum and amplitudes, GS for phase-shift parametrization)

- This allows for a prediction of FV effects beyond chiral perturbation theory given that the phase-shift parametrization captures all relevant effects (can be checked against lattice data)

- This method is now being employed by ETMC, Mainz, and RBC/UKQCD.

First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:



$E_\rho = 0.766(21)$ GeV (PDG 0.77549(34) GeV)
$\Gamma_\rho = 0.139(18)$ GeV (PDG 0.1462(7) GeV)

GSL finite-volume results compared to sQED and lattice

Results for light-quark isospin-symmetric connected contribution:

- FV difference between $a_\mu(L = 6.22 \text{ fm}) - a_\mu(L = 4.66 \text{ fm}) = 12.2 \times 10^{-10}$ (sQED), $21.6(6.3) \times 10^{-10}$ (lattice QCD), $20(3) \times 10^{-10}$ (GSL)
- GSL prediction agrees with actual FV effect measured on the lattice, sQED is in slight tension, two-loop FV ChPT to be compared next Bijnens and Relefors 2017
- Use GSL to update FV correction of Phys. Rev. Lett. 121, 022003 (2018): $a_\mu(L \to \infty) - a_\mu(L = 5.47 \text{ fm}) = 16(4) \times 10^{-10}$ (sQED), $22(1) \times 10^{-10}$ (GSL); sQED error estimate based on Bijnens and Relefors 2017, table 1.

Improved statistics and systematics – Bounding Method

BMW/RBC/UKQCD 2016

The correlator in finite volume

$$C(t) = \sum_n |\langle 0|V|n\rangle|^2 e^{-E_n t}\,.$$

We can bound this correlator at each t from above and below by the correlators

$$\tilde{C}(t;T,\tilde{E}) = \begin{cases} C(t) & t < T\,, \\ C(T)e^{-(t-T)\tilde{E}} & t \geq T \end{cases}$$

for proper choice of $\tilde{E}$. We can chose $\tilde{E} = E_0$ (assuming $E_0 < E_1 < \ldots$) to create a strict upper bound and any $\tilde{E}$ larger than the local effective mass to define a strict lower bound.

Improved Bounding Method RBC/UKQCD 2018

Therefore if we had precise knowledge of the lowest $n = 0, \ldots, N$ values of $|\langle 0|V|n\rangle|$ and E_n, we could define a new correlator

$$C^N(t) = C(t) - \sum_{n=0}^{N} |\langle 0|V|n\rangle|^2 e^{-E_n t}$$

which we could bound much more strongly through the larger lowest energy $E_{N+1} \gg E_0$. New method: do a GEVP study of FV spectrum to perform this subtraction.

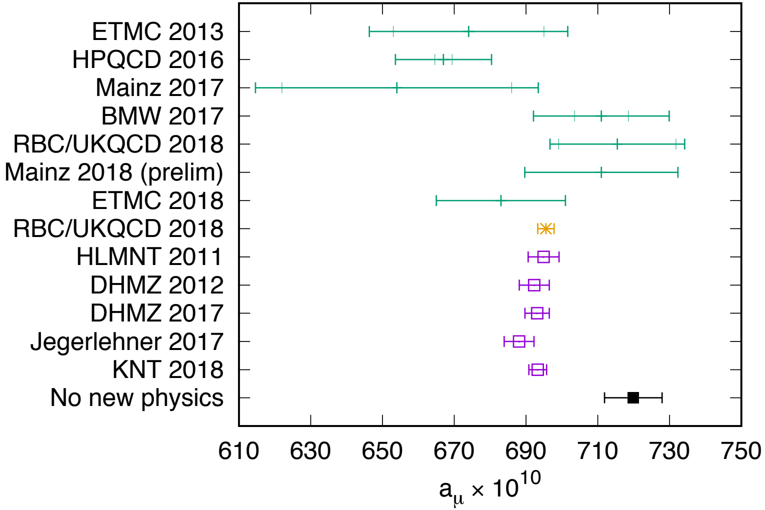
Reduces statistical error of RBC/UKQCD 2018 light quark result from 10×10^{-10} to approximately 3×10^{-10}

Conclusions and Outlook

- Target precision for HVP is of $O(1 \times 10^{-10})$ in a few years; for now consolidate error at $O(3 \times 10^{-10})$
- Dispersive result from $e^+e^- \to$ hadrons right now is at 3×10^{-10} but limited by experimental tensions
- Two-pion channel from DHMZ17, KNT18 (e^+e^-) and DHMYZ13 (τ) are scattered by 12.5×10^{-10}

  Experimental updates and first-principles calculation of isospin-breaking corrections desirable. Combination of dispersive and lattice results can in short term lessen dependence on contested experimental data.
- Lattice efforts by many groups, results at physical pion mass, QED, SIB corrections available. New methods to reduce statistical and systematic errors.
- By end of this year, first-principles lattice result could have error of $O(5 \times 10^{-10})$
- In a few years, new spacelike measurements from MUonE experiment (t-channel scattering) may be available

Status of HVP determinations



Green: LQCD, Orange: LQCD+Dispersive, Purple: Dispersive

Backup

We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass $m_{\rm light}$ and a heavy quark with mass $m_{\rm heavy}$ tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant α as well as $\Delta m_{\rm up,\ down} = m_{\rm up,\ down} - m_{\rm light}$, and $\Delta m_{\rm strange} = m_{\rm strange} - m_{\rm heavy}$. We write

$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\rm QED}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \rm m_f}(t) + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2)\,.$$

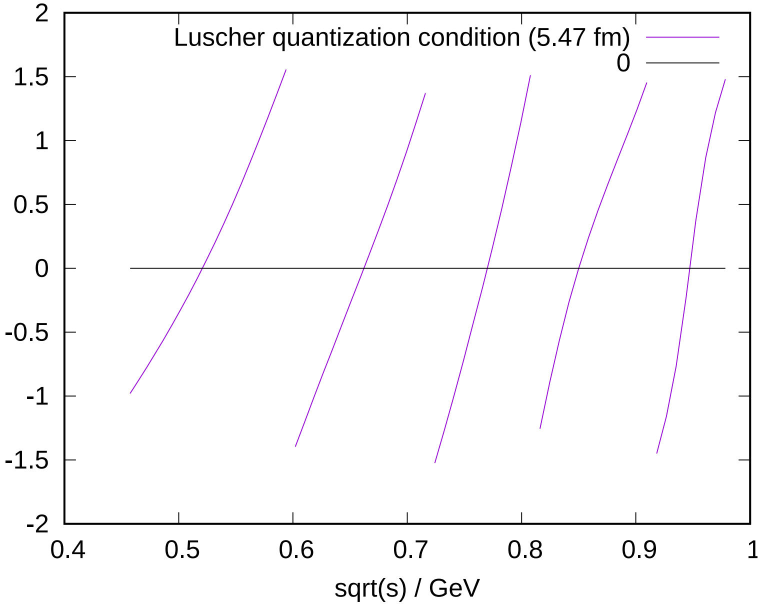
The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to a_μ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than 0.3×10^{-10}.

We tune the bare up, down, and strange quark masses $m_{\rm up}$, $m_{\rm down}$, and $m_{\rm strange}$ such that the π^0, π^+, K^0, and K^+ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the Ω^- mass to its experimental value.

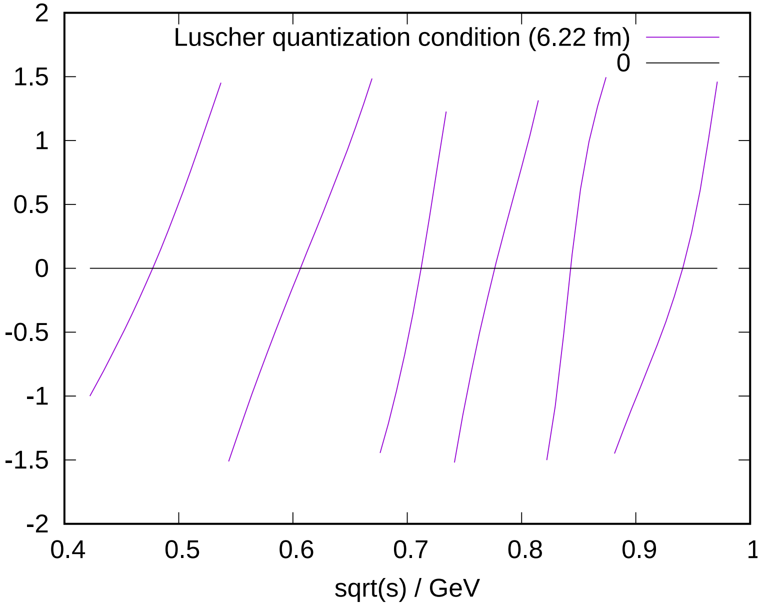
We perform the lattice calculations for the light quark contributions using RBC/UKQCD's 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\rm up} = -0.00050(1)$, $\Delta m_{\rm down} = 0.00050(1)$, and $\Delta m_{\rm strange} = -0.0002(2)$.

The shift of the Ω^- mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.



Luscher quantization condition (5.47 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV



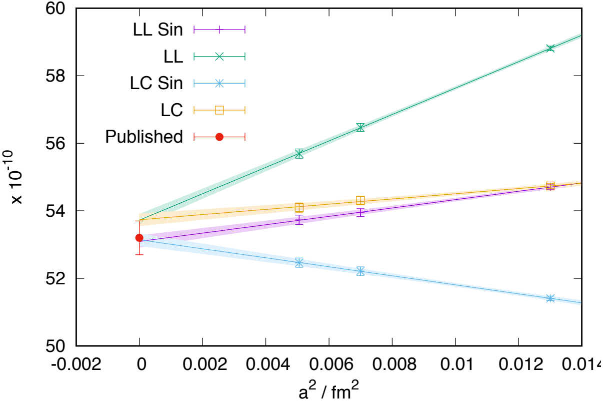
Luscher quantization condition (6.22 fm)
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.4
0.5
0.6
0.7
0.8
0.9
1
sqrt(s) / GeV

Consolidate continuum limit

Adding a finer lattice

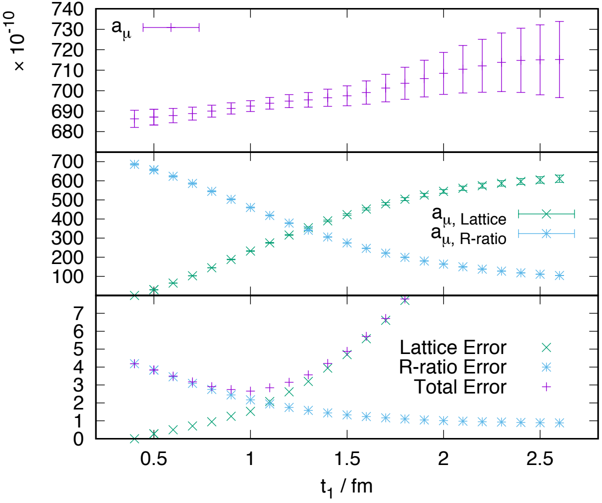
Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Proposed for early science time at Summit Machine at Oak Ridge later this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

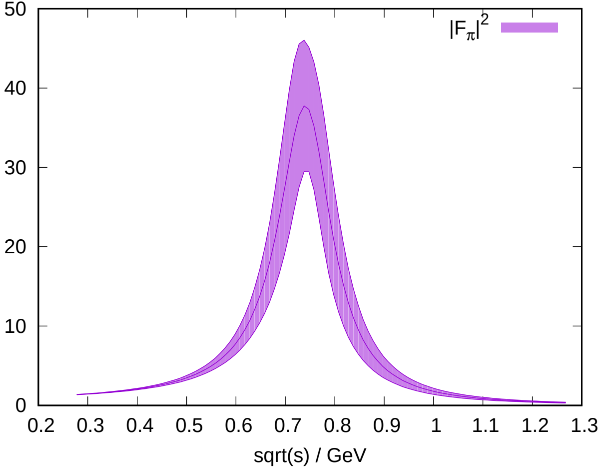
Window method with fixed $t_0 = 0.4$ fm



For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!

Can use this to check experimental data sets; see my KEK talk for more details

Predicts $|F_\pi(s)|^2$:



We can then also predict matrix elements and energies for our other lattices; successfully checked!