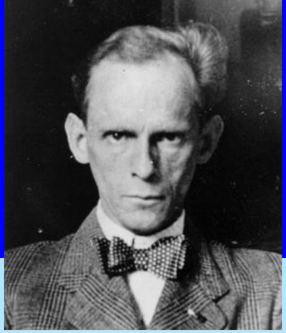
Muon g-2: The History

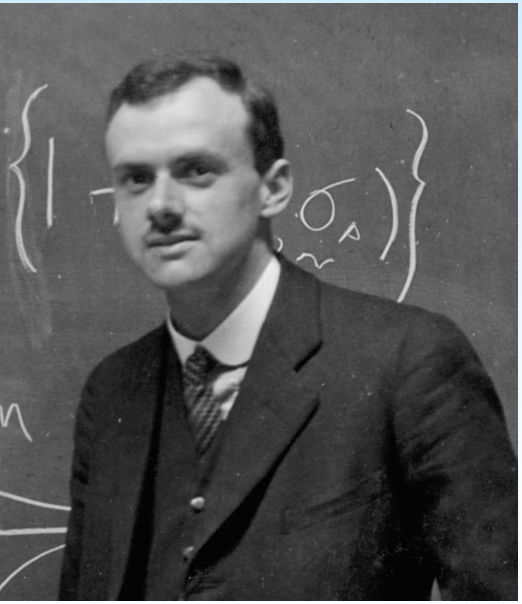
Thompson

e	μ
ν_e	ν_μ



Kunze

Anderson, Neddermeyer



Dirac

Thomas

G.E. Uhlenbeck
S. Goudsmit

Schwinger

Lee Roberts
Department of Physics
Boston University

BOSTON UNIVERSITY

Outline

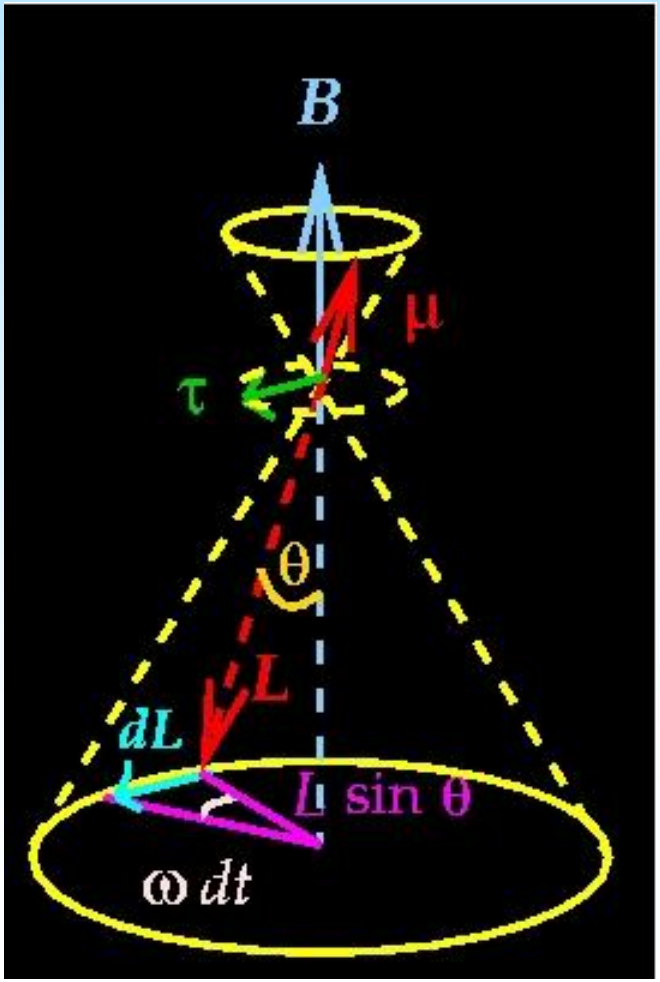
- A few words about magnetic moments
- The beginning: $g_e \cong 2$
- The discovery that $g_e > 2$
- The discovery that $g_\mu \cong 2$
- The discovery that $g_\mu > 2$
- The CERN $(g_\mu - 2)$ Experiments
- The BNL Experiment E821
- Final remarks

Magnetic Dipole Moments

Dipole in a B field:

$$Torque: \quad \vec{N} = \vec{\mu} \times \vec{B}$$

$$Energy: \quad H = -\vec{\mu} \cdot \vec{B}$$



μ_s for a particle with spin:

$$\vec{\mu}_S = g_S \left(\frac{Qe}{2m} \right) \vec{s}, \quad e > 0$$

Larmor precession

G.E. Uhlenbeck S. Goudsmit postulated electron spin to explain fine-structure spectra

Klein, U & G

Naturwissenschaften **13**, 953 (1925)
Nature **117**, 264 (1926)

Letters to the Editor.

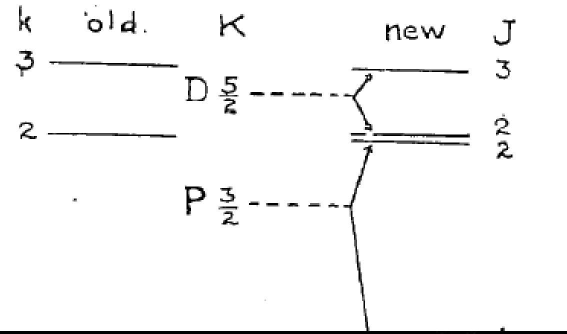
[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of rejected manuscripts intended for this or any other part of NATURE. *No notice is taken of anonymous communications.]*

Spinning Electrons and the Structure of Spectra.

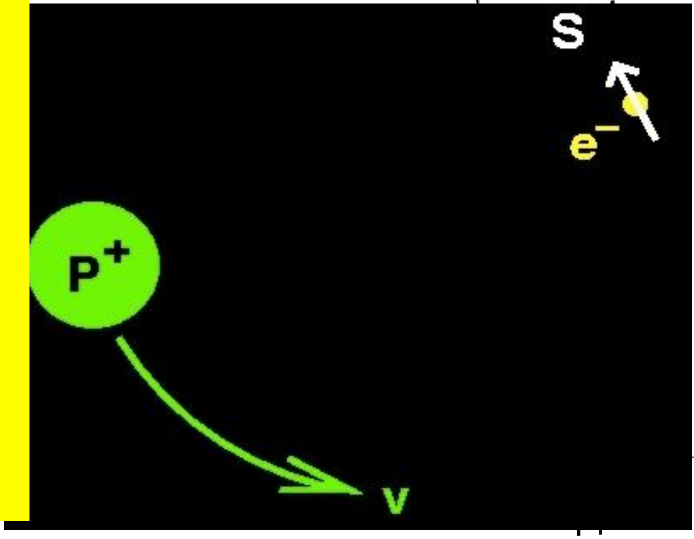
So far as we know, the idea of a quantised spinning of the electron was put forward for the first time by A. K. Compton (*Journ. Frankl. Inst.*, Aug. 1921, p. 145), who pointed out the possible bearing of this idea on the origin of the natural unit of magnetism. Without being aware of Compton's suggestion, we have directed attention in a recent note (*Naturwissenschaften*, Nov. 20, 1925) to the possibility of applying the spinning electron to interpret a number of features of the quantum theory of the Zeeman effect, which were brought to light by the work especially of van Lohuizen, Sommerfeld, Landé and

this moment of momentum is given by $Kh/2\pi$, where $K = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. The total angular momentum of the atom is $Jh/2\pi$, where $J = 1, 2, 3$. The symbols K and J correspond to those used by Landé in his classification of the Zeeman effects of the optical multiplets. The letters S, P, D also relate to the analogy with the structure of optical spectra which we consider below. The dotted lines represent the position of the energy levels to be expected in the absence of the spin of the electron. As the arrows indicate, this spin now splits each level into two, with the exception of the level $K = \frac{1}{2}$, which is only displaced.

In order to account for the experimental facts, the resulting levels must fall in just the same places as the levels given by the older theory. Nevertheless, the two schemes differ fundamentally. In particular, the new theory explains at once the occurrence of certain components in the fine structure of the hydrogen spectrum and of the helium spark spectrum



On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus.

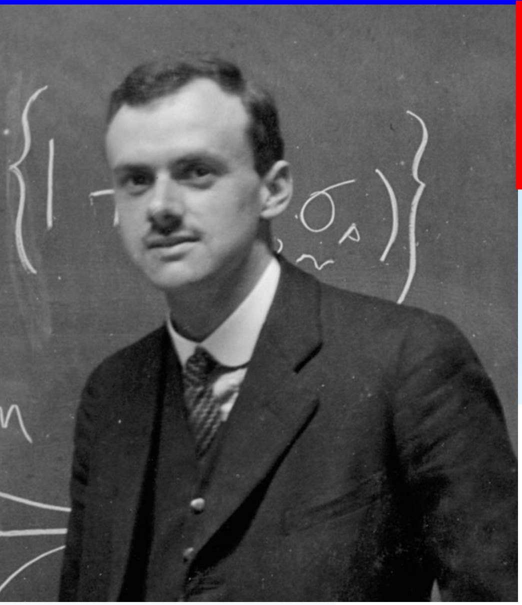


Thomas later told Goudsmit that Pauli ridiculed the idea when he first heard the suggestion from Kronig.

L.H. Thomas

I think you and Uhlenbeck have been very lucky to get your spinning electron published and talked about before Pauli heard of it. It appears that more than a year ago Kronig believed in the spinning electron and worked out something; the first person he showed it to was Pauli. Pauli ridiculed the whole thing so much that the first person become also the last and no one else heard anything of it. Which all goes to show that the infallibility of the Deity does not extend to his self-styled vicar on earth.

Theory of Magnetic and Electric Dipole Moments



$$i(\partial_\mu - ieA_\mu(x))\gamma^\mu\psi(x) = m\psi(x)$$

$$g \equiv 2$$

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Proc. R. Soc. (London) **A117**, 610 (1928)

Dirac was surprised when his theory gave the correct magnetic moment of the electron, i.e. g=2

"an unexpected bonus for me, completely unexpected."

quoted by A. Pais in *Paul Dirac: The Man and His Work*, P. Goddard, ed., Cambridge U. Press, New York (1998).

Non-relativistic reduction of the Dirac Equation for an electron in a weak magnetic field.

$$-i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{\nabla^2}{2m} + \frac{e}{2m}\left(\vec{L} + 2\vec{S}\right)\cdot\vec{B}\right]\Psi$$

$$g_L = 1 \quad g_S = 2$$

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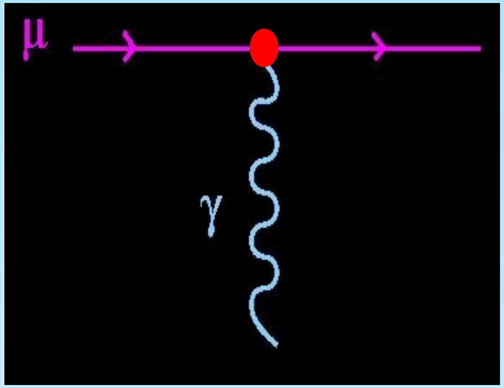
Magnetic and Electric Dipole Moments

$$\Gamma_\beta = eF_1 \bar{\psi}_R \gamma_\beta \psi_R + \frac{ie}{2m} F_2 \bar{\psi}_R \sigma_{\beta\delta} q^\delta \psi_L + HC$$



- Muon Magnetic Dipole Moment a_μ

$$\bar{u}_\mu [eF_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu} F_2(q^2)\sigma_{\beta\delta} q^\delta] u_\mu$$

$$F_1(0) = 1 \quad F_2(0) = a_\mu$$

chiral-changing flavor-conserving interaction

- Electric Dipole Moment d_μ

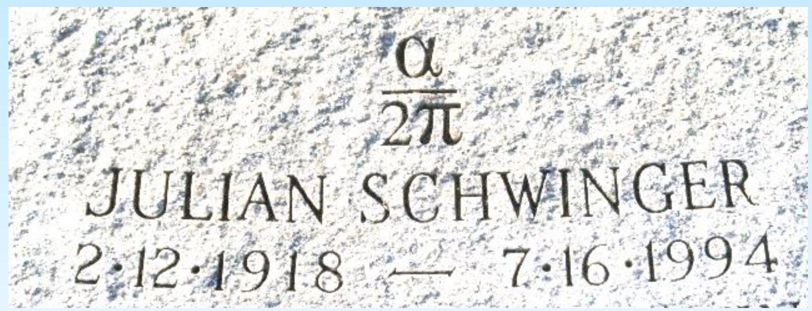
$$\bar{u}_\mu \left[\frac{iQe}{2m_\mu} F_2(q^2) - F_3(q^2)\gamma_5 \right] \sigma_{\beta\delta} q^\delta u_\mu$$

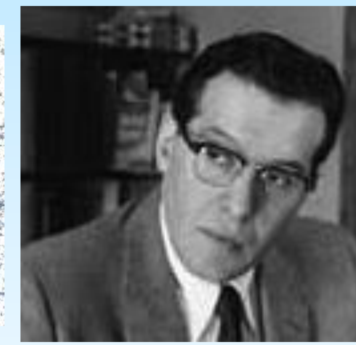
$$F_2(0) = a_\mu \quad F_3(0) = d_\mu; \ \mathrm{EDM}$$

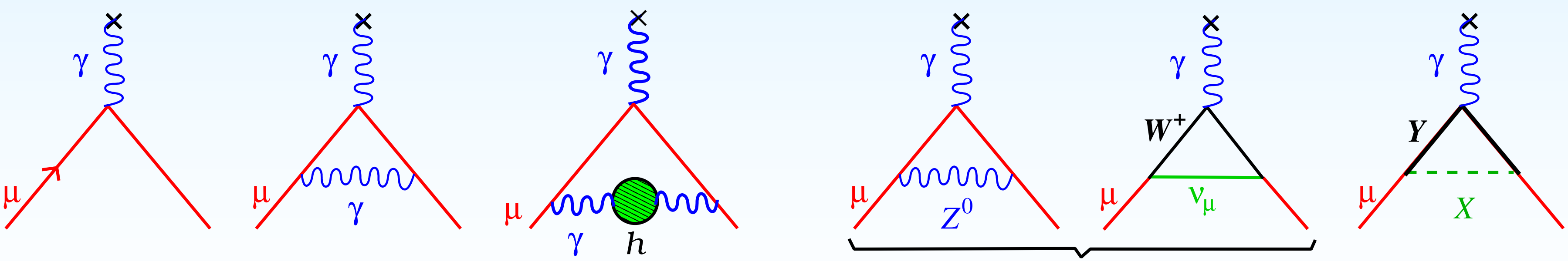
Radiative Corrections

The first loop calculation:

$$a = \left(\frac{\alpha}{2\pi}\right)$$







$$g = 2(\ 1 \quad + .00116\ldots \quad + .00000006951\ldots \quad \underbrace{+ .000000001536}_{} \quad +?\ )$$

(a) (b) (c) (d) (e)

Measuring the Muon Magnetic Dipole Moment

$$\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}$$

$$g = 2(1+a) \qquad a = \frac{(g-2)}{2}$$

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PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*

AND

In the decay processes

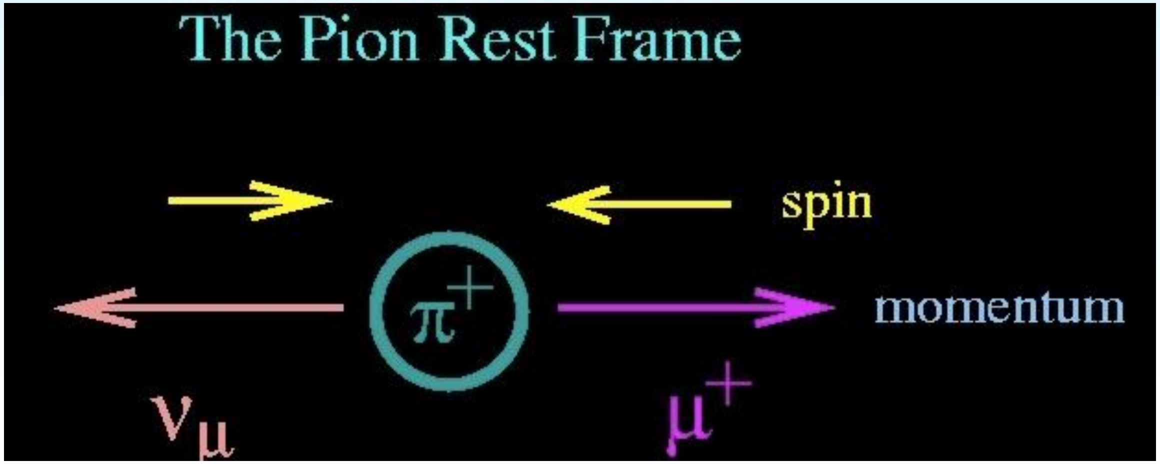
$$\pi \rightarrow \mu + \nu, \tag{5}$$

$$\mu \rightarrow e + \nu + \nu, \tag{6}$$

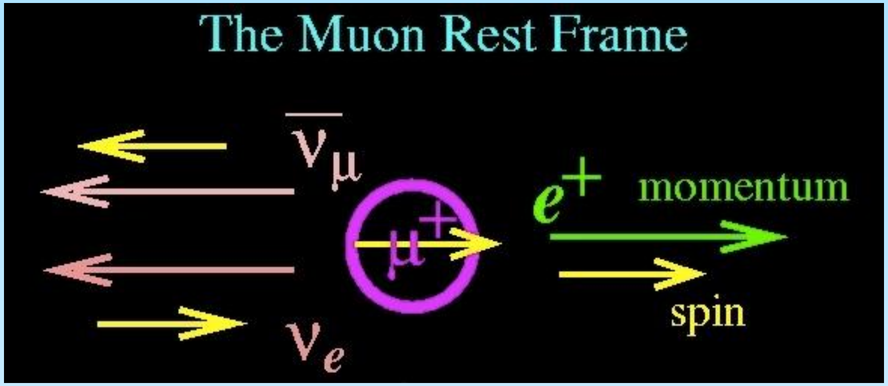
starting from a π meson at rest, one could study the distribution of the angle θ between the μ-meson momentum and the electron momentum, the latter being in the center-of-mass system of the μ meson. If parity is conserved in neither (5) nor (6), the distribution will not in general be identical for θ and $\pi - \theta$. To understand this, consider first the orientation of the muon spin. If (5) violates parity conservation, the muon would be in

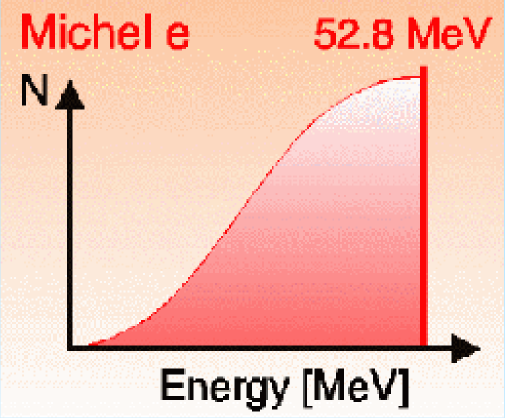
Muon Production: $\pi^+ \to \mu^+ + \nu_\mu$

- In the rest frame:
  - Initial spin is 0
  - Final spin is 0 – but the neutrino is left-handed so the muon is polarized!
- In the Lab Frame with a pion beam:
  - the very forward muons (highest energy) are highly polarized
  - the very backward muons (lowest energy) are highly polarized



Muon decay $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$





- The highest energy e+, are correlated with the muon spin direction

Garwin, Lederman, Weinrich, PR 105,1415 (1957)

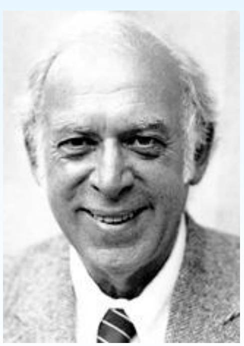
Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

RICHARD L. GARWIN,† LEON M. LEDERMAN, AND MARCEL WEINRICH

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)

Friedman, Telegdi PR105, 1681 (1957)



Nuclear Emulsion Evidence for Parity Nonconservation in the Decay Chain $\pi^+-\mu^+-e^{+}$*†

JEROME I. FRIEDMAN AND V. L. TELEGDI

Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 17, 1957)

Lee and Yang explained that if parity is violated

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

- produces polarized muons along the muon momentum, and the decay

$$\mu^- \to e^- + \bar{\nu}_e \nu_\mu$$

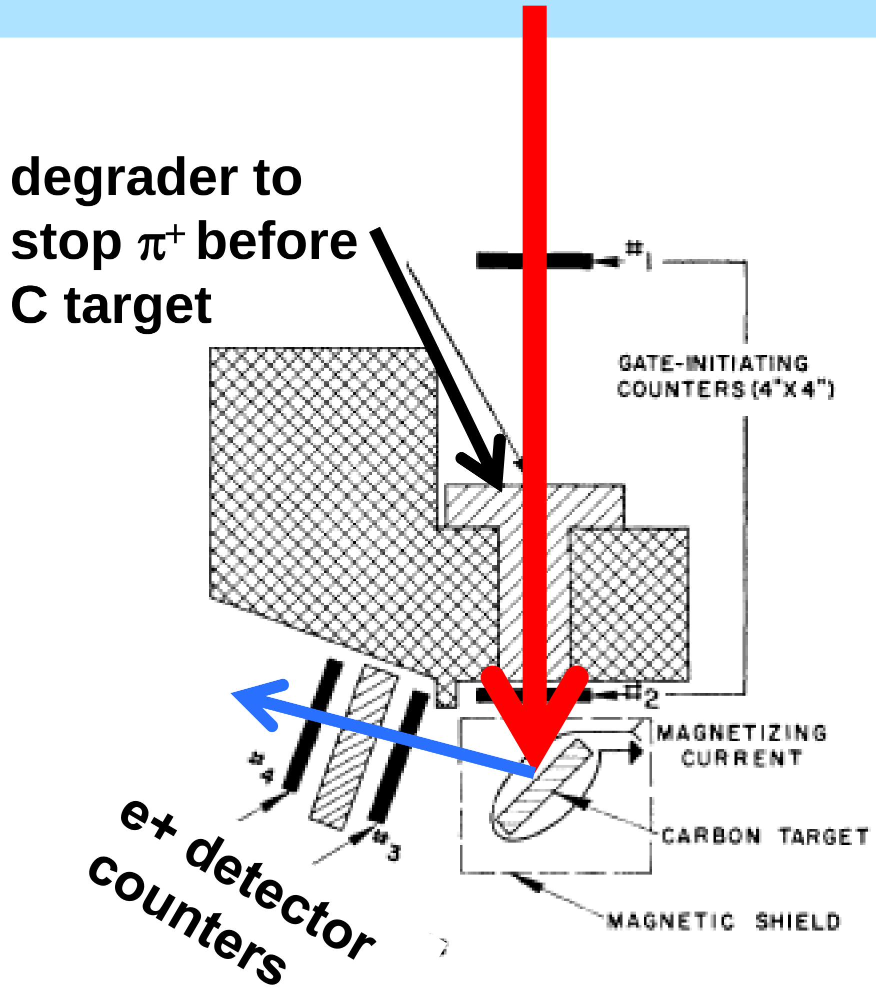
- analyzes the spin orientation at the decay time

"They also point out that the longitudinal polarization of the muon offers a natural way of determining a magnetic moment."
– Garwin, Lederman, Weinrich

The Nevis Experiment

85 MeV π^+, μ^+

- Produce a beam of π^+,μ^+
- Stop π^+ in a carbon degrader, permitting muons to stop in a carbon target
- Use a simple telescope to detect e^+ with $E_e > 25$ MeV
- count e^+ between 0.75 – 2 μs
- look for the angular distribution $(1 + a\, P \cos\theta)$
- However, the counter only samples e^+ at 100º
- Use a B field to rotate the spin, so <u>for a small time window</u>, the angular distribution is turned into a distribution of counts vs. magnet current



The first muon spin rotation experiment

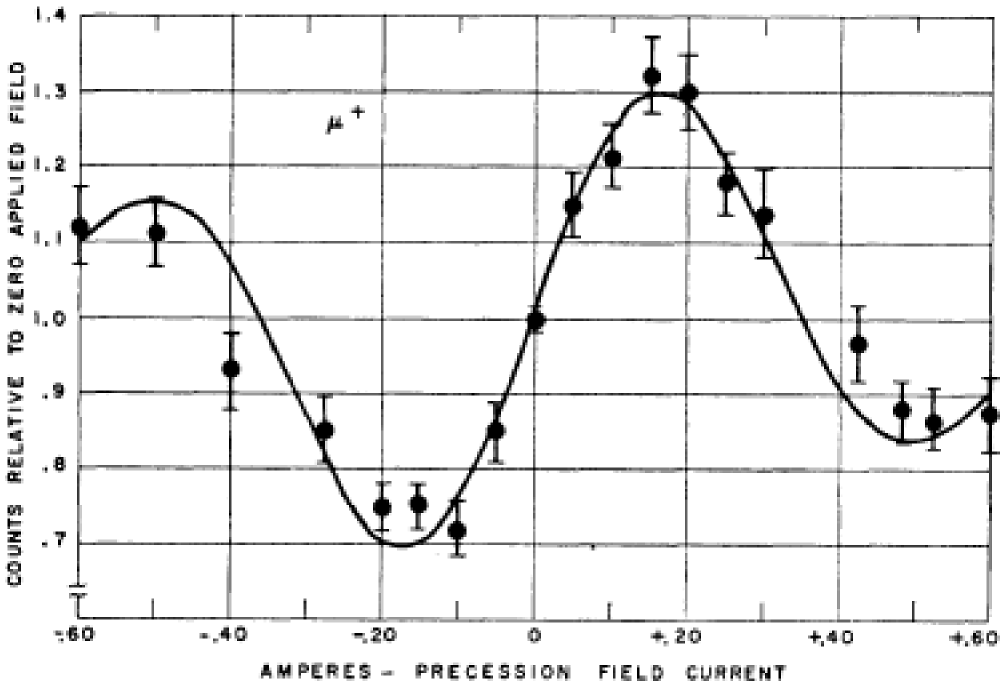


FIG. 2. Variation of gated 3–4 counting rate with magnetizing current. The solid curve is computed from an assumed electron angular distribution $1-\frac{1}{3}\cos\theta$, with counter and gate-width resolution folded in.

Fit to

$$\left(1+\frac{1}{3}\cos\theta\right)$$

$$g = 2 \quad \pm 10\%$$

n.b. The number of details that must be understood is <u>inversely proportional to the error</u>!

Cassels, et al. (Liverpool)
Stopped $\vec{\mu}^+$ from π^+ decay

- Counted e^+ decays vs. time in a 100.9 G B field.

$$g_\mu = 2.004 \pm 0.014$$

$$g_\mu = 2 \pm 0.7\%$$

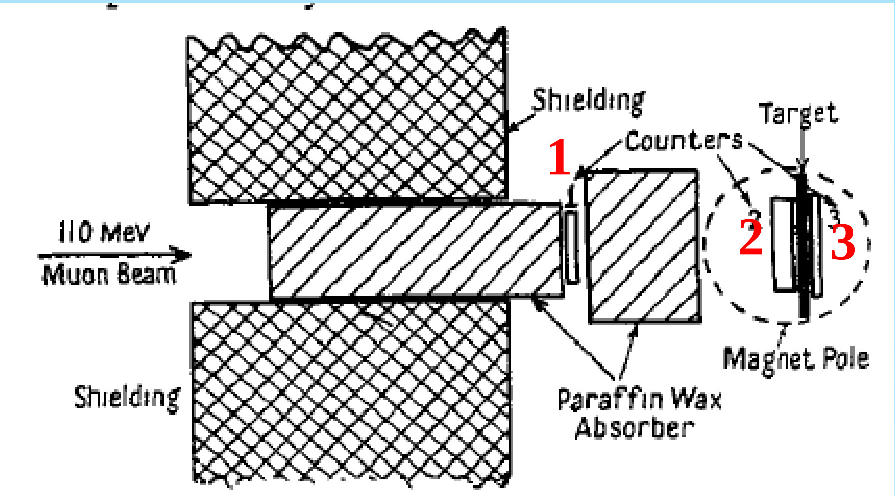
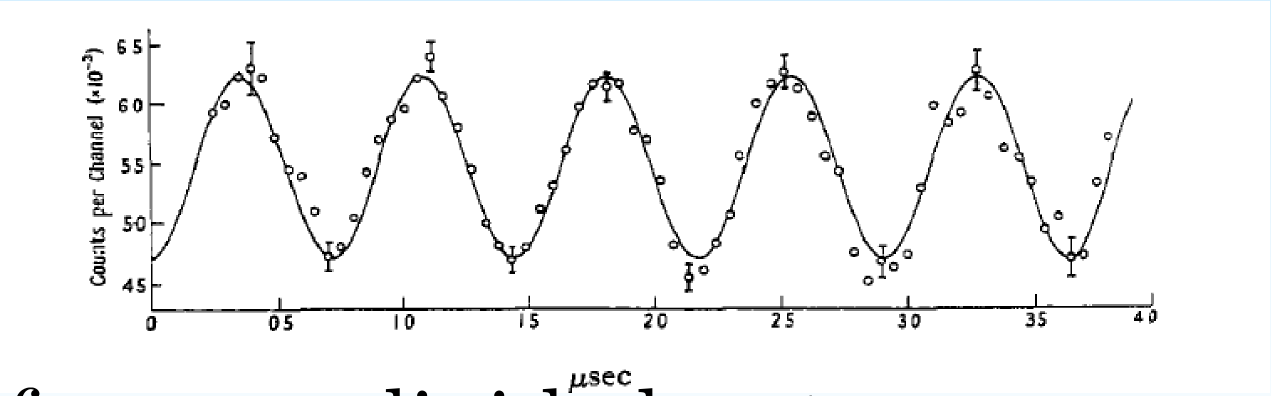


Figure 1 Layout of experimental apparatus

stopped μ then decay → e^+ $\quad 1 \cdot 2 \cdot \bar{3}$ followed by $3 \cdot \bar{2}$

J M Cassels et al
1957 Proc. Phys. Soc. A 70 543



$$\text{exponential from } \tau_\mu \text{ divided out}$$

"the value of g itself should be sought in a comparison of the precession and cyclotron frequencies of muons in a magnetic field. The two frequencies are expected to differ only by the radiative correction"

Final Nevis Experiment: PR 118, 271 (1959)

PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1,

Accurate Determination of the μ^+ Magn... ...ment*

R. L. Garwin,† D. P. Hutchinson, S. Penm... ... Shapiro§
Columbia University, New Yor...
(Received August ...

$$\frac{\alpha}{2\pi} = 0.00116\ldots$$

Using a precession technique, the magnetic mome... ...ative mu meson is determined to an accuracy of 0.007%. Muons are brought to rest in a br... ...get situated in a homogeneous magnetic field, oriented at right angles to the initial muon sp... . The precession of the spin about the field direction, together with the asymmetric decay of ... produces a periodic time variation in the probability distribution of electrons emitted in a ... ...atory direction. The period of this variation is compared with that of a reference oscillator ... ...t phase measurements of the "beat note" between the two. The magnetic field at which th... . and reference frequencies coincide is measured with reference to a proton nuclear magnetic ... ...agnetometer. The ratio of the muon precession frequency to that of the proton in the same ... ...d is thus determined to be 3.1834±0.0002. Using a re-evaluated lower limit to the muon m... ...own to yield a lower limit on the muon g factor of 2(1.00122±0.00008), in agreement with t... ...s of quantum electrodynamics.

In a magnetic field the muon behaves like a heavy electron!

Note adde... proof

$$g_\mu = 2\left(1.00113^{+0.00016}_{-0.00012}\right)$$

...easured to ~140 × 10^{-6} (140 ppm)

a_μ to 42 × 10^{-3} (12.4%)

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Spin Motion in a Magnetic Field

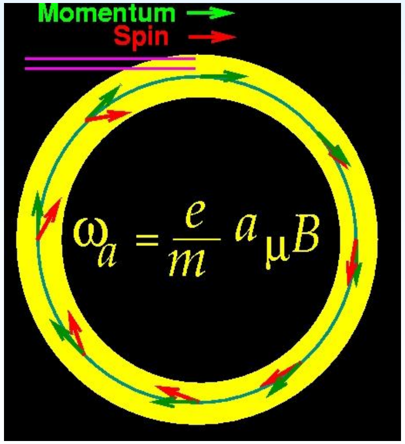
Particle: q = Qe moving in a magnetic field:

momentum turns with ω_C, spin turns with ω_S

$$\omega_C = -\frac{QeB}{m\gamma}; \qquad \omega_S = -g\frac{QeB}{2m} - (1-\gamma)\frac{QeB}{\gamma m}$$

Spin turns relative to the momentum with ω_a <u>independent of</u> p

$$\omega_a = \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{Qe}{m}B = -a\frac{Qe}{m}B$$



At rest, Larmor precession:

$$\vec{\omega}_S = \vec{\omega}_L = g\left(\frac{Qe}{2m}\right)\vec{B}$$

The featues that make the experiment possible:

- Parity violation
- The 2.2μs μ lifetime is practically forever.
- All a_μ experiments, except at Nevis and Liverpool, used the rate at which the spin turns relative to the momentum, which only depends on the anomaly and B field.

$$\vec{\omega}_a = \omega_S - \omega_C = -\frac{Qe}{m} a_\mu \vec{B}$$

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CERN 1 at the SC: Search for new physics - 1961

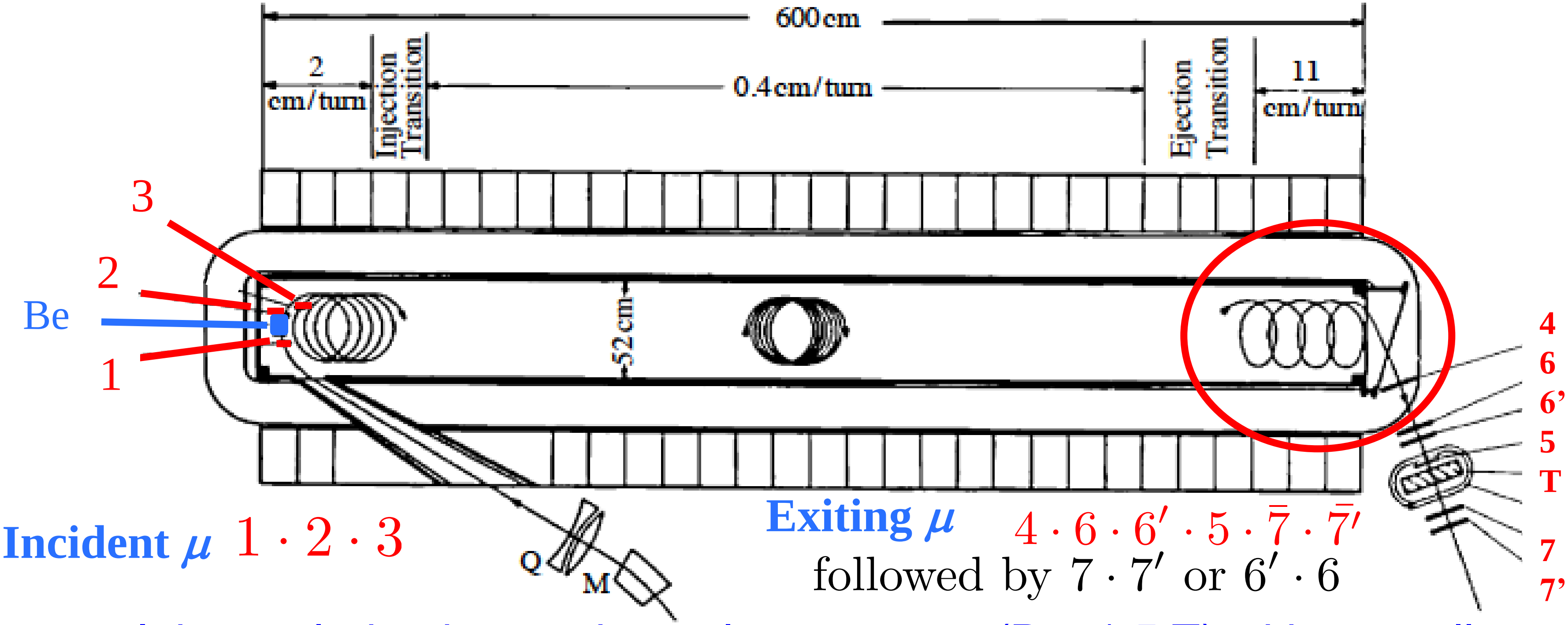
Measurement of the anomalous magnetic moment of the muon

G. Charpak, F.J.M. Farley, R. Garwin, T Muller, J.C. Sens, V.L. Telegdi and A. Zichichi, PRL 6, 128 (1961)

A "breakdown of quantum electrodynamics," for instance a cutoff on the photon propagator at energy $\Lambda m_\mu c^2$, will modify Eq. (2) to (approximately)[7]

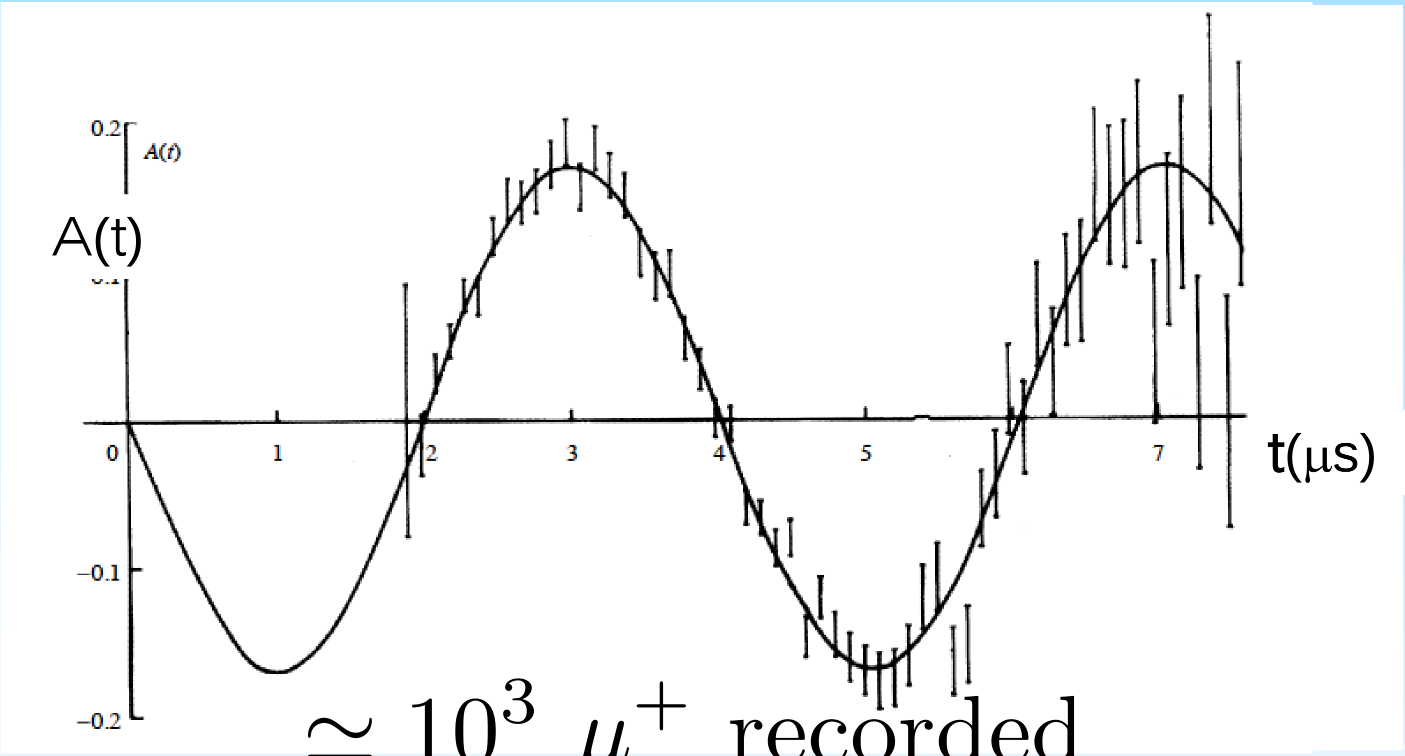
$$a = (\alpha/2\pi)(1 - \tfrac{2}{3}\Lambda^{-2}) + \cdots, \qquad (3)$$

CERN 1 at the SC: Search for new physics - 1961



- Inject polarized muon into a long magnet (B ≈ 1.5 T) with a small gradient – particles drift in circular orbits to the other end: 7.5 μs = 1600 turns
- Extract muons with a large gradient into a polarization monitor where they stopped
- Time in the magnetic field was measured by counters
- Measure the time dependent forward-backward decay asymmetry

A portion of the CERN data and the final answer



$$\simeq 10^3\ \mu^+ \text{ recorded}$$

$$a_\mu = 0.001\,162(5)\ (0.43\%)$$ (4300 ppm)

sensitive to $C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2$

- Limitations:
  - not enough data (1 muon/second in analyzer)
  - muon lifetime too short

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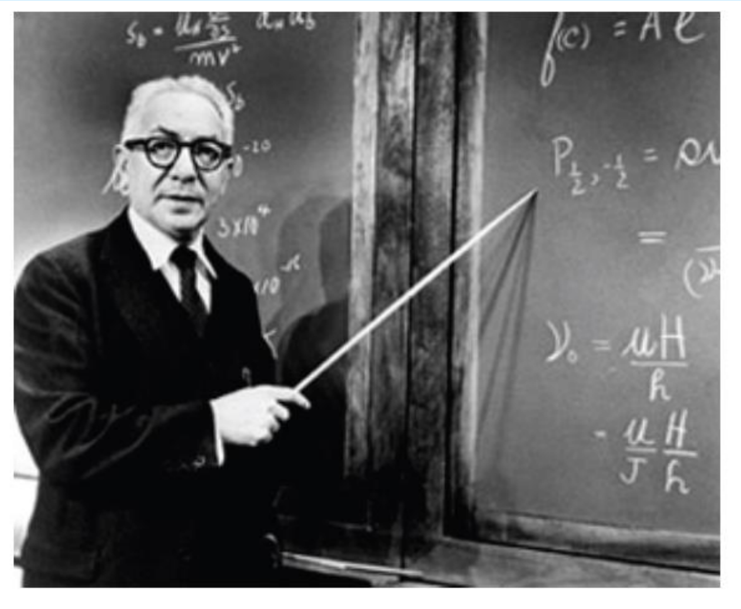
You need to measure B and ω_a

- The magnetic field is normalized to the Larmor frequency of a free proton ω_p using NMR.

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

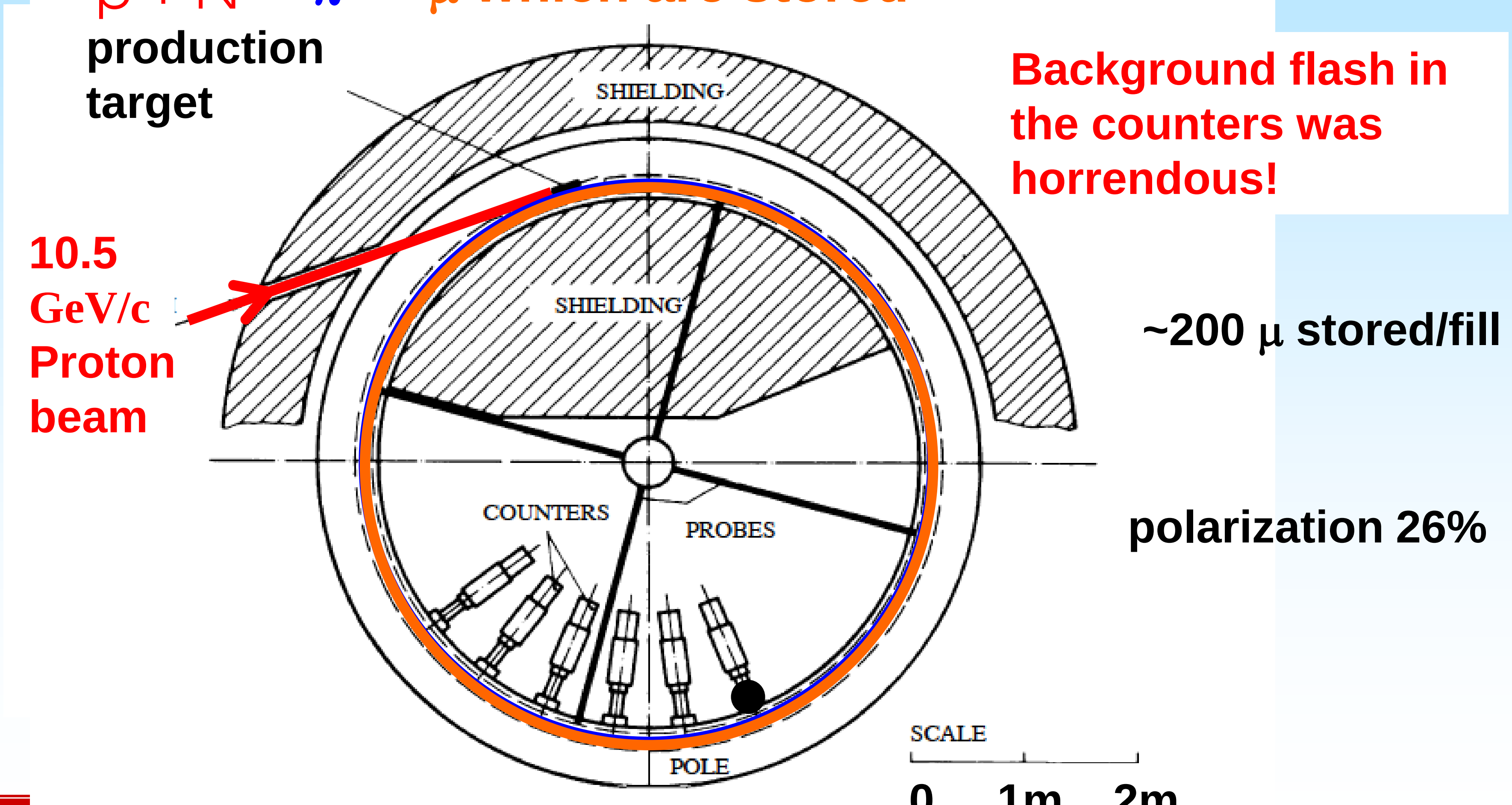


- Remember what I.I. Rabi said:

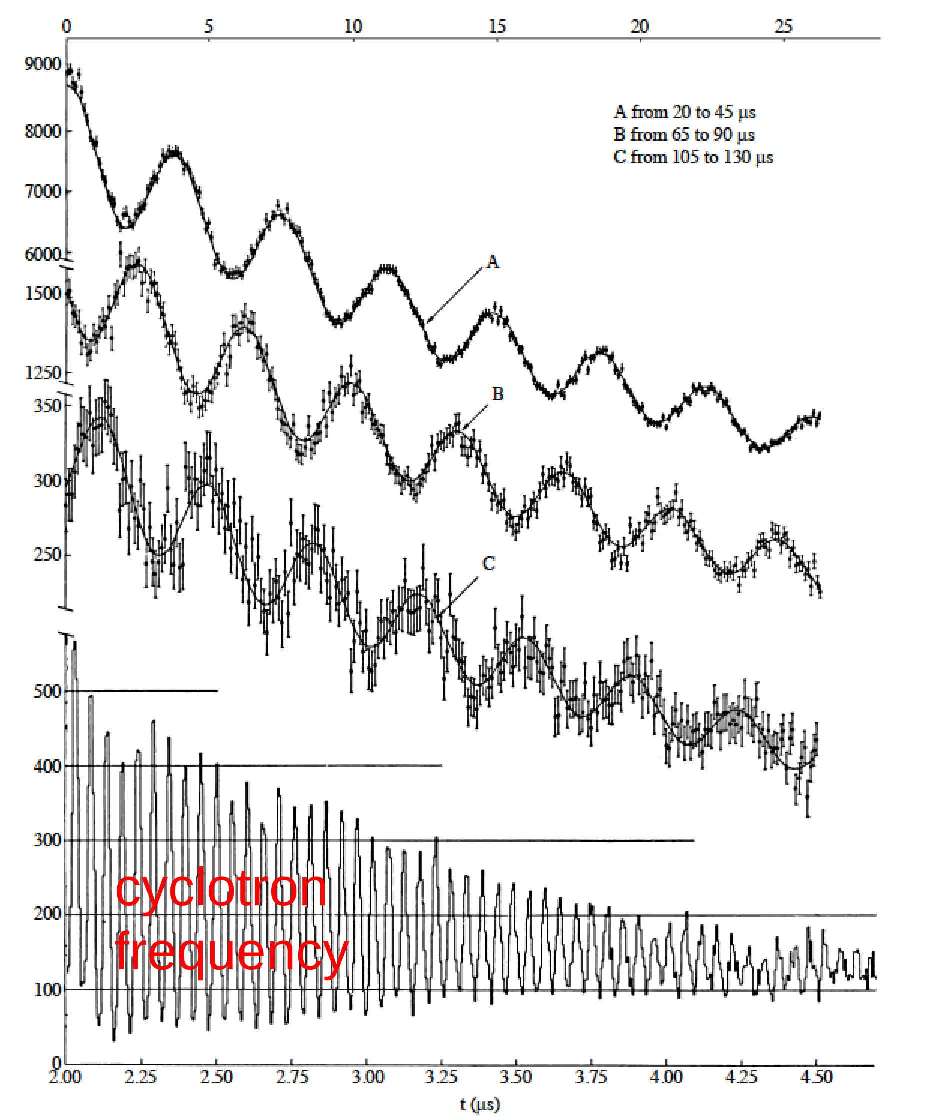
  Always measure frequencies.

The first CERN storage ring; π production target inside

- Go to p_μ = 1.27 GeV/c, γ_μ = 12; $\gamma\tau$ = 27 μs;
- using a weak-focusing magnetic storage ring; B_z = 1.71 T; n = 0.13 (weak quadrupole); $\tau_a \approx$ 3.7 μs.
- $p + N \to \pi \to \mu$ **which are stored** $\to e$ **which are detected**



Arrival time spectrum for E_e > 830 MeV



$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau \sqrt{N}}$$

Sensitive to:

$$C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3$$

n.b. There were a number of mistakes in the QED calculations that were found post g-2 measurements!

$$a_\mu = (116\,616 \pm 31) \times 10^{-8} \quad (266\mathrm{ppm})$$

To get better precision, a number of things needed:

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau_\mu \sqrt{N}}$$

- Longer muon lifetime (more wiggles) (higher momentum)
- More muons stored
- To decrease the uncertainty on $\langle B \rangle$, since

$$\omega_a = - - a\frac{Qe}{m}\langle B \rangle_{muon-dist}$$

- With gradients in the field, you have to know the muon trajectories very well to determine $\langle B \rangle$
- Find some other way besides magnetic gradients to keep the muons stored.
- What about using an electric quadrupole field to provide vertical focusing?

Beam Dynamics: Weak Focusing Betatron

$$\text{Field index}: n = \frac{R_0}{\beta B_0}\frac{dE_r}{dr} = \frac{\kappa R_0}{\beta B_0} \simeq 0.135$$

$$\text{or } \frac{dB}{dr}$$ in CERN 2

If the quadrupole filed is uniform around the ring, get simple harmonic motion in x and y.

$$f_y = f_C\sqrt{n} \simeq 0.37 f_C; \qquad \lambda_y \simeq 2.7(2\pi R)$$

$$f_x = f_C\sqrt{1-n} \simeq 0.929 f_C \qquad \lambda_x \simeq 1.08(2\pi R)$$

Must adjust the field index to avoid resonances.

Full spin equation:

$$\vec{\omega}_a = -\frac{Qe}{m}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

0

- we must correct for these extra terms
  - Pitch correction because of vertical betatron motion.
  - Radial Electric field
- Use γ_{magic} = 29.3 (3.09 GeV/c), which minimizes the *E*-field term.
- These corrections were at the 0.2 and 0.5 ppm level respectively in E821.
- To calculate them we need the muon distribution

$$C_p = -\frac{n}{4}\frac{\langle y^2\rangle}{R_0^2}$$

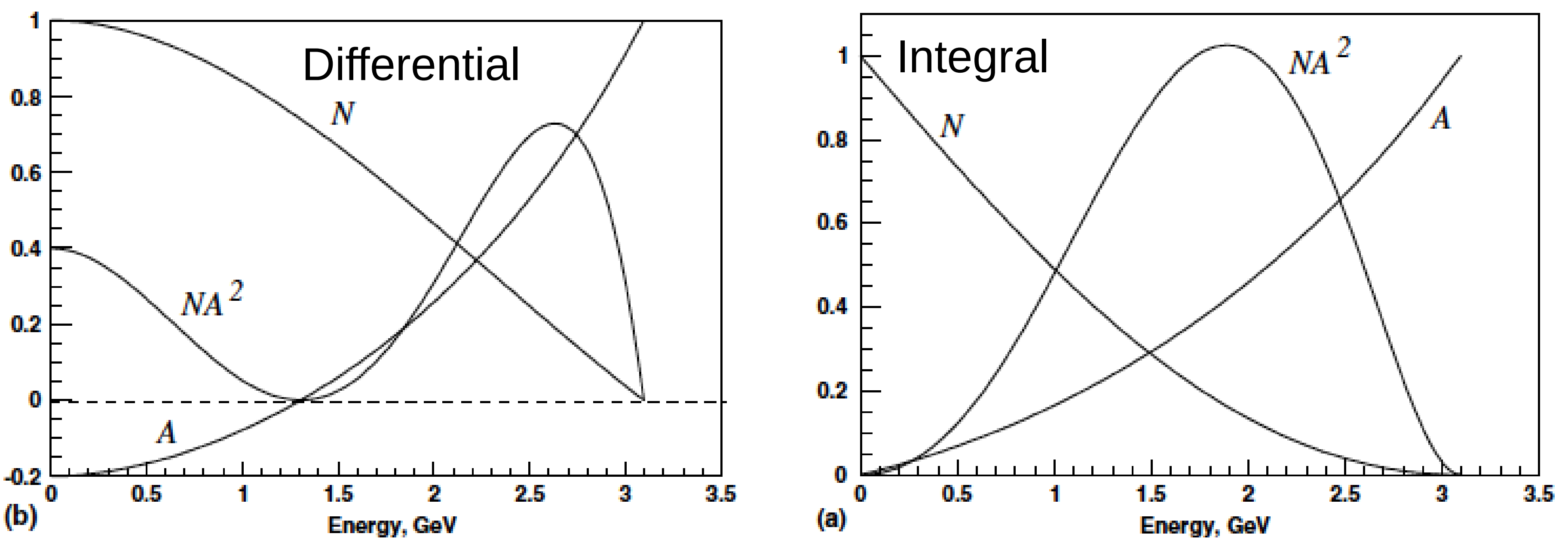
Field index $n = \frac{\kappa R_0}{\beta B_0} \simeq 0.135$

$$C_E = \frac{\Delta\omega}{\omega} = -2n(1-n)\beta^2\frac{\langle x_e^2\rangle}{R_0^2}$$

Muon decay in flight

- μ-e decay asymmetry depends on p_e, and the μ beam polarization P.
- For a single energy threshold on the $e^{\pm}$ detectors, the figure of merit, NA^2 peaks at ~0.65 E_μ.

For p_μ = 3 GeV/c; differential and integral N, NA^2 **and** A

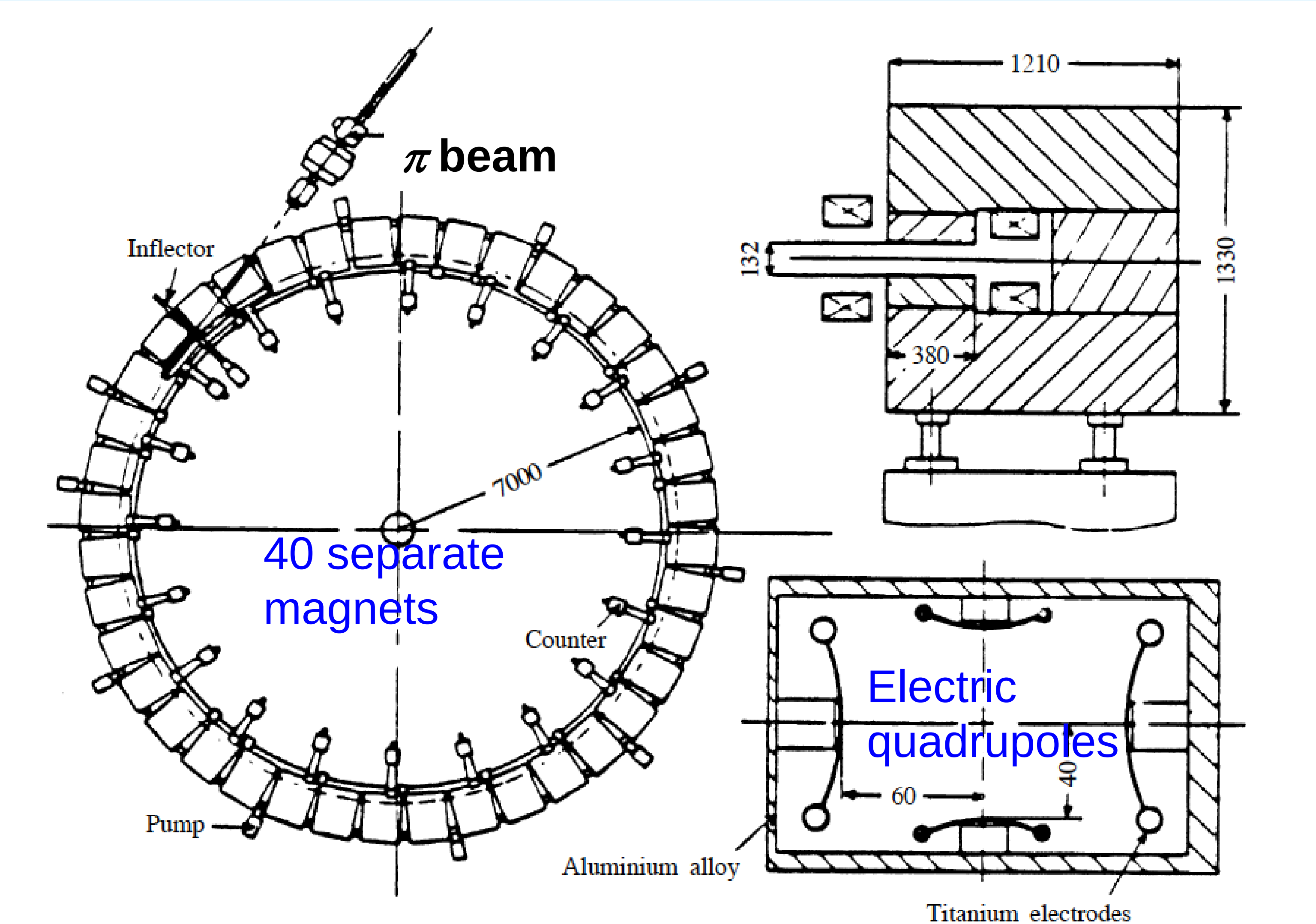


Threshold Energy

J. Miller, E. de Rafael, BL Roberts, Rep. Prog. Phys. 70 (2007) 795–881

The Third CERN Experiment; The magic γ
NPB 150, 1 (1979)

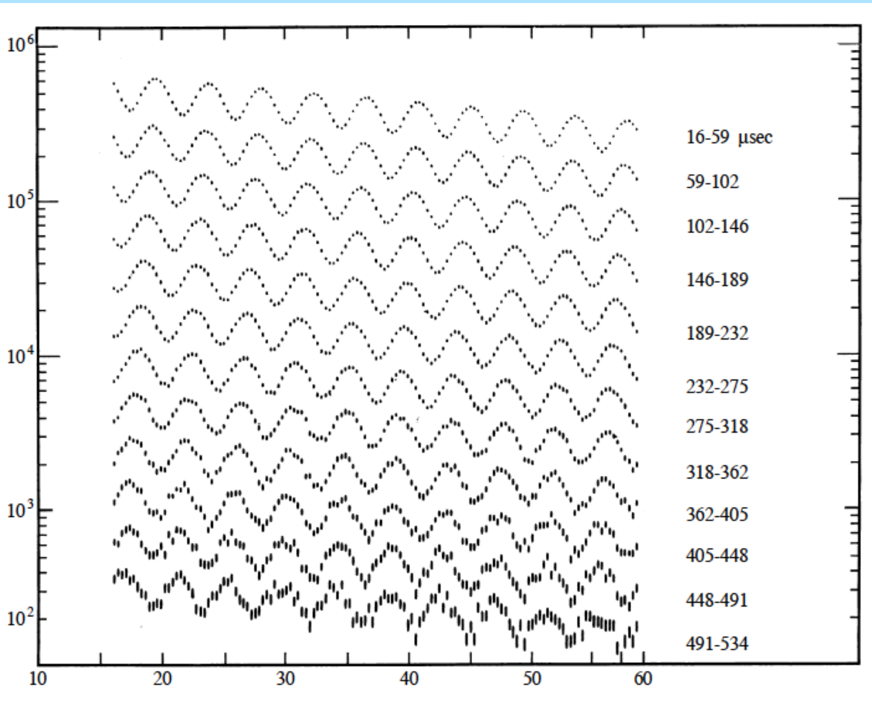
- Inject pions
- Use $\pi \rightarrow \mu$ decay to kick muons onto stable orbits

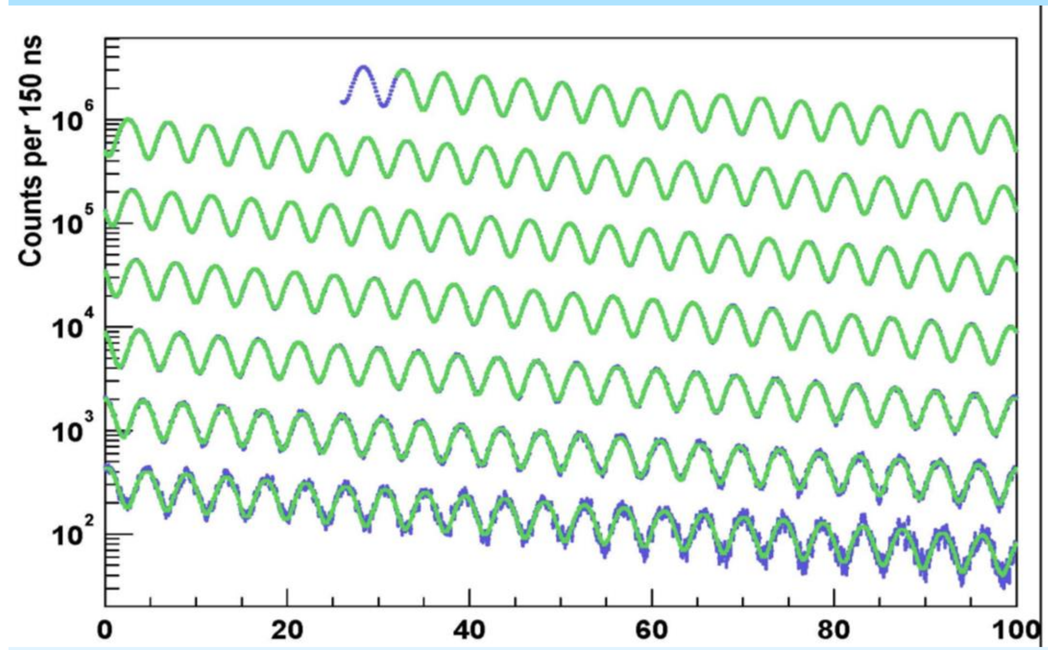


Still have pion flash at injection!

Not as bad as for CERN2

CERN 3 results and a comparison with BNL





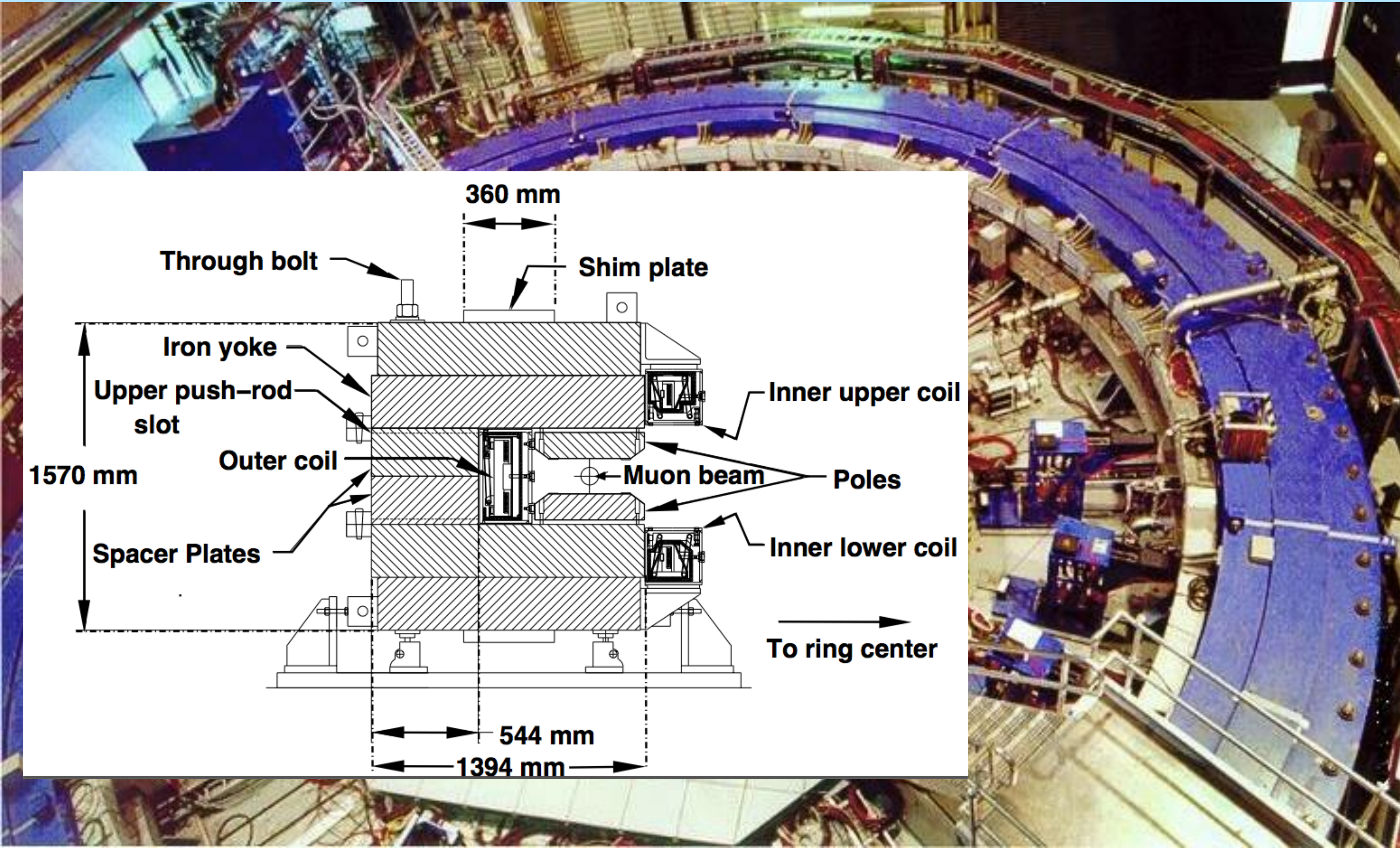
A portion of the CERN 3 data

The 2001 μ^- BNL E821 data
$\simeq 9 \times 10^9$ e^- detected.
BNL did 14 times better.

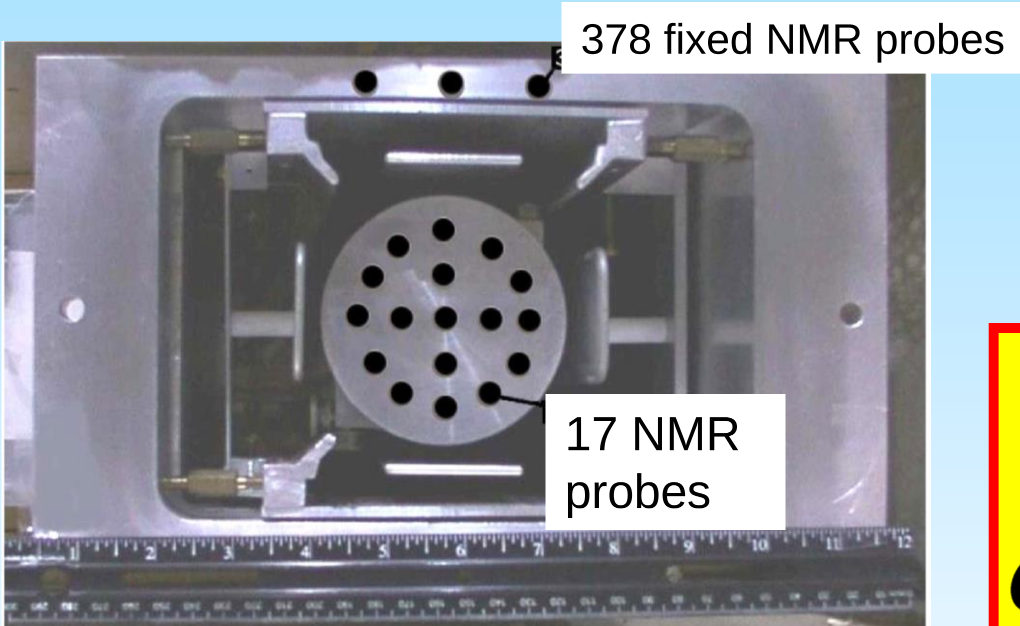
$$a_{\mu^\pm} = (1\,165\,923 \pm 8.5) \times 10^{-9} \quad (7.3\ \mathrm{ppm})$$

$$C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + a_\mu^{Had}$$

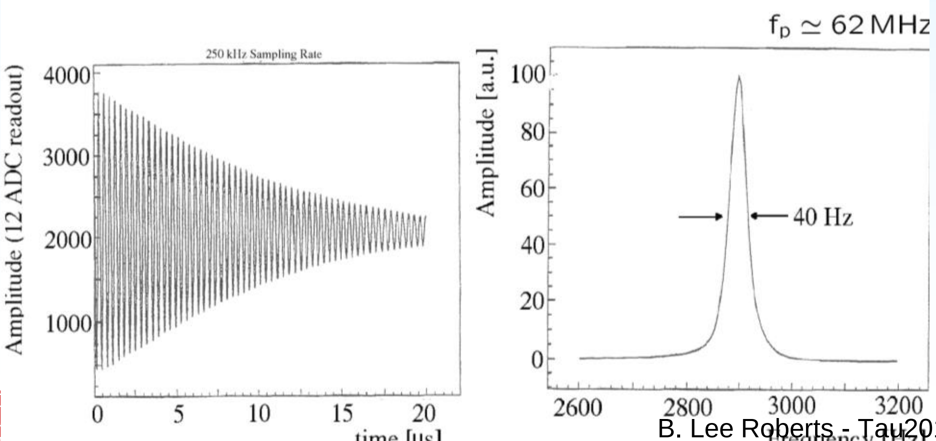
The Brookhaven/Fermilab Storage Ring Magnet: Direct muon injection, with a fast kicker that left no remnant B field after 20 μs



The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



Free induction decay signals:



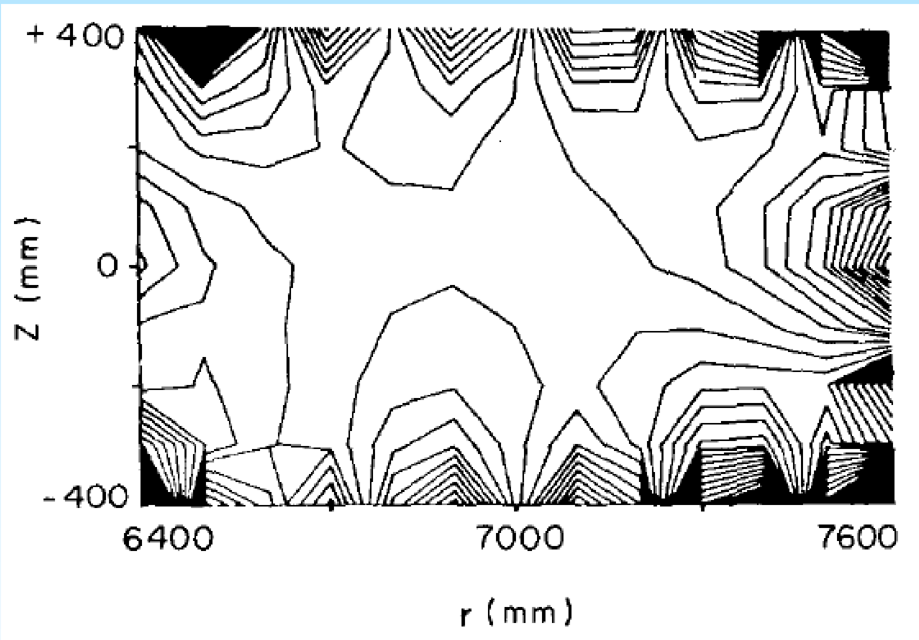
Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.

$$a_\mu = \frac{\dfrac{\omega_a}{\omega_p}}{\dfrac{\mu_\mu}{\mu_p} - \dfrac{\omega_a}{\omega_p}}$$

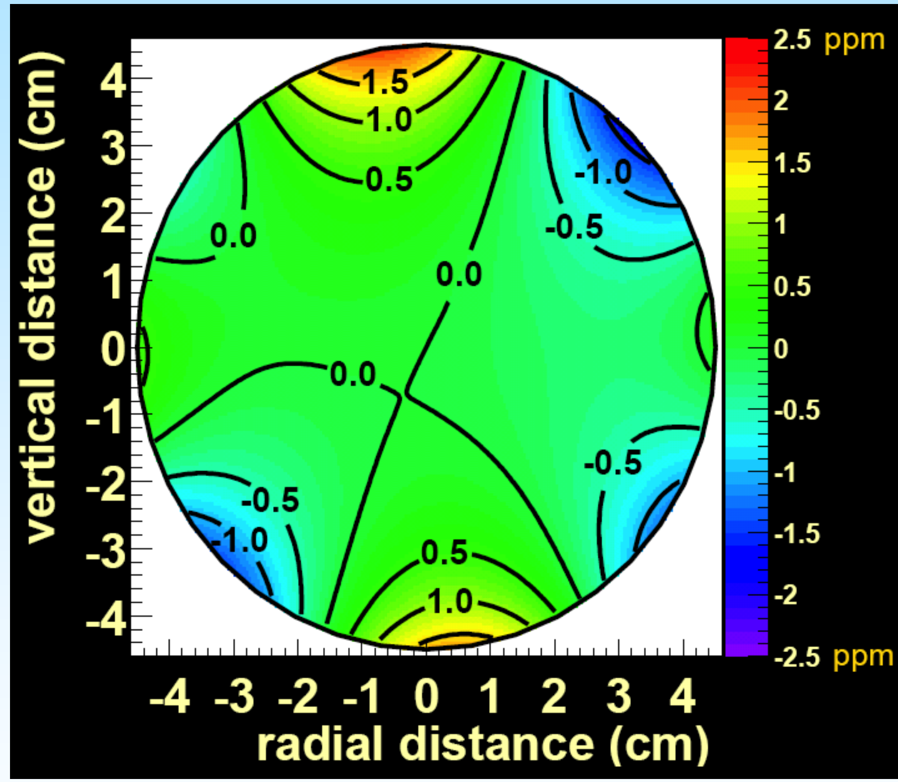
$\dfrac{\mu_\mu}{\mu_p}$ from $\mu^+ e^-$ atom

B field averaged over azimuth

CERN: 2 ppm contours



E821: 0.5 ppm contours



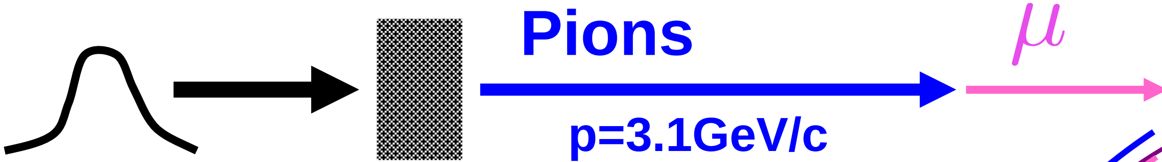
$$\langle B \rangle_{\mu-\mathrm{dist}} = \int M(r,\theta) B(r,\theta) r dr d\theta$$

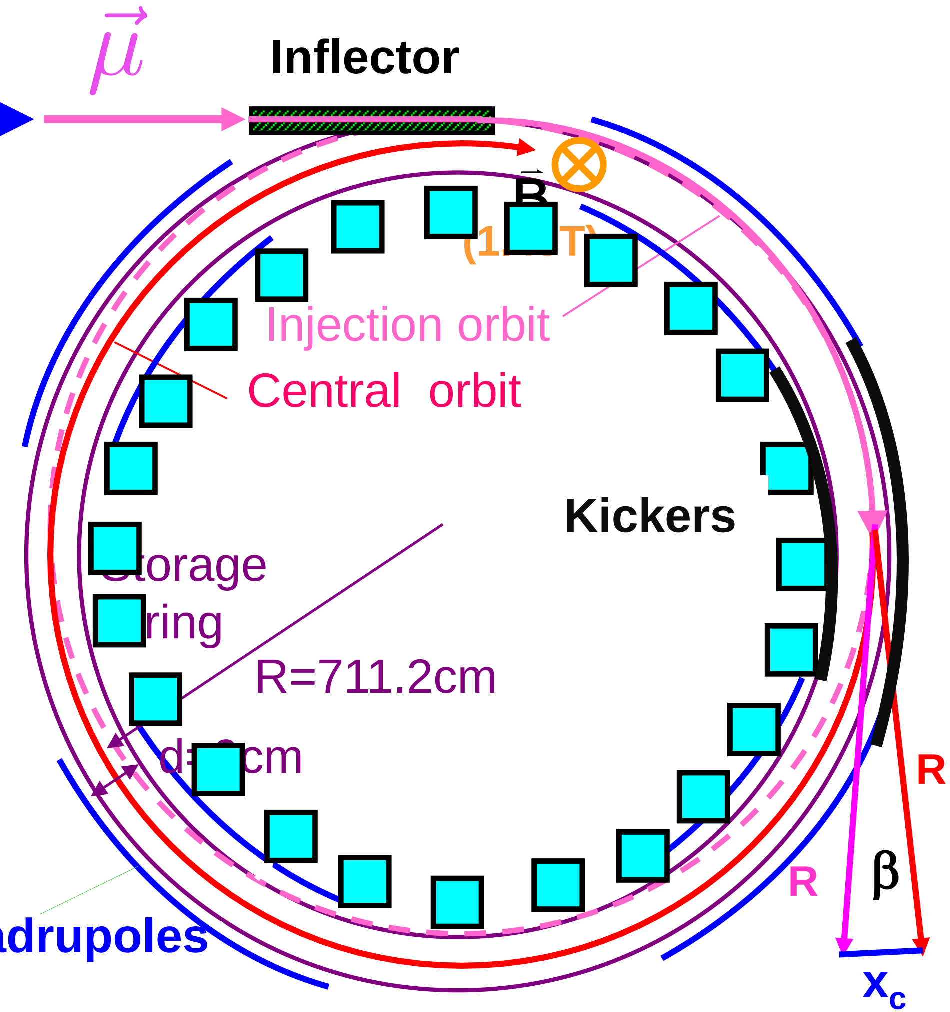
E821 Technique: Direct μ injection

Muon lifetime	$\gamma\tau_\mu$ = 64.4 μs
(g-2) period	τ_a = 4.37 μs
Cyclotron period	τ_C = 149 ns

narrow bunch of protons

$x_c \approx 77$ mm
$\beta \approx 10$ mrad
B·dl ≈ 0.1 Tm

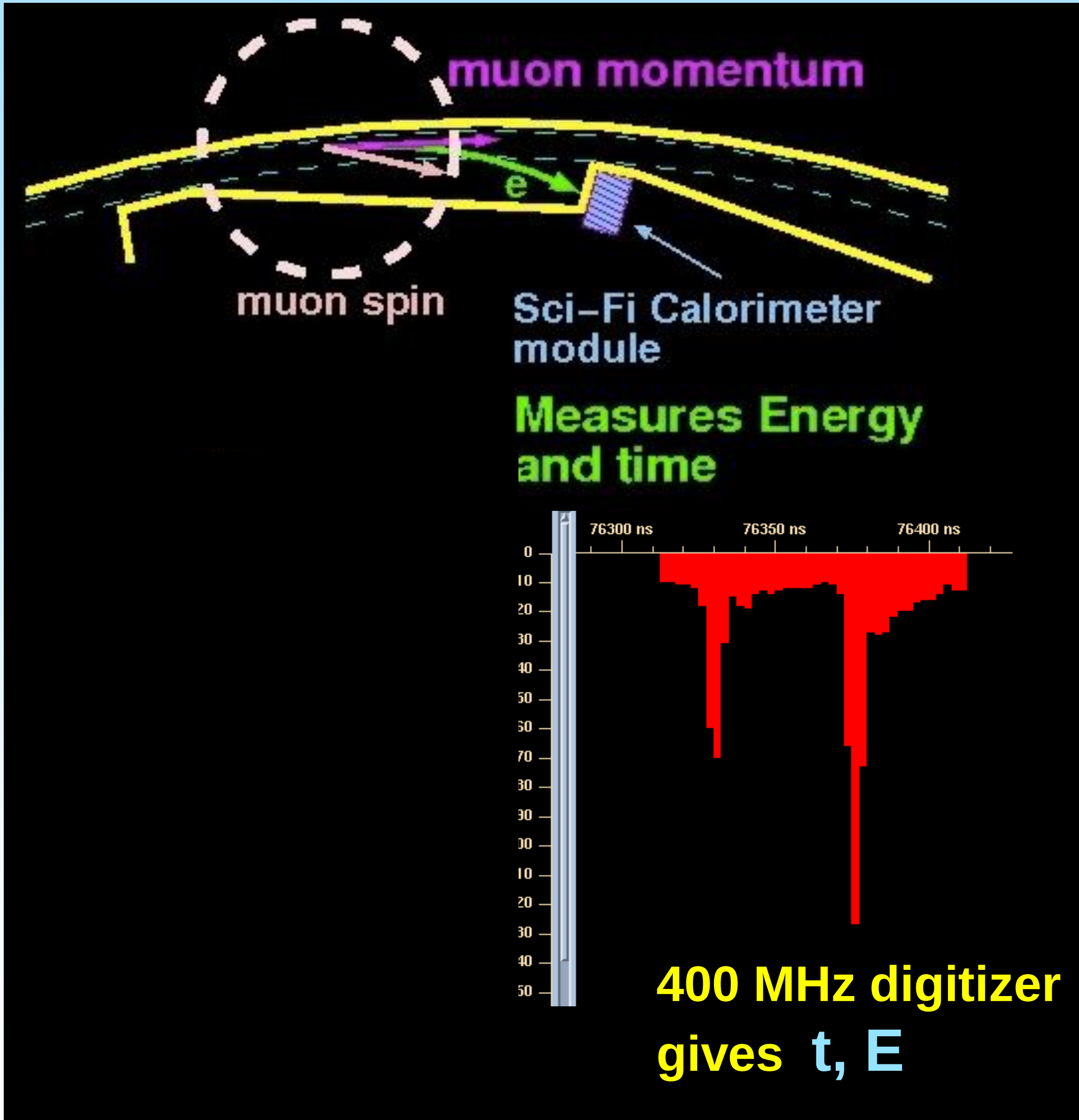




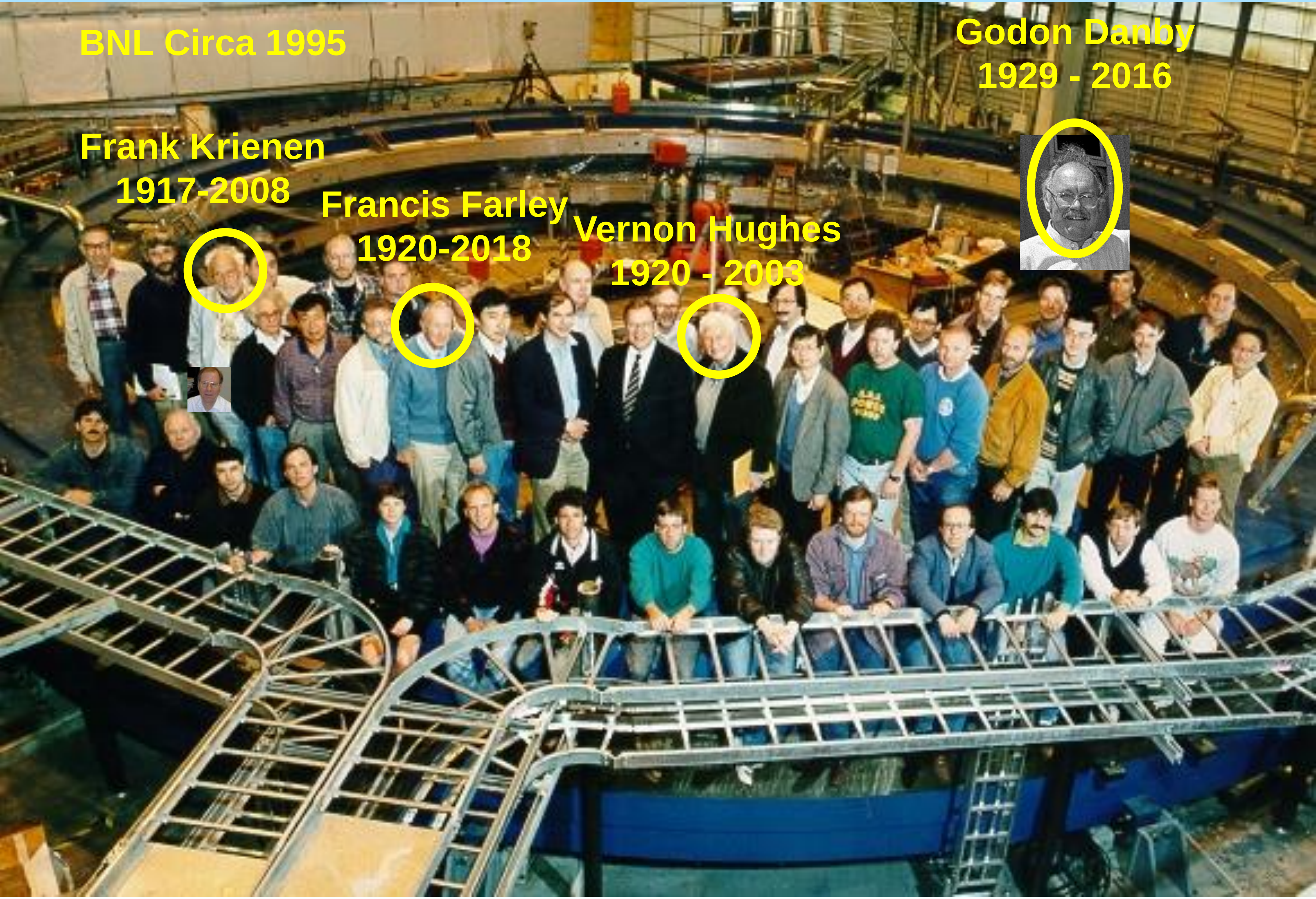
- Muon storage ring – weak focusing betatron
- Muon polarization
- Injection & kicking
- Focus with electric quads
- 24 electron calorimeters (E, t)

$$\vec{\omega}_a = -\frac{Qe}{m} a_\mu \vec{B}$$

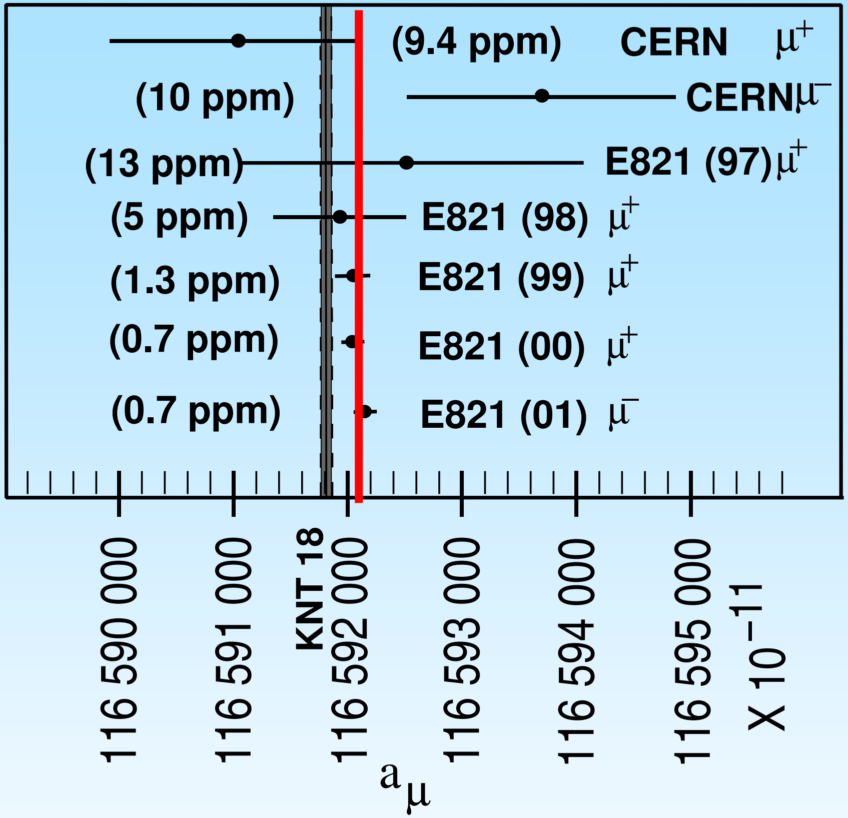
To measure ω_a, we used Pb-scintillating fiber calorimeters.



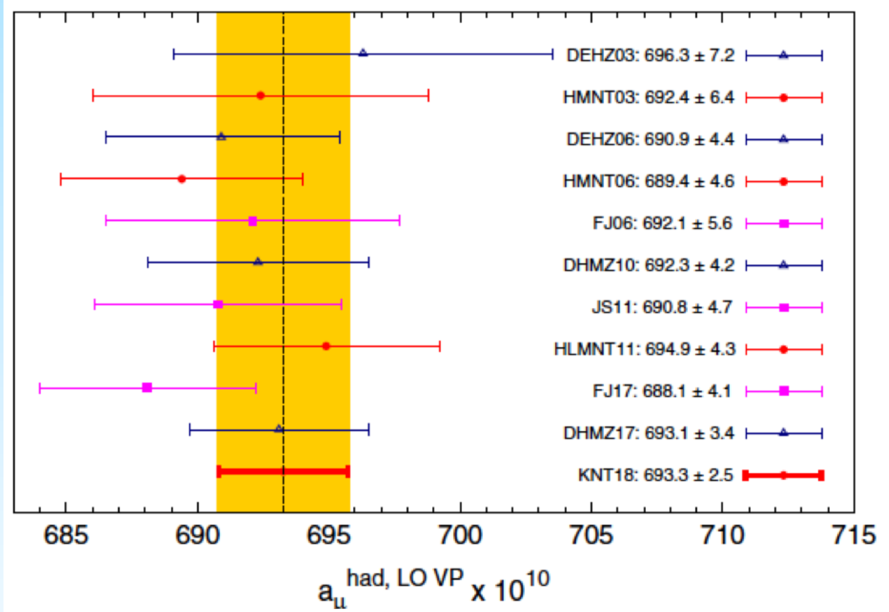
Count number of e^- with $E_e \geq 1.8$ GeV


BNL Circa 1995
Godon Danby
1929 - 2016
Frank Krienen
1917-2008
Francis Farley
1920-2018
Vernon Hughes
1920 - 2003

Where E821 was at the end of 2001 data taking?



Value of LOH contribution has been steady since 2003, which the error decreased significantly



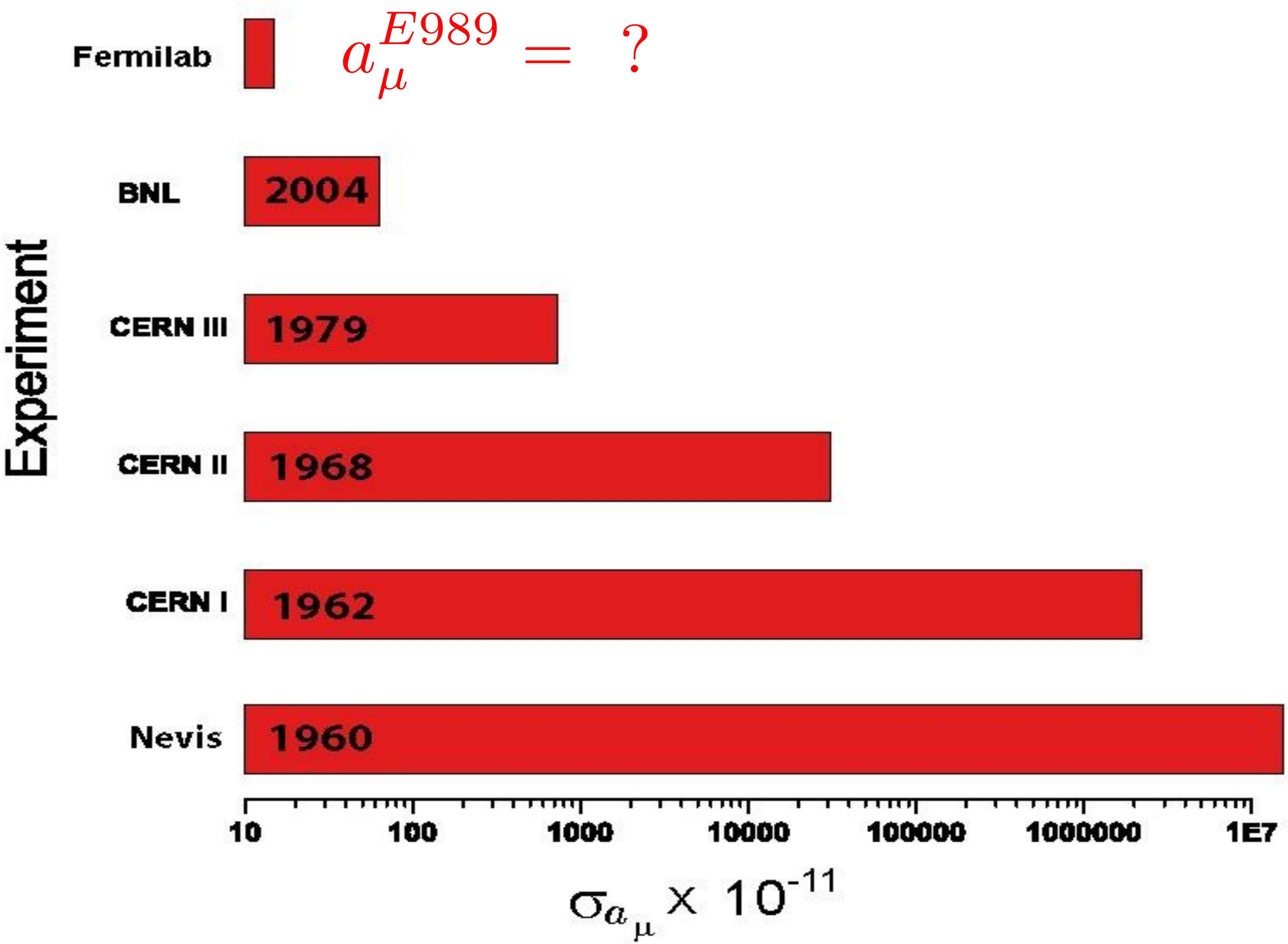
From KNT, PRD 97, 114025 (2018)

$$a_\mu^{exp} = 116\,592\,089(54)_{st}(33)_{sy}(63)_{tot} \times 10^{-11}$$

$$a_\mu^{SM} = 116\,591\,820.4(35.6) \times 10^{-11}$$

$$\Delta = (270.6 \pm 72.6) \times 10^{-11} \quad 3.7\sigma$$

$$a_\mu^{E989} = \ ?$$



Fermilab
BNL
2004
CERN III
1979
CERN II
1968
CERN I
1962
Nevis
1960
Experiment
10
100
1000
10000
100000
1000000
1E7
$\sigma_{a_\mu} \times 10^{-11}$

Acknowledgements

- I have borrowed heavily from
- Charpak G et al 1961 Phys. Rev. Lett. 6 128
- Bailey et al, Nuclear Physics B150 (1979)
- Bailey J et al 1968 Phys. Lett. B 28 287
- Miller, de Rafael, Roberts, Rep. Prog. Phys. 70 (2007) 795-881
- Farley and Semertzidis, Prog. Nucl. Part. Phys. 52 (2004) 1-83.
- Bennett et al., Phys. Rev. D 73, 072003 (2006).
- The web, including the AIP Segre archives and the Nobel website for photos.
- BNL photo archives