Experimental Input for $a_\mu$ Light-by-Light

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Muon Anomaly

- Muon anomaly: \( a_\mu = \frac{g_\mu - 2}{2} = \frac{\alpha}{2\pi} + \ldots = 0.0011659 \ldots \)
- Experimental: \( a_\mu^{\text{exp}} = 1165.920 \pm 8.9 (6.3) \times 10^{-10} (0.54 \text{ ppm}) \)

  [BNL-E821: Phys. Rev. D 73 072003]

- Standard Model prediction: \( a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}} \)

  \[
  = 1165.918 \pm 2.3 (4.3) \times 10^{-10}
  \]


\[ a_\mu^{\text{exp}} - a_\mu^{\text{SM}}: 26.8 \pm 7.6 (3.5\sigma) \]
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[HVP

HLbL]
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- Model calculations
- Data-driven approach
  - Reduce model dependency (goal:10..20%)
  - Reliable error estimations
- Lattice calculation

[BNL-E821: Phys. Rev. D 73 072003]

[Colangelo et al '14; '15; '17; Pauk, Vanderhaeghen '14; 15'; RBC–UKQCD; Mainz]
Experimental Input for HLbL

Transition form factors, Helicity amplitudes

BaBar, Belle, BESIII, CELLO, CLEO, ...

A2, BESIII, NA60, CMD-2, SND, ...
Experimental Input for HLbL

Transition form factors, Helicity amplitudes

CELLO

CLEO

BESIII

KLOE-2

SND

CMD-2

\[ IF(q^2, q'^2) = \text{Experimental Input for HLbL} \]

\[ \gamma^* \rightarrow \pi^0, \eta, \eta' \]

\[ \gamma^* \rightarrow \pi^+ \pi^- \]

\[ e^+ e^- \rightarrow \pi^0, \eta, \eta' \]

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**Space-like TFFs**

**Untag:**
- Only tag the hadron products, $P_t$-balance
- $F[Q_{1,2}^2 \sim 0]$

**Single tag:**
- Tag the hadron products
- Tag only one lepton, missing momentum direction
- $F[Q_1^2, Q_2^2 \sim 0]$

**Double tag:**
- Tag the hadron products
- Tag both leptons
- $F[Q_1^2, Q_2^2]$
Existing Data: Space-like

\[ e^+ e^- \rightarrow e^+ e^- \pi^0 \]

\[ e^+ e^- \rightarrow e^+ e^- \eta \]

\[ e^+ e^- \rightarrow e^+ e^- \eta' \]

- Recent results from BABAR and BELLE: \( Q^2 > 4 \text{ GeV}^2 \)

- CLEO: \( Q^2 > 1.5 \text{ GeV}^2 \)

- CELLO: \( Q^2 < 1.5 \text{ GeV}^2 \), very poor accuracy

Low \( Q^2 \) range not covered/precise

Relevant $Q^2$ Region

$$a_{\mu}^{\text{HLBL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i W_i(Q_1, Q_2) f_i(Q_1, Q_2)$$

Form factor dependent

Universal weight functions

Relevant $Q^2$ region:

$< 1.5 \text{ GeV}^2$

Peak around 0.25 GeV$^2$

Double virtual form factor needed!

$e^+e^- \rightarrow e^+e^- \pi^0$ at BESIII

$151$ R-scan samples:
$2.0 \sim 4.6$ GeV

$\sim 17.5$ fb$^{-1}$
$e^+e^- \rightarrow e^+e^-\pi^0$ at BESIII

Event Selection:

- Exactly one lepton candidate, $E/P > 0.8$
- At least two, max four photons
- Cut on angle of the missing momentum
- Helicity angle $\cos \theta_H < 0.8$
- Kinematic cuts to reject ISR background

- 2.93 fb$^{-1}$
- Signal: EKHARA2.1 generator

151 R-scan samples: $2.0 \sim 4.6$ GeV

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$e^+e^- \rightarrow e^+e^- \pi^0$ at BESIII
**Form Factor Calculation**

Strategy: Count $\pi^0$ yield in bins of $Q^2$ → $d\sigma/dQ^2$ → Form factor $F(Q^2)$
Form Factor Calculation

**Strategy:**
- Count \( \pi^0 \) yield in bins of \( Q^2 \)
- \( d\sigma/dQ^2 \)
- Form factor \( F(Q^2) \)

- Fit invariant mass distribution for each \( Q^2 \) bin
  - Exclude peak region from fit
  - Count number of events in peak region above fitted background

![Graphs showing invariant mass distributions for different \( Q^2 \) bins](graph.png)
Form Factor Calculation

Strategy: Count $\pi^0$ yield in bins of $Q^2$ → $d\sigma/dQ^2$ → Form factor $F(Q^2)$

- Normalize background subtracted data to detection efficiency and luminosity
Form Factor Calculation

Strategy:

- Count $\pi^0$ yield in bins of $Q^2$
- Divide by point-like cross section to obtain TFF

- Normalize background subtracted data to detection efficiency and luminosity

Form factor $F(Q^2)$
Form Factor Calculation

Strategy: Count $\pi^0$ yield in bins of $Q^2$ $\rightarrow$ $d\sigma/dQ^2$ $\rightarrow$ Form factor $F(Q^2)$

- Normalize background subtracted data to detection efficiency and luminosity
- Divide by point-like cross section to obtain TFF

- First Measurement below 0.5 GeV$^2$
- Unprecedented accuracy below 1.5 GeV$^2$
- Competitive accuracy up to 3.1 GeV$^2$

Comparison to Theory

Models:

- Parameters fixed according to publications
- Agreement with result:

\[ \chi^2_{\text{VMD}} = 8.48 \]
\[ \chi^2_{\text{LMD+V}} = 8.62 \]
\[ \chi^2_{1\text{-Octet}} = 9.54 \]
\[ \chi^2_{2\text{-Octet}} = 24.14 \]
\[ \chi^2_{3\text{-Octet}} = 5.94 \]

\[ n.d.f = 18 \]

\[ F_{\text{VMD}}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} \frac{M_V^2}{M_V^2 + Q^2} \]
\[ F_{\text{LMD+V}}(Q^2) = -\frac{F_\pi}{3} \frac{h_1 Q^4 - h_3 Q^2 + h_7}{(M_{V1}^2 + Q^2)(M_{V1}^2 + Q^2)M_{V1}^2 M_{V2}^2} \]
\[ F_{n=1,2\text{-Octet}}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} + \sum_{i=1}^{n} \frac{4\sqrt{2}h_{V1}f_{V1}}{3F_\pi} Q^2(D_{\rho_i} - D_{\omega_i}) \]
\[ F_{3\text{-Octet}}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} + \sum_{i=1}^{3} \frac{4\sqrt{2}h_{V1}f_{V1}}{3F_\pi} Q^2(D_{\rho_i} + F_{\omega_i}H_{\omega_i} + A_{\phi_i}^2 F_{\phi_i} + D_{\phi_i}) \]

[Czyz et al. Phys. Rev. D55 094010 (2012)]
[Czyz et al. Phys. Rev. D97 016006 (2018)]
Comparison to Theory

Data-driven Approaches:

- Construction of space-like TFF using time-like experimental results in dispersive calculations
- Agreement with result: $\chi^2_{\text{center}} = 11.52$

[Hoferichter et al., Phys. Rev. Lett. 121 112002 (2018)]

- Padé approximants
  - Fit previous measurements
  - Model independent
  - Estimation of systematic uncertainty
- Agreement with result: $\chi^2_{\text{center}} = 5.74$

[Masjuan et al., Phys. Rev. D 86 094021]
Space-like TFF: $\eta / \eta'$

MC only, $\psi(3770)$ statistics: 2.93 fb$^{-1}$

- Results competitive to previous measurement
- More data and more decay modes $\rightarrow$ order of magnitude improvement

\[ \eta \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \pi^+ \pi^- \gamma \gamma \]

\[ \eta' \rightarrow \pi^+ \pi^- \eta \rightarrow \pi^+ \pi^- \gamma \gamma \]
Time-like TFF: $\eta'$

$$\frac{d\Gamma(\eta' \rightarrow \gamma l^+l^-)}{dq^2 \Gamma(\eta' \rightarrow \gamma\gamma)} = [\text{QED}(q^2)] \times |F(q^2)|^2$$

Point-like

Vector meson dominance model:

$$|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2 + \gamma^2)}{(\Lambda^2 - q^2)^2 + \Lambda^2\gamma^2}$$

$\Lambda^2 = (1.60 \pm 0.17 \pm 0.08) \text{ GeV}^{-2}$

Agrees with VMD prediction

Space-like result

Space-like $\pi^+\pi^-$

- First single-tag measurement, 7.5fb$^{-1}$ data sample
- Event selection similar to pesudoscalar analysis
- Dominate background:
  - $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$
    - QED process, dedicated $\mu/\pi$ separation, remaining estimated with precise MC simulation BdkRc + DIAG36-ABC
  - $e^+e^-\rightarrow e^+e^-\pi^+\pi^-$ (not two photon process)
    - Same final states as signal, radiative Bhabha scattering couples to vector resonance ($\rho$), extract using fit method
Available region

- Extract cross section in bins of $W$, $Q^2$ and $\cos\theta^*$

Access to: $Q^2$ region: 0.1 - 4.0 GeV$^2$

$W$: threshold - 2.0 GeV/$c^2$

$|\cos\theta^*|$: 0.0 - 1.0
Conclusion and Outlook

Input for HLbL:

- Transition form factors of $\pi^0$, $\eta$, $\eta'$ in relevant $Q^2$ region
  - Unprecedented accuracy for $Q^2<1.5$ GeV$^2$

- First measurement of single tag $\gamma\gamma\rightarrow\pi^+\pi^-$
  - Start from $2\pi$ invariant mass threshold, access to low $Q^2$, cover full helicity angle
  - To be extended to neutral pion pair final state

- Double tag study of $\pi^0$ possible using BESIII statistics

Thank you for your attention!
BEPCII: \( \tau \)-charm factory
Beam energy: 1-2.3 GeV
Design luminosity: \( 1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1} \) (April 2016)
Data taking from 2009 to present

MUC: 9/8 layer RPC, \( \sigma_{R\Phi} \): 2 cm
Magnet yoke
TOF: (\( \sigma_T \))
80 ps / 110 ps
Beam pipe
MDC:
\( \sigma_p/p \): 0.5% at 1 GeV/c
\( \Delta E/E \): 2.5% / 5.0% at 1 GeV;
\( \sigma_Z \): 0.6 cm/\( \sqrt{E} \)

SC Magnet: 1 Tesla

Csl calorimeter:
Previous Measurement

- **MarkII**: 209 fb\(^{-1}\) @ 29 GeV cover W from 0.35 to 1.60 GeV  
  [PRD42, 5, 1990]

- **Cello**: 86 fb\(^{-1}\) cover W from 0.75 to 1.9 GeV  
  [Z.Phys.C56, 381, 1992]

- **Belle**: 85.9 fb\(^{-1}\) @ 10.52-10.58 GeV cover W from 0.8 to 1.5 GeV  
  [PRD75, 051101(R), 2007]

- All in two real photon case: \(\gamma\gamma \rightarrow \pi^+\pi^-\)

- In low mass region, only measurement come from MarkII
### Systematic Uncertainties

Error propagation: 
\[ \Delta|F(Q^2)|_i = \frac{1}{2} \frac{1}{\sqrt{|F(Q^2)|^2}} \Delta(|F(Q^2)|^2)_i \]

<table>
<thead>
<tr>
<th>Source</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>External</strong></td>
<td></td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.25%</td>
</tr>
<tr>
<td>Photon detection efficiency</td>
<td>1%</td>
</tr>
<tr>
<td>Luminosity</td>
<td>0.25%</td>
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<tr>
<td><strong>Analysis</strong></td>
<td></td>
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<tr>
<td>( q_{\text{tag}} \cdot \cos \theta_{\text{miss}} &lt; -0.99 )</td>
<td>0.1% – 3.1%</td>
</tr>
<tr>
<td>( \cos \theta_H &lt; 0.8 )</td>
<td>0.2% – 4.5%</td>
</tr>
<tr>
<td>(</td>
<td>\Delta \phi_{\gamma\gamma}</td>
</tr>
<tr>
<td>(</td>
<td>\Delta \theta_{\gamma\gamma} - 0.01q_{\text{tag}}</td>
</tr>
<tr>
<td>( R_{\gamma} &lt; 0.05 )</td>
<td>1.0% – 7.7%</td>
</tr>
<tr>
<td>Reconstruction efficiency</td>
<td>1.6% – 17.2%</td>
</tr>
<tr>
<td><strong>Background subtraction</strong></td>
<td></td>
</tr>
<tr>
<td>Signal shape</td>
<td>0.1% – 1.9%</td>
</tr>
<tr>
<td>Event counting</td>
<td>0.1% – 11.1%</td>
</tr>
<tr>
<td>Background shape</td>
<td>0.2% – 21.0%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>3.9% – 30.0%</td>
</tr>
</tbody>
</table>
**Time-like TFF: A2**

4.0×10^5 dalitz decays; $a_\pi = 0.030(10)_{\text{tot}}$

[Phys. Rev. C 95 025202 (2017)]

$\Lambda^{-2} = 1.97(13)_{\text{tot}}$ GeV^{-2}

[Phys. Rev. C 95 035208 (2017)]