INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

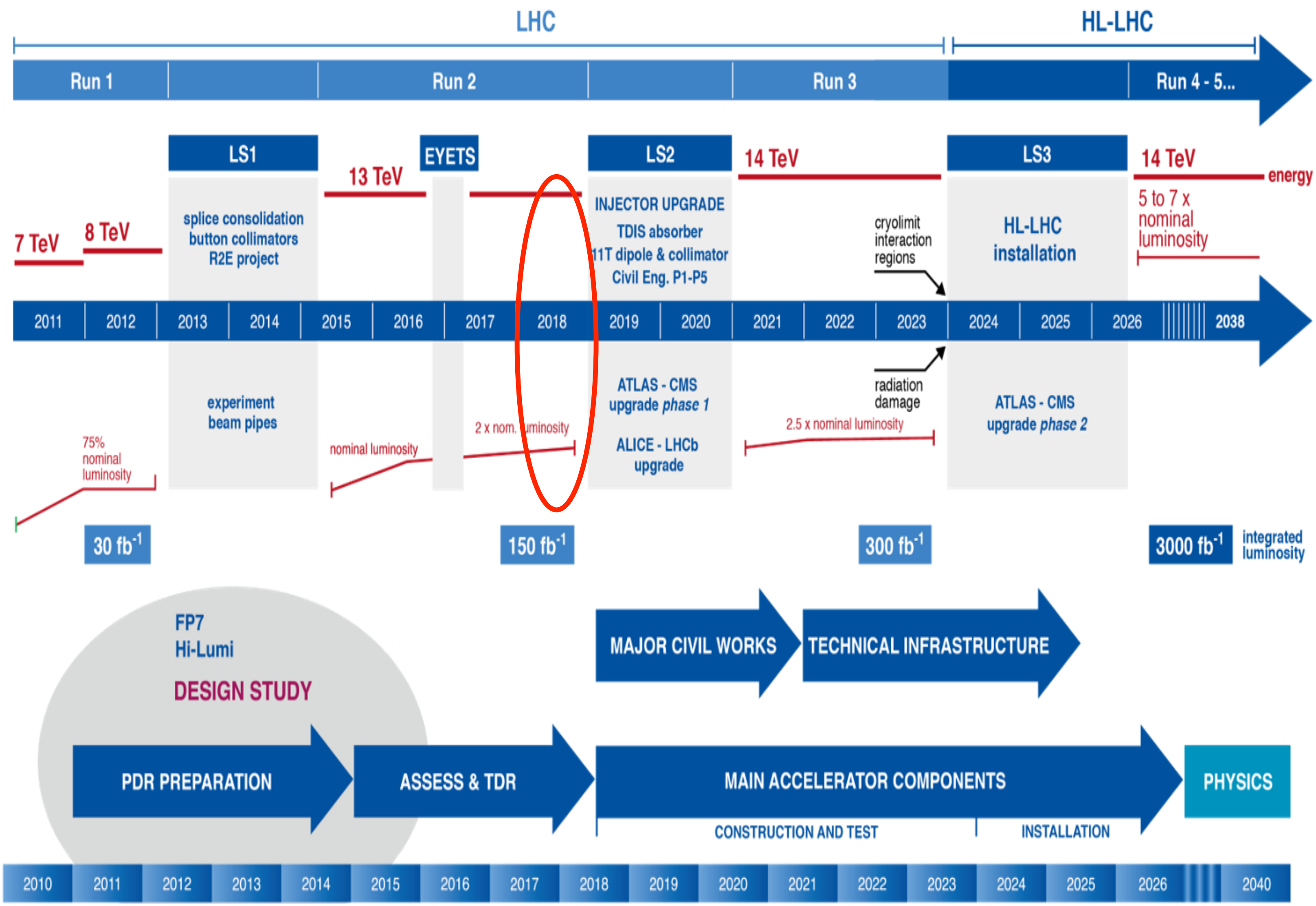
Tau Physics (theory) at the High Luminosity LHC

Emilie Passemar
Indiana University/Jefferson Laboratory

The 15th International Workshop on Tau Lepton Physics
Amsterdam, September 28, 2018

Based on CERN Yellow Report on the Physics Potential of HL/HE-LHC
WG4: Opportunities in Flavour Physics
Conveners: Jorge Martin Camalich, Jure Zupan (Th), Alex Cerri (ATLAS), Sandra Malvezzi (CMS), Vladimir Gligorov (LHCb)
HL/HE questions for taus, Authors for Theory
M. Gonzalez-Alonso, Vincenzo Cirigliano, Adam Falkowski, Emilie Passemar

High Luminosity LHC



High Luminosity LHC

S. Malvezzi, A. Cerri

- Center-of-mass energy of 14 TeV for a total integrated luminosity of ~3000 fb^{-1} in 2035 ⇨ $6 \times 10^{14}\ \tau$
- 200 proton-proton interactions in each collision
- In this regime, experimental sensitivity to new physics enhanced
- Good place for flavour physics but some difficulties:
  - low momenta of typical flavour signatures
  - high pile-up which might affect the precision of the measurements
- Some advantages: Phase II GPD upgrades
  - new inner tracker
  - muon system improvements
  - topological trigger capabilities
  - possibility to use tracking in early stages of the trigger chain

⇨ good detection potential, good pile-up mitigation and in some cases improved performance.

High Luminosity LHC

Observable	Current LHCb	LHCb 2025	Belle II	Upgrade II	GPDs Phase II
EW Penguins					
R_K $(1 < q^2 < 6\,\mathrm{GeV}^2c^4)$	0.1 [255]	0.022	0.036	0.006	–
R_{K^*} $(1 < q^2 < 6\,\mathrm{GeV}^2c^4)$	0.1 [254]	0.029	0.032	0.008	–
R_ϕ, R_{pK}, R_π	–	0.07, 0.04, 0.11	–	0.02, 0.01, 0.03	–
CKM tests					
γ, with $B_s^0 \to D_s^+ K^-$	$(^{+17}_{-22})^\circ$ [123]	4°	–	1°	–
γ, all modes	$(^{+5.0}_{-5.8})^\circ$ [152]	1.5°	1.5°	0.35°	–
$\sin 2\beta$, with $B^0 \to J/\psi K_S^0$	0.04 [569]	0.011	0.005	0.003	–
ϕ_s, with $B_s^0 \to J/\psi\phi$	49 mrad [32]	14 mrad	–	4 mrad	22 mrad [570]
ϕ_s, with $B_s^0 \to D_s^+ D_s^-$	170 mrad [37]	35 mrad	–	9 mrad	–
$\phi_s^{s\bar{s}s}$, with $B_s^0 \to \phi\phi$	150 mrad [571]	60 mrad	–	17 mrad	Under study [572]
a_{sl}^s	33×10^{-4} [193]	10×10^{-4}	–	3×10^{-4}	–
$\lvert V_{ub}\rvert/\lvert V_{cb}\rvert$	6% [186]	3%	1%	1%	–
$\boldsymbol{B_s^0, B^0 \to \mu^+\mu^-}$					
$\mathcal{B}(B^0 \to \mu^+\mu^-)/\mathcal{B}(B_s^0 \to \mu^+\mu^-)$	90% [244]	34%	–	10%	21% [573]
$\tau_{B_s^0 \to \mu^+\mu^-}$	22% [244]	8%	–	2%	–
$S_{\mu\mu}$	–	–	–	0.2	–
$\boldsymbol{b \to c l^- \bar{\nu}_l}$ **LUV studies**					
$R(D^*)$	9% [199, 202]	3%	2%	1%	–
$R(J/\psi)$	25% [202]	8%	–	2%	–
Charm					
$\Delta A_{CP}(KK - \pi\pi)$	8.5×10^{-4} [574]	1.7×10^{-4}	5.4×10^{-4}	3.0×10^{-5}	–
A_Γ $(\approx x \sin\phi)$	2.8×10^{-4} [222]	4.3×10^{-5}	3.5×10^{-5}	1.0×10^{-5}	–
$x \sin\phi$ from $D^0 \to K^+\pi^-$	13×10^{-4} [210]	3.2×10^{-4}	4.6×10^{-4}	8.0×10^{-5}	–
$x \sin\phi$ from multibody decays	–	$(K3\pi)$ 4.0×10^{-5}	$(K_S^0\pi\pi)$ 1.2×10^{-4}	$(K3\pi)$ 8.0×10^{-6}	–

What about τ?

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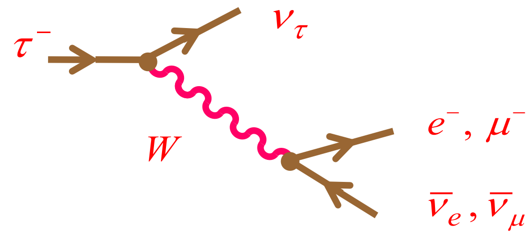
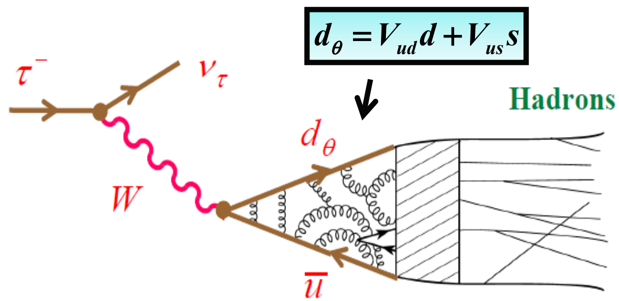
τ ->3μ is used as a benchmark of CMS muon detector upgrade performance

- τ rich phenomenology



- Leptonic decays:
  - Lepton Universality
  - Michel parameters

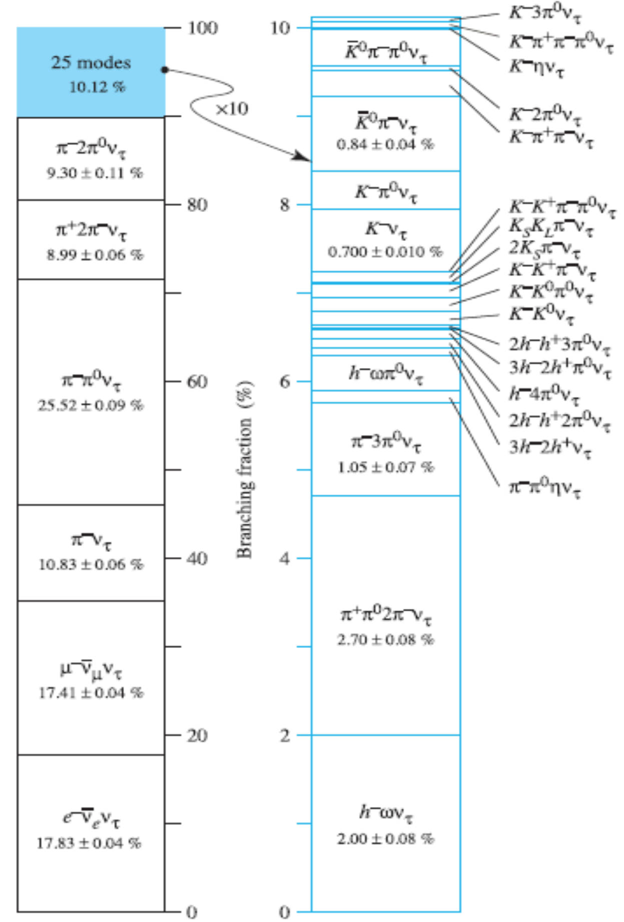


- Hadronic decays:
  - Inclusive τ decays

$\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇒ $\alpha_s(m_\tau)$, $|V_{us}|$, m_s

  - Exclusive τ decays

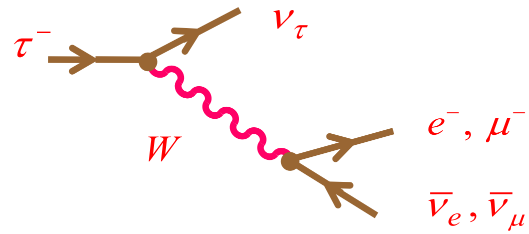
$\boxed{\tau \to (PP, PPP, \ldots)\nu_\tau}$ ⇒ FFs,
resonance parameters
Hadronization of QCD currents
Hadronic contribution to muon g-2
CP violation in Kπ



- Charged lepton flavour violation, Electromagnetic dipole moments
- Muon g-2, 2 photon physics
- Precision EW tests

Role of HL LHC?

- τ rich phenomenology



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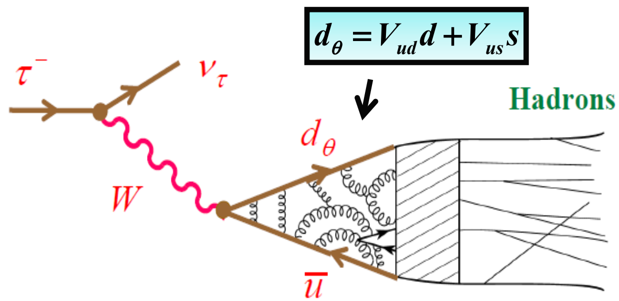

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  - Exclusive τ decays

$$\boxed{\tau \to (PP, PPP, \dots)\nu_\tau} \Rightarrow$$

FFs,
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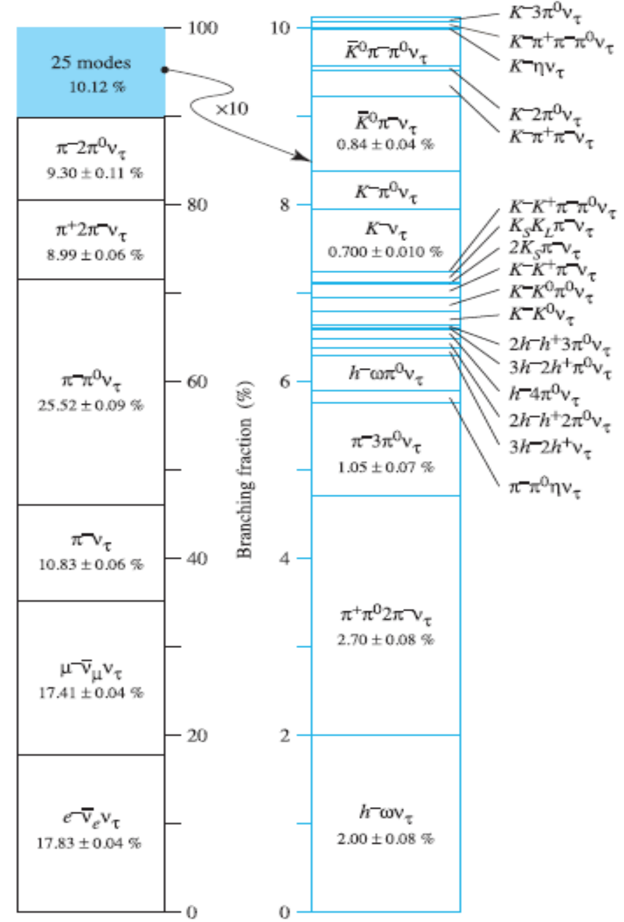

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- Muon g-2, 2 photon physics
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Role of HL LHC?

1. Charged Lepton-Flavour Violation

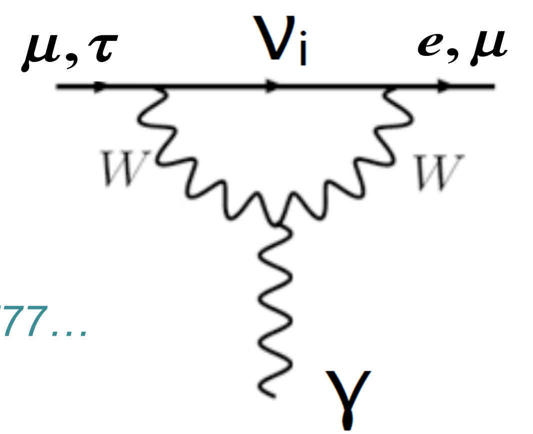
1.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the SM (m_ν=0)

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br\left(\mu \to e\gamma\right) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br\left(\tau \to \mu\gamma\right) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

- In New Physics models: seazible effects
Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

1.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

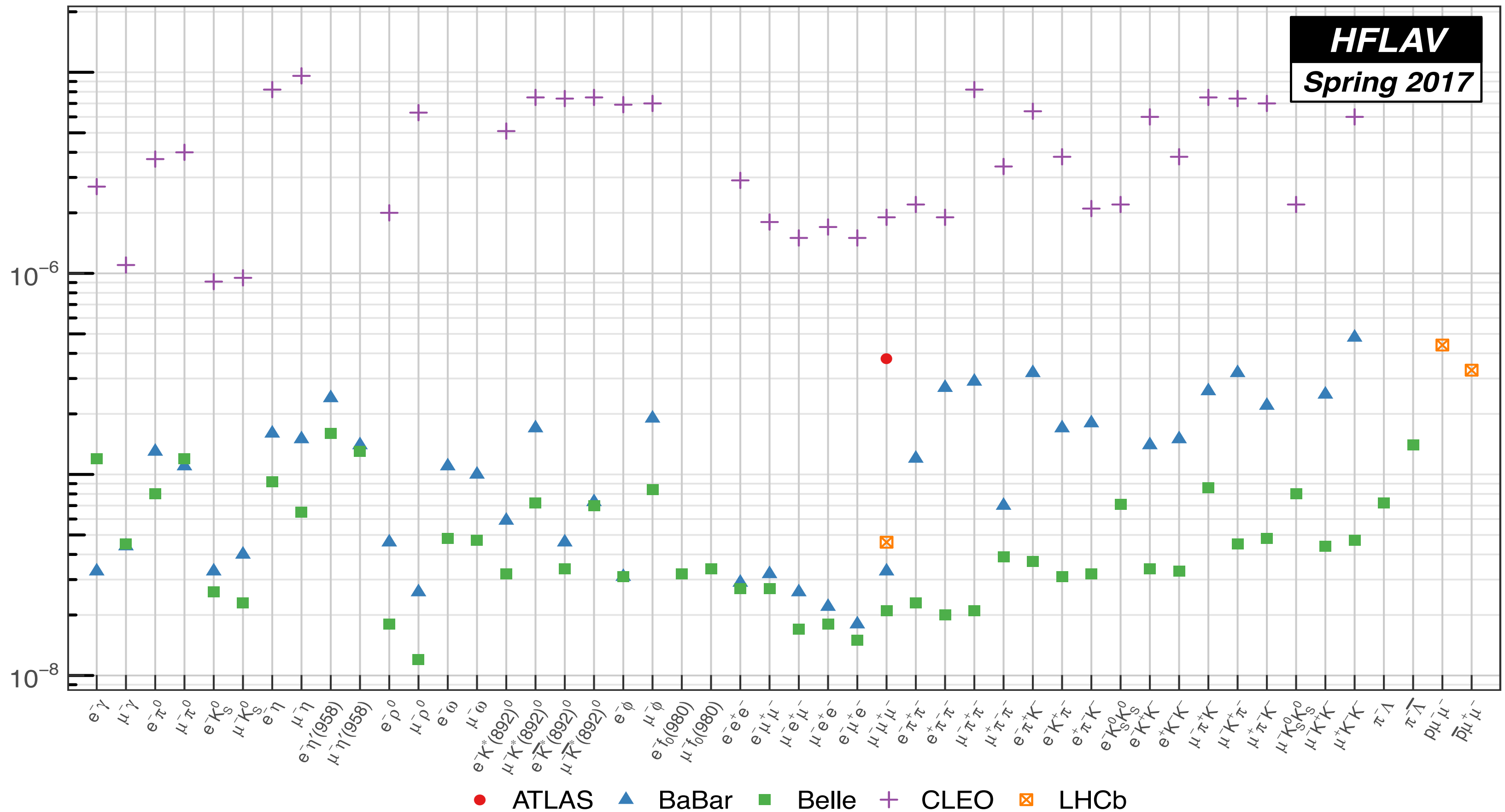
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

1.2 Tau LFV

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ↖ $P,\ S,\ V,\ P\bar{P}, \ldots$

90% CL upper limits on τ LFV decays



- 48 LFV modes studied at Belle and BaBar

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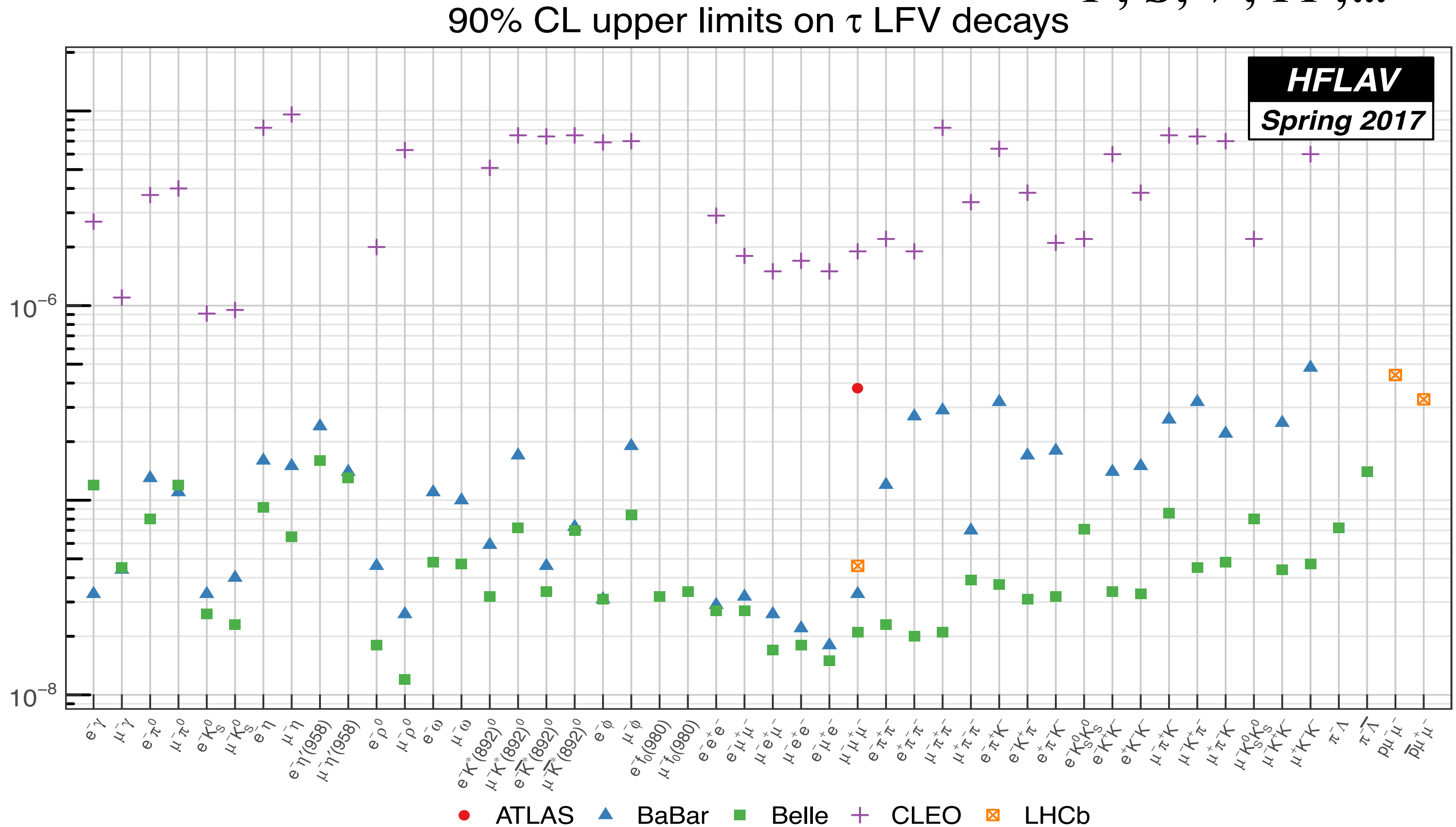
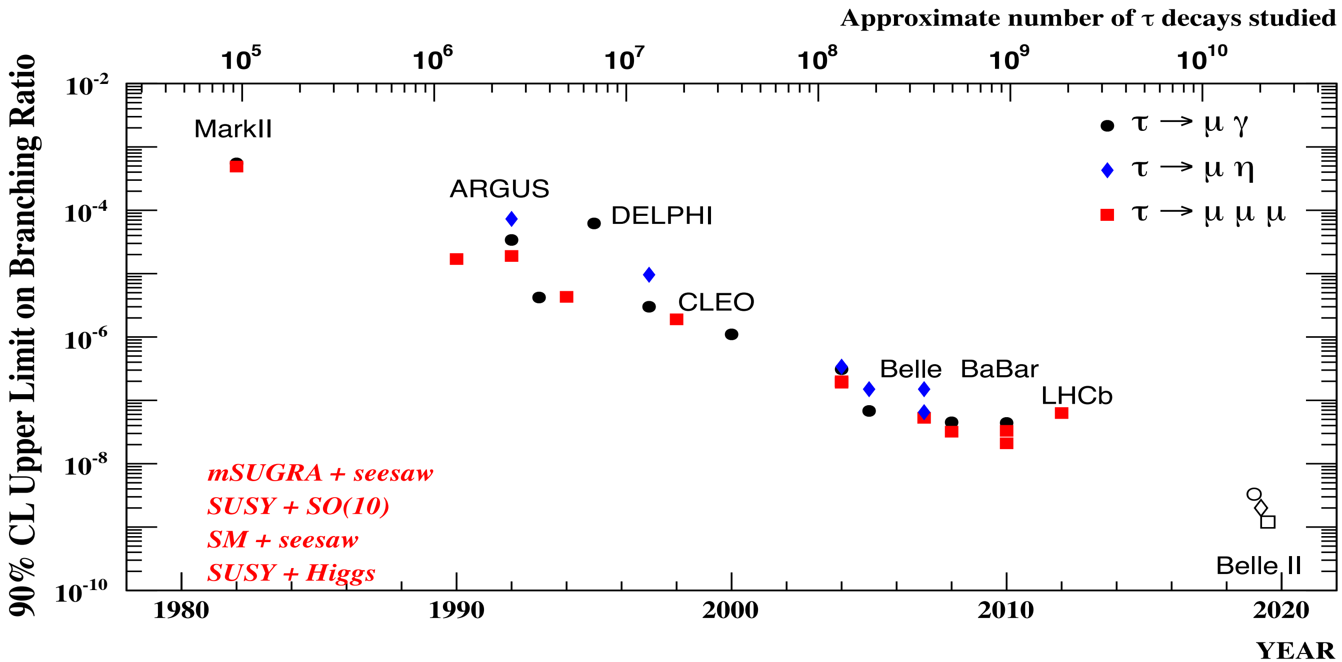
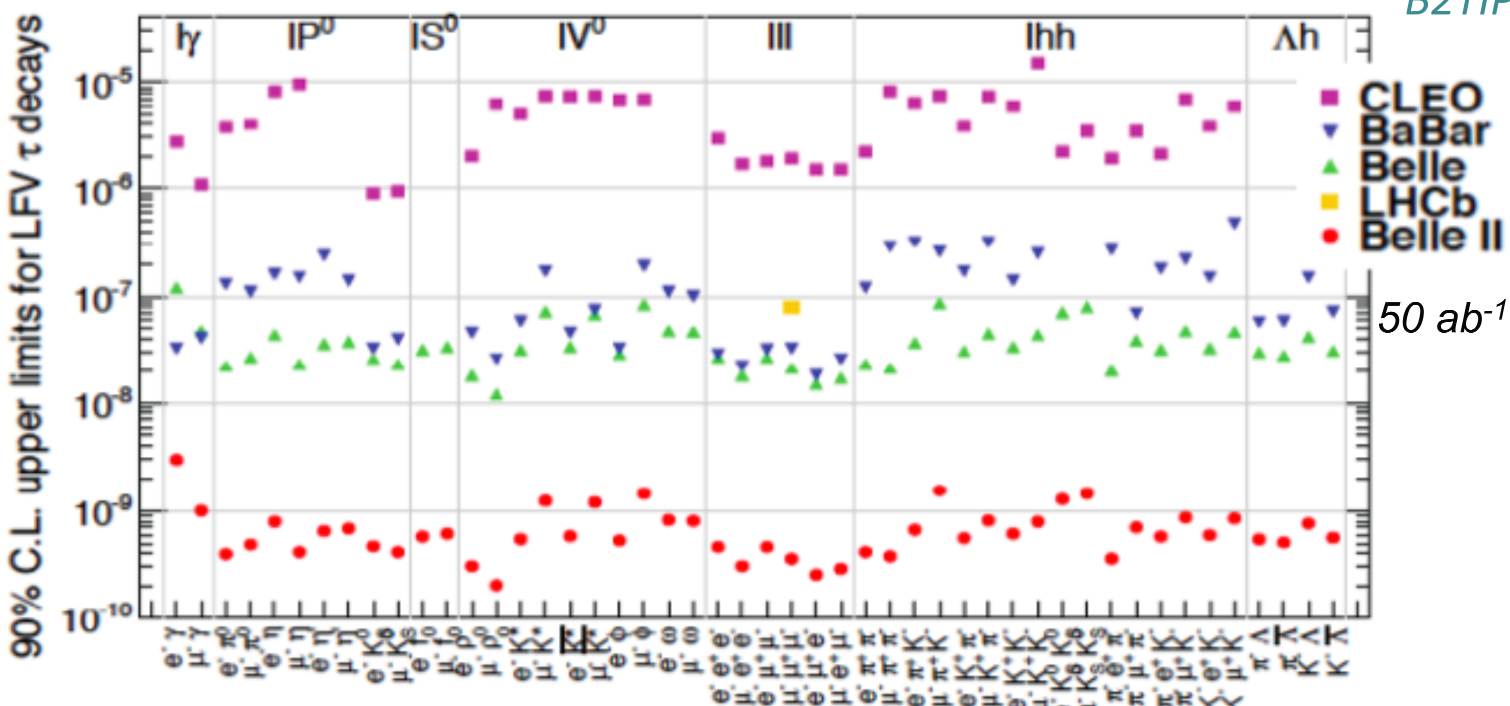


- Expected sensitivity 10^{-9} or better at *LHCb, ATLAS, CMS, Belle II, HL-LHC?*

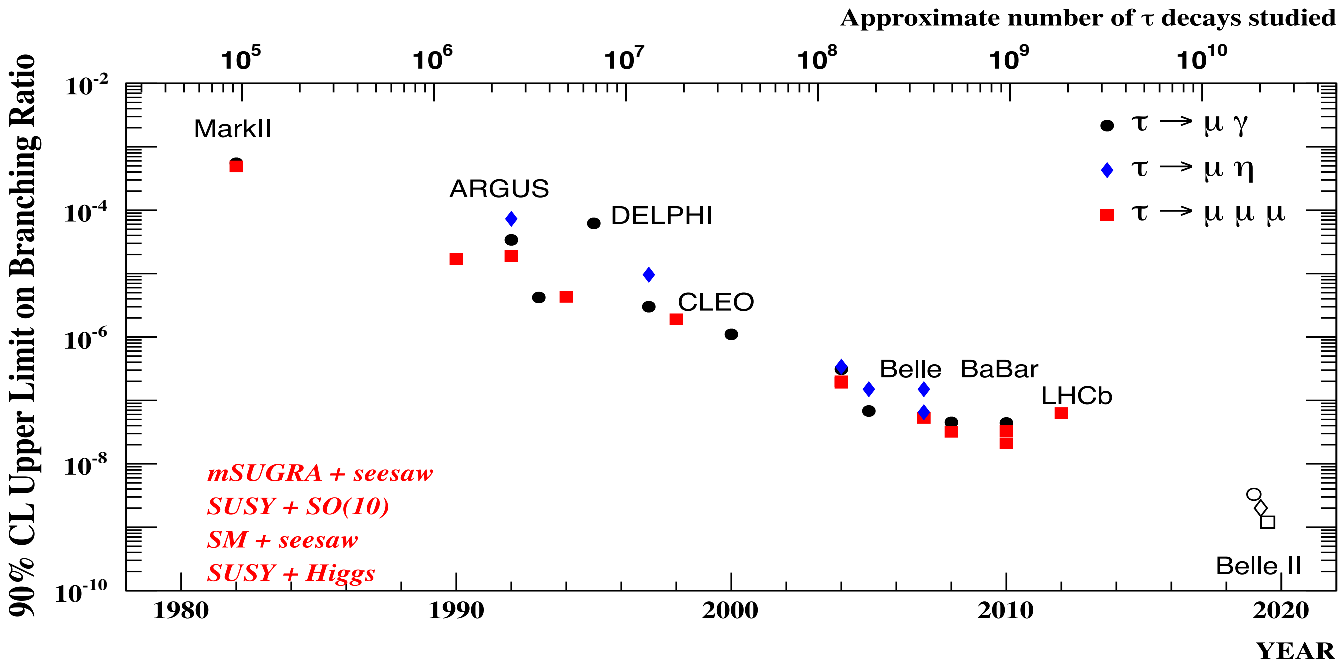


S. Banerjee’17

B2TIP’18



50 ab^{-1} Luminosity



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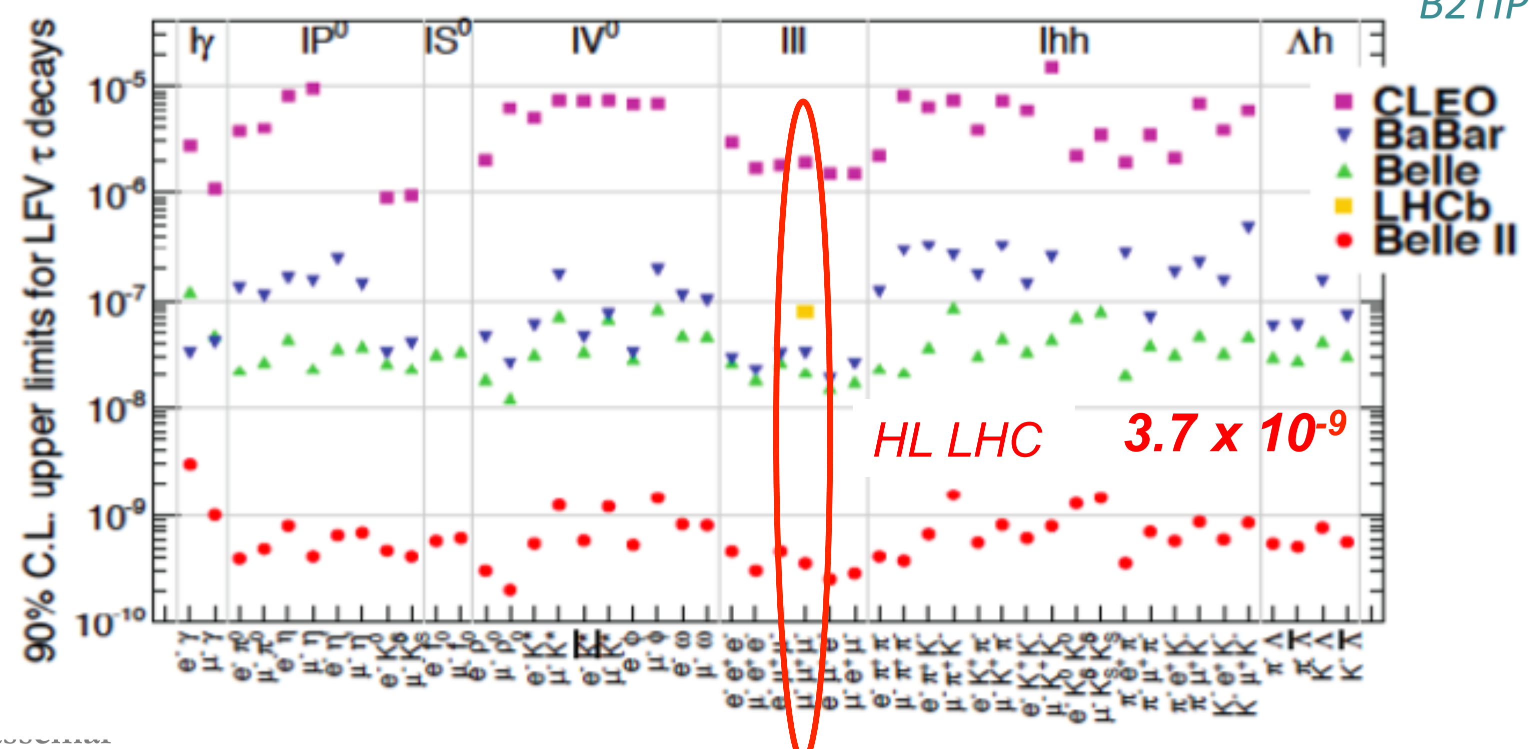

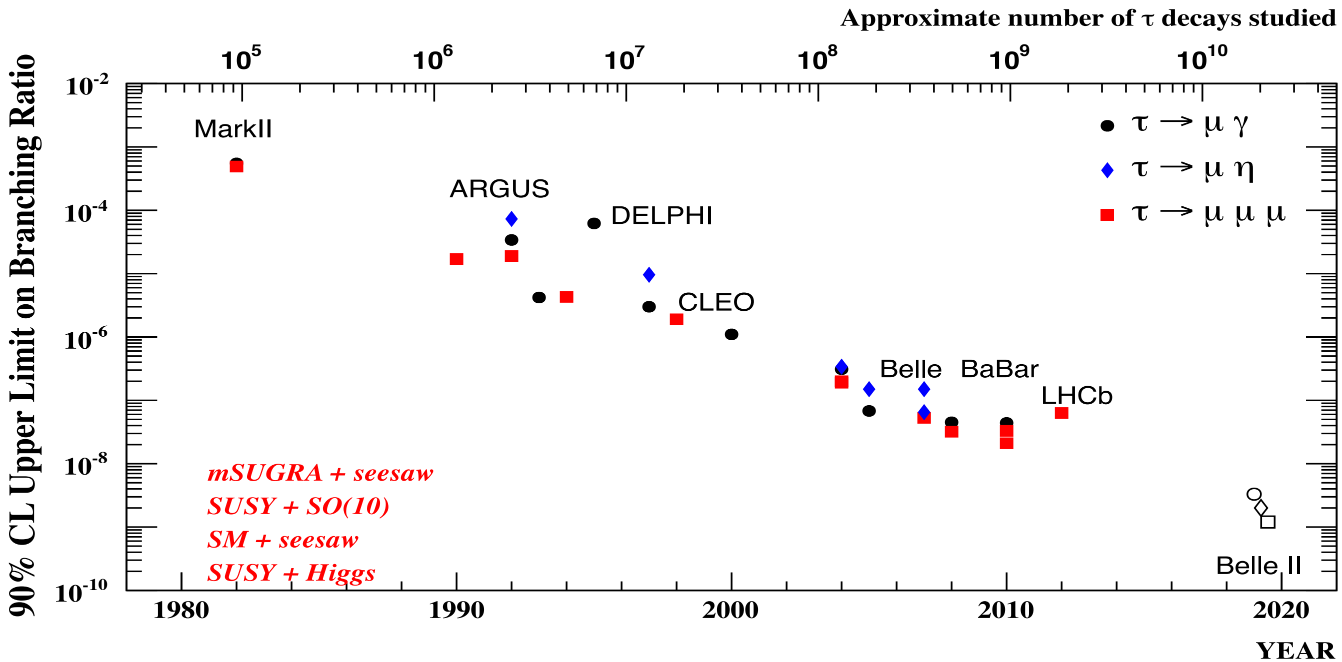

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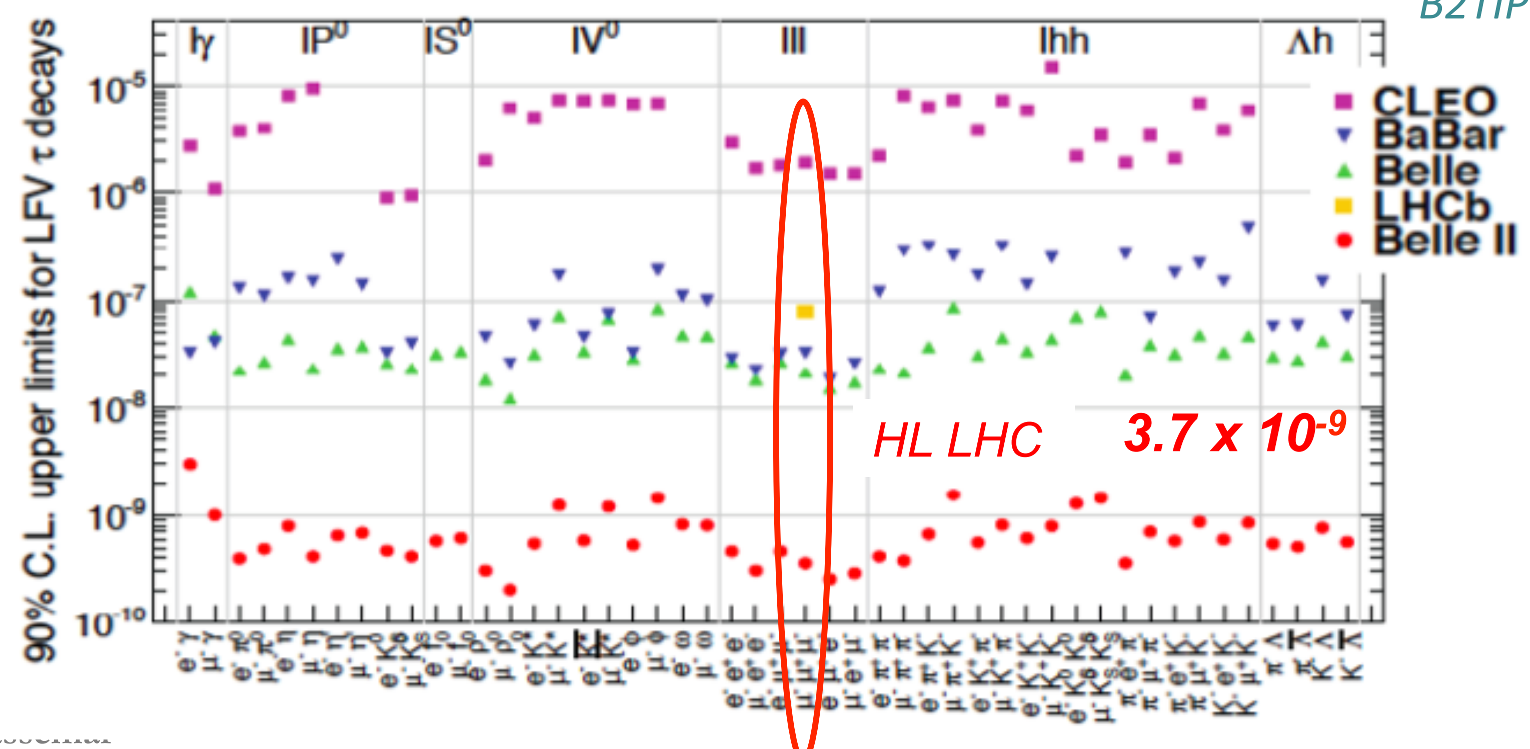


Slide from Talk by *Jian Wang*
On Tuesday

τ->3μ @ HL-LHC

Signal and background yields in [1.55, 2.00] GeV, assuming Br(τ->3μ)=2x10^-8

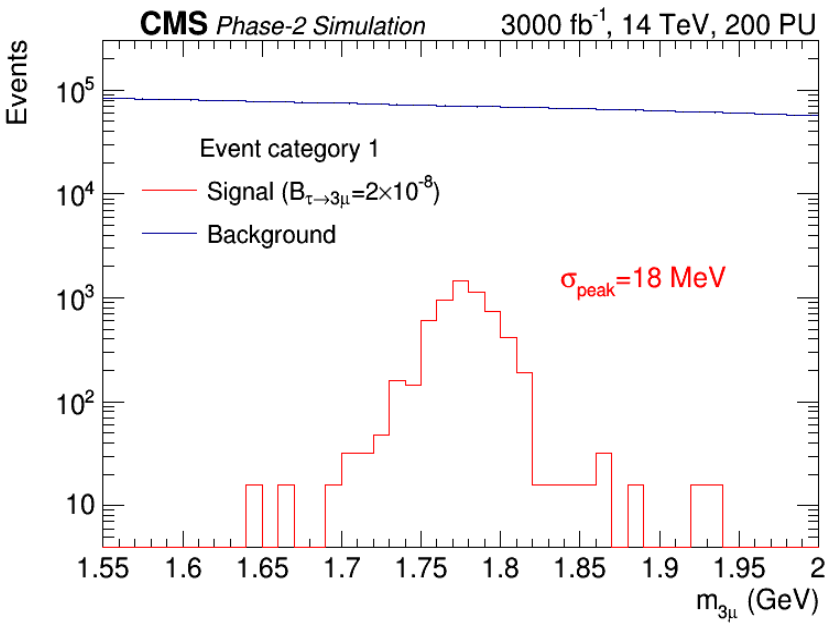
	Category 1	Category 2
Number of background events	2.4×10^6	2.6×10^6
Number of signal events	4 580	3 640
Trimuon mass resolution	18 MeV	31 MeV
$B(\tau \to 3\mu)$ limit per event category	4.3×10^{-9}	7.0×10^{-9}
$B(\tau \to 3\mu)$ 90%C.L. limit	3.7×10^{-9}	

MC simulation study

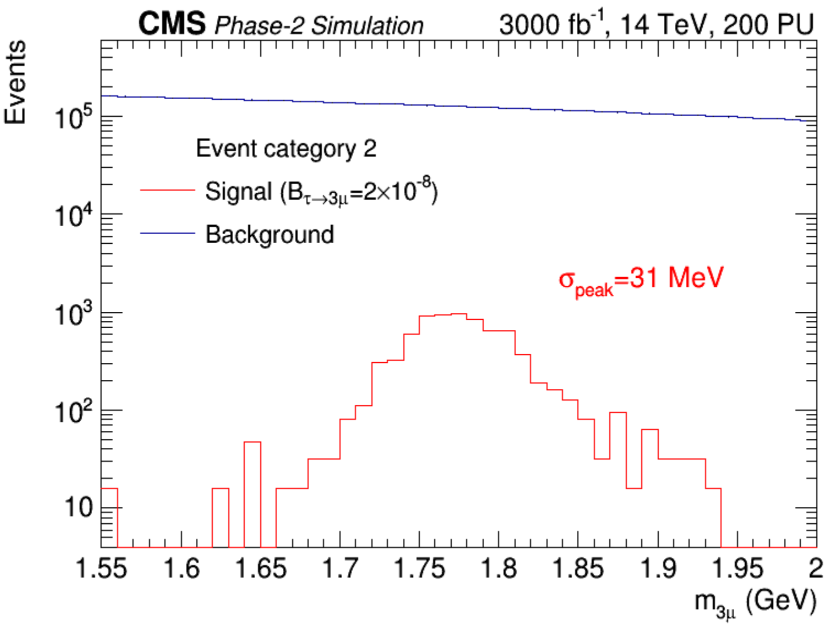
Projected to 3000 fb^{-1}

Adding ME0 detector gains 15% sensitivity

Category 1: Events without using ME0



Category 2: Events with at least one muon tagged by ME0



Note: ME0 reconstruction software was not yet optimised at the time of this study

1.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

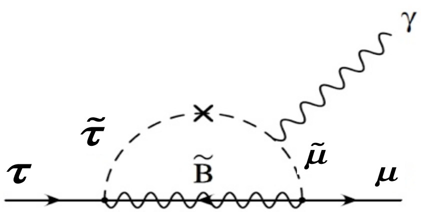
See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:

  - Dipole:

$$\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$$

e.g.



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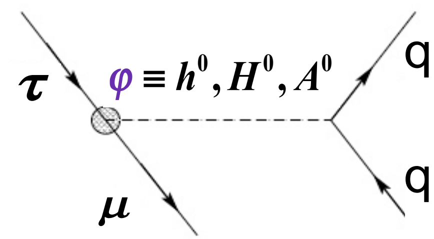
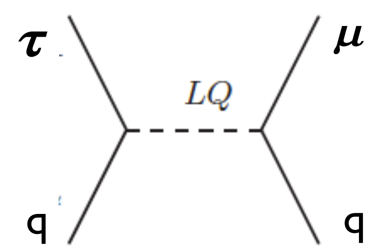


$\boxed{\Gamma \equiv 1}$



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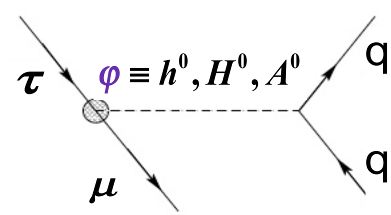
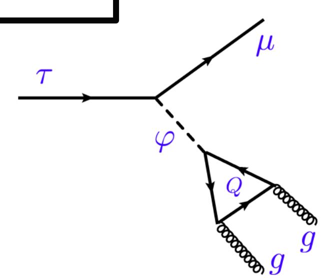
  - Integrating out heavy quarks generates *gluonic operator*

$$\boxed{\frac{1}{\Lambda^2}\bar{\mu}P_{L,R}\tau Q\bar{Q}} \rightarrow \boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu}P_{L,R}\tau\; G_{\mu\nu}^{a} G_{a}^{\mu\nu}}$$





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  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C^{4\ell}_{S,V}}{\Lambda^2} \bar{\mu}\,\Gamma P_{L,R}\tau\, \bar{\mu}\,\Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

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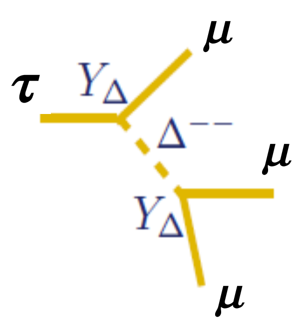

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  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^{a} G_{a}^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu}$
- Each UV model generates a *specific pattern* of them

$$\boxed{\Gamma \equiv 1\,, \gamma^{\mu}}$$

1.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

1.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O_V^q	–	–	✓ (I=1)	✓(I=0,1)	–	–
O_S^q	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O_A^q	–	–	–	–	✓ (I=1)	✓ (I=0)
O_P^q	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Form factors for $\tau \to \mu(e)\pi\pi$ determined using *dispersive techniques*
- Hadronic part:

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13
Celis, Cirigliano, E.P.'14

$$\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} |0\rangle = \left(Lorentz\ \mathrm{struct.}\right)^i_\mu F_i(s)}$$

with

$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

- 2-channel unitarity condition is solved with I=0 S-wave ππ and KK scattering data as input

$$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$$n = \pi\pi, K\bar{K}$$

1.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*
- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*
- *It would be good to be able to constrain $\tau \to \mu\pi\pi$ at HL-LHC!*

1.5 Handles

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state: $\boxed{\frac{dBR(\tau \to \mu\mu\mu)}{d\sqrt{s}}}$
- Benchmarks:
  - Dipole model: $C_D \neq 0$, $C_{else} = 0$
  - Scalar model: $C_S \neq 0$, $C_{else} = 0$
  - Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
  - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

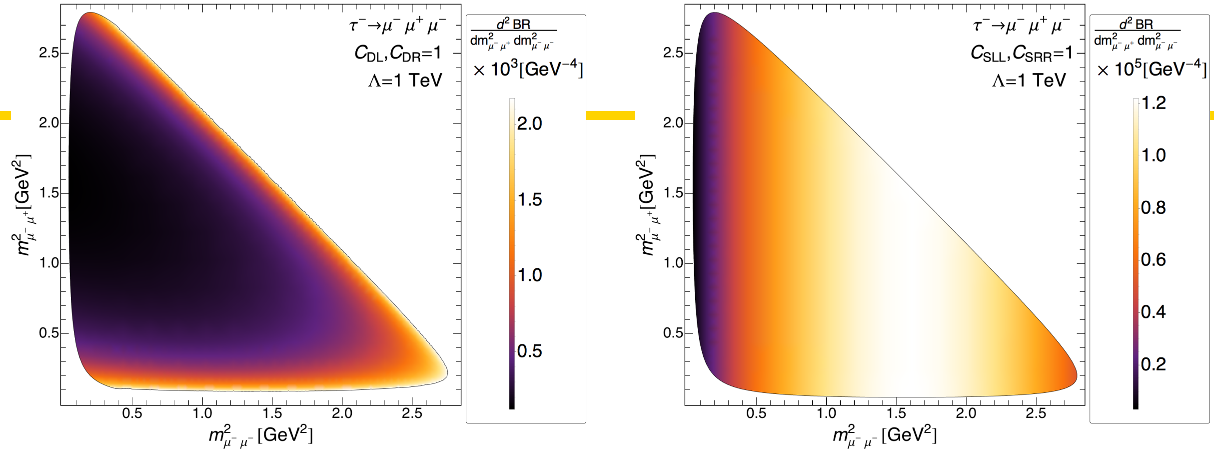
1.6 Model discriminating of BRs

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

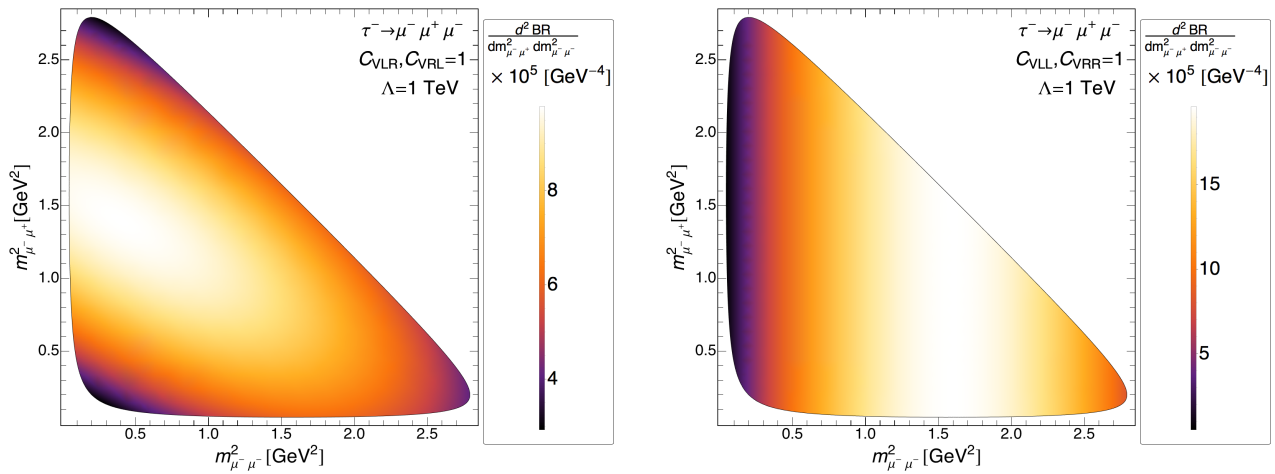
		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

Benchmark



Dassinger, Feldman, Mannel, Turczyk' 07
Celis, Cirigliano, E.P.'14

Figure 3: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of $C_{DL,DR} = 1$ (left) and $C_{SLL,SRR} = 1$ (right), taking $\Lambda = 1$ TeV in both cases. Colors denote the density for $d^2BR/(dm^2_{\mu^-\mu^+}dm^2_{\mu^-\mu^-})$, small values being represented by darker colors and large values in lighter ones. Here $m^2_{\mu^-\mu^+}$ represents m^2_{12} or m^2_{23}, defined in Sec. 3.1.*



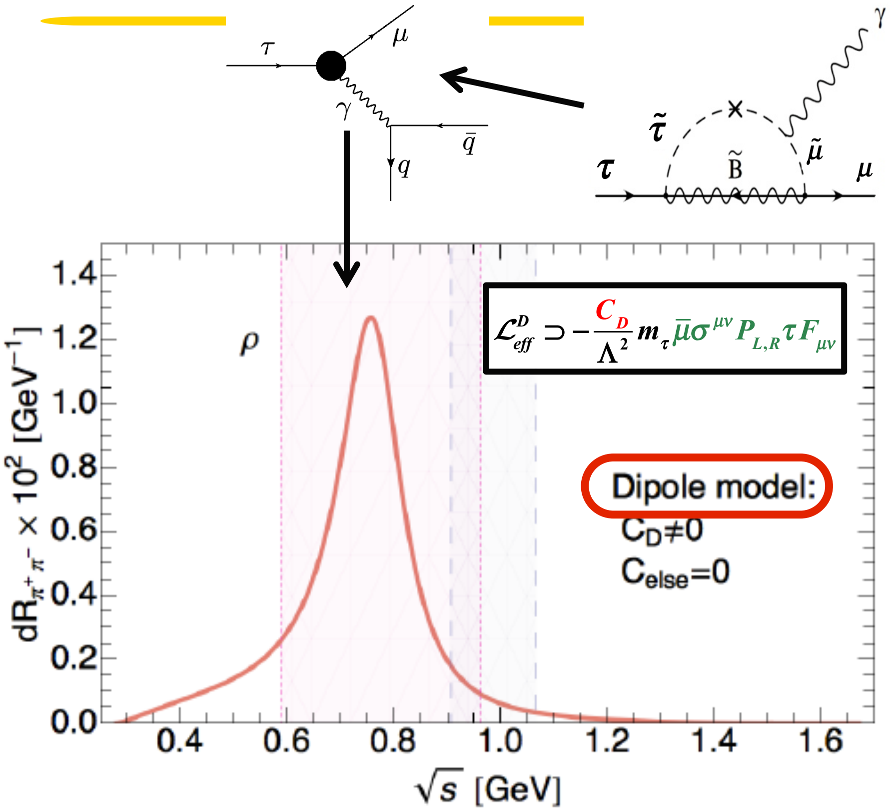
Angular analysis with polarized taus

Dassinger, Feldman, Mannel, Turczyk' 07

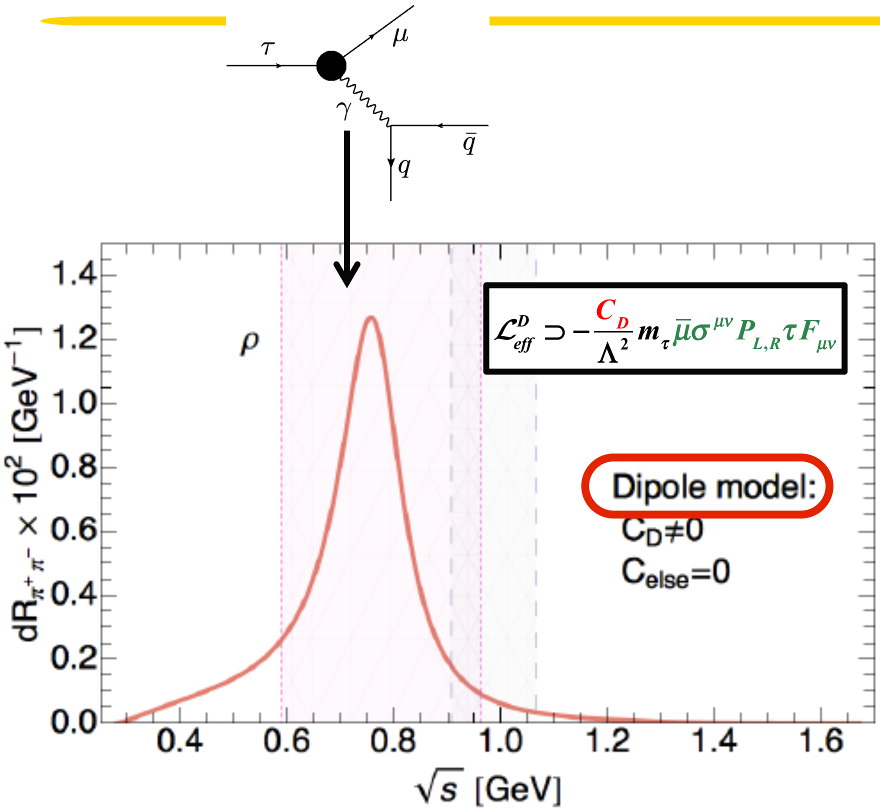
Figure 4: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of $C_{VRL,VLR} = 1$ (left) and $C_{VLL,VRR} = 1$ (right), taking $\Lambda = 1$ TeV in both cases. Colors are defined as in Fig. 3.*

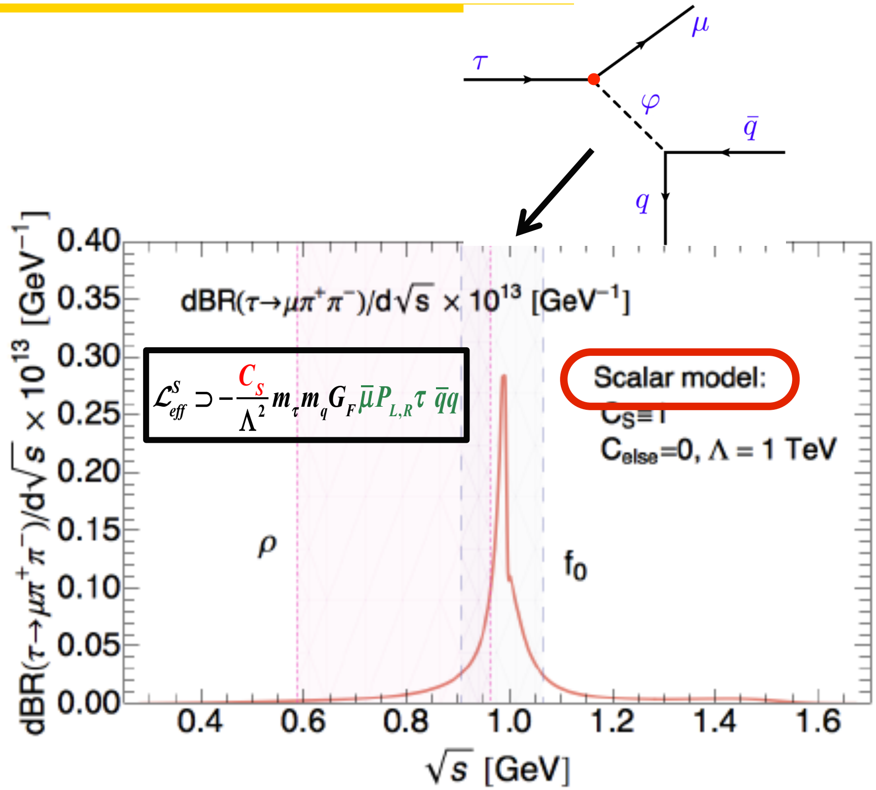
1.7 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays

Celis, Cirigliano, E.P.'14

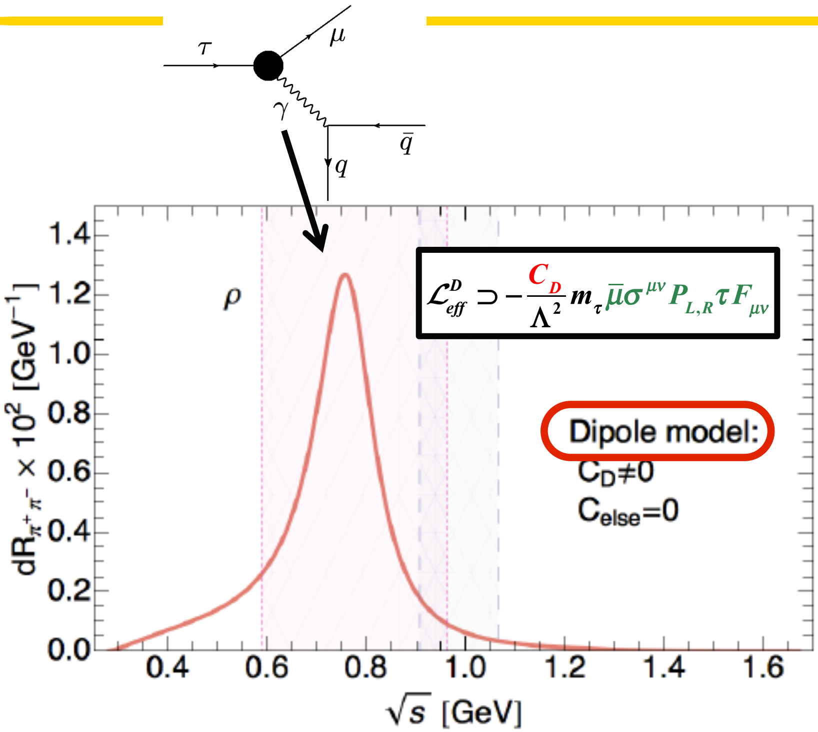


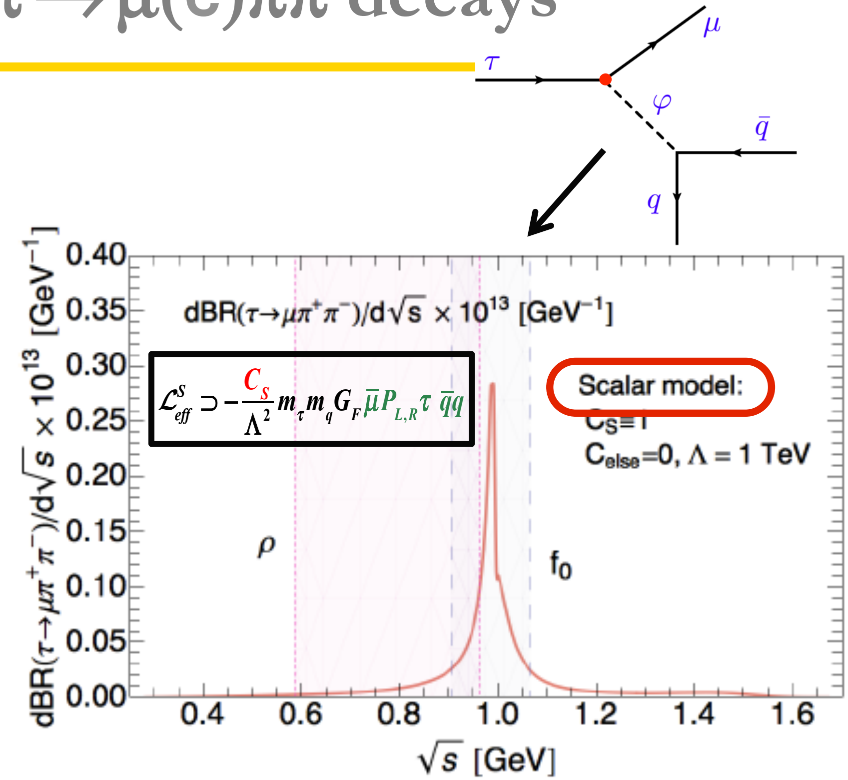
1.7 Discriminating power of τ→μ(e)ππ decays

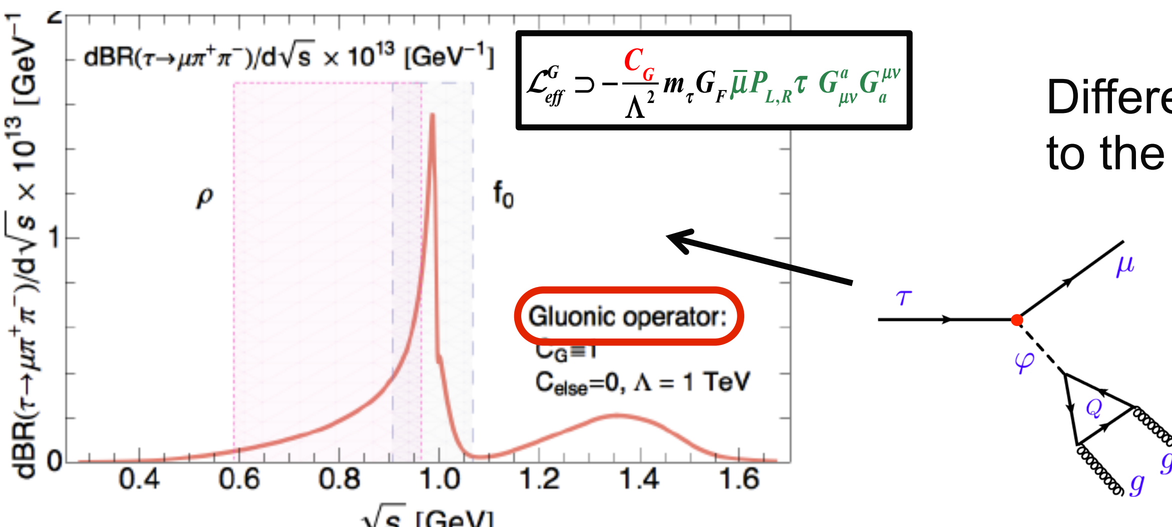




1.7 Discriminating power of $\tau \to \mu(e)\pi\pi$ decays



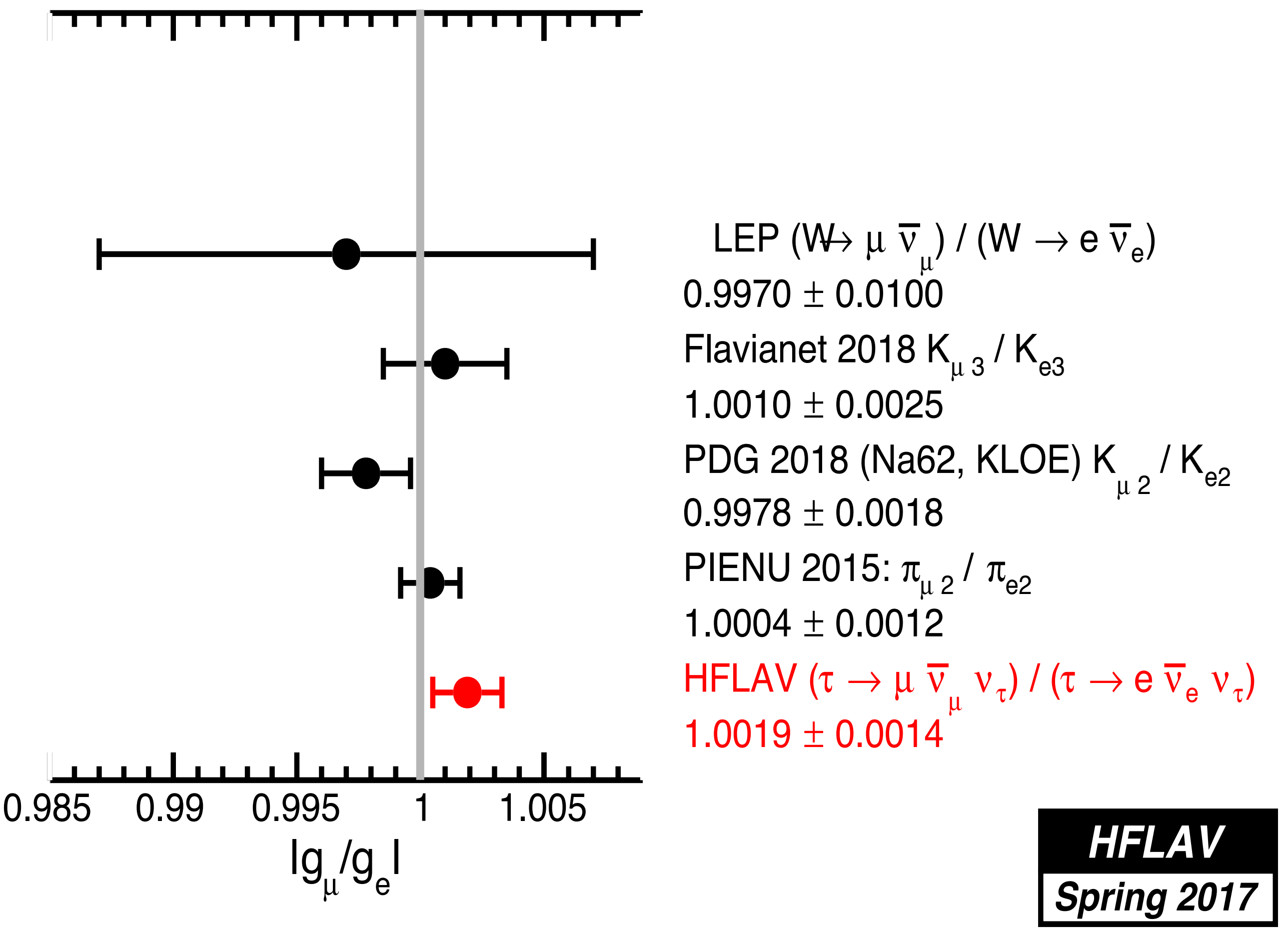


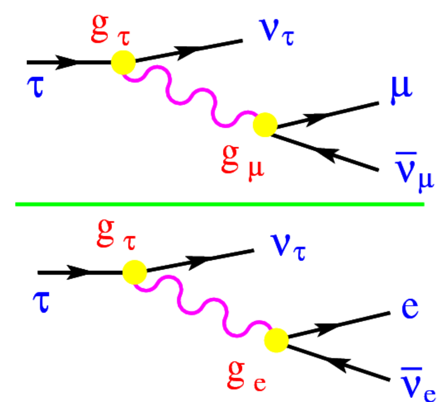


Different distributions according to the *operator!*

2. Lepton Universality tests with τ physics

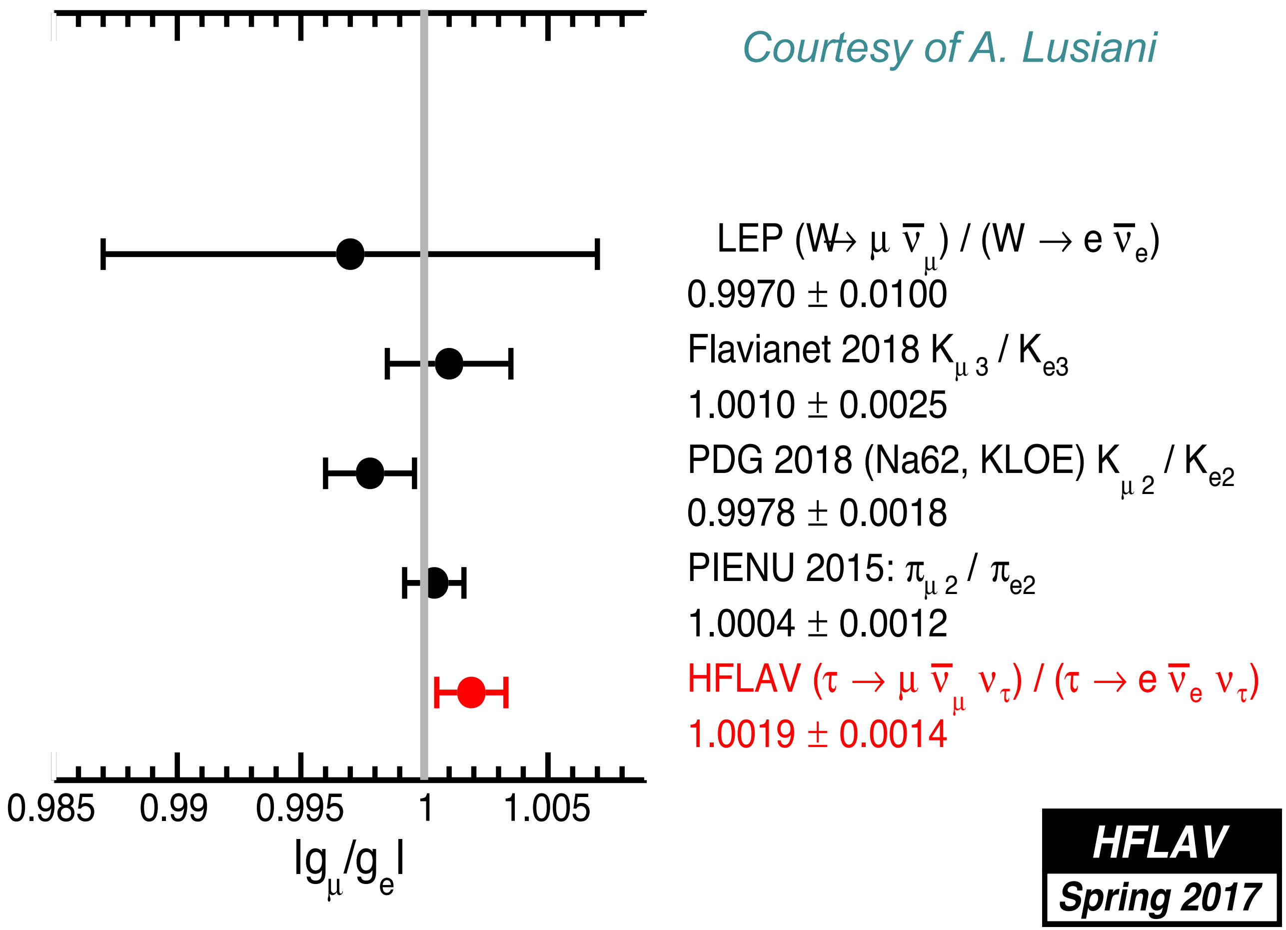
2.1 Test of μ/e universality

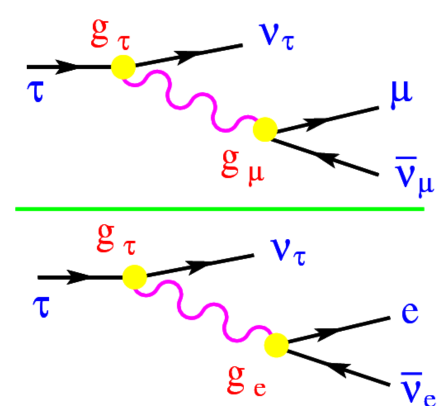




- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)

2.1 Test of μ/e universality



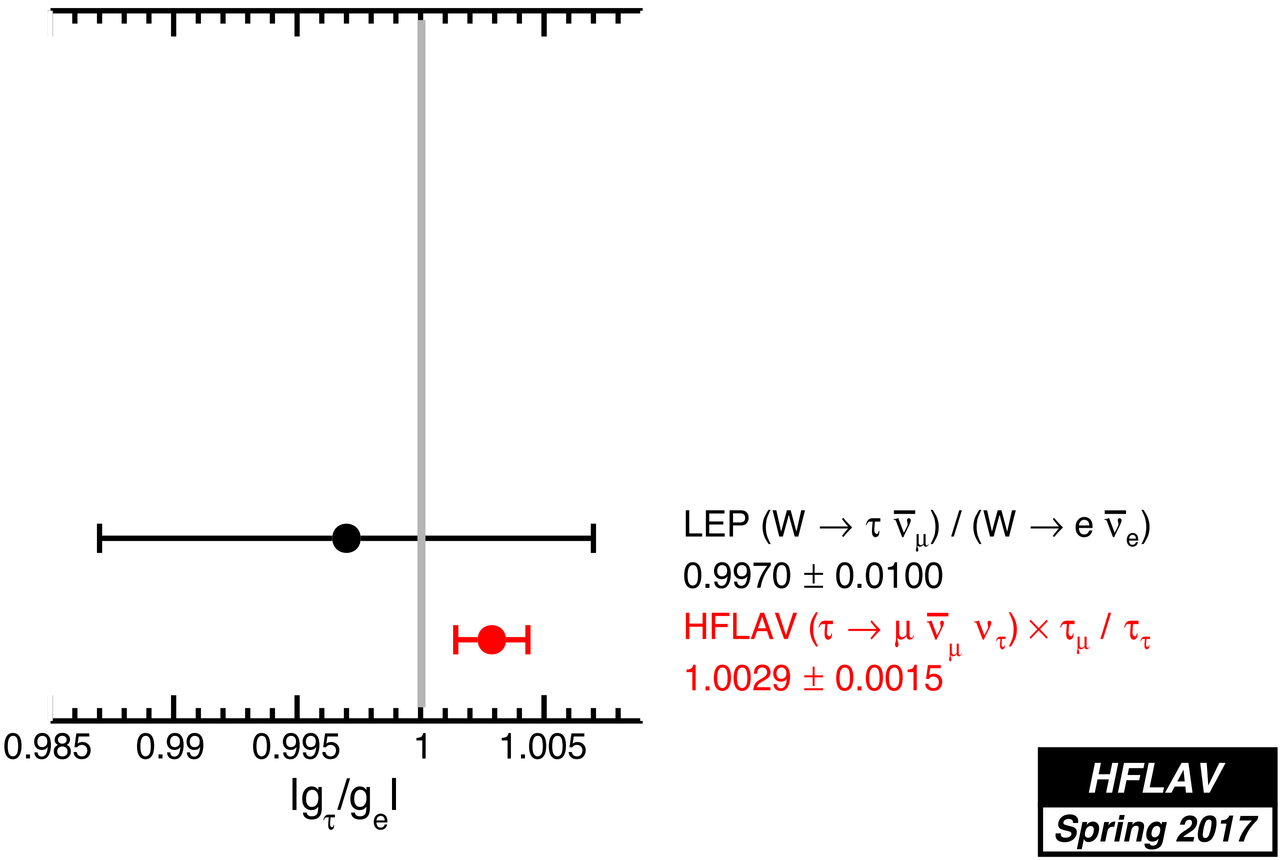


- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)
- What about the *third family*?

2.2 Test of τ/e universality

- What about the *third family*?

Courtesy of A. Lusiani

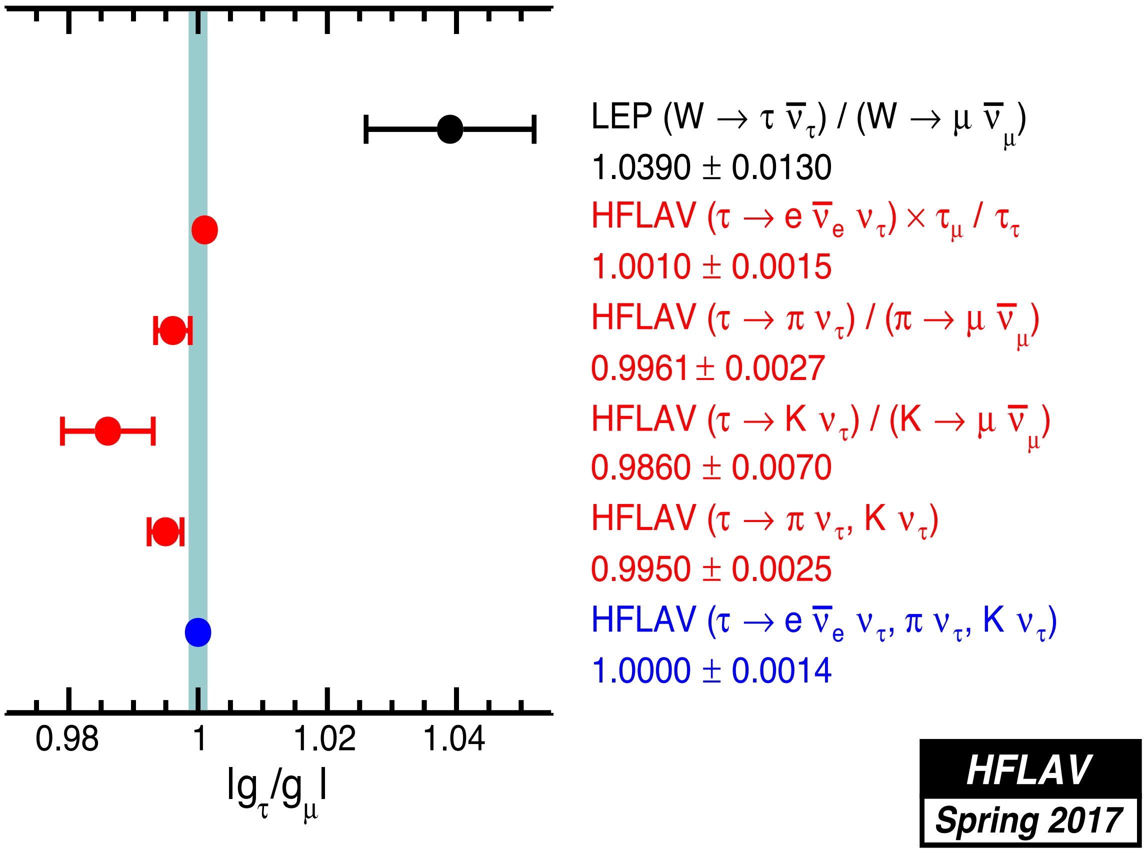


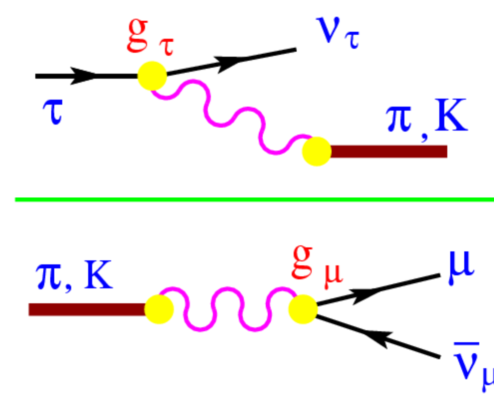
- Universality tested at 0.15% level and ~2σ except for
  - W decay old anomaly
  - B decays ⇒ *See talks on Tuesday morning*

2.3 Test of τ/μ universality

- What about the *third family*?

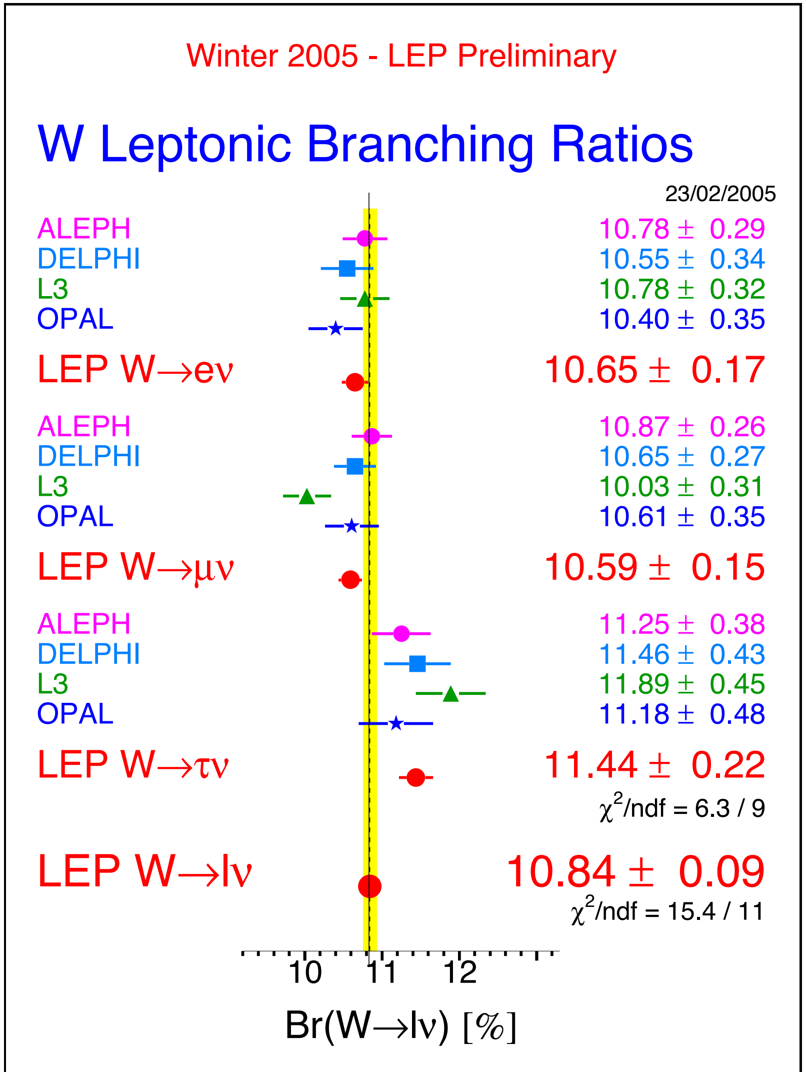
Courtesy of A. Lusiani





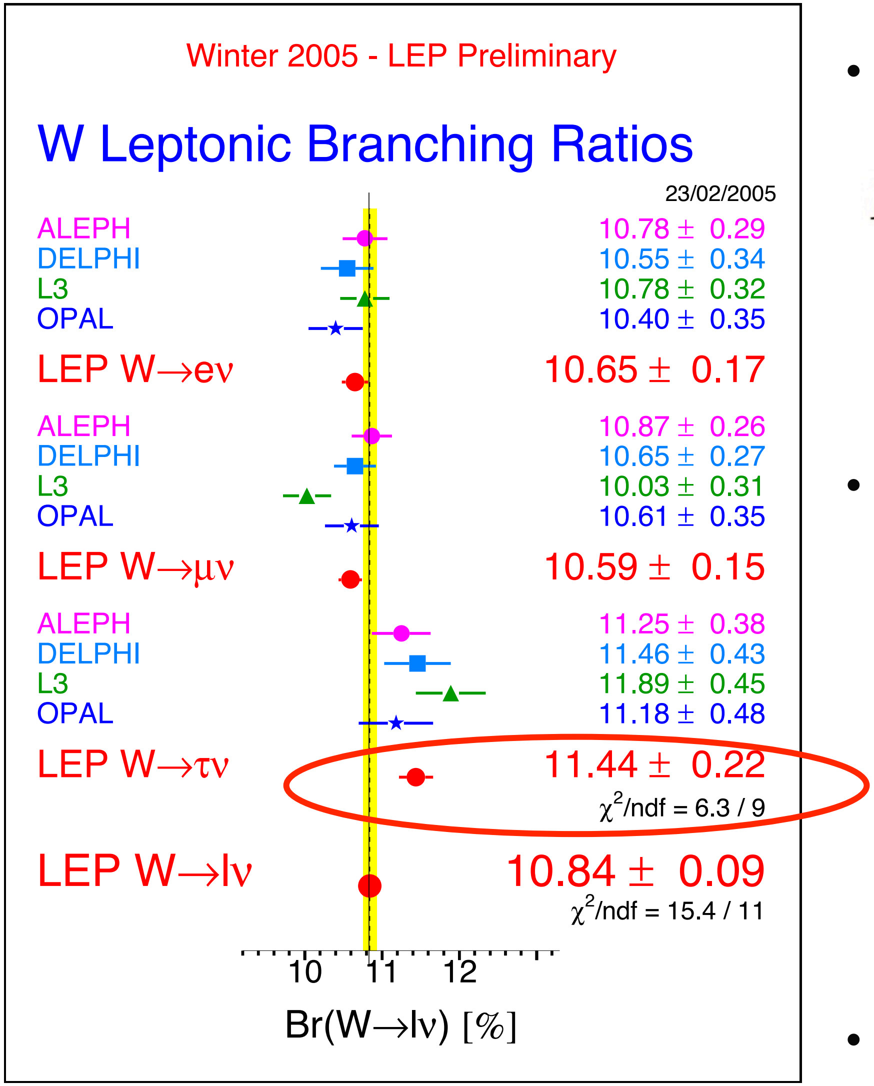
- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays ⇨ *See talks on Tuesday morning*

2.4 Lepton Flavour Universality anomaly $W \to \tau\nu_\tau$



- Old LEP anomaly

2.4 Lepton Flavour Universality anomaly $W \to \tau\nu_\tau$



- Old LEP anomaly

$$R^W_{\tau\ell} = \frac{2\,\mathrm{BR}\,(W \to \tau\,\bar{\nu}_\tau)}{\mathrm{BR}\,(W \to e\,\bar{\nu}_e) + \mathrm{BR}\,(W \to \mu\,\bar{\nu}_\mu)} = 1.077(26)$$

2.8σ away from SM!

- New physics?

  Some models:

  Li & Ma'05, Park'06, Dermisek'08

  Try to explain with SM EFT approach with $[U(2)xU(1)]^5$ flavour symmetry

  ⇨ Very difficult to explain without modifying any other observables

  Filipuzzi, Portoles, Gonzalez-Alonso'12

- Would be great to have another measurement by LHC

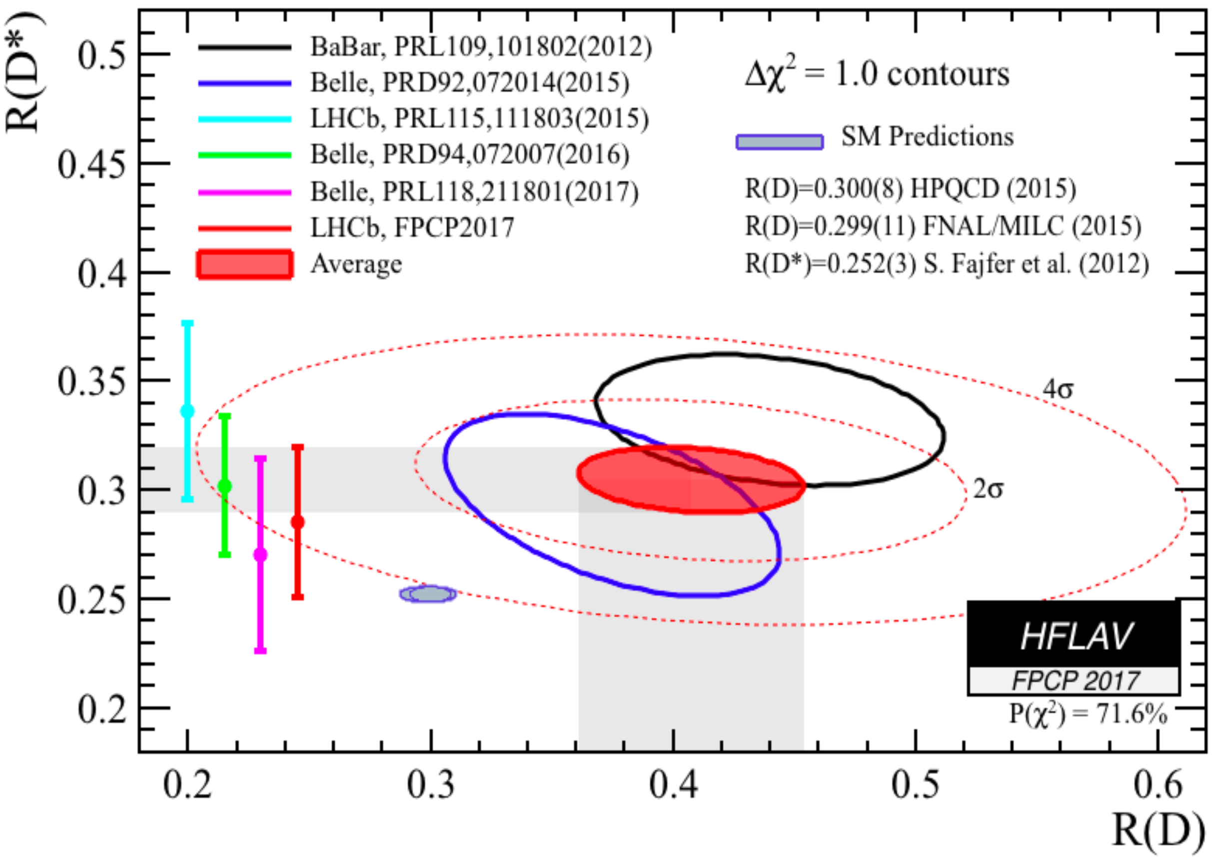
3. B physics anomalies & Charged Lepton-Flavour Violation

3.1 B physics anomalies

- Hint from B physics anomalies?

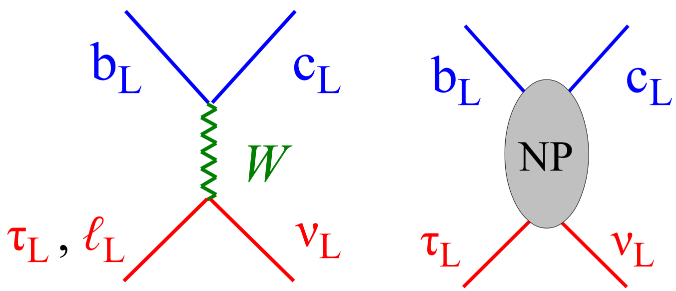
  b → c charged currents:
  τ vs. light leptons (μ, e) [R(D), R(D*)]

$$R(X) = \frac{\Gamma(B \to X\tau\bar{\nu})}{\Gamma(B \to X\ell\bar{\nu})}$$



- New Physics explanations

e.g.



3.2 Key role of τ physics observables

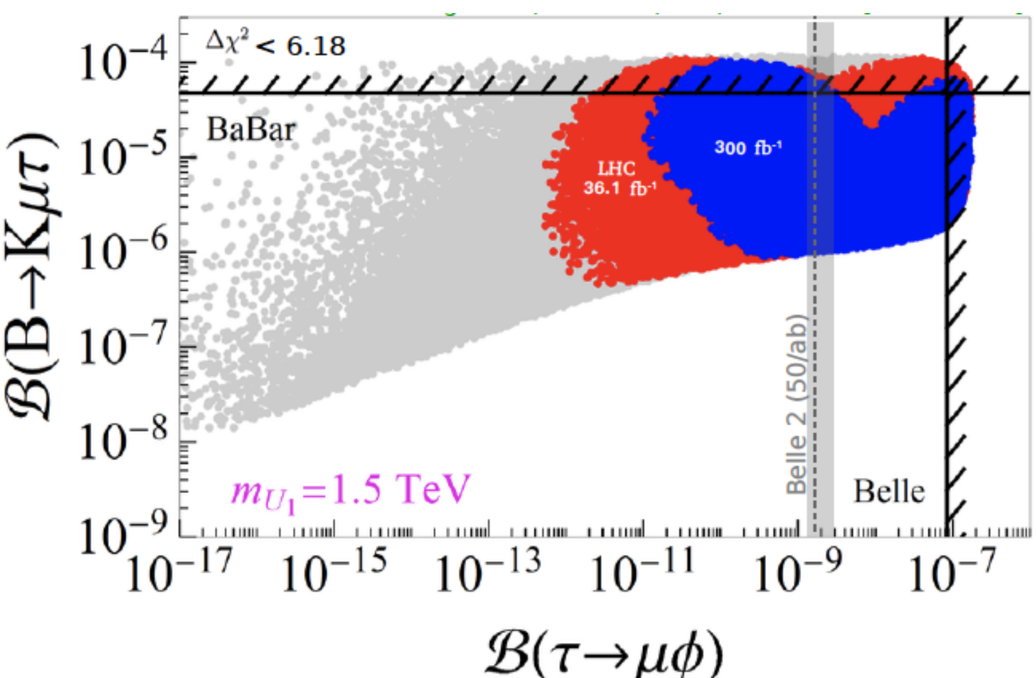
Isidori@CKM '18

- If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables involving τ

- Large $\tau \to \mu$ LFV transitions in many realistic set-up

  Glashow, Guadagnoli, Lane '15

- Ex: Leptoquark scenario:



	Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
Scalars	$S_1 = (\mathbf{3},\mathbf{1})_{-1/3}$	✗	✓	✗
	$R_2 = (\mathbf{3},\mathbf{2})_{7/6}$	✗	✓	✗
	$\widetilde{R}_2 = (\mathbf{3},\mathbf{2})_{1/6}$	✗	✗	✗
	$S_3 = (\mathbf{3},\mathbf{3})_{-1/3}$	✓	✗	✗
Vectors	$U_1 = (\mathbf{3},\mathbf{1})_{2/3}$	✓	✓	✓
	$U_3 = (\mathbf{3},\mathbf{3})_{2/3}$	✓	✗	✗

Angelescu, Bečirević, Faroughy & Sumensari'18

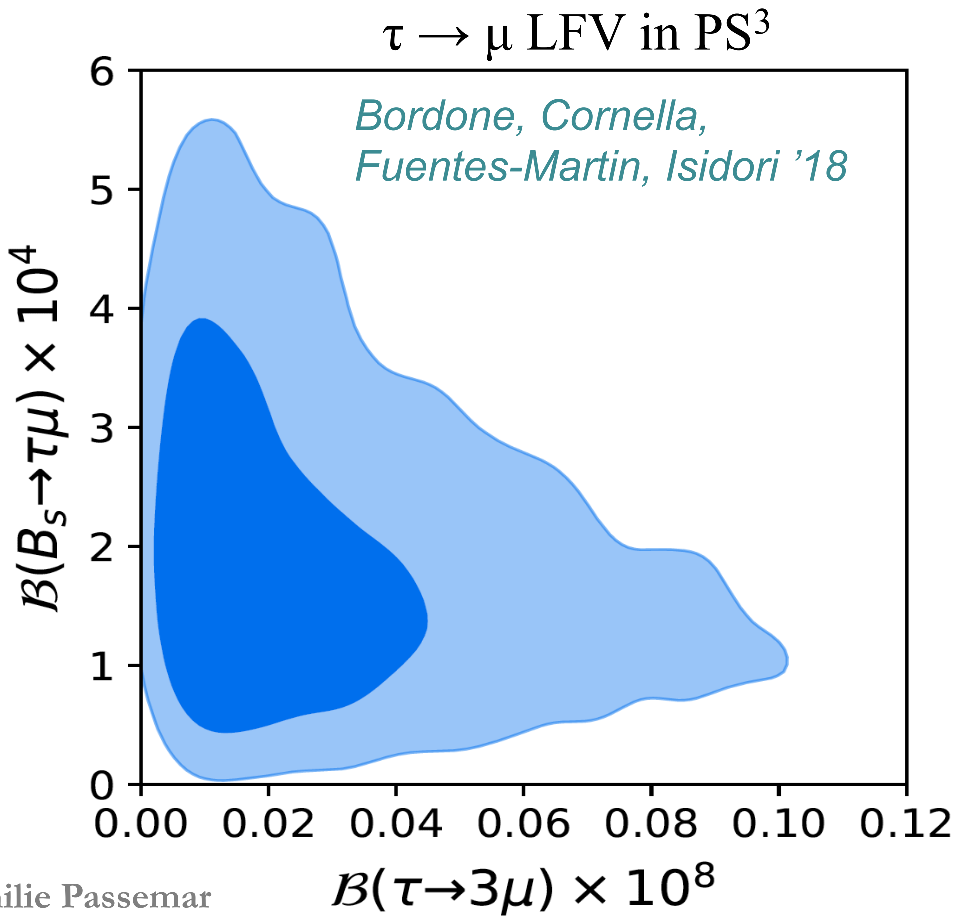
- Possibility to constrain $\tau \to \mu\phi$ and $B \to K\mu\tau$ at HL LHC?

3.2 Key role of τ physics observables

Isidori@CKM '18

- If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables involving τ
- Large $\tau \to \mu$ LFV transitions in many realistic set-up

Glashow, Guadagnoli, Lane '15



- PS³ model:

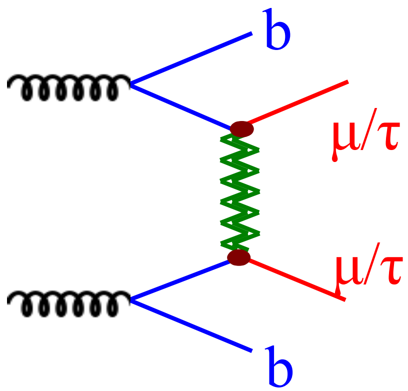
  $[\ \mathrm{PS}\ ]^3 = [\ SU(4) \times SU(2)_L \times SU(2)_R\ ]^3$

  At high energies the 3 families are charged under 3 independent gauge groups
  - Light LQ coupled mainly to 3rd gen.
  - Accidental U(2)5 flavor symmetry
  - Natural structure of SM Yukawa couplings

- Possibility to constrain $\tau \to 3\mu$ and $B_s \to \tau\mu$ at HL LHC?

3.2 Key role of τ physics observables

- Important constraints from $pp \to \tau\tau$



Isidori@CKM '18

- Constraints from $pp \to \tau\mu$

3.3 Lepton Flavour Violating h and Z decays

- HL-LHC can improve the bounds on LFV Z decays

Br(Z→τμ) < 1.2 x 10^{-5} *DELPHI@LEP’97*

Br(Z→τμ) < 1.3 x 10^{-5} *ATLAS’18, 8+13 TeV*

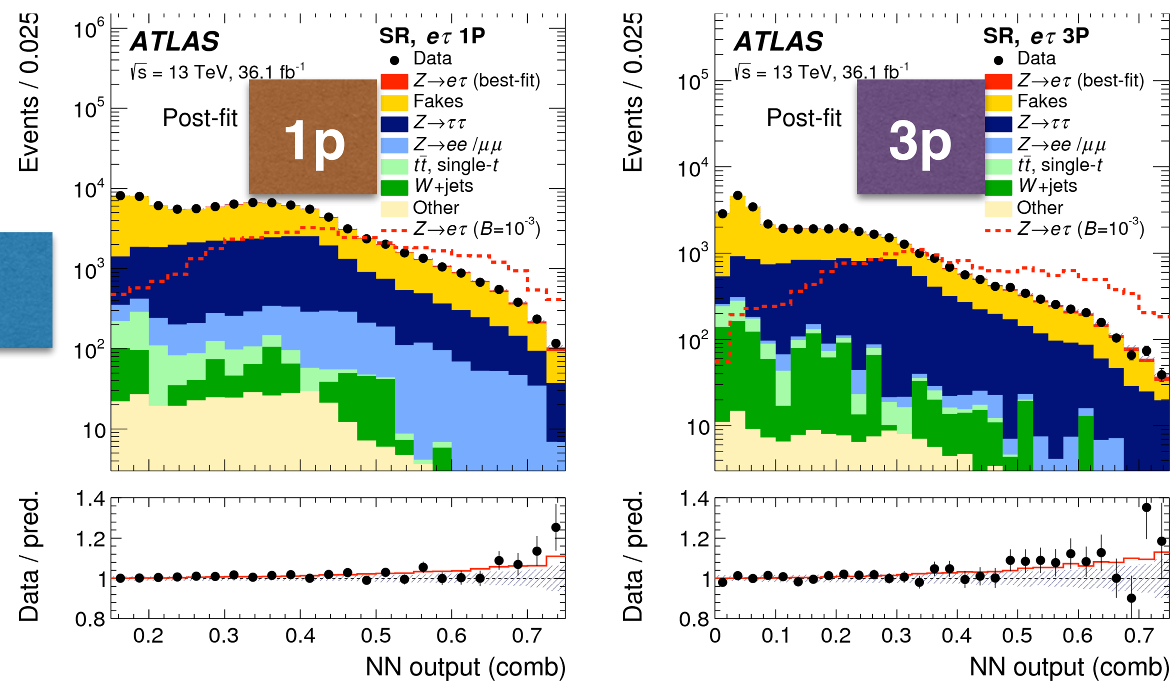
Br(Z→τe) < 1.0 x 10^{-5} *OPAL@LEP’95*

Br(Z→τe) < 5.8 x 10^{-5} *ATLAS’18, 13 TeV*

Slight excess for τe
(2.3σ over background)

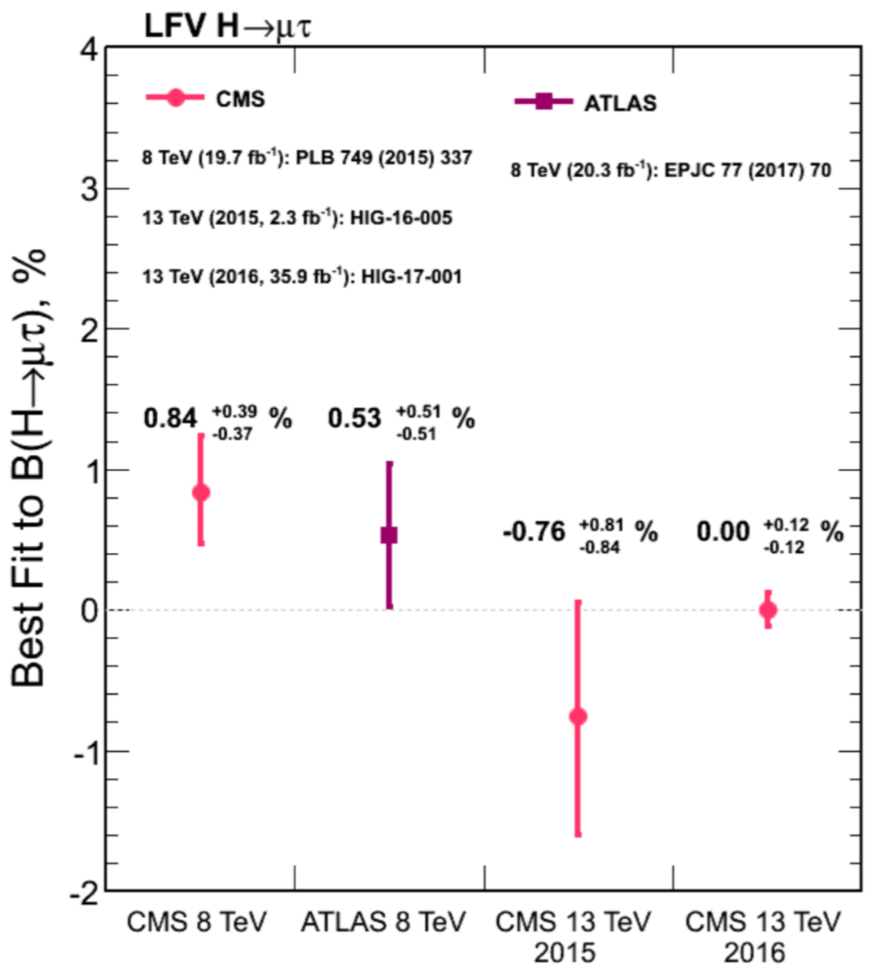
See Talk by *Brian Le*
On Tuesday

eτ



3.3 Lepton Flavour Violating h and Z decays

- HL-LHC can improve the bounds on LFV Higgs decays



$Br(h \to \tau e) < 0.61\%$

$Br(h \to \tau \mu) < 0.25\%$

Previous excess in $h \to \tau\mu$ not confirmed with new data

See Talk by *Jian Wang* On Tuesday

4. Conclusion and outlook

Conclusion and outlook

- HL-LHC will produce many more taus than any other running machines but measurements in Tau physics very difficult

- Tau physics dominated by e^+e^- machine measurements: CLEO, LEP, BaBar, Belle, BES and more to come with Belle II

- What can be done at HL-LHC for taus:
  - LFV: $\tau \to 3\mu$, What about $\tau \to \mu\phi$ and $\tau \to \mu(e)\pi\pi$?
  - LFV: $Z \to \tau\mu/e$, $h \to \tau\mu/e$
  - Lepton Universality: W decay old anomaly?
  - What about hadronic tau decays?

- Correlations with UV models explaining the B physics anomalies

- Very rich phenomenology: new ideas are welcome!

5. Back-up

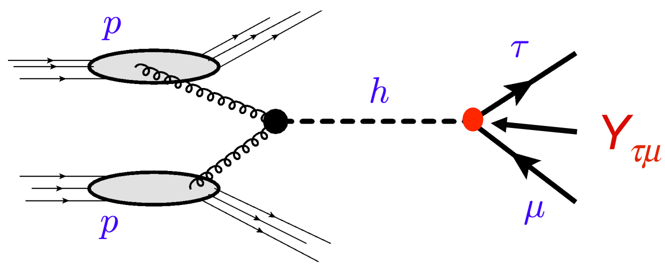
2.8 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right)h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

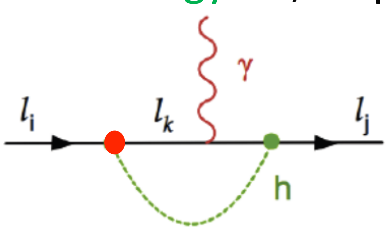
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

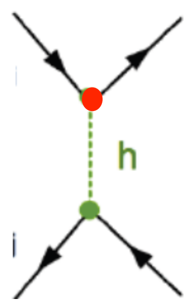
- High energy : LHC

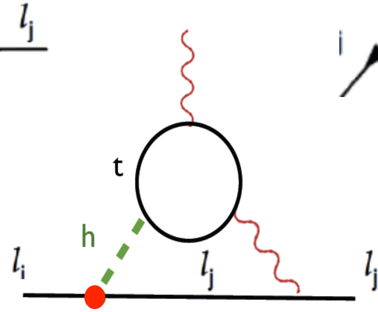


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







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Goudelis, Lebedev, Park'11
Davidson, Grenier'10
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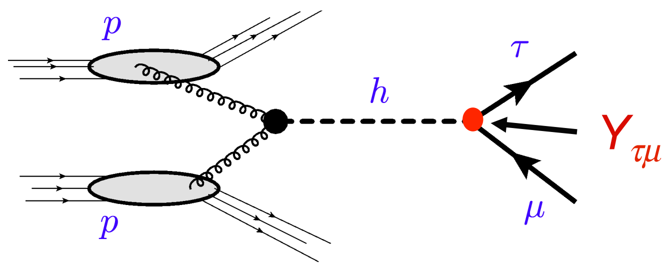
Arhrib, Cheng, Kong'12

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

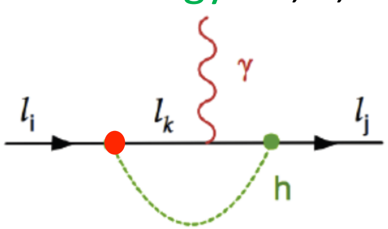
- High energy : LHC

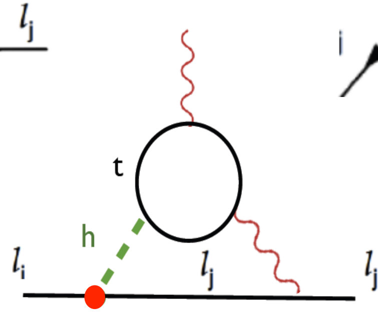


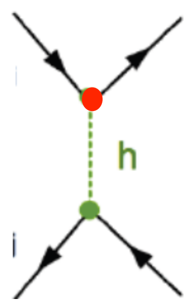
Hadronic part treated with perturbative QCD

Reverse the process

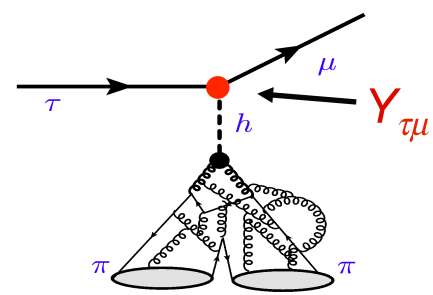
- Low energy : D, S, G operators







+



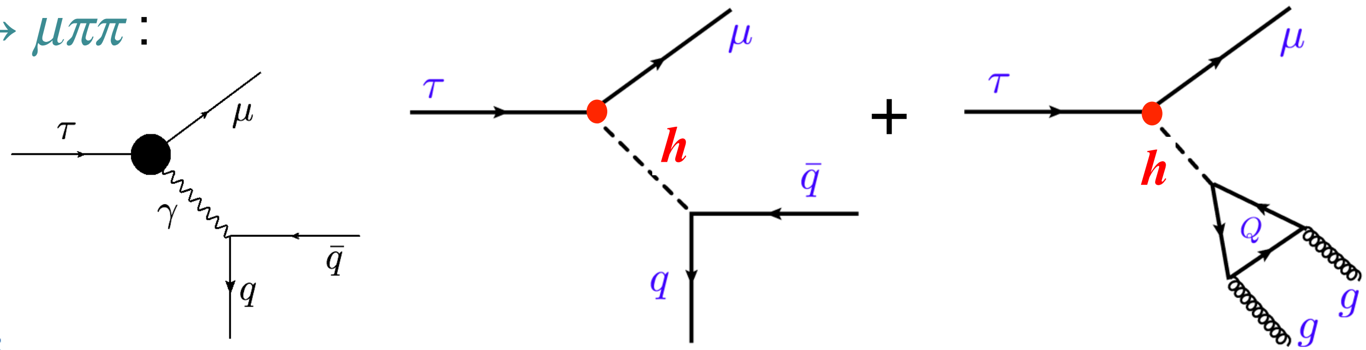
Hadronic part treated with non-perturbative QCD

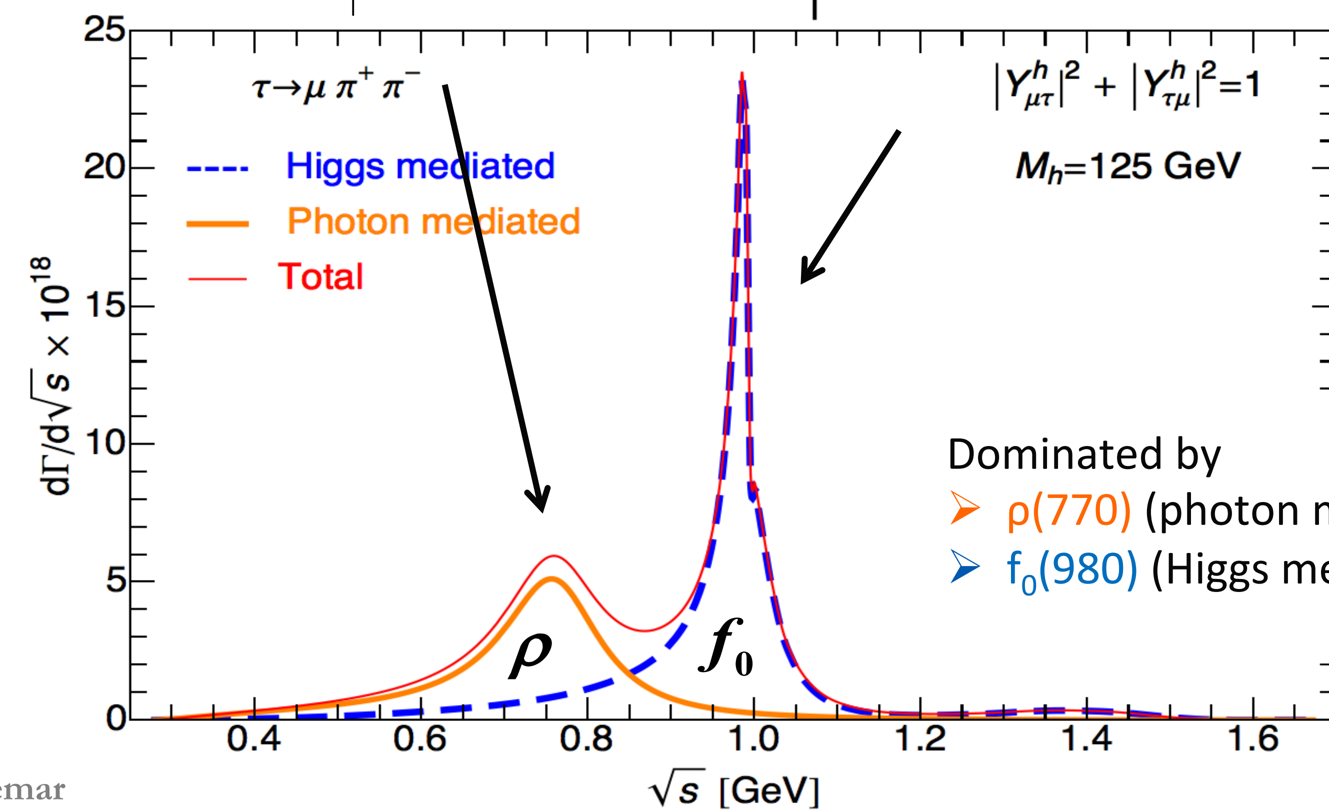
Constraints in the τμ sector

- At low energy

Cirigliano, Celis, E.P.'14

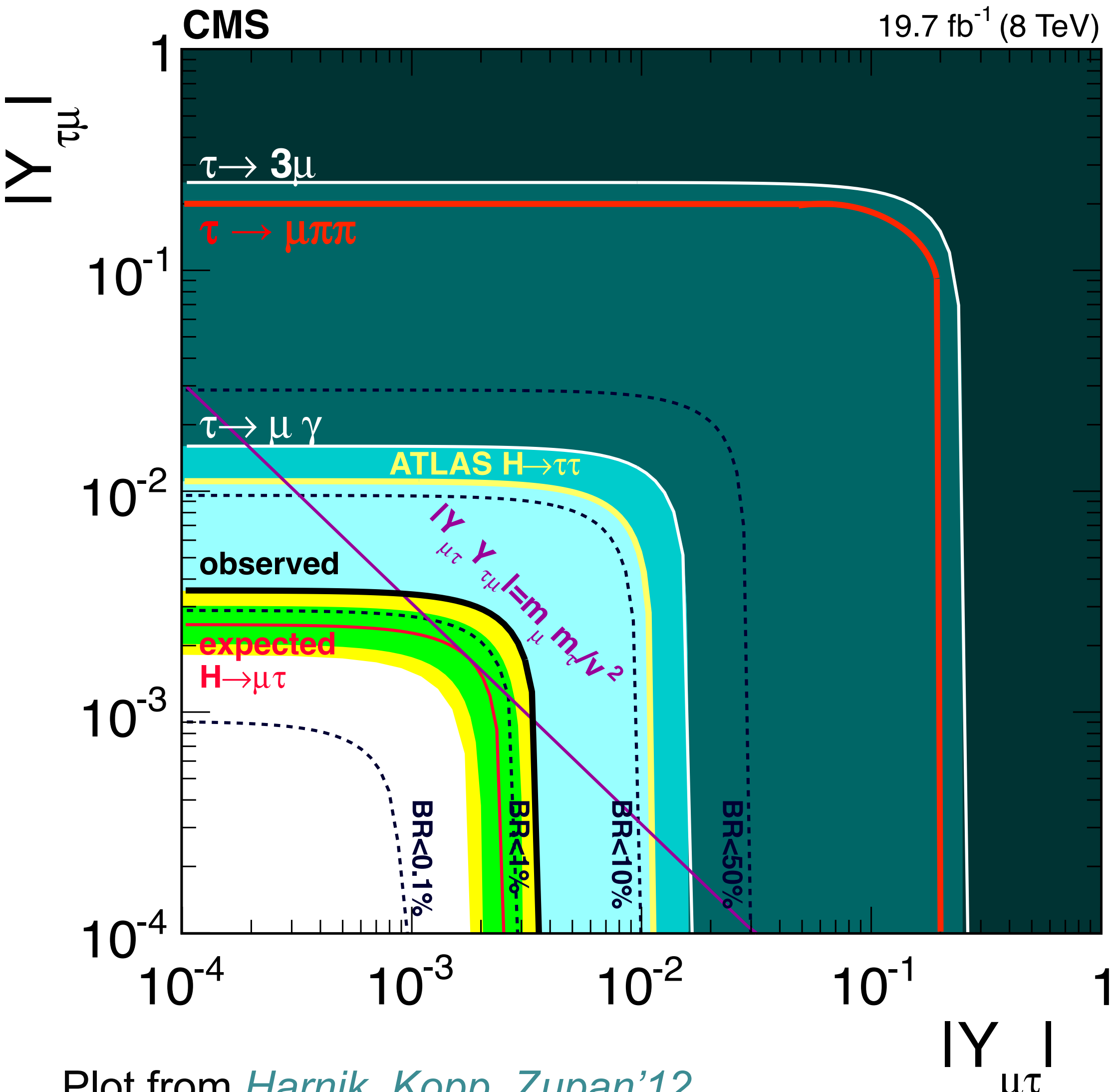
  - $\tau \to \mu\pi\pi$:





Dominated by
- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12*
updated *by CMS'15*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    - ⇨ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    - ⇨ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

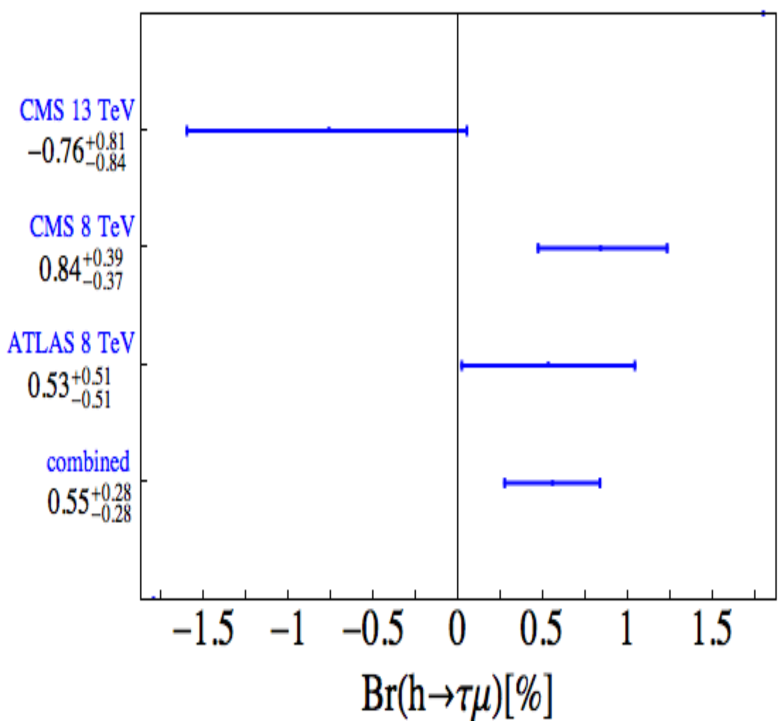
⇨

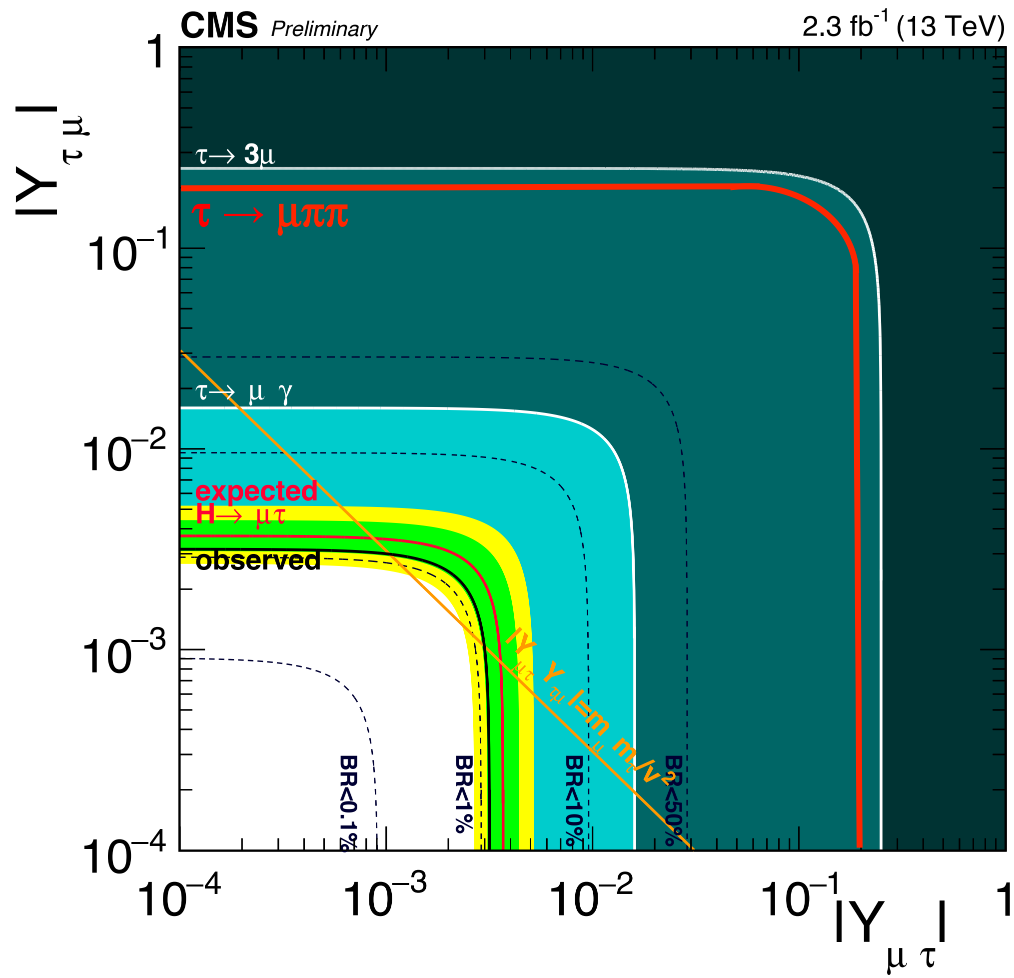
$$\boxed{BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}}$$

$$\boxed{BR(\tau \to \mu\pi\pi) < 1.5 \times 10^{-11}}$$

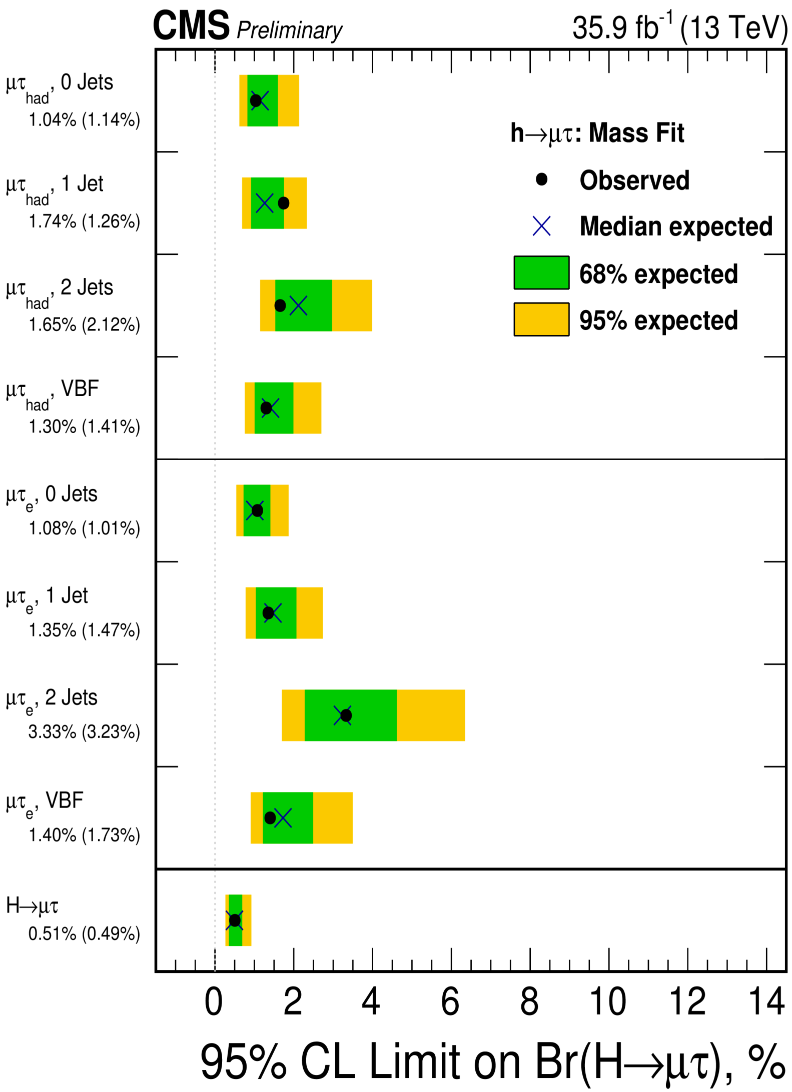
Hint of New Physics in h → τμ?

CMS'16

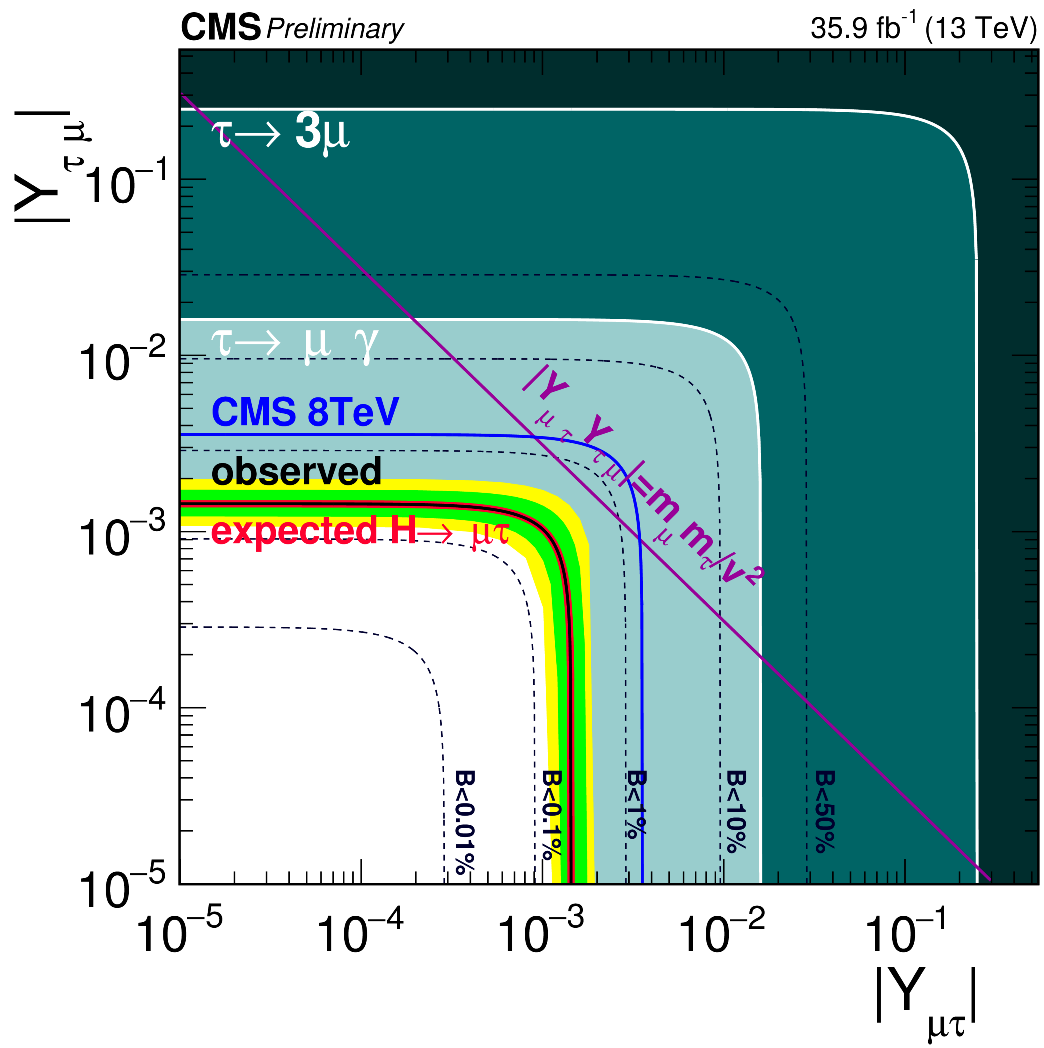




Hint of New Physics in h → τμ?



CMS’17



2.6 Model discriminating of BRs

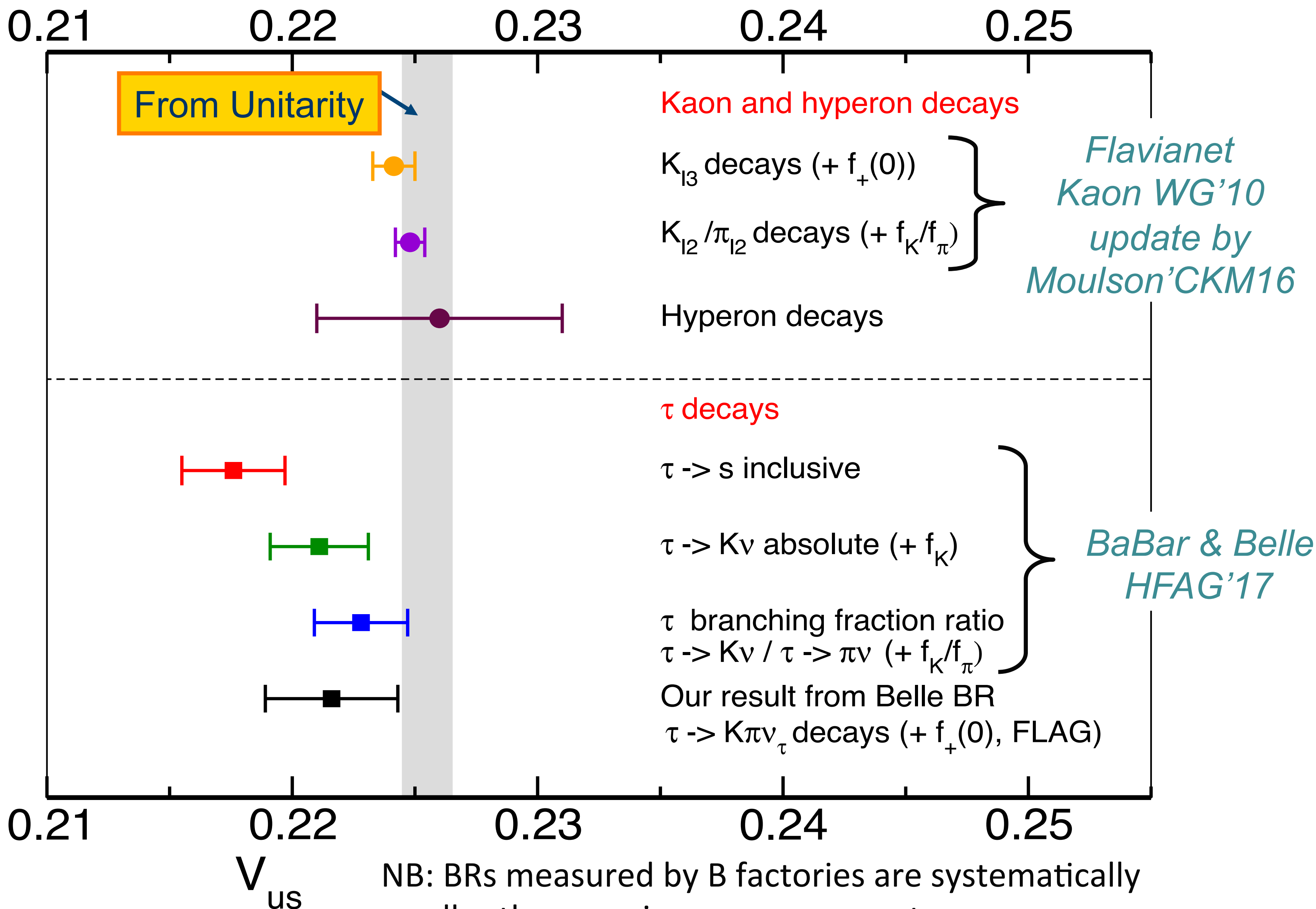
- Studies in specific models

Buras et al.'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 0.1$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.02\ldots 0.04$	$0.03\ldots 1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots 2$	~ 5	$0.3\ldots 0.5$	$1.5\ldots 2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots 1.6$	~ 0.2	$5\ldots 10$	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots 0.15$	$10^{-12}\ldots 26$

⇒ Disentangle the *underlying dynamics* of NP

3.2 V_{us} determination



NB: BRs measured by B factories are systematically smaller than previous measurements

3.2 V_{us} determination

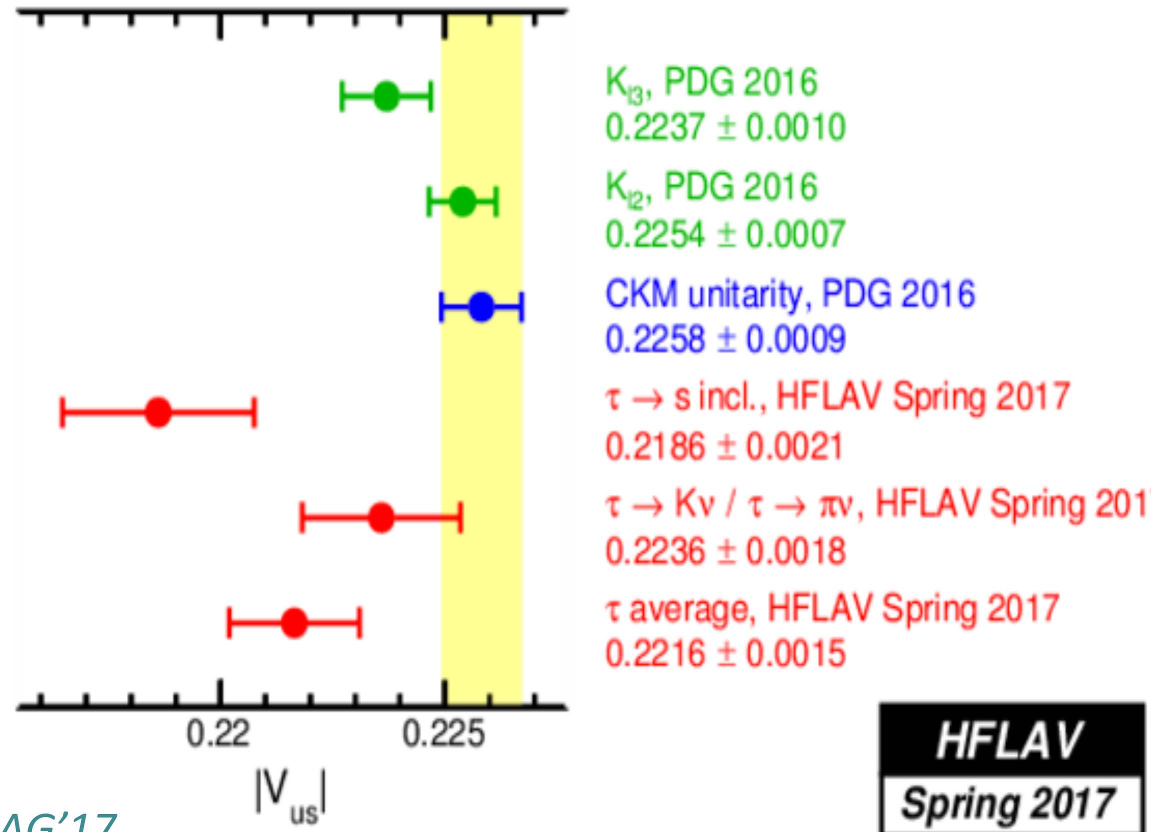
- Longstanding inconsistencies between inclusive τ and kaon decays in extraction of V_{us}
- Inclusive τ decays:

$$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$

Gamiz et al'07, Maltman'11



$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFAG'17

$$R_{\tau,S} = 0.1633(28)$$

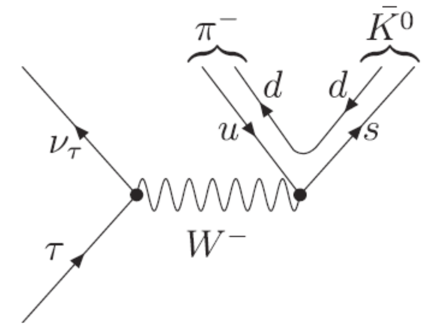
$$R_{\tau,NS} = 3.4718(84)$$

$$|V_{ud}| = 0.97417(21)$$

$$|V_{us}| = 0.2186 \pm 0.0019_{exp} \pm 0.0010_{th}$$

3.1σ away from unitarity!

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$= |p|^2 - |q|^2 \quad \boxed{\approx (0.36 \pm 0.01)\%}$$ in the SM

Bigi & Sanda'05
Grossman & Nir'11

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$$\boxed{\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}(\varepsilon_K)}$$

- Experimental measurement : *BaBar'11*

$$\boxed{A_{Q\,\exp} = \left(-0.36 \pm 0.23_{\rm stat} \pm 0.11_{\rm syst}\right)\%} \Rightarrow \boxed{2.8\sigma}$$ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

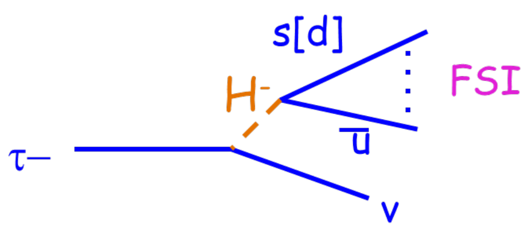
Grossman & Nir'11

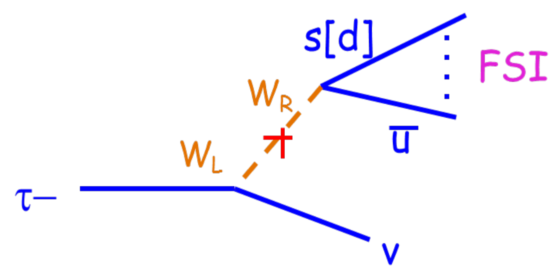
$$\boxed{A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)}} = (-0.54 \pm 0.14)\%$$

Belle, Babar, CLEO, FOCUS

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14, Cirigliano, Crivellin, Hoferichter'17*)?





Bigi'Tau12

Very difficult to explain!

- Need to investigate how large can be the prediction in realistic new physics models: it looks like *a tensor interaction* can explain the effect but in conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17

⇨ light BSM physics?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha'14
Cirigliano, Crivellin, Hoferichter'17

- We need a tensor interaction to get some interference:

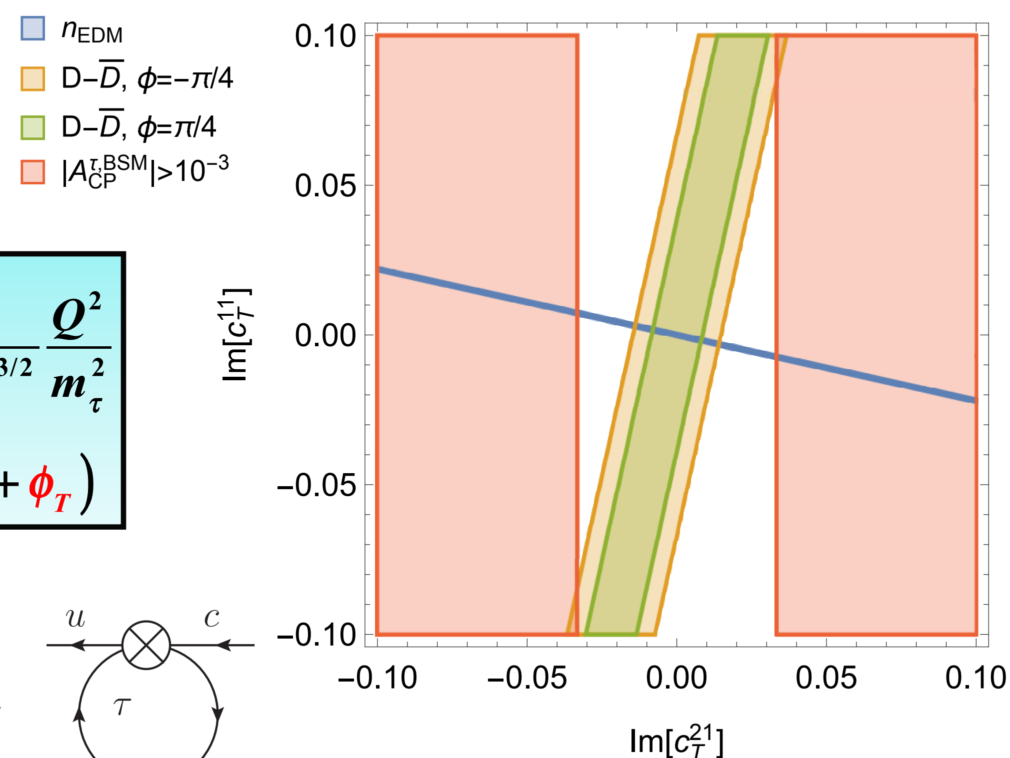
$$\boxed{\mathcal{H}_T^{eff} \equiv G'(\bar{s}\sigma_{\mu\nu}u)(\bar{\nu}_\tau(1+\gamma_5)\sigma^{\mu\nu}\tau)}$$

with $G' = \frac{G_F}{\sqrt{2}} C_T$, $\quad C_T = |C_T| e^{i\phi_T}$

- When integrating the interference term between vector and tenson does not vanish:

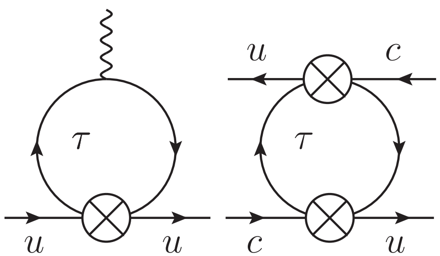
$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma_{SM}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$

$$\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2\theta_C \frac{m_\tau^3}{32\pi^3}\left(\frac{m_\tau^2 - Q^2}{m_\tau^2}\right)^2 \frac{q_1^3}{(Q^2)^{3/2}} \frac{Q^2}{m_\tau^2} \times |C_T||F_V(s)||F_T(s)|\cos(\delta_T(s) - \delta_V(s) + \phi_T)$$

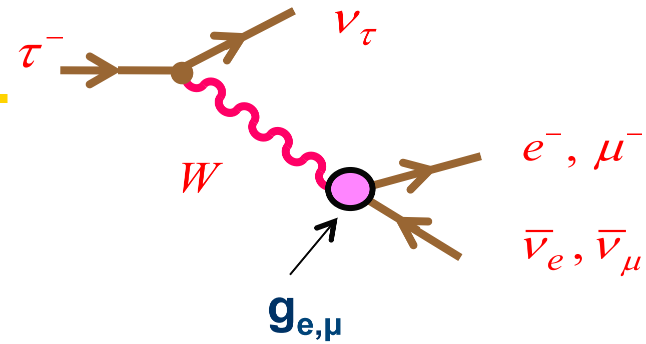


In conflict with bounds from neutron EDM and D$\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17



3.1 Lepton Universality



- The leptonic decay width:

$$\Gamma(\tau \to \nu_\tau\, l\, \bar{\nu}_l) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\, f(m_l^2/m_\tau^2)\, \left(1+\delta_{\rm RC}\right)$$

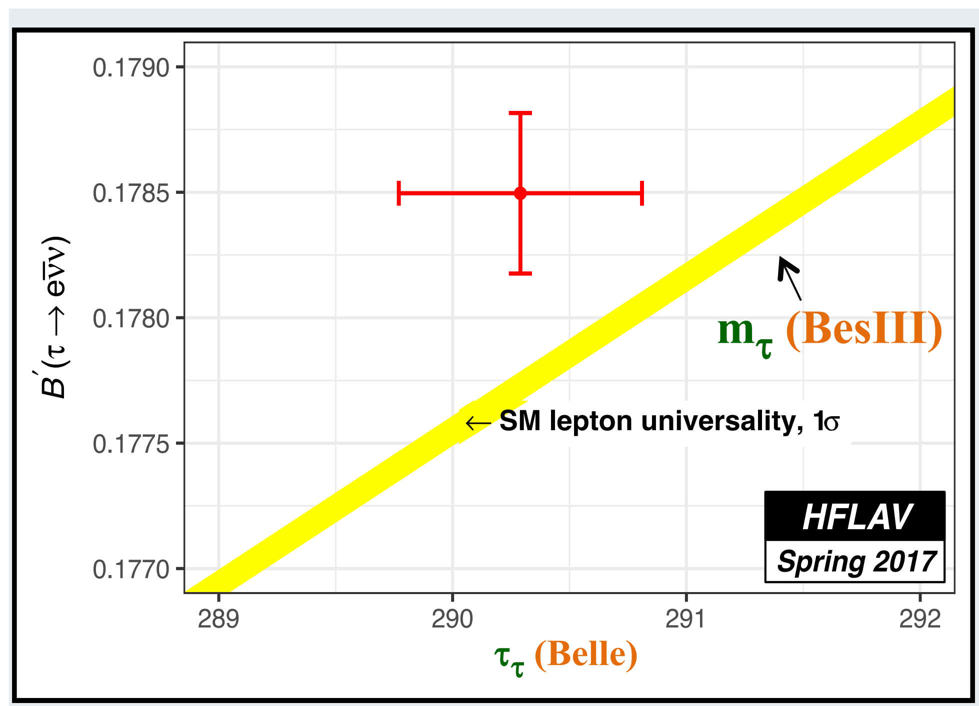
Experimental inputs:

$\Gamma(\tau_{l3})$ Rates with well-determined treatment of radiative decays

- Branching ratios
- Tau lifetimes

Inputs from theory: *Marciano'88*

δ_{RC} Radiative corrections



- Test of μ/e universality:

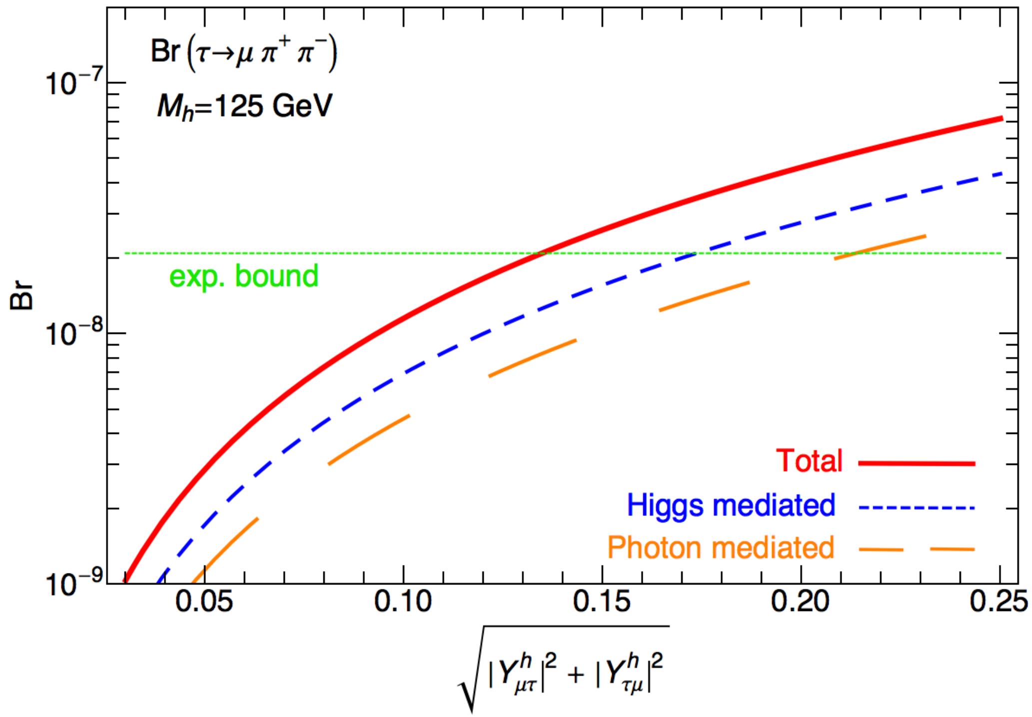
$$\left(B_\mu / B_e\right)_{\rm exp} = 0.9761 \pm 0.0028$$

Non-BF: 0.9725 ± 0.0039

BaBar '10: 0.9796 ± 0.0039

⇨ $$\boxed{B_e^{univ} = (17.818 \pm 0.0022)}$$

3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2+\left|Y^h_{\tau\mu}\right|^2}\leq 0.13}$$

Less stringent but more robust handle on LFV Higgs couplings

Process	$(\mathrm{BR}\times10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2+\lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau\to\mu\gamma$	<4.4 [88]	<0.016	Dipole
$\tau\to\mu\mu\mu$	<2.1 [89]	<0.24	Dipole
$\tau\to\mu\pi^+\pi^-$	<2.1 [86]	<0.13	Scalar, Gluon, Dipole
$\tau\to\mu\rho$	<1.2 [85]	<0.13	Scalar, Gluon, Dipole
? $\tau\to\mu\pi^0\pi^0$	$<1.4\times10^3$ [87]	<6.3	Scalar, Gluon

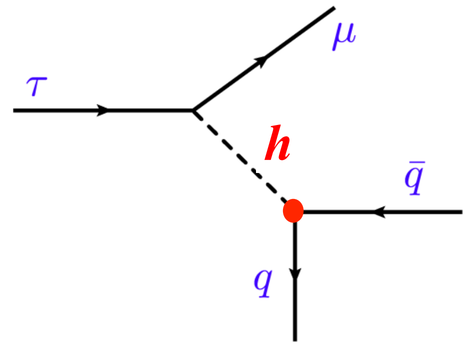
Belle'08'11'12 except last from *CLEO'97*

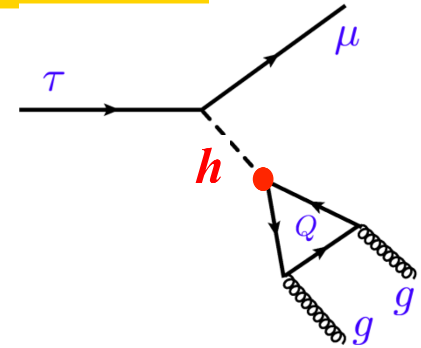
3.5 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!





- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

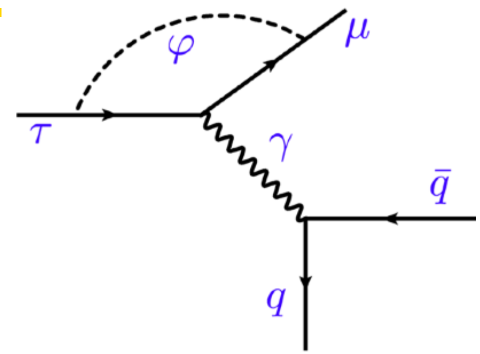
$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

below present experimental limits!

- If discovered ⇨ among other things *upper limit* on $Y_{u,d,s}$*!*

⇨ Interplay between high-energy and low-energy constraints!

3.1 Constraints from $\tau \to \mu\pi\pi$

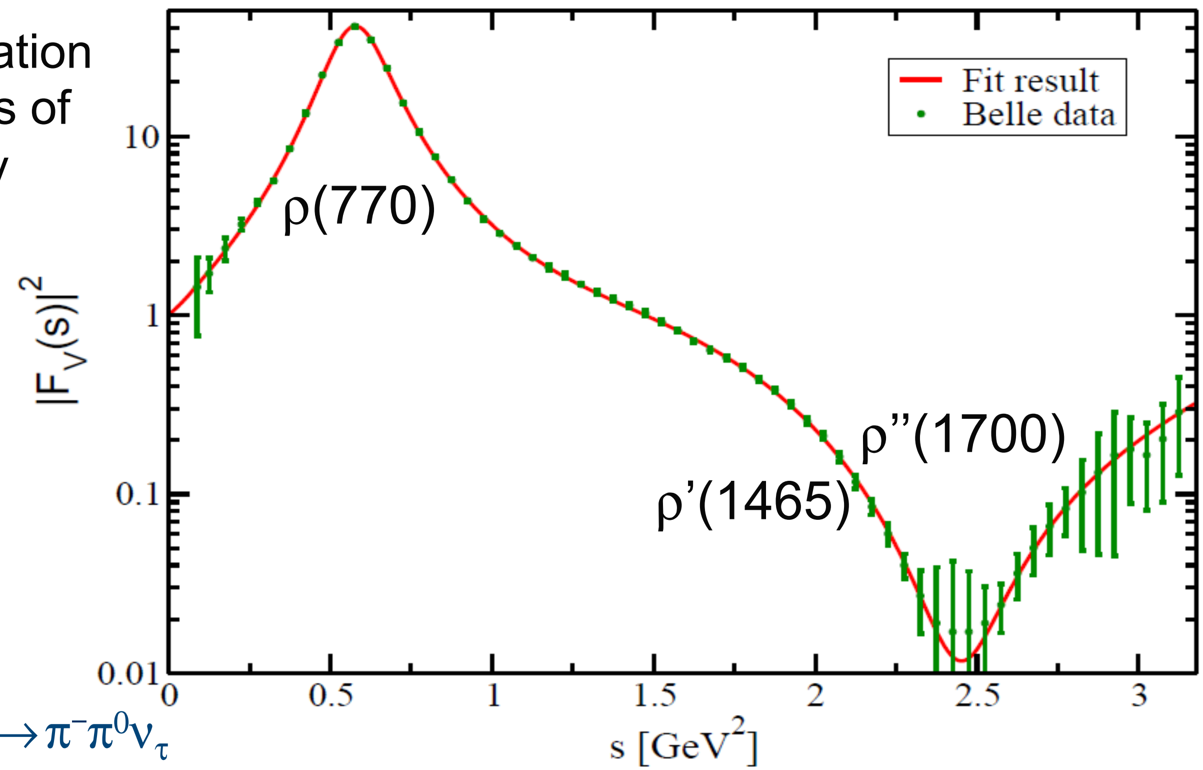


- Photon mediated contribution requires the pion vector form factor:

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \left| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \right| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

  Gasser, Meißner´91
  Guerrero, Pich'97
  Oller, Oset, Palomar´01
  Pich, Portolés ´08
  Gómez Dumm&Roig'13
  ...



- Determined from a fit to the Belle data on $\tau^- \to \pi^-\pi^0\nu_\tau$

Celis, Cirigliano, E.P.'14

Determination of $F_V(s)$

- Vector form factor

  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

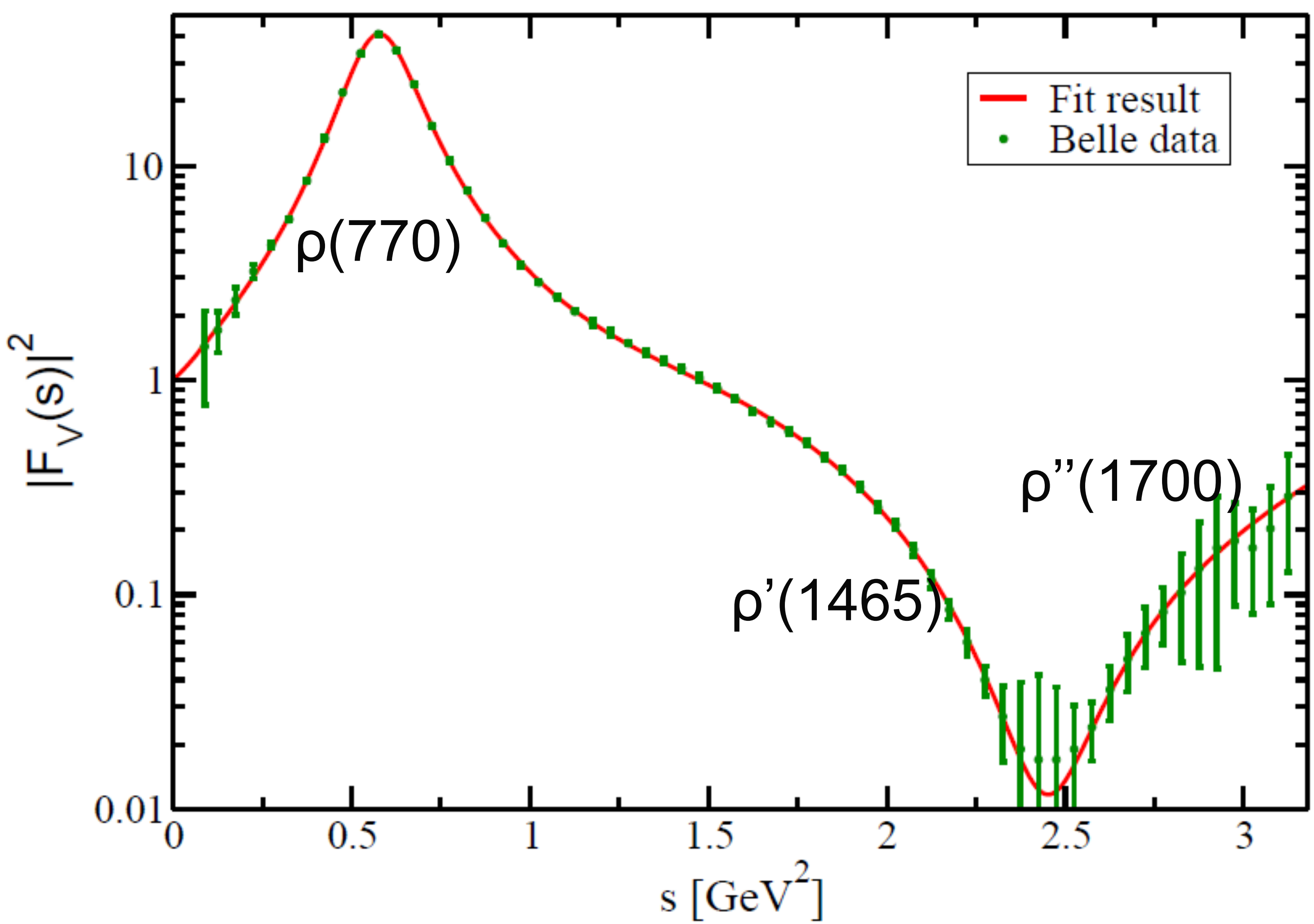
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]$$

Extracted from a model including 3 resonances ρ(770), ρ'(1465) and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

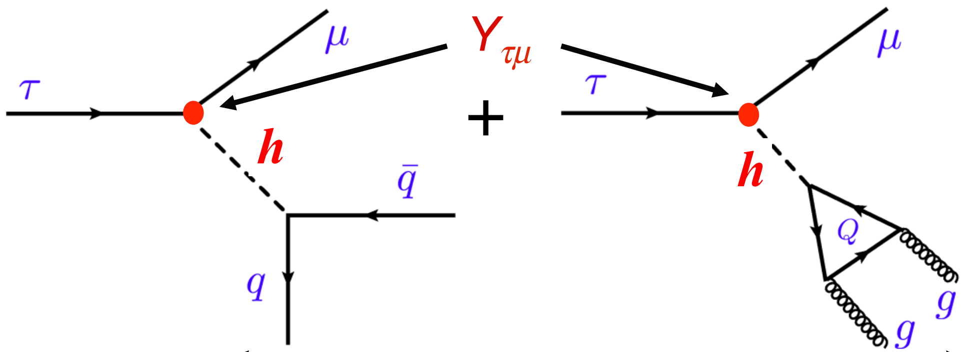
Determination of $F_V(s)$

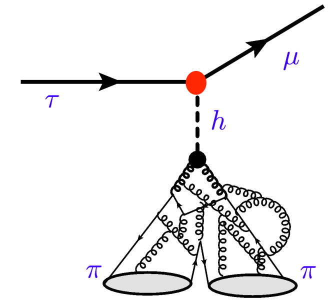


Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange





$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\left\langle \pi^+\pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s) \qquad \left\langle \pi^+\pi^- \left| \theta_\mu^\mu \right| 0 \right\rangle \equiv \theta_\pi(s)$$

Voloshin'85

$$\left\langle \pi^+\pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta_\mu^\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2}$$

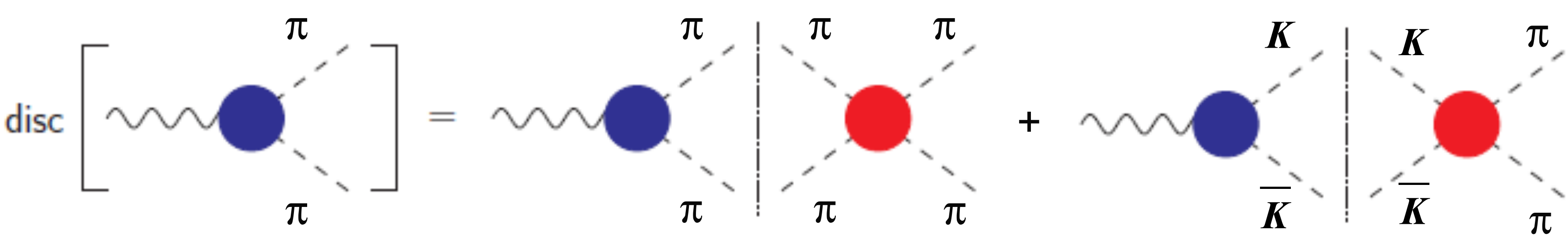
$$f\left(y_q^h\right)$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs ⇒ *Coupled channel analysis*
  up to $\sqrt{s}\sim$1.4 GeV
  Inputs: I=0, S-wave $\pi\pi$ and $K\bar{K}$ data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:

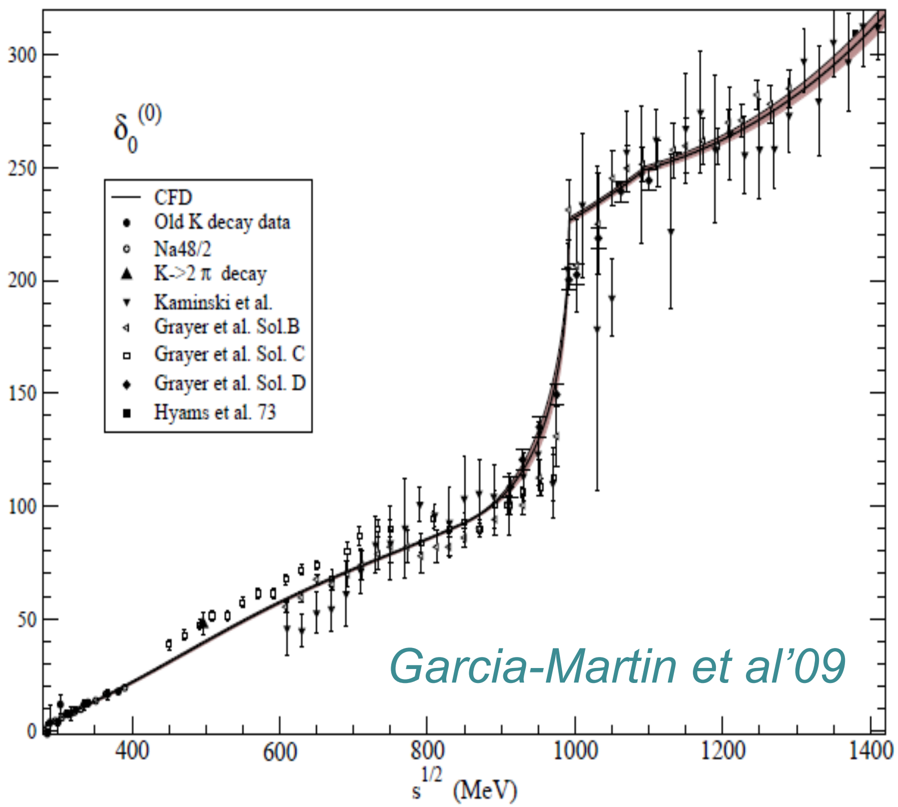


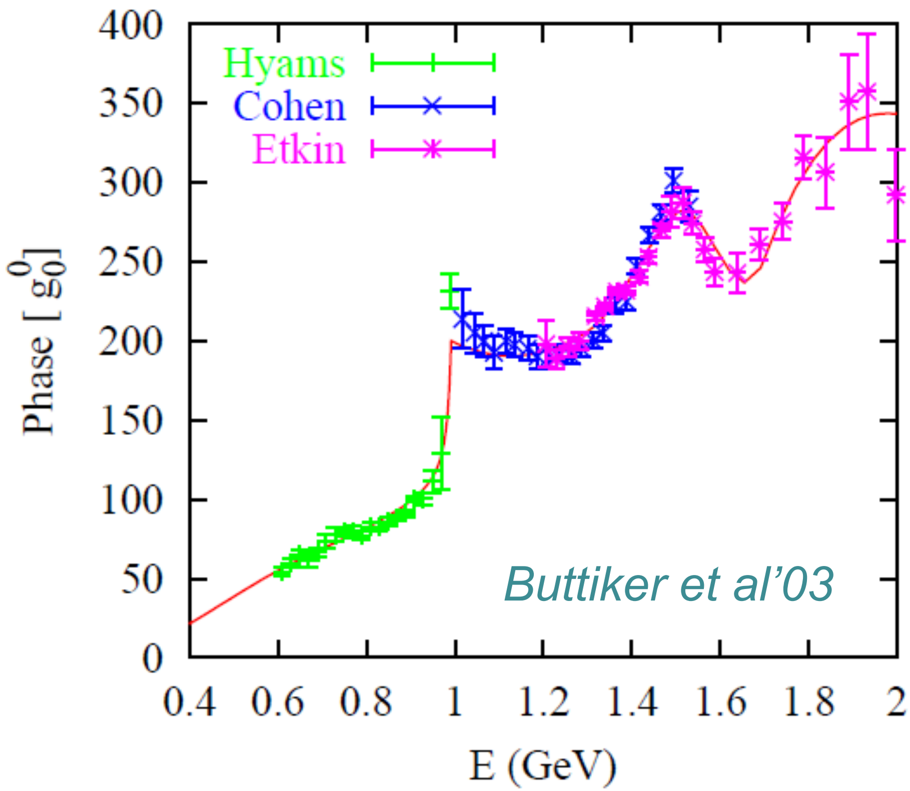
⇒ $$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$ $n = \pi\pi, K\bar{K}$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T_{nm}^{*}\sigma_m(s)X_m^{(N)}\right\} \longrightarrow \mathrm{Re}X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s}\mathrm{Im}X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

  Feynman-Hellmann theorem: ⇒

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

⇒

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$\begin{aligned} M^2_{\pi^+} &= (m_u + m_d)\, B_0 + O(m^2) \\ M^2_{K^+} &= (m_u + m_s)\, B_0 + O(m^2) \\ M^2_{K^0} &= (m_d + m_s)\, B_0 + O(m^2) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  $\Rightarrow$ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\ M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

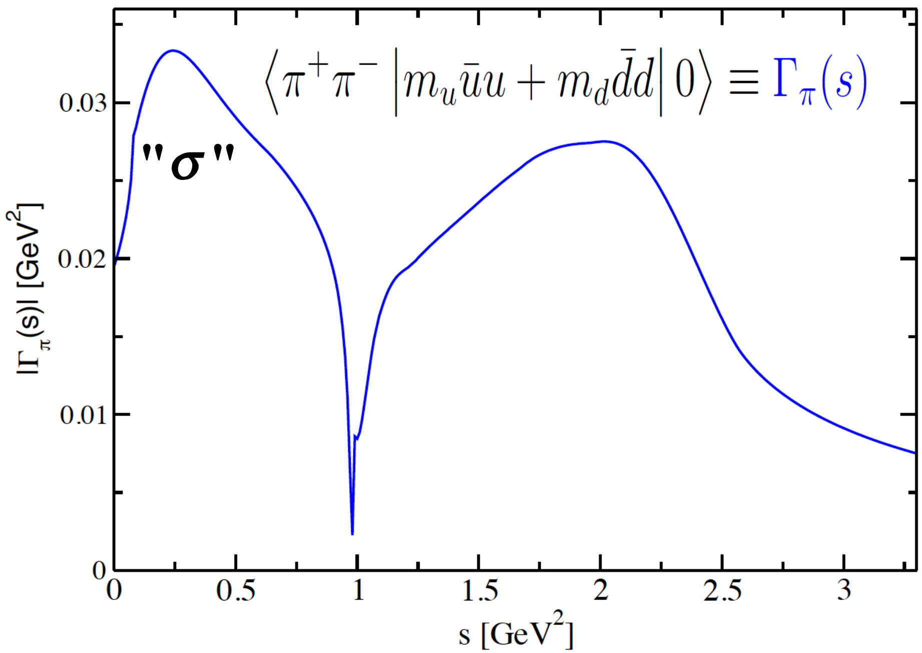
- General solution

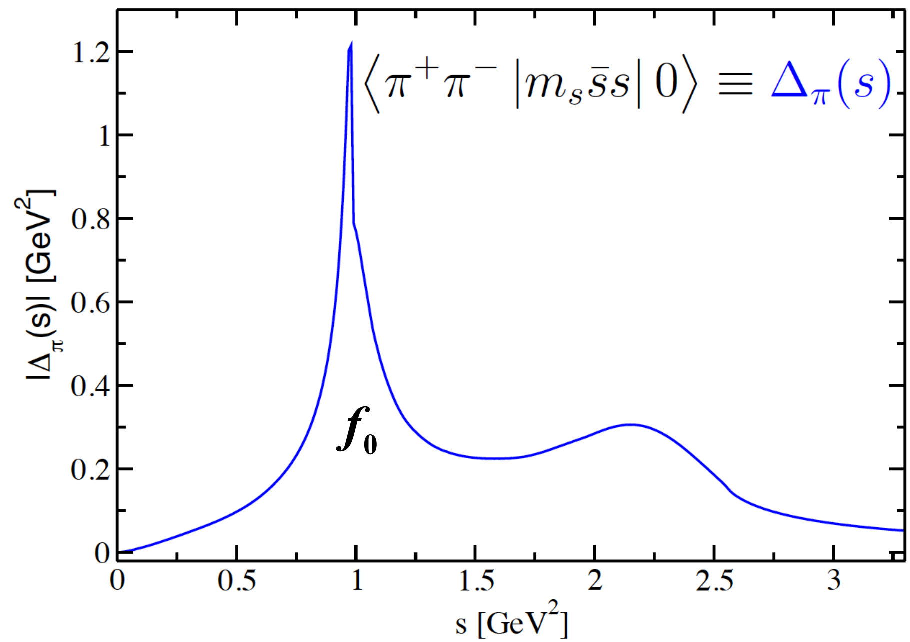
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

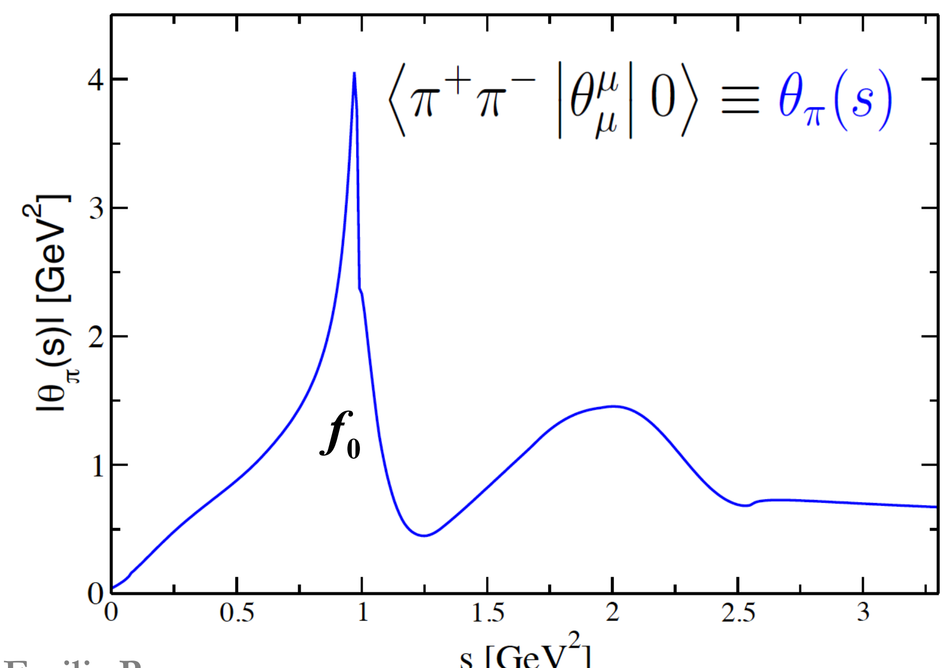
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

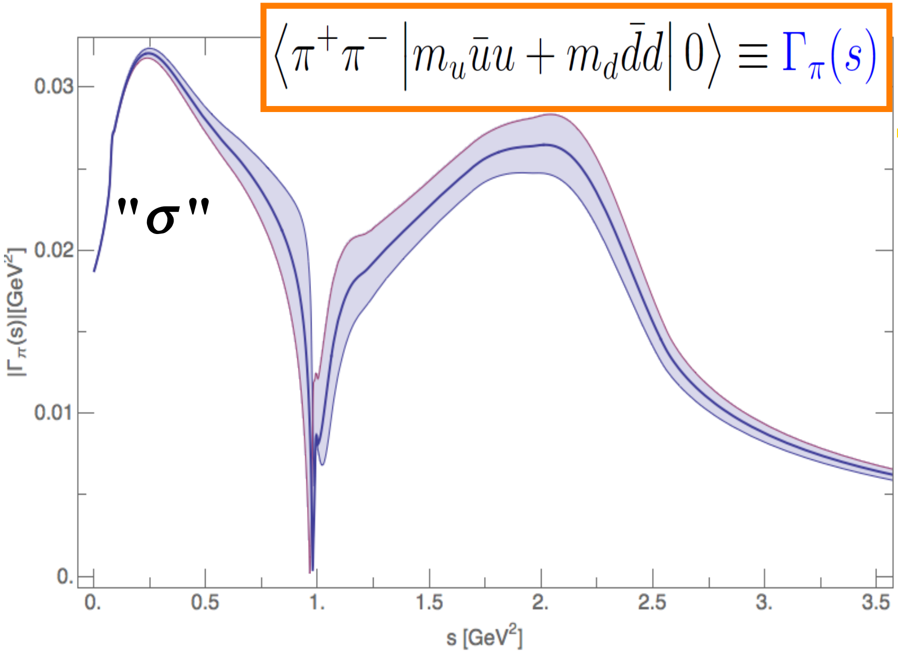
⇨ Relax the constraints and match to ChPT

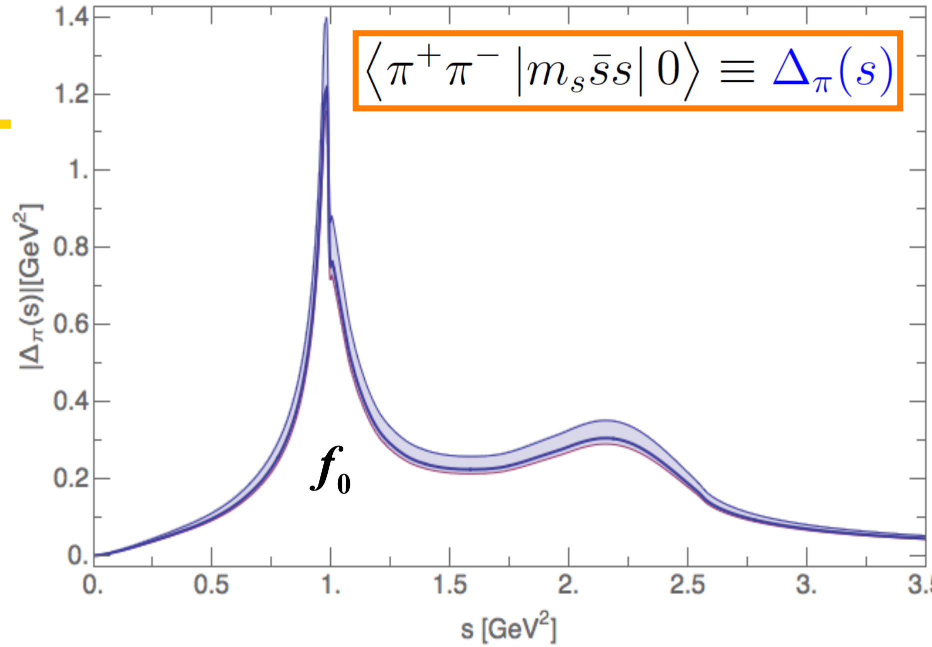
$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$

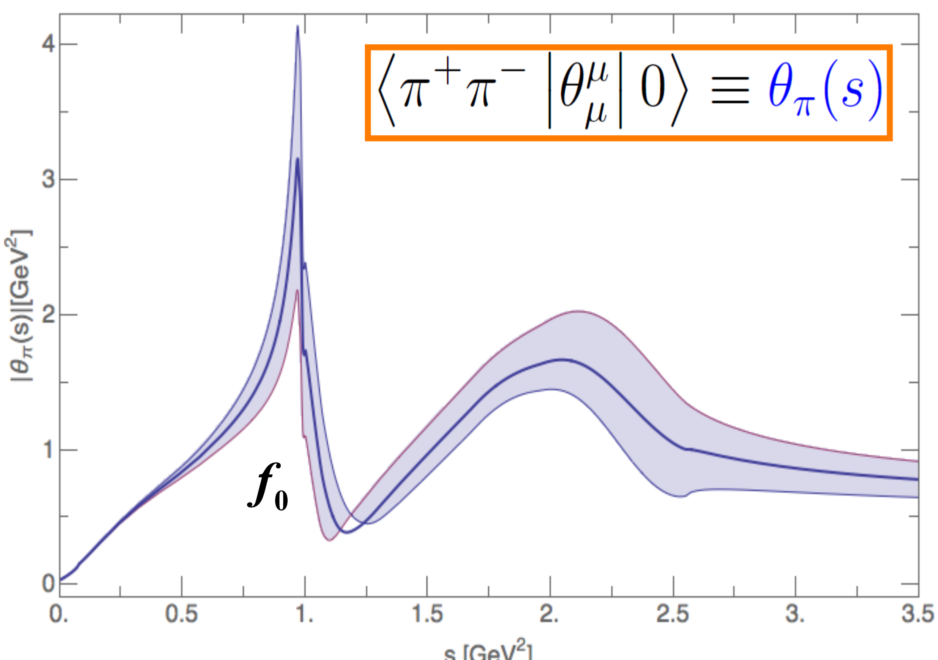

$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$
"σ"
$|\Gamma_\pi(s)|$ [GeV^2]
s [GeV^2]
0
0.01
0.02
0.03
0.5
1
1.5
2
2.5
3


$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
f_0
$|\Delta_\pi(s)|$ [GeV^2]
s [GeV^2]
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5
3

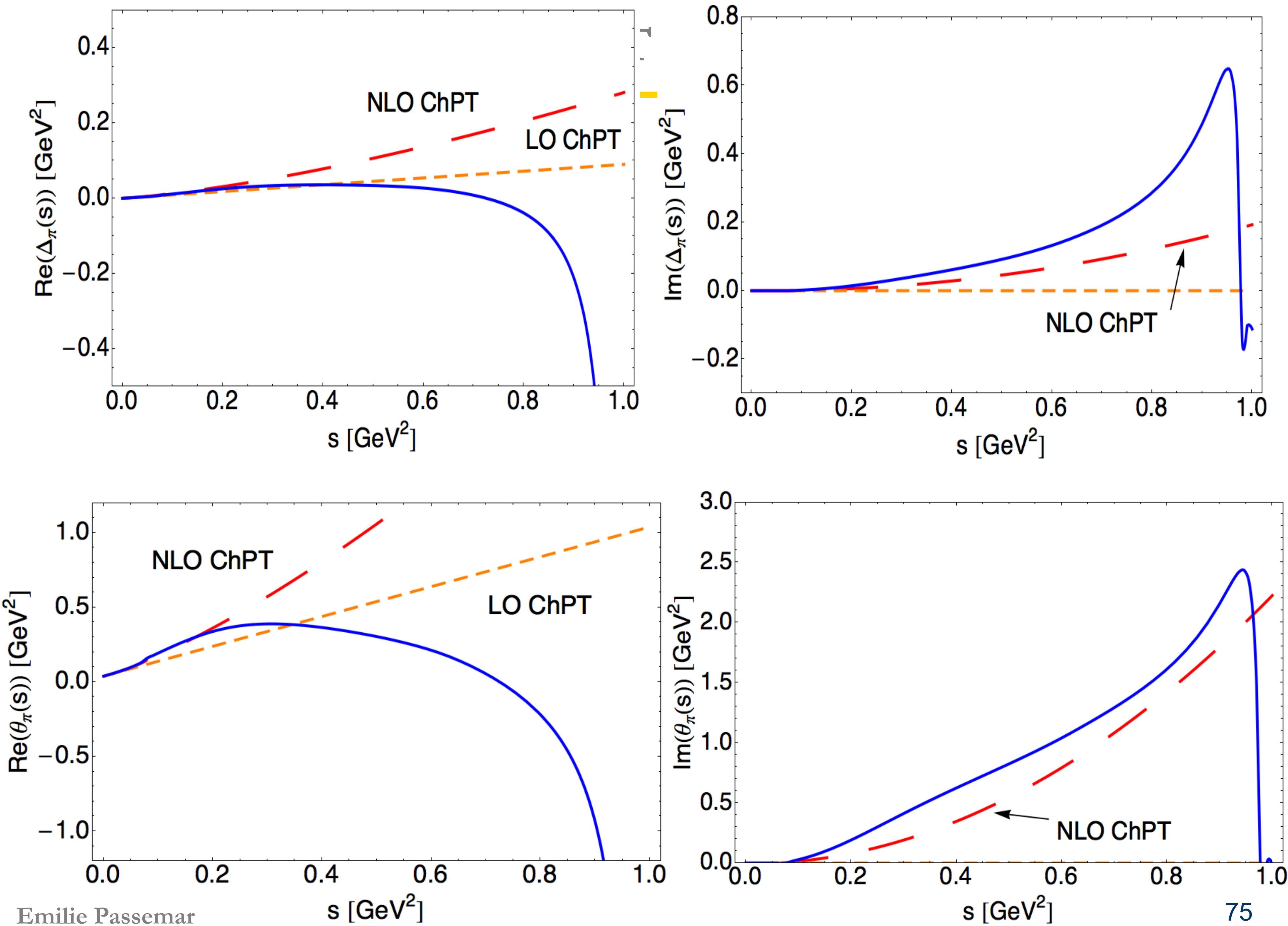

$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
f_0
$|\theta_\pi(s)|$ [GeV^2]
s [GeV^2]
0
1
2
3
4
0.5
1.5
2.5







- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs


NLO ChPT
LO ChPT
Re(Δπ(s)) [GeV2]
s [GeV2]
Im(Δπ(s)) [GeV2]
NLO ChPT
s [GeV2]
NLO ChPT
LO ChPT
Re(θπ(s)) [GeV2]
s [GeV2]
Im(θπ(s)) [GeV2]
NLO ChPT
s [GeV2]

3.1 Lepton Universality

- What about the *third family*?

A. Pich@KEKFF'15

updated on *HFAG'17*

	$\left\| g_\tau / g_\mu \right\|$
$B_{\tau\to e} \quad \tau_\mu / \tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.9962 ± 0.0027
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.9858 ± 0.0070
$B_{W\to\tau} / B_{W\to\mu}$	1.034 ± 0.013

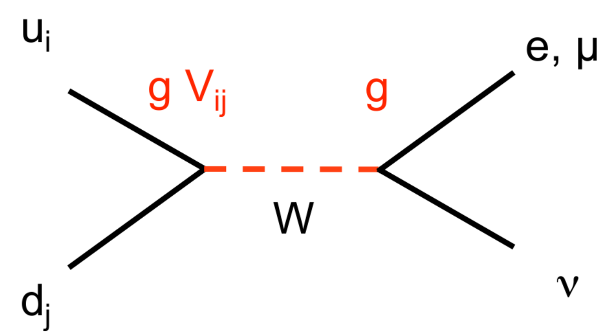
	$\left\| g_\tau / g_e \right\|$
$B_{\tau\to\mu} \, \tau_\mu / \tau_\tau$	1.0029 ± 0.0015
$B_{W\to\tau} / B_{W\to e}$	1.031 ± 0.013

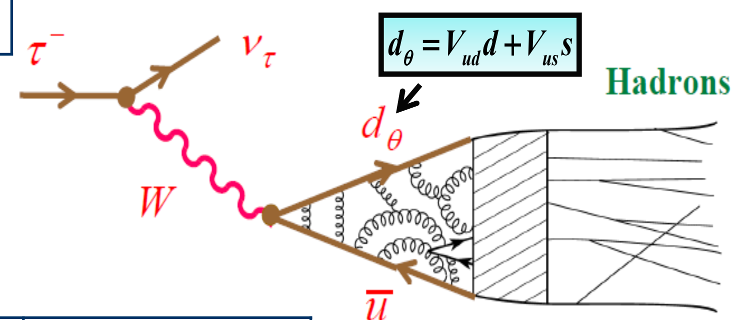
- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays

2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$





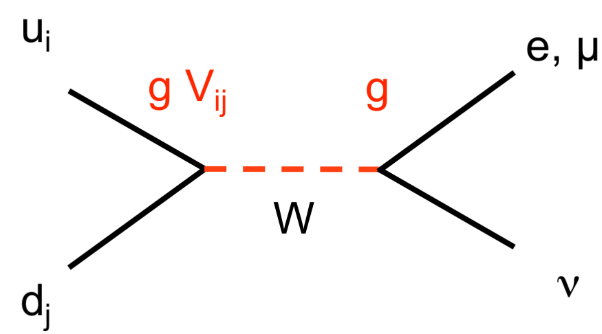
- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

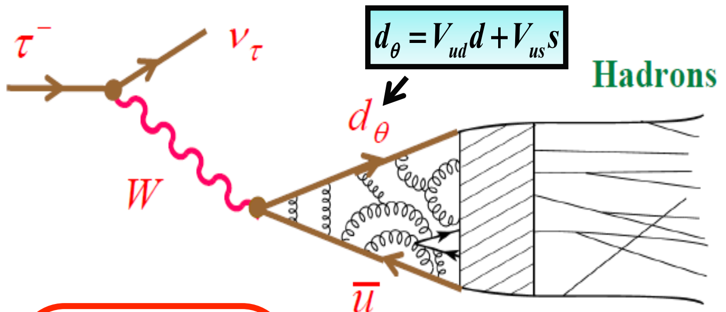
2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

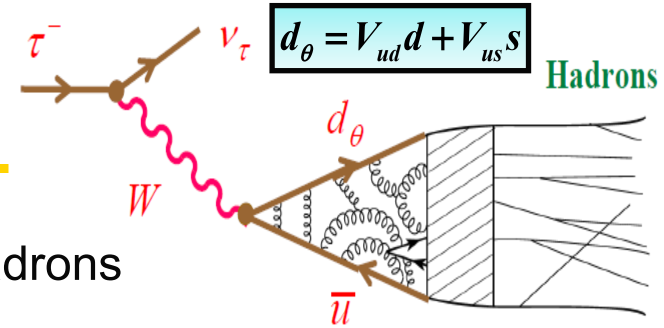


- From τ decays (crossed channel)



V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

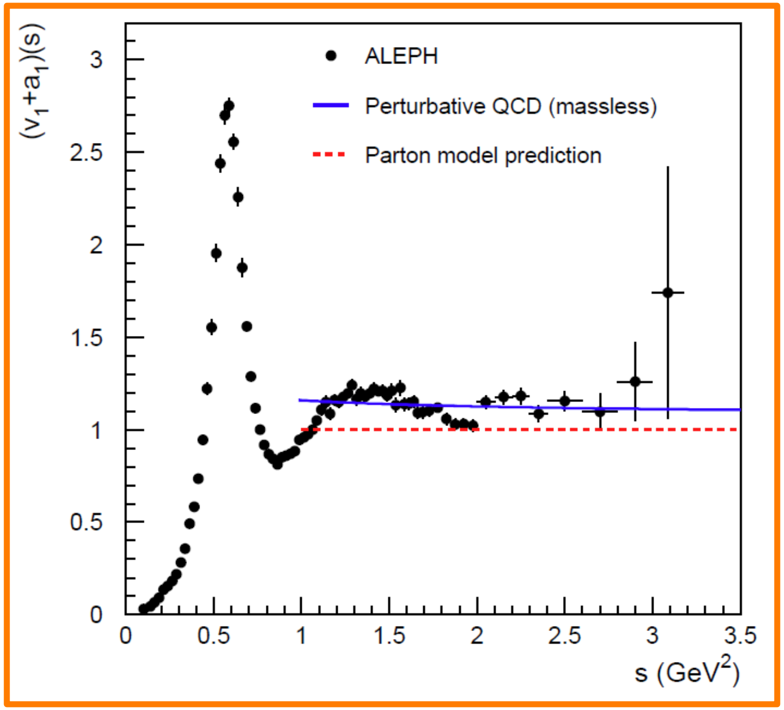
2.3 V_{us} from inclusive measurement



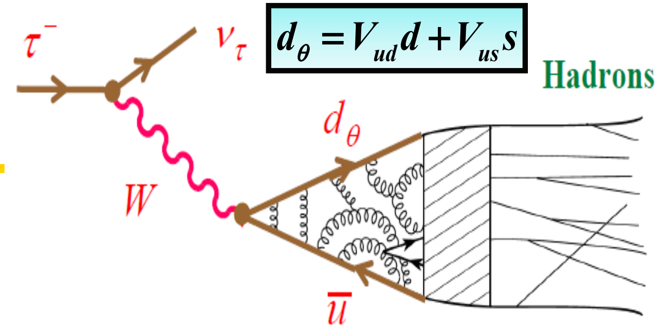
- Tau, the only lepton heavy enough to decay into hadrons
- $\boxed{m_\tau \sim 1.77\,\text{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*
- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $\left(\alpha_s(m_\tau),\ |V_{us}|,\ m_s\right)$
- We consider $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S=0}\right)}$

  $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S\neq 0}\right)}$
- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!
- Observable studied: $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$

Davier et al'13



2.4 Theory



- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- $$\boxed{\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} = \frac{R_\tau^{S}}{R_\tau^{NS}}} \Rightarrow \left|V_{us}\right|$$

QCD switch

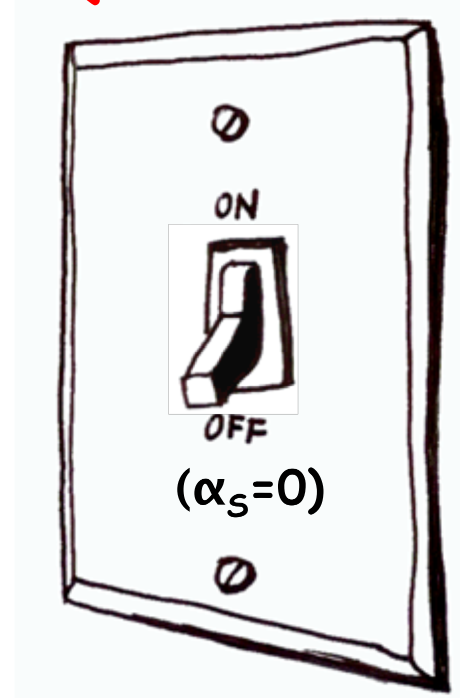
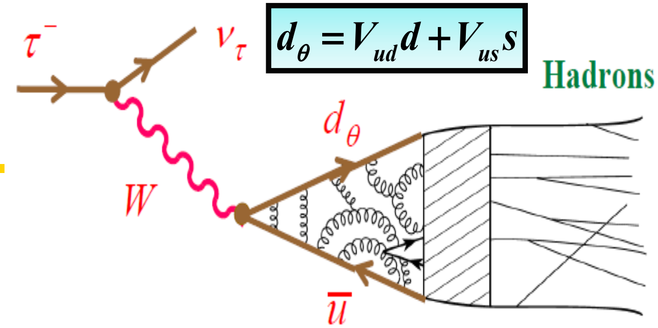
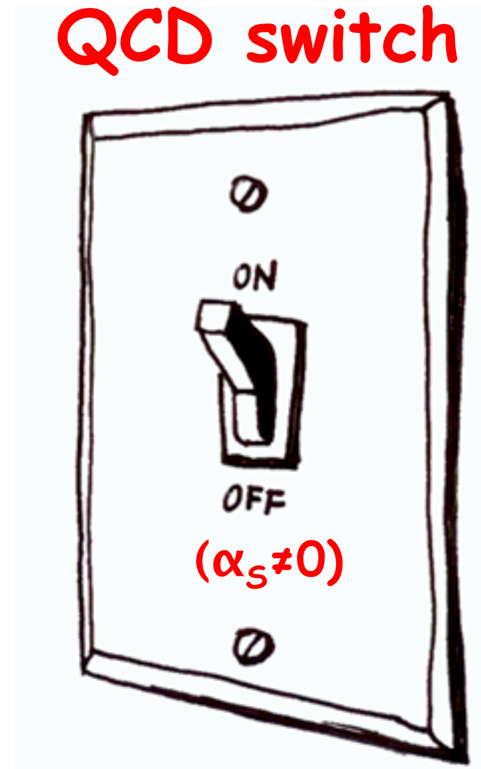


Figure from M. González Alonso'13

2.4 Theory



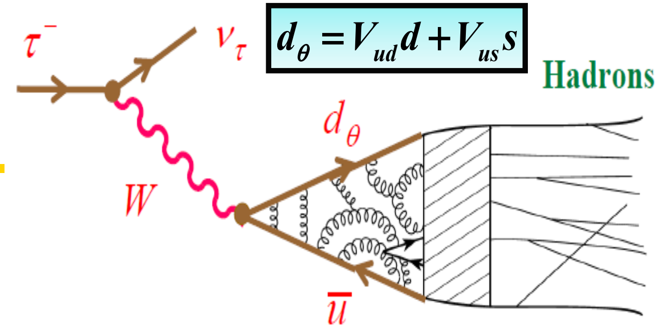
- $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$ parton model prediction



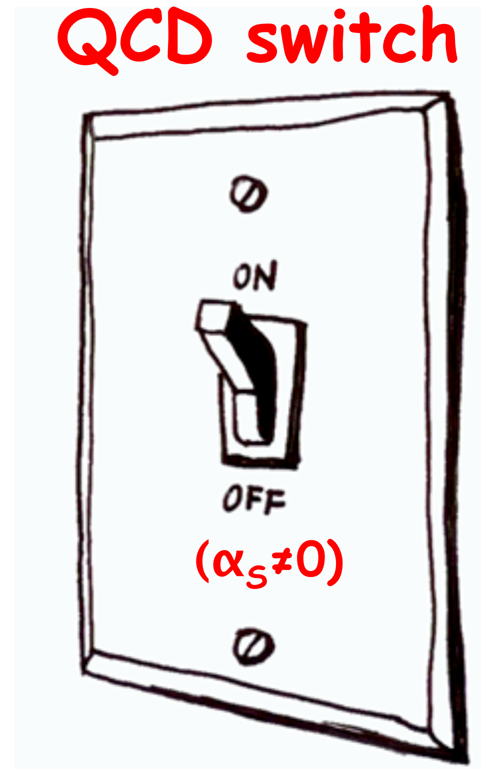
- $R_\tau = R_\tau^{NS} + R_\tau^S \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$

- Experimentally: $\boxed{R_\tau = \dfrac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

2.4 Theory



- $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$ parton model prediction

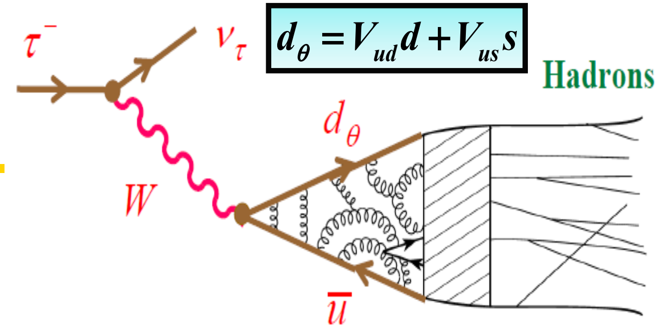


- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- Experimentally: $\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

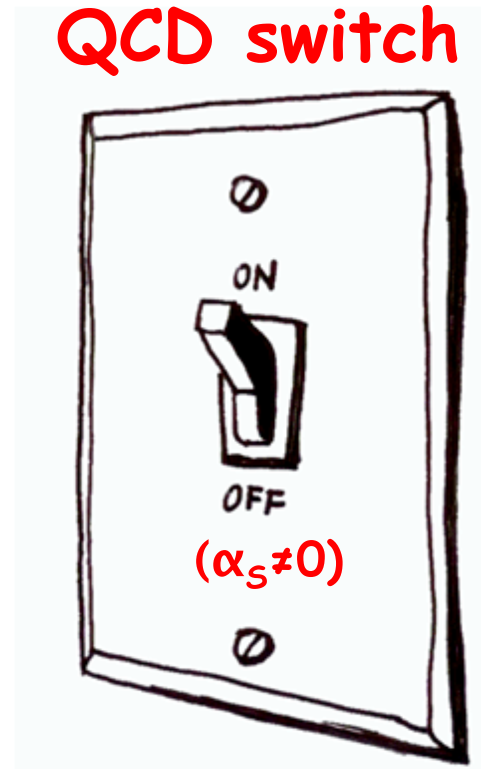
- Due to *QCD corrections*: $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$

2.4 Theory



- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $$\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C}$$ naïve QCD prediction



- Extraction of the strong coupling constant :

$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_S) \Rightarrow \alpha_S$$

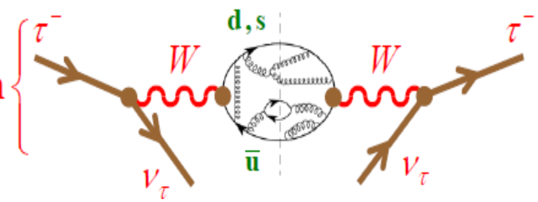
measured calculated

- Determination of V_{us} : $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_S)}$$

- Main difficulty: compute the QCD corrections with the best accuracy

2.5 Calculation of the QCD corrections

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\Big\{$$



$$\Big\}$$

- Calculation of R_τ:

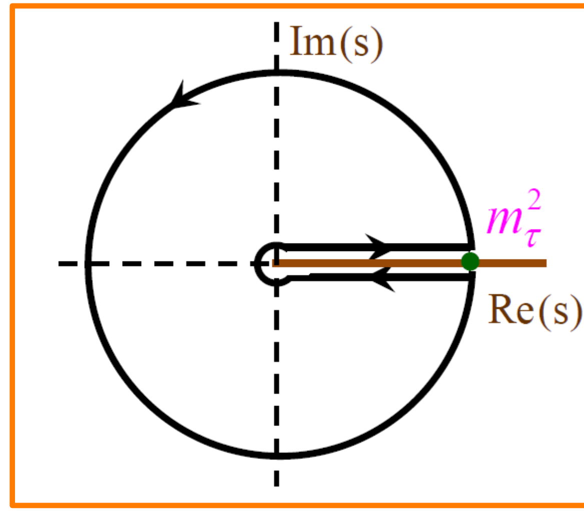
$$\boxed{R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon)+\text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$\boxed{R_\tau(m_\tau^2) = 6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]}$$



- We are now at sufficient energy to use OPE:

$$\boxed{\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots}\frac{1}{(-s)^{D/2}}\sum_{\dim O=D}C^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

2.5 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\; S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\; a_\tau^2 + 26\; a_\tau^3 + 127\; a_\tau^4 + \ldots \approx 20\%$ $\qquad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\;(\propto m_u, m_d)$ but not for $R_\tau^{S}\;(\propto m_s)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

2.5 Calculation of the QCD corrections

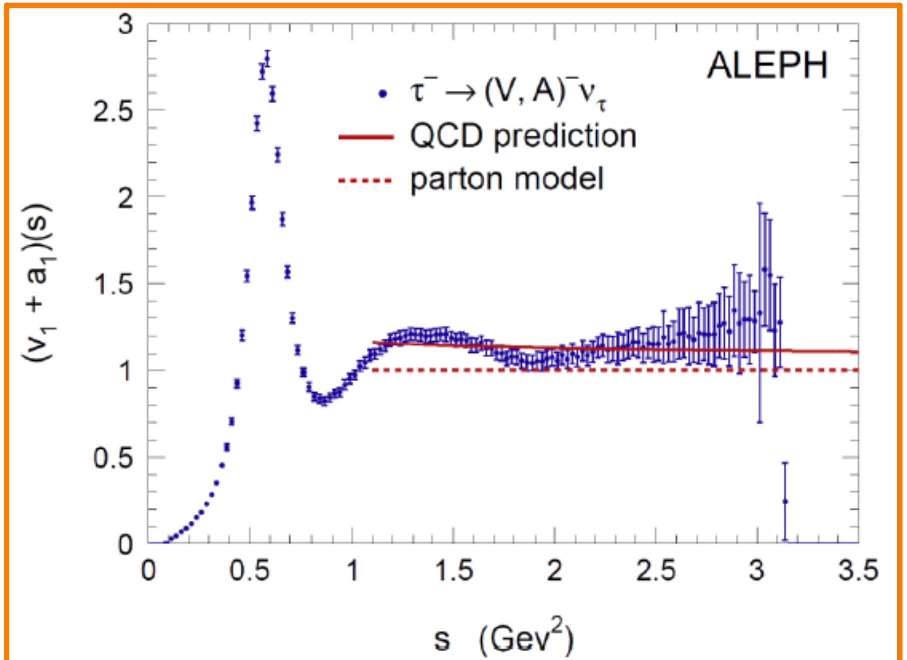
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

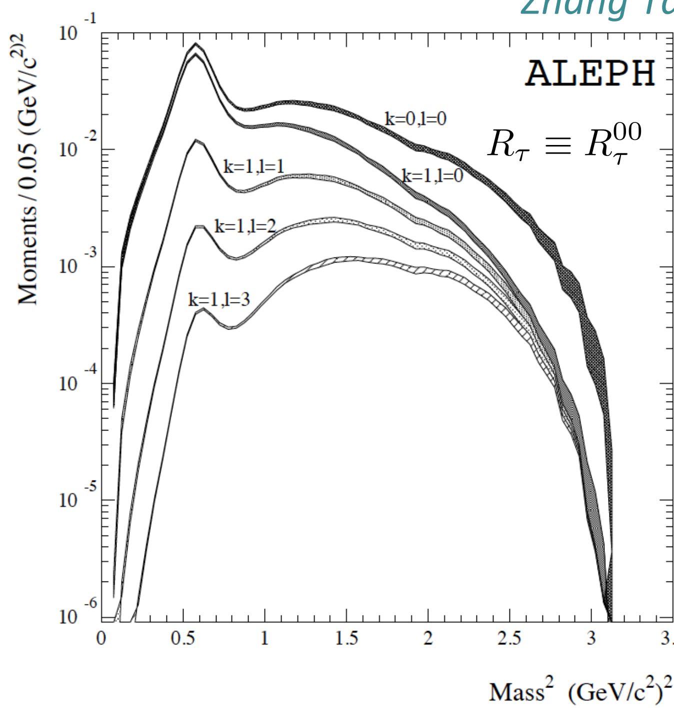
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R^{k\ell}_{\tau,U}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14

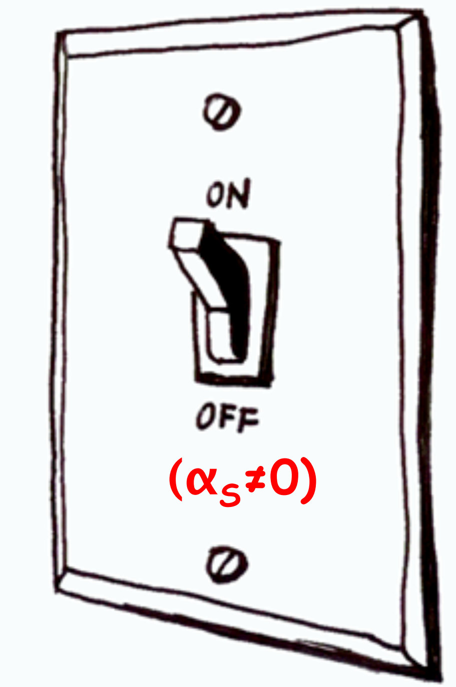


2.5 Inclusive determination of V_{us}

- With QCD on: $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$$

- Use OPE: $$\boxed{R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1+\delta_P+\delta_{NP}^{ud}\right)}$$
$$\boxed{R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1+\delta_P+\delta_{NP}^{us}\right)}$$

QCD switch



$(\alpha_s \neq 0)$

- $$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$ *Gamiz et al'07, Maltman'11*

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

HFAG'17

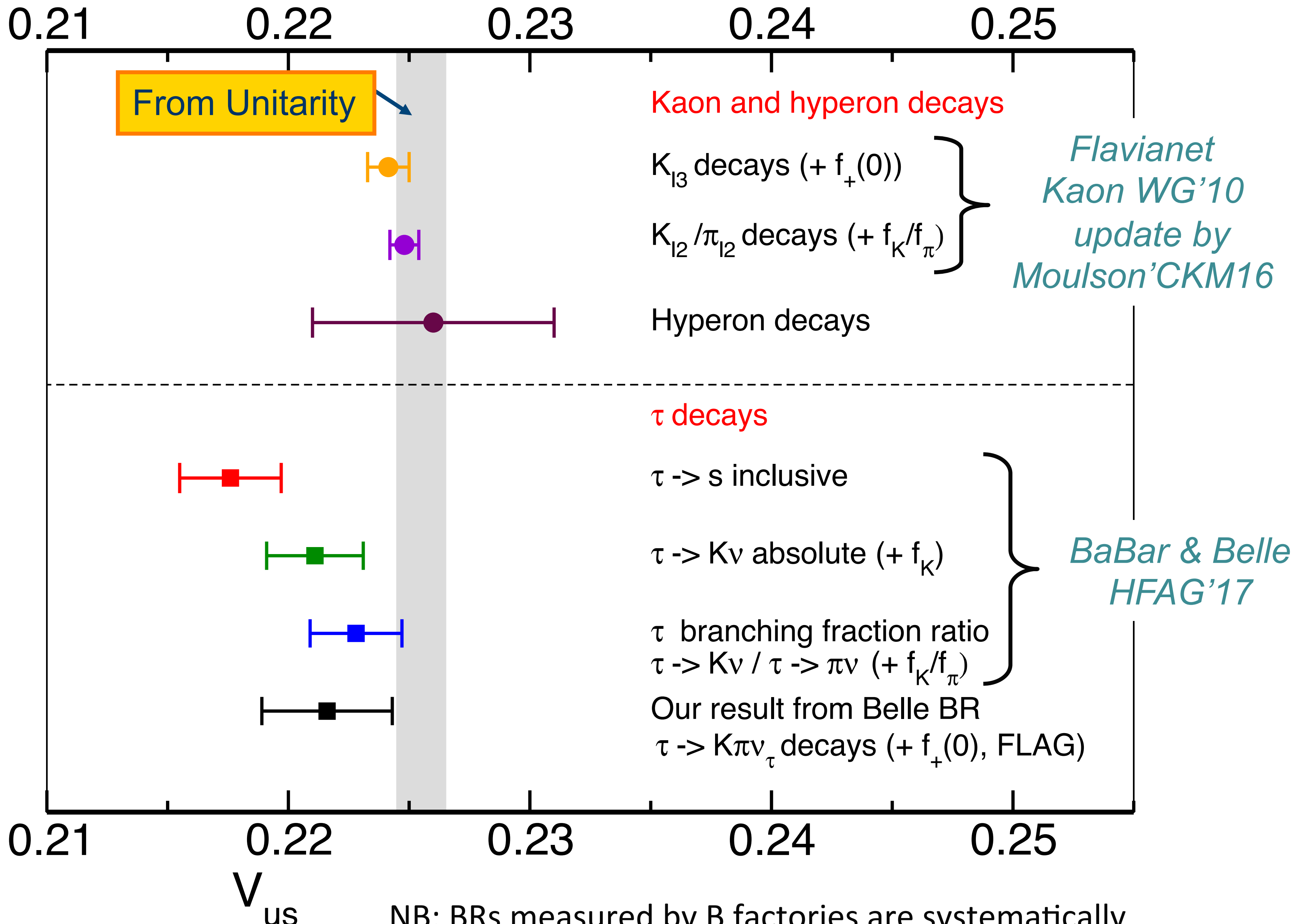
$$R_{\tau,S} = 0.1633(28)$$

$$R_{\tau,NS} = 3.4718(84)$$

$$|V_{ud}| = 0.97417(21)$$

⇒ $$\boxed{|V_{us}| = 0.2186 \pm 0.0019_{\rm exp} \pm 0.0010_{\rm th}}$$

3.1σ away from unitarity!



NB: BRs measured by B factories are systematically smaller than previous measurements

2.6 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	➡ **(0.713 ± 0.003)%**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	➡ **(0.471 ± 0.018)%**
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	➡ **(0.857 ± 0.030)%**
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	➡ **(2.967 ± 0.060)%**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us}

  ➡ Recent studies

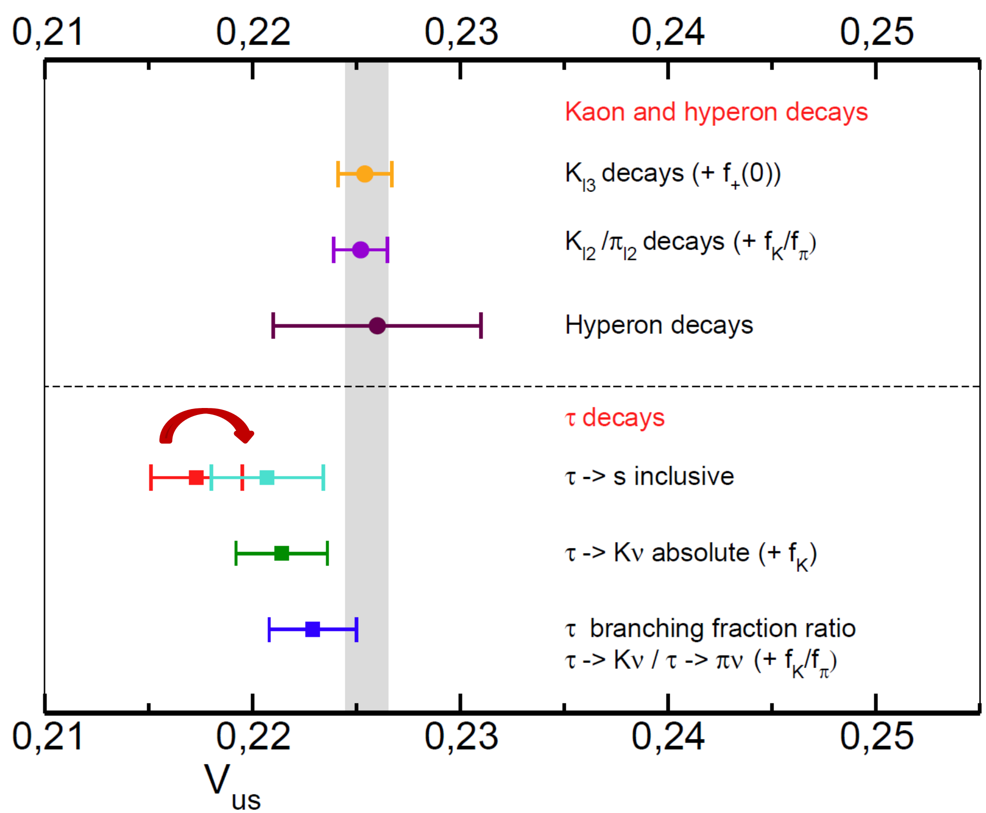
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:

  $\tau \to K\pi\nu_\tau$ Br + spectrum

$$|V_{us}| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$

➡ need new data



2.6 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	➡ **(0.713 ± 0.003)%**
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	➡ **(0.471 ± 0.018)%**
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	➡ **(0.857 ± 0.030)%**
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	➡ **(2.967 ± 0.060)%**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us}
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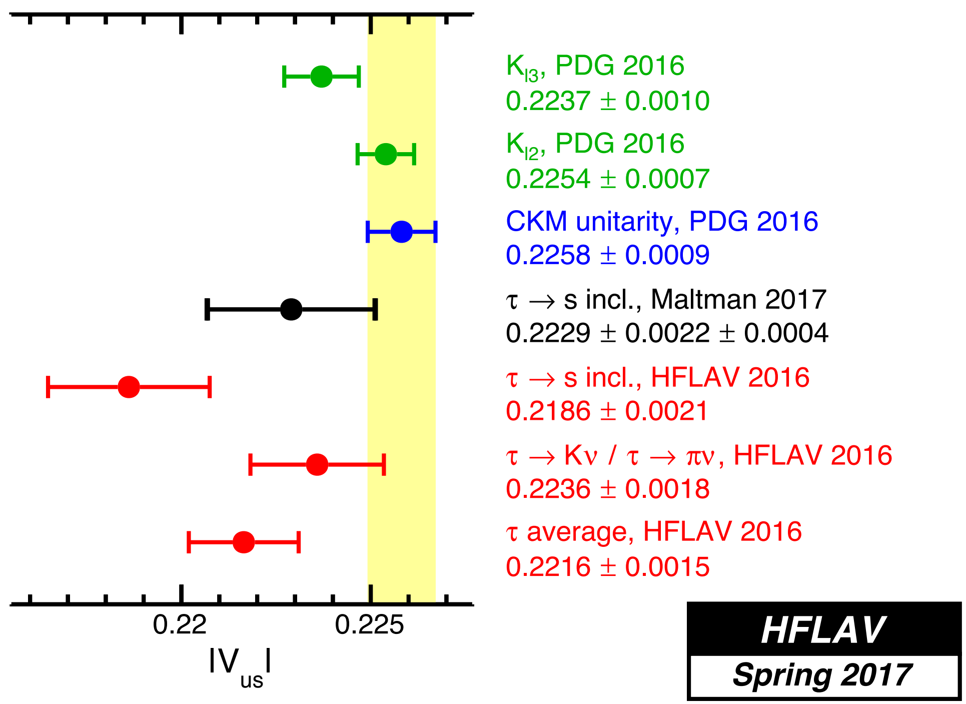


➡ need new data

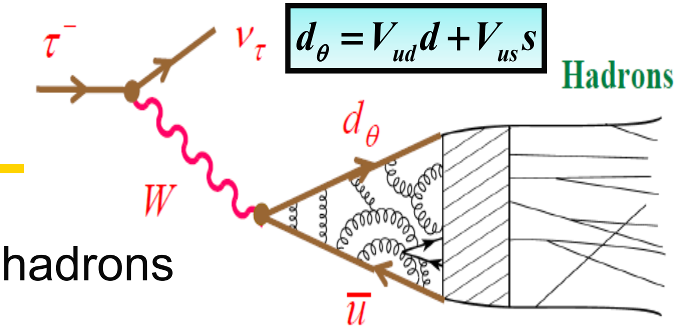
Very good prospect from Belle II, BES?

4.2 Outlook

- 45 billion $\tau^+\tau^-$ pairs in full dataset from $\sigma(\tau^+\tau^-)_{E=\Upsilon(4S)}$= 0.9 nb *@Belle II*
- B2TiP initiative: define the first set of measurements to be performed at *Belle II*
  https://confluence.desy.de/display/BI/B2TiP+WebHome
- *Golden/Silver modes* for the Tau, Low Multiplicity and EW working group

	Process	Observable	Theory	Sys. limit (Discovery) [ab^{-1}]	vs LHCb/BESIII	vs Belle	Anomaly	NP
●	$\tau \to \mu\gamma$	$Br.$	⋆⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆	⋆⋆⋆
●	$\tau \to lll$	$Br.$	⋆⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆	⋆⋆⋆
●	$\tau \to K\pi\nu$	A_{CP}	⋆⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆⋆	⋆⋆
●	$e^+e^- \to \gamma A'(\to \text{invisible})$	σ	⋆⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆	⋆⋆⋆
●	$e^+e^- \to \gamma A'(\to \ell^+\ell^-)$	σ	⋆⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆	⋆⋆⋆
●	π form factor	$g-2$	⋆⋆	-	⋆⋆⋆	⋆⋆	⋆⋆	⋆⋆⋆
●	ISR $e^+e^- \to \pi\pi$ g-2	$g-2$	⋆⋆	-	⋆⋆⋆	⋆⋆⋆	⋆⋆	⋆⋆⋆

3.1 Introduction



- Tau, the only lepton heavy enough to decay into hadrons

- $m_\tau \sim 1.77\text{GeV} > \Lambda_{QCD}$ ⇨ use *perturbative tools: OPE…*

- Inclusive τ decays : $\tau \to (\bar{u}d, \bar{u}s)\nu_\tau$ ⇨ fund. SM parameters $(\alpha_s(m_\tau), |V_{us}|, m_s)$

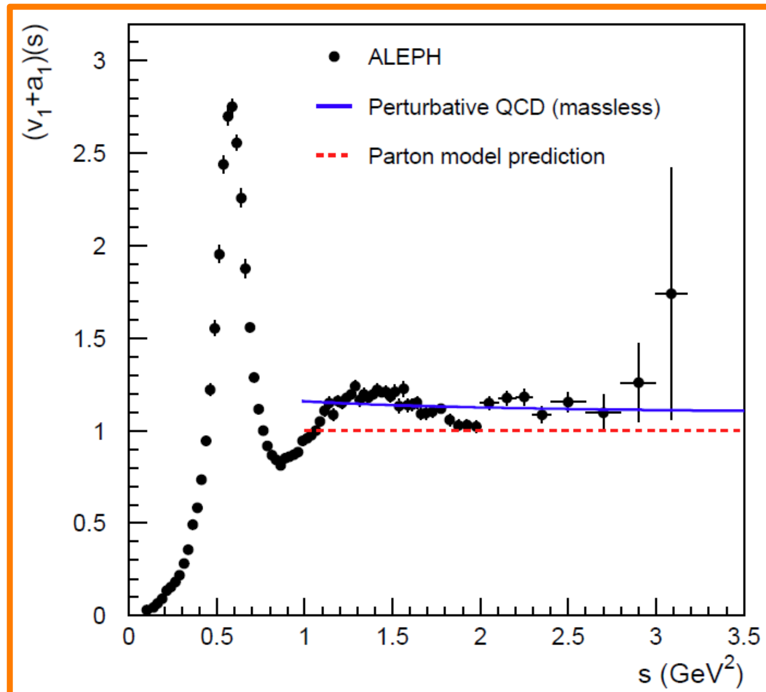
- We consider $\Gamma(\tau^- \to \nu_\tau + \text{hadrons}_{S=0})$

  $\Gamma(\tau^- \to \nu_\tau + \text{hadrons}_{S\neq0})$

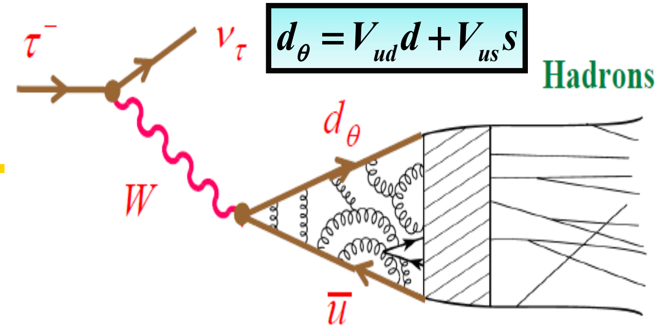
- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!

- Observable studied: $R_\tau \equiv \dfrac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$

Davier et al'13



3.2 Theory



- $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$ parton model prediction

- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$

- $\boxed{\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} = \frac{R_\tau^S}{R_\tau^{NS}}} \Rightarrow \left|V_{us}\right|$

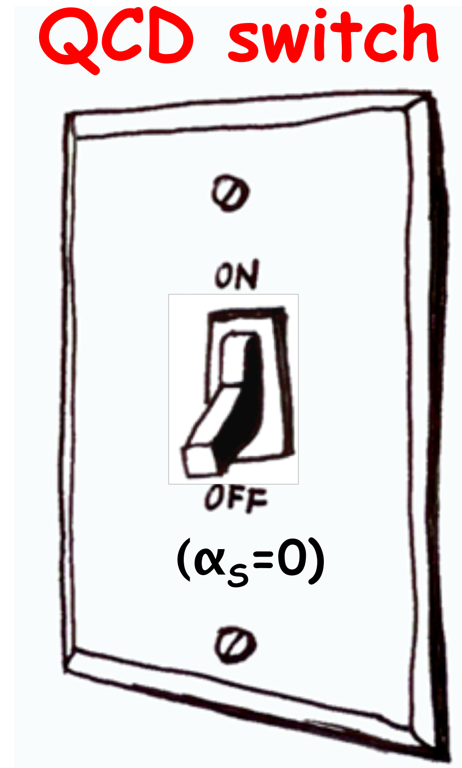
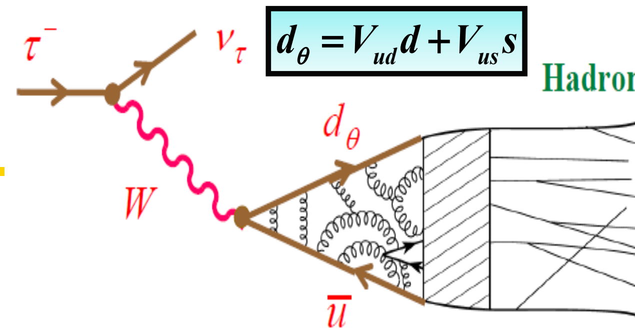
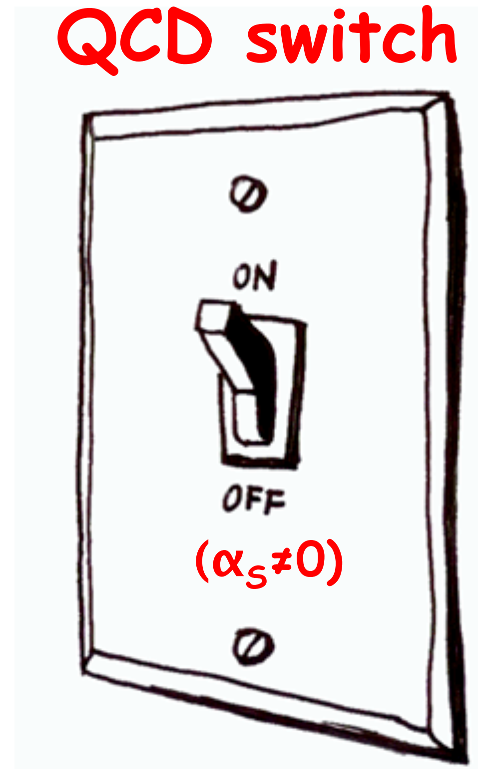


Figure from M. González Alonso'13

3.2 Theory



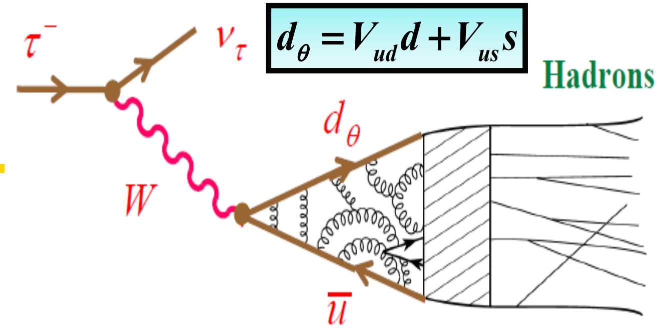
- $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \mathbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$ parton model prediction



- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- Experimentally: $\boxed{R_\tau = \dfrac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

3.2 Theory

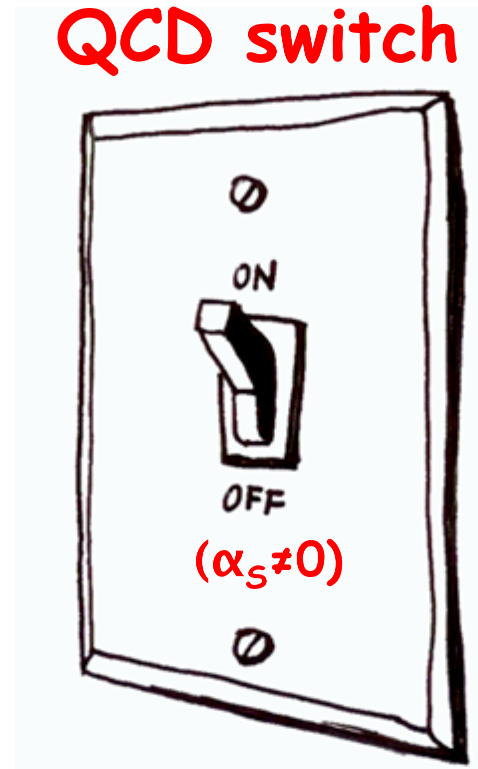


- $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$ parton model prediction

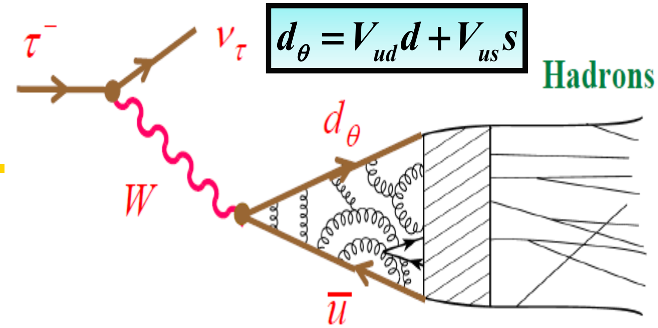
- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$

- Experimentally: $\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

- Due to *QCD corrections*: $R_\tau = \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C + O\left(\alpha_S\right)$

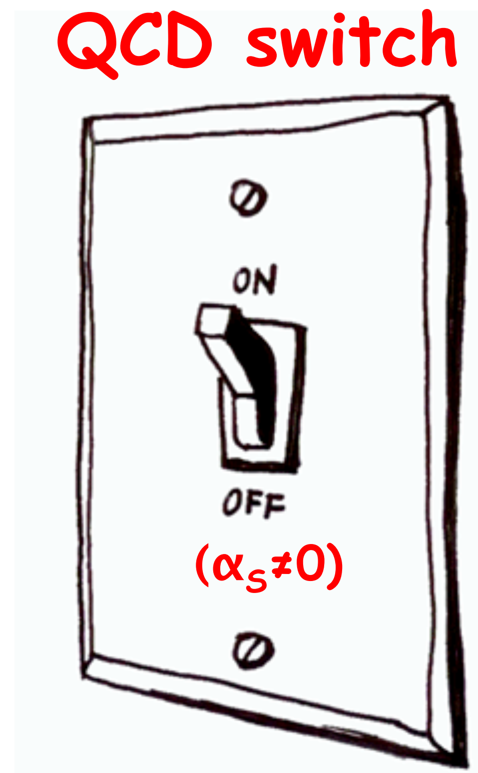


3.2 Theory



- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $$\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C}$$ naïve QCD prediction



- Extraction of the strong coupling constant :

$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_S) \Rightarrow \alpha_S$$

measured calculated

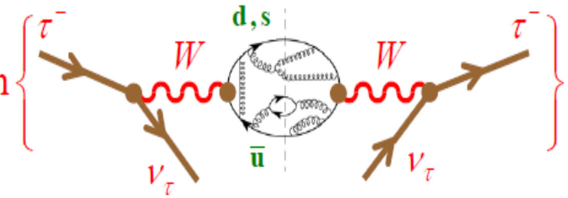
- Determination of V_{us} : $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_S)}$$

- Main difficulty: compute the QCD corrections with the best accuracy

3.3 Calculation of the QCD corrections

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}\{\ \}$$



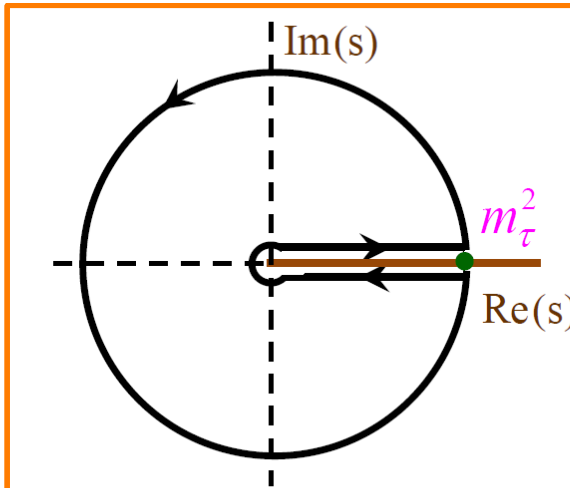
Braaten, Narison, Pich'92

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$\boldsymbol{R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]}$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

3.3 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%$ $\qquad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ (\propto m_u, m_d)$ but not for $R_\tau^{S}\ (\propto m_s)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

3.3 Calculation of the QCD corrections

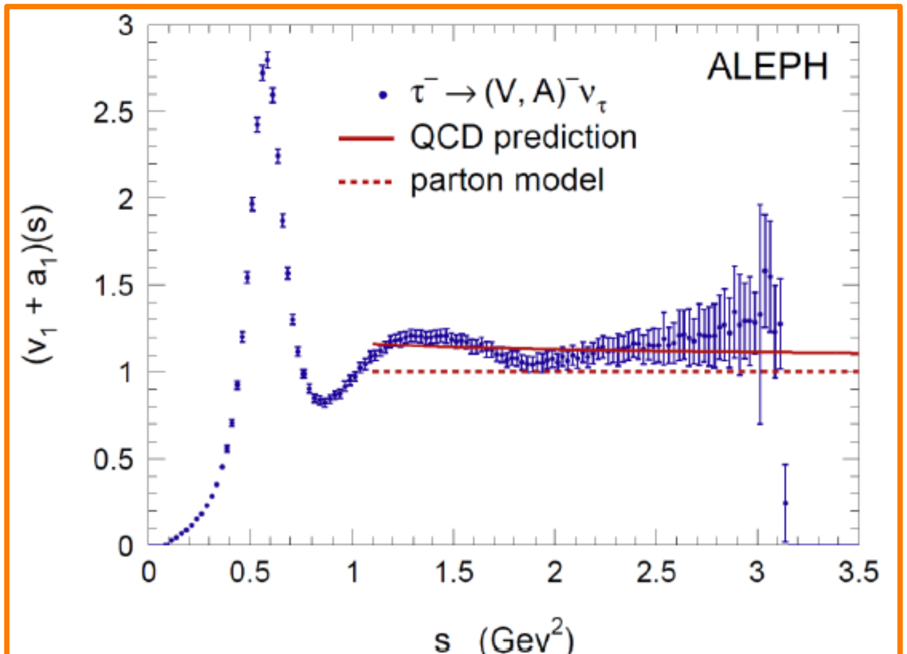
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

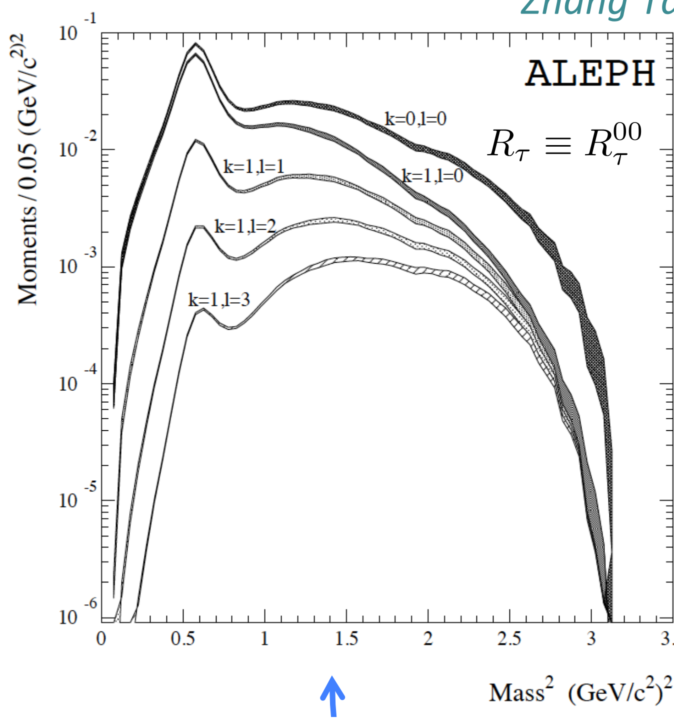
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

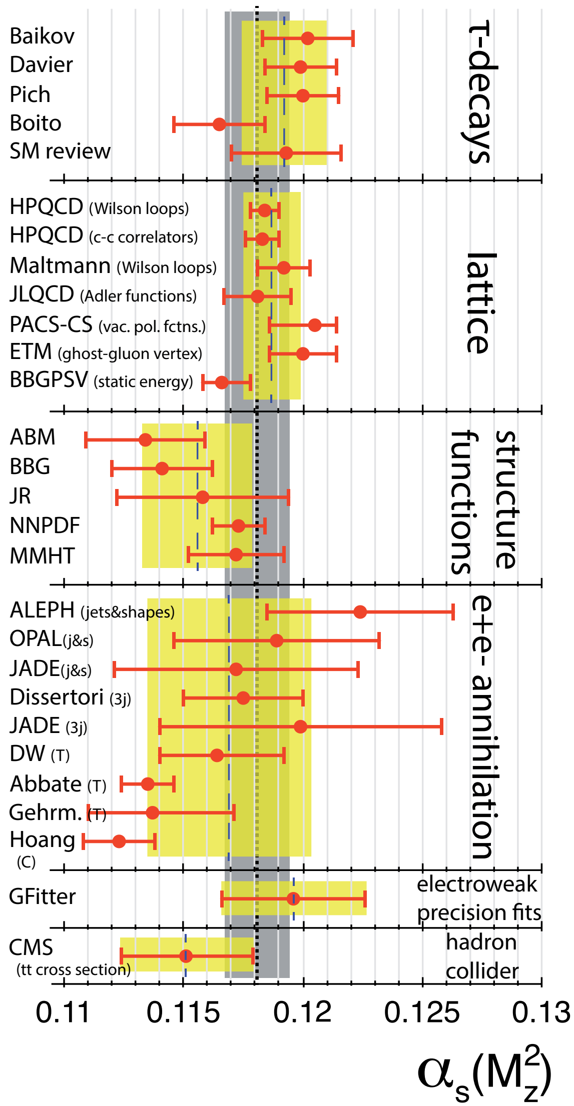


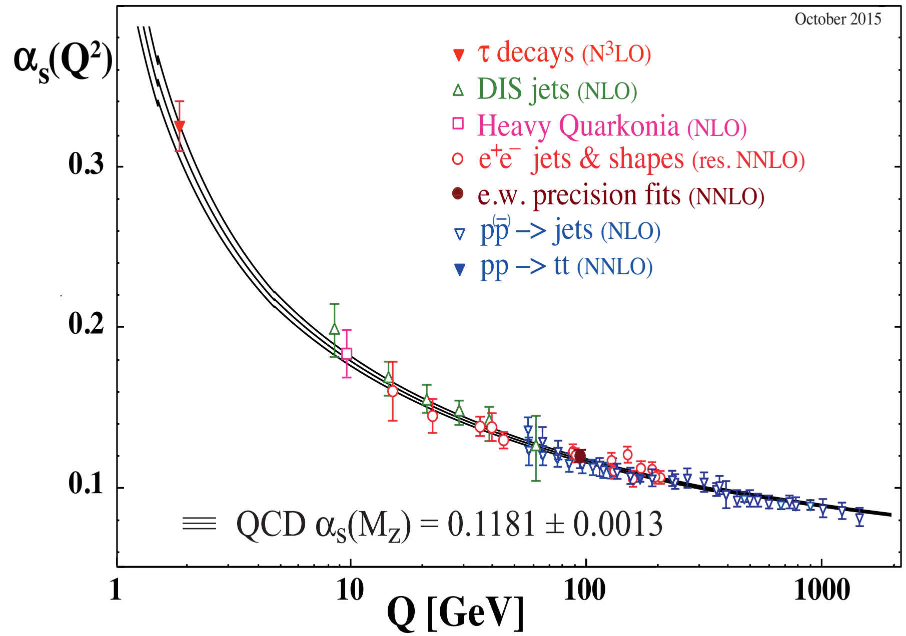
Zhang'Tau14



3.4 Extraction of α_s

Bethke, Dissertori, Salam, PDG'15





- *Extraction of α_s* from hadronic τ very interesting : Moderate precision at the τ mass ⇨ very good precision at the Z mass

- Beautiful test of the QCD running

3.4 Extraction of α_s

- Several delicate points:
  - How to compute the perturbative part: CIPT vs. FOPT?
  - How to estimate the non perturbative contribution? Where do we truncate the expansion, what is the role of higher order condensates?
  - Which weights should we use?
  - What about duality violations?

⇨ A MITP topical workshop in Mainz: March 7-12, 2016

Determination of the fundamental parameters of QCD

A session on Tuesday afternoon

- New data on spectral functions needed to help to answer some of these questions