Tau Physics (theory) at the High Luminosity LHC

Emilie Passemar
Indiana University/Jefferson Laboratory

The 15th International Workshop on Tau Lepton Physics
Amsterdam, September 28, 2018

Based on CERN Yellow Report on the Physics Potential of HL/HE-LHC
WG4: Opportunities in Flavour Physics
Conveners: Jorge Martin Camalich, Jure Zupan (Th), Alex Cerri (ATLAS), Sandra Malvezzi (CMS), Vladimir Gligorov (LHCb)
HL/HE questions for taus, Authors for Theory
M. Gonzalez-Alonso, Vincenzo Cirigliano, Adam Falkowski, Emilie Passemar
High Luminosity LHC

LHC

Run 1 | Run 2 | Run 3 | Run 4 - 5...

LS1 | EYETS | LS2 | LS3

splice consolidation button collimators R2E project

13 TeV

7 TeV 8 TeV


experiment beam pipes

14 TeV

LHC

HL-LHC

14 TeV

ATLAS - CMS upgrade phase 1

ALICE - LHCb upgrade

5 to 7 x nominal luminosity

30 fb⁻¹ 150 fb⁻¹ 300 fb⁻¹ 3000 fb⁻¹

75% nominal luminosity

2 x nominal luminosity

2.5 x nominal luminosity

Radiation damage

Cryomodule interaction regions

Energy

Luminosity

Major Civil Works

Technical Infrastructure

Main Accelerator Components

Construction and Test

Installation

Physics

PDR Preparation

Assess & TDR

Design Study

FP7 Hi-Lumi

Emilie Passemard
Center-of-mass energy of 14 TeV for a total integrated luminosity of \(~3000 \text{ fb}^{-1}\) in 2035 \(6 \times 10^{14} \tau\)

- 200 proton-proton interactions in each collision
- In this regime, experimental sensitivity to new physics enhanced
- Good place for flavour physics but some difficulties:
  - low momenta of typical flavour signatures
  - high pile-up which might affect the precision of the measurements
- Some advantages: Phase II GPD upgrades
  - new inner tracker
  - muon system improvements
  - topological trigger capabilities
  - possibility to use tracking in early stages of the trigger chain
good detection potential, good pile-up mitigation and
in some cases improved performance.
## High Luminosity LHC

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\(\tau \to 3\mu\) is used as a benchmark of CMS muon detector upgrade performance
• \( \tau \) rich phenomenology

• Leptonic decays:
  – Lepton Universality
  – Michel parameters

• Hadronic decays:
  – Inclusive \( \tau \) decays

\[
\tau \to (\bar{u}d, \bar{u}s)\nu_\tau
\]

\[
\alpha_s(m_\tau), |V_{us}|, m_s
\]

  – Exclusive \( \tau \) decays

\[
\tau \to (PP, PPP, \ldots)\nu_\tau
\]

FFs,
resonance parameters
Hadronization of QCD currents
Hadronic contribution to muon g-2
CP violation in \( K\pi \)

• Charged lepton flavour violation, Electromagnetic dipole moments
• Muon g-2, 2 photon physics
• Precision EW tests

**Role of HL LHC?**
• τ rich phenomenology

• Leptonic decays:
  – *Lepton Universality*
  – Michel parameters

• Hadronic decays:
  – Inclusive τ decays
    \[ \tau \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau \]
  
  \[ \alpha_s(m_\tau), |V_{us}|, m_s \]

  – Exclusive τ decays
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• **Charged lepton flavour violation**, Electromagnetic dipole moments

• Muon g-2, 2 photon physics

• Precision EW tests

*Role of HL LHC?*
1. Charged Lepton-Flavour Violation
1.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the SM ($m_\nu=0$)

- In the SM with massive neutrinos effective CLFV vertices are tiny due to GIM suppression → unobservably small rates!

E.g.:

\[
\mu \rightarrow e\gamma
\]

\[
Br(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m_{1i}^2}{M_W^2} \right|^2 < 10^{-54}
\]

Petcov’77, Marciano & Sanda’77, Lee & Shrock’77…

\[
\left[ Br(\tau \rightarrow \mu\gamma) < 10^{-40} \right]
\]

- Extremely clean probe of beyond SM physics

- In New Physics models: seazible effects
  Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic
1.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

| Model Type                  | Coupling Parameters                                                                 | Sensitivity
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<td>SM + heavy Maj νR</td>
<td>Cvetic, Dib, Kim, Kim, PRD66 (2002) 034008</td>
<td>10^{-9} 10^{-10}</td>
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<td>Non-universal Z'</td>
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- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic
1.2 Tau LFV

- Several processes: $\tau \rightarrow \ell \gamma$, $\tau \rightarrow \ell \bar{\ell}_\alpha \ell_\beta$, $\tau \rightarrow \ell Y$ $P$, $S$, $V$, $P\bar{P}$, ...

90% CL upper limits on $\tau$ LFV decays

- 48 LFV modes studied at Belle and BaBar
1.2 Tau LFV

- Several processes: \( \tau \to \ell \gamma, \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta, \tau \to \ell Y \)

90% CL upper limits on \( \tau \) LFV decays

- Expected sensitivity \( 10^{-9} \) or better at \( \text{LHCb, ATLAS, CMS, Belle II, HL-LHC?} \)
Overview of LFV physics

Evolution of LFV limits

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<th>Year</th>
<th>Decays Studied</th>
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MarkII  ARGUS  DELPHI  CLEO  Belle  BaBar  LHCb

mSUGRA + seesaw  SUSY + SO(10)  SM + seesaw  SUSY + Higgs

90% CL Upper Limit on Branching Ratio

\( \tau \rightarrow \mu \gamma \)  \( \tau \rightarrow \mu \eta \)  \( \tau \rightarrow \mu \mu \mu \)

![Graph showing approximate number of tau decays studied](image)

Belle II can reduce most of these limits by 1~2 orders of magnitude.

LFV is suppressed in SM → a few models predict enhancements within Belle II’s reach.

S. Banerjee’17

B2TIP’18

50 ab\(^{-1}\) Luminosity
Evolution of LFV limits

- 1980
- 1990
- 2000
- 2010
- 2020

Approximate number of τ decays studied

90% CL Upper Limit on Branching Ratio

- τ → μ γ
- τ → μ η
- τ → μ μ μ

mSUGRA + seesaw
SUSY + SO(10)
SM + seesaw
SUSY + Higgs

MarkII
ARGUS
DELPHI
CLEO
Belle
BaBar
LHCb
Belle II

S. Banerjee’17

B2TIP’18

HL LHC
3.7 x 10^{-9}
Evolution of LFV limits

- **MarkII**
- **ARGUS**
- **DELPHI**
- **CLEO**
- **Belle**
- **BaBar**
- **LHCb**

Recent progress includes:
- **Belle II**
  - *mSUGRA + seesaw*
  - *SUSY + SO(10)*
  - *SM + seesaw*
  - *SUSY + Higgs*

**S. Banerjee’17**

**B2TIP’18**

**CERN LHCC 2017-003**

**HL LHC** $3.7 \times 10^{-9}$
Slide from Talk by Jian Wang
On Tuesday

\[ \tau \rightarrow 3\mu @ HL-LHC \]

---

### Category 1: Events without using ME0

- Number of background events: \(2.4 \times 10^6\)
- Number of signal events: 4580
- Trimuon mass resolution: 18 MeV
- \(B(\tau \rightarrow 3\mu)\) limit per event category: \(4.3 \times 10^{-9}\)
- \(B(\tau \rightarrow 3\mu) 90\%\text{C.L. limit}\): \(3.7 \times 10^{-9}\)

### Category 2: Events with at least one muon tagged by ME0

- Number of background events: \(2.6 \times 10^6\)
- Number of signal events: 3640
- Trimuon mass resolution: 31 MeV
- \(B(\tau \rightarrow 3\mu)\) limit per event category: \(7.0 \times 10^{-9}\)

**Signal and background yields in \([1.55, 2.00]\) GeV, assuming \(Br(\tau \rightarrow 3\mu) = 2 \times 10^{-8}\)**

**Note:** ME0 reconstruction software was not yet optimised at the time of this study
1.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_{i} \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots \]

- Build all D>5 LFV operators:

➢ Dipole:

\[ \mathcal{L}_{eff}^{D} \sim -\frac{C_{D}}{\Lambda^2} m_{\tau} \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu} \]

- See e.g.
  - Black, Han, He, Sher’02
  - Brignole & Rossi’04
  - Dassinger et al.’07
  - Matsuzaki & Sanda’08
  - Giffels et al.’08
  - Crivellin, Najjari, Rosiek’13
  - Petrov & Zhuridov’14
  - Cirigliano, Celis, E.P.’14
### 1.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \ldots \]

- **Build all D>5 LFV operators:**
  - **Dipole:**
    \[ \mathcal{L}_{dip}^{\text{eff}} \supset - \frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu} \]
  - **Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):**
    \[ \mathcal{L}_{dip}^{S,V} \supset - \frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q \]

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Emilie Passemar
1.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots \]

- Build all D>5 LFV operators:
  - Dipole: \[ \mathcal{L}^D_{\text{eff}} \supset \frac{C_D}{\Lambda^2} m_{\tau} \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu} \]
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):
  
  - Integrating out heavy quarks generates gluonic operator

\[ \frac{1}{\Lambda^2} \bar{\mu} P_{L,R} \tau Q \bar{Q} \rightarrow \mathcal{L}^G_{\text{eff}} \supset \frac{-C_G}{\Lambda^2} m_{\tau} G_F \bar{\mu} P_{L,R} \tau G_{a_{\mu\nu}} \]

See e.g. Black, Han, He, Sher’02
Brignole & Rossi’04
Dassinger et al.’07
Matsuzaki & Sanda’08
Giffels et al.’08
Crivellin, Najjari, Rosiek’13
Petrov & Zhuridov’14
Cirigliano, Celis, E.P.’14

Emilie Passemard
1.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots \]

- Build all D>5 LFV operators:
  - Dipole: \( \mathcal{L}_{\text{eff}}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} p_{L,R} \tau F_{\mu\nu} \)
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): \( \mathcal{L}_{\text{eff}}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \tau \Gamma p_{L,R} \bar{q} \Gamma q \)
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): \( \mathcal{L}_{\text{eff}}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu} \Gamma p_{L,R} \bar{\mu} \Gamma p_{L,R} \mu \)

See e.g.
Black, Han, He, Sher’02
Brignole & Rossi’04
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Giffels et al.’08
Crivellin, Najjari, Rosiek’13
Petrov & Zhuridov’14
Cirigliano, Celis, E.P.’14

\[ Y_{\Delta}, \Delta^{--}, Y_{\Delta}, \mu \]

\( \Gamma \equiv 1, \gamma^\mu \)
1.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots \]

- Build all D>5 LFV operators:
  - Dipole:
    \[ \mathcal{L}^D_{\text{eff}} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu} \]
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):
    \[ \mathcal{L}^S_{\text{eff}} \supset -\frac{C_S}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q \]
  - Lepton-gluon (Scalar, Pseudo-scalar):
    \[ \mathcal{L}^G_{\text{eff}} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau G^a_{\mu\nu} G^a_{\mu\nu} \]
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector):
    \[ \mathcal{L}^{4\ell}_{\text{eff}} \supset -\frac{C^{4\ell}_{S,V}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu \]

Each UV model generates a \textit{specific pattern} of them
### 1.4 Model discriminating power of Tau processes

**Summary table:**

<table>
<thead>
<tr>
<th>$\tau \rightarrow 3\mu$</th>
<th>$\tau \rightarrow \mu\gamma$</th>
<th>$\tau \rightarrow \mu\pi^+\pi^-$</th>
<th>$\tau \rightarrow \mu K\bar{K}$</th>
<th>$\tau \rightarrow \mu\pi$</th>
<th>$\tau \rightarrow \mu\eta(\prime)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{S,V}^{4\mu}$</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$O_D$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>$O_{\eta}^{q}$</td>
<td>-</td>
<td>-</td>
<td>✓ (I=0)</td>
<td>✓ (I=0,1)</td>
<td>-</td>
</tr>
<tr>
<td>$O_{S}^{q}$</td>
<td>-</td>
<td>✓ (I=0)</td>
<td>✓ (I=0,1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$O_{GG}$</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$O_{A}^{q}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓ (I=0)</td>
<td>✓ (I=0)</td>
</tr>
<tr>
<td>$O_{P}^{q}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓ (I=1)</td>
<td>✓ (I=0)</td>
</tr>
<tr>
<td>$O_{GG}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
</tr>
</tbody>
</table>

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!
- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. $f_\eta$, $f_\eta'$)
1.4 Model discriminating power of Tau processes

- **Summary table:**

<table>
<thead>
<tr>
<th>( \tau \rightarrow 3\mu )</th>
<th>( \tau \rightarrow \mu \gamma )</th>
<th>( \tau \rightarrow \mu \pi^+\pi^- )</th>
<th>( \tau \rightarrow \mu K\bar{K} )</th>
<th>( \tau \rightarrow \mu \pi )</th>
<th>( \tau \rightarrow \mu \eta^{(i)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O^4_{S,V} )</td>
<td>✓</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( O_D )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>–</td>
</tr>
<tr>
<td>( O^q_V )</td>
<td>–</td>
<td>–</td>
<td>✓ (I=1)</td>
<td>✓ (I=0,1)</td>
<td>–</td>
</tr>
<tr>
<td>( O^q_S )</td>
<td>–</td>
<td>–</td>
<td>✓ (I=0)</td>
<td>✓ (I=0,1)</td>
<td>–</td>
</tr>
<tr>
<td>( O_{GG} )</td>
<td>–</td>
<td>–</td>
<td>✓</td>
<td>✓</td>
<td>–</td>
</tr>
<tr>
<td>( O^q_A )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>✓ (I=1)</td>
</tr>
<tr>
<td>( O^q_P )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>✓ (I=1)</td>
</tr>
<tr>
<td>( O_{GG} )</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

- **Form factors for** \( \tau \rightarrow \mu(e)\pi\pi \) **determined using** *dispersive techniques*

- **Hadronic part:**

\[
H_\mu = \langle \pi\pi | (V_\mu - A_\mu) e^{i\eta_{qcd}} | 0 \rangle = (\text{Lorentz struct.})_\mu^i F_i(s)
\]

*with*

\[
s = (p_{\pi^+} + p_{\pi^-})^2
\]

- **2-channel unitarity condition is solved with** I=0 S-wave \( \pi\pi \) **and** KK scattering data as input

\[
\text{Im} F_n(s) = \sum_{m=1}^{2} T_{nm}(s)\sigma_m(s)F_m(s)
\]

Celis, Cirigliano, E.P.’14

Celis, Cirigliano, E.P.’14

Donoghue, Gasser, Leutwyler’90

Moussallam’99

Daub et al’13

Emilie Passemar
### 1.4 Model discriminating power of Tau processes

**Summary table:**

<table>
<thead>
<tr>
<th>Process</th>
<th>( \tau \to 3\mu )</th>
<th>( \tau \to \mu \gamma )</th>
<th>( \tau \to \mu \pi^+ \pi^- )</th>
<th>( \tau \to \mu K\bar{K} )</th>
<th>( \tau \to \mu \pi )</th>
<th>( \tau \to \mu \eta^{(0)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O^{4_L}_{S,V} )</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( O_D )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( O^q_V )</td>
<td>-</td>
<td>-</td>
<td>✓ (I=1)</td>
<td>✓ (I=0,1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( O^q_S )</td>
<td>-</td>
<td>-</td>
<td>✓ (I=0)</td>
<td>✓ (I=0,1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( O_{GG} )</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( O^q_A )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓ (I=1)</td>
<td>✓ (I=0)</td>
</tr>
<tr>
<td>( O^q_P )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓ (I=1)</td>
<td>✓ (I=0)</td>
</tr>
<tr>
<td>( O_{GG} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
</tr>
</tbody>
</table>

- The notion of "best probe" (process with largest decay rate) is *model dependent*.
- If observed, compare rate of processes as a key handle on *relative strength* between operators and hence the underlying mechanism.
- *It would be good to be able to constrain* \( \tau \to \mu \pi \pi \) *at HL-LHC!*
1.5 Handles

• Two handles:
  - Branching ratios: \( R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)} \) with \( F_M \) dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

• Benchmarks:
  - Dipole model: \( C_D \neq 0, C_{\text{else}} = 0 \)
  - Scalar model: \( C_S \neq 0, C_{\text{else}} = 0 \)
  - Vector (gamma,Z) model: \( C_V \neq 0, C_{\text{else}} = 0 \)
  - Gluonic model: \( C_{GG} \neq 0, C_{\text{else}} = 0 \)
1.6 Model discriminating of BRs

- Two handles:
  - Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)}$ with $F_M$ dominant LFV mode for model M

<table>
<thead>
<tr>
<th></th>
<th>$\mu\pi^+\pi^-$</th>
<th>$\mu\rho$</th>
<th>$\mu f_0$</th>
<th>$3\mu$</th>
<th>$\mu\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$R_{F,D}$</td>
<td>$0.26 \times 10^{-2}$</td>
<td>$0.22 \times 10^{-2}$</td>
<td>$0.13 \times 10^{-3}$</td>
<td>$0.22 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>BR</td>
<td>$&lt; 1.1 \times 10^{-10}$</td>
<td>$&lt; 9.7 \times 10^{-11}$</td>
<td>$&lt; 5.7 \times 10^{-12}$</td>
<td>$&lt; 9.7 \times 10^{-11}$</td>
</tr>
<tr>
<td>S</td>
<td>$R_{F,S}$</td>
<td>1</td>
<td>0.28</td>
<td>0.7</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>BR</td>
<td>$&lt; 2.1 \times 10^{-8}$</td>
<td>$&lt; 5.9 \times 10^{-9}$</td>
<td>$&lt; 1.47 \times 10^{-8}$</td>
<td>-</td>
</tr>
<tr>
<td>V(γ)</td>
<td>$R_{F,V(γ)}$</td>
<td>1</td>
<td>0.86</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>BR</td>
<td>$&lt; 1.4 \times 10^{-8}$</td>
<td>$&lt; 1.2 \times 10^{-8}$</td>
<td>$&lt; 1.4 \times 10^{-9}$</td>
<td>-</td>
</tr>
<tr>
<td>Z</td>
<td>$R_{F,Z}$</td>
<td>1</td>
<td>0.86</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>BR</td>
<td>$&lt; 1.4 \times 10^{-8}$</td>
<td>$&lt; 1.2 \times 10^{-8}$</td>
<td>$&lt; 1.4 \times 10^{-9}$</td>
<td>-</td>
</tr>
<tr>
<td>G</td>
<td>$R_{F,G}$</td>
<td>1</td>
<td>0.41</td>
<td>0.41</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>BR</td>
<td>$&lt; 2.1 \times 10^{-8}$</td>
<td>$&lt; 8.6 \times 10^{-9}$</td>
<td>$&lt; 8.6 \times 10^{-9}$</td>
<td>-</td>
</tr>
</tbody>
</table>

\textit{Benchmark Emiline Passemar}

Celis, Cirigliano, E.P.’14
Figure 3: Dalitz plot for $\tau^{-}\to\mu^{-}\mu^{+}\mu^{-}$ decays when all operators are assumed to vanish with the exception of $C_{DL,DR}=1$ (left) and $C_{SLL,SRR}=1$ (right), taking $\Lambda=1$ TeV in both cases. Colors denote the density for $d^{2}BR/(dm_{\mu^{+}\mu^{-}}dm_{\mu^{\pm}})$, small values being represented by darker colors and large values in lighter ones. Here $m_{\mu^{+}\mu^{-}}$ represents $m_{12}^{2}$ or $m_{23}^{2}$, defined in Sec. 3.1.

Figure 4: Dalitz plot for $\tau^{-}\to\mu^{-}\mu^{+}\mu^{-}$ decays when all operators are assumed to vanish with the exception of $C_{VRL,VLR}=1$ (left) and $C_{VLL,VRR}=1$ (right), taking $\Lambda=1$ TeV in both cases. Colors are defined as in Fig. 3.
1.7 Discriminating power of $\tau \rightarrow \mu (e) \pi \pi$ decays

Celis, Cirigliano, E.P.'14

\[ \mathcal{L}_{\text{eff}}^D = -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu \nu} P_{L,R} \tau F_{\mu \nu} \]

Dipole model:
- $C_D \neq 0$
- $C_{\text{else}} = 0$
1.7 Discriminating power of $\tau \to \mu (e) \pi \pi$ decays

Two basic handles:

1. Spectra in $\geq 2$ body decays
2. Spin and isospin of the hadronic operator leave imprint in the spectrum

$\mathcal{L}_{\text{eff}}^{D} \supset -\frac{C_{D}}{\Lambda^{2}} m_{\tau} \bar{\mu} \sigma_{\mu \nu} P_{L,R} \tau F_{\mu \nu}$

Dipole model:
- $C_{D} \neq 0$
- $C_{\text{else}} = 0$

Scalar model:
- $C_{S} = 1$
- $C_{\text{else}} = 0$, $\Lambda = 1$ TeV

Emilie Passemar
1.7 Discriminating power of $\tau \rightarrow \mu(\pi)\pi\pi$ decays

Different distributions according to the operator!
2. Lepton Universality tests with $\tau$ physics
2.1 Test of $\mu/e$ universality

- Tested at 0.14% from Tau leptonic Brs! (0.28% in Z decays)

![Diagram showing charged current universality](image)
2.1 Test of $\mu/e$ universality

Tested at 0.14\% from Tau leptonic $\text{Brs}$! (0.28\% in $\text{Z}$ decays)

- What about the \textit{third family}?
2.2 Test of $\tau/e$ universality

- What about the *third family?*  

\[
\begin{align*}
\text{LEP (} W \to \tau \bar{\nu}_\mu) / (W \to e \bar{\nu}_e) & \quad 0.9970 \pm 0.0100 \\
\text{HFLAV (} \tau \to \mu \bar{\nu}_\mu \nu_\tau) \times \tau_\mu / \tau_\tau & \quad 1.0029 \pm 0.0015
\end{align*}
\]

- Universality tested at 0.15% level and $\sim 2\sigma$ except for
  - $W$ decay old anomaly
  - $B$ decays

*See talks on Tuesday morning*
2.3 Test of $\tau/\mu$ universality

- What about the third family?  

- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays

\[
\begin{align*}
\text{LEP (} W \to \tau \bar{\nu}_\tau \text{) / (} W \to \mu \bar{\nu}_\mu \text{)} & \quad 1.0390 \pm 0.0130 \\
\text{HFLAV (} \tau \to e \bar{\nu}_e \nu_\tau \text{) / } \tau & \quad 1.0010 \pm 0.0015 \\
\text{HFLAV (} \tau \to \pi \nu_\tau \text{) / (} \pi \to \mu \bar{\nu}_\mu \text{)} & \quad 0.9961 \pm 0.0027 \\
\text{HFLAV (} \tau \to K \nu_\tau \text{) / (} K \to \mu \bar{\nu}_\mu \text{)} & \quad 0.9860 \pm 0.0070 \\
\text{HFLAV (} \tau \to \pi \nu_\tau, K \nu_\tau \text{)} & \quad 0.9950 \pm 0.0025 \\
\text{HFLAV (} \tau \to e \bar{\nu}_e \nu_\tau, \pi \nu_\tau, K \nu_\tau \text{)} & \quad 1.0000 \pm 0.0014
\end{align*}
\]

See talks on Tuesday morning
2.4 Lepton Flavour Universality anomaly $W \rightarrow \tau \nu_{\tau}$

- Old LEP anomaly
2.4 Lepton Flavour Universality anomaly $W \rightarrow \tau \nu_\tau$

- **Old LEP anomaly**

$$R^W_{\tau\ell} = \frac{2 \cdot BR(W \rightarrow \tau \bar{\nu}_\tau)}{BR(W \rightarrow e \bar{\nu}_e) + BR(W \rightarrow \mu \bar{\nu}_\mu)} = 1.077(26)$$

2.8σ away from SM!

- **New physics?**

Some models:

*Li & Ma’05, Park’06, Dermisek’08*

Try to explain with SM EFT approach with [U(2)xU(1)]\(^5\) flavour symmetry

Very difficult to explain without modifying any other observables

*Filipuzzi, Portoless, Gonzalez-Alonso’12*

- **Would be great to have another measurement by LHC**
3. B physics anomalies & Charged Lepton-Flavour Violation
3.1 B physics anomalies

- Hint from B physics anomalies?
  - $b \rightarrow c$ charged currents:
    - $\tau$ vs. light leptons ($\mu$, e) [R(D), R(D*)]

\[
R(X) = \frac{\Gamma(B \rightarrow X\tau \bar{\nu})}{\Gamma(B \rightarrow X\ell \bar{\nu})}
\]

- New Physics explanations e.g.
  - $b_L$\,\!$c_L$
  - $\tau_L$, $\ell_L$
  - $\nu_L$
  - $W$
  - NP

\[
\Delta \chi^2 = 1.0 \text{ contours}
\]

- Consistent results by 3 different exps. → SM predic*on solid:
  - RH or scalar amplitudes disfavored
  - $\chi^2$ uncertainty cancel (to a good extent...)

- Kaon Physics: the next step

- Recent data show some EFT-type considerations from physics
- Simultaneous explanations of B-physics anomalies

- 4σ excess over SM (combining D and D*)
### 3.2 Key role of $\tau$ physics observables

- If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables involving $\tau$

- Large $\tau \rightarrow \mu$ LFV transitions in many realistic set-up
  
  *Glashow, Guadagnoli, Lane '15*

- Ex: Leptoquark scenario:

  
  ![Diagram](image)

  *Angelescu, Bečirević, Faroughy & Sumensari'18*

- Possibility to constrain $\tau \rightarrow \mu \phi$ and $B \rightarrow K\mu\tau$ at HL LHC?
3.2 Key role of $\tau$ physics observables

- If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables involving $\tau$.
- Large $\tau \rightarrow \mu$ LFV transitions in many realistic set-up.

$\tau \rightarrow \mu$ LFV transitions in PS$^3$

Bordone, Cornella, Fuentes-Martin, Isidori '18

- PS$^3$ model:

$$[\text{PS}]^3 = [SU(4) \times SU(2)_L \times SU(2)_R]^3$$

At high energies the 3 families are charged under 3 independent gauge groups:
- Light LQ coupled mainly to 3$^{rd}$ gen.
- Accidental U(2)$^5$ flavor symmetry
- Natural structure of SM Yukawa couplings

- Possibility to constrain $\tau \rightarrow 3\mu$ and $B_S \rightarrow \tau\mu$ at HL LHC?
3.2 Key role of $\tau$ physics observables

- Important constraints from $pp \rightarrow \tau\tau$

- Constraints from $pp \rightarrow \tau\mu$

If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables and even more striking signals at high-$p_T$.

Interesting constraints for models addressing $R_K \rightarrow$ Allanach, Marzocca, but still large room.

Situation more problematic for models addressing $R_D \rightarrow$ Faroughy:

Unambiguous prediction of large $pp \rightarrow \tau\tau$ in models with LH currents and "nice" flavor structure (independently of the mediator), which starts to be in tension with present data.

Model predicting colorless companions of the LQ (such as the "coloron"), starts to be in tension because of $pp \rightarrow \tau\tau$. 

Isidori@CKM ’18
3.3 Lepton Flavour Violating h and Z decays

- HL-LHC can improve the bounds on LFV Z decays
  \[ \text{Br}(Z \rightarrow \tau\mu) < 1.2 \times 10^{-5} \quad \text{DELPHI@LEP'97} \]
  \[ \text{Br}(Z \rightarrow \tau\mu) < 1.3 \times 10^{-5} \quad \text{ATLAS’18, 8+13 TeV} \]
  \[ \text{Br}(Z \rightarrow \tau e) < 1.0 \times 10^{-5} \quad \text{OPAL@LEP’95} \]
  \[ \text{Br}(Z \rightarrow \tau e) < 5.8 \times 10^{-5} \quad \text{ATLAS’18, 13 TeV} \]

Slight excess for $\tau e$ (2.3\(\sigma\) over background)

See Talk by Brian Le
On Tuesday
3.3 Lepton Flavour Violating h and Z decays

- HL-LHC can improve the bounds on LFV Higgs decays

\[ \text{Br}(h \rightarrow \tau e) < 0.61\% \]
\[ \text{Br}(h \rightarrow \tau \mu) < 0.25\% \]

Previous excess in \( h \rightarrow \tau \mu \) not confirmed with new data

See Talk by Jian Wang
On Tuesday
4. Conclusion and outlook
Conclusion and outlook

- HL-LHC will produce many more taus than any other running machines but measurements in Tau physics very difficult

- Tau physics dominated by $e^+e^-$ machine measurements: CLEO, LEP, BaBar, Belle, BES and more to come with Belle II

- What can be done at HL-LHC for taus:
  - LFV: $\tau \rightarrow 3\mu$, What about $\tau \rightarrow \mu\phi$ and $\tau \rightarrow \mu(e)\pi\pi$?
  - LFV: $Z \rightarrow \tau\mu/e$, $h \rightarrow \tau\mu/e$
  - Lepton Universality: $W$ decay old anomaly?
  - What about hadronic tau decays?

- Correlations with UV models explaining the B physics anomalies

- Very rich phenomenology: new ideas are welcome!
5. Back-up
2.8 Non standard LFV Higgs coupling

\[ \Delta \mathcal{L}_y = -\frac{\lambda_{ij}}{\Delta^2} \left( \bar{f}_{L}^i f_{R}^j H \right) H^\dagger H \]

- High energy : LHC

In the SM: \[ Y_{ij}^{h_{SM}} = \frac{m_i}{\upsilon} \delta_{ij} \]

Hadronic part treated with perturbative QCD

- Low energy : D, S operators

Goudelis, Lebedev, Park’11
Davidson, Grenier’10
Harnik, Kopp, Zupan’12
Blankenburg, Ellis, Isidori’12
McKeen, Pospelov, Ritz’12
Arhrib, Cheng, Kong’12

Emilie Passemar
2.8 Non standard LFV Higgs coupling

- \[ \Delta L_y = -\frac{\lambda_{ij}}{\Delta^2} \left( f_L^i f_R^j H \right) H^+ H \]

- High energy: LHC

- Low energy: D, S, G operators

\[ Y_{\tau\mu}^{h_{SM}} = \frac{m_i}{v} \delta_{ij} \]

In the SM:

Hadronic part treated with perturbative QCD

Reverse the process

Hadronic part treated with non-perturbative QCD

Goudelis, Lebedev, Park’11
Davidson, Grenier’10
Harnik, Kopp, Zupan’12
Blankenburg, Ellis, Isidori’12
McKeen, Pospelov, Ritz’12
Arhrib, Cheng, Kong’12
Constraints in the $\tau\mu$ sector

- At low energy
  - $\tau \rightarrow \mu \pi\pi$:
    - Dominated by $\rho(770)$ (photon mediated)
    - $f_0(980)$ (Higgs mediated)

$|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2 = 1$

$M_h = 125 \text{ GeV}$

Dominated by
- $\rho(770)$ (photon mediated)
- $f_0(980)$ (Higgs mediated)
Constraints in the $\tau\mu$ sector

- **Constraints from LE:**
  - $\tau \to \mu\gamma$: best constraints but loop level sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams robust handle on LFV

- **Constraints from HE:**
  - $LHC$ wins for $\tau\mu$!
  - Opposite situation for $\mu e$!

- For LFV Higgs and nothing else: LHC bound

$$BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}$$

$$BR(\tau \to \mu\pi\pi) < 1.5 \times 10^{-11}$$

**Summary**

The first direct search for lepton-flavour-violating decays of a Higgs boson to a $\mu - t$ pair, based on the full 8 TeV data set collected by CMS in 2012 is presented. It improves upon previously published indirect limits by an order of magnitude. A slight excess of events with a significance of 2.4 $\sigma$ is observed, corresponding to a $p$-value of 0.010. The best fit branching fraction is $B(\mathcal{H} \to \mu t) = (0.84^{+0.39}_{-0.37})\%$. A constraint of $B(\mathcal{H} \to \mu t) < 1.51\%$ at 95% confidence level is set. The limit is used to constrain the Yukawa couplings, $|Y_{\mu t}|^2 + |Y_{t\mu}|^2 < 3.6 \times 10^{-3}$. It improves the current bound by an order of magnitude.

**Acknowledgments**

We congratulate our colleagues in the CERN accelerator departments for the excellent performance of the LHC and thank the technical and administrative staffs at CERN and at other CMS institutes for their contributions to the success of the CMS effort. In addition, we gratefully acknowledge the computing centres and personnel of the Worldwide LHC Computing Grid for delivering so effectively the computing infrastructure essential to our analyses. Finally, we acknowledge the enduring support for the construction and operation of the LHC and the CMS detector provided by the following funding agencies: BMWFW and FWF (Austria); FNRS and FWO (Belgium); CNPq, CAPES, FAPERJ, and FAPESP (Brazil); MES (Bulgaria); CERN; CAS, MoST, and NSFC (China); COLCIENCIAS (Colombia); MSES and CSF (Croatia); RPF (Cyprus); MoER, ERC IUT and ERDF (Estonia); Academy of Finland, MEC, and HIP (Finland); CEA and CNRS/IN2P3 (France); BMBF, DFG, and HGF (Germany); GSRT (Greece); OTKA and NIH (Hungary); DAE and DST (India); IPM (Iran); SFI (Ireland); INFN (Italy); MSIP and NRF (Republic of Korea); BMBF and MPG (Germany); DOE and NSF (USA).
Hint of New Physics in $h \rightarrow \tau \mu$?

Figure 5: Constraints on the flavour violating Yukawa couplings, $|Y_{\mu t}|$ and $|Y_{t \mu}|$. The black dashed lines are contours of $BR(HT^\mu t)$ for reference. The expected limit (red solid line) with one standard deviation (green) and two standard deviation (yellow) bands, and observed limit (black solid line) are derived from the limit on $BR(HT^\mu t)$ from the present analysis. The shaded regions are derived constraints from null searches for $t \rightarrow 3 \mu$ (dark green) and $t \rightarrow \mu g$ (lighter green). The light blue region indicates the additional parameter space excluded by our result. The purple diagonal line is the theoretical naturalness limit $Y_{ij}Y_{ji} \leq m_i m_j / v^2$.

9 Conclusions
A direct search for lepton flavour violating decays of the Higgs boson in the $H^\mu t$ channel is described. The data sample used in the search was collected in proton-proton collisions at $p_s = 13$ TeV with the CMS experiment at the LHC and corresponds to an integrated luminosity of 2.3 fb$^{-1}$. No excess is observed. The best-fit branching fraction is $BR(H^\mu t) = 0.76^{+0.81}_{-0.84}$% and an upper limit of $BR(H^\mu t) < 1.20$% (1.62% expected) is set at 95% CL.

At $p_s = 8$ TeV a small excess was observed, corresponding to 2.4 $\sigma$, with an analysis based on an integrated luminosity of 19.7 fb$^{-1}$ that yielded an expected 95% CL limit on the branching fraction of 0.75%. More data are needed to make definitive conclusions on the origin of that excess.
Hint of New Physics in $h \rightarrow \tau \mu$?

CMS Preliminary

35.9 fb$^{-1}$ (13 TeV)

$\mu_{\text{had}^0}$ Jets
1.04% (1.14%)

$\mu_{\text{had}^1}$ Jet
1.74% (1.26%)

$\mu_{\text{had}^2}$ 2 Jets
1.65% (2.12%)

$\mu_{\text{had}^V}$ VBF
1.30% (1.41%)

$\mu_{e^0}$, 0 Jets
1.08% (1.01%)

$\mu_{e^1}$, 1 Jet
1.35% (1.47%)

$\mu_{e^2}$, 2 Jets
3.33% (3.23%)

$\mu_{e^V}$ VBF
1.40% (1.73%)

$H \rightarrow \mu\tau$
0.51% (0.49%)

95% CL Limit on $\text{Br}(H \rightarrow \mu\tau)$, %

CMS'17

CMS Preliminary

35.9 fb$^{-1}$ (13 TeV)

$\tau \rightarrow 3\mu$

$\tau \rightarrow \mu\gamma$

CMS 8TeV

observed

expected $H \rightarrow \mu\tau$
2.6 Model discriminating of BRs

- Studies in specific models

Buras et al.’10

<table>
<thead>
<tr>
<th>ratio</th>
<th>LHT</th>
<th>MSSM (dipole)</th>
<th>MSSM (Higgs)</th>
<th>SM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\text{Br}(\mu^-\rightarrow e^- e^+ e^-)}{\text{Br}(\mu\rightarrow e\gamma)})</td>
<td>0.02...1</td>
<td>(\sim 6 \cdot 10^{-3})</td>
<td>(\sim 6 \cdot 10^{-3})</td>
<td>0.06...2.2</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow e^- e^+ e^-)}{\text{Br}(\tau\rightarrow e\gamma)})</td>
<td>0.04...0.4</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>0.07...2.2</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow \mu^- \mu^+ \mu^-)}{\text{Br}(\tau\rightarrow \mu\gamma)})</td>
<td>0.04...0.4</td>
<td>(\sim 2 \cdot 10^{-3})</td>
<td>0.06...0.1</td>
<td>0.06...2.2</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow e^- \mu^+ \mu^-)}{\text{Br}(\tau\rightarrow e\gamma)})</td>
<td>0.04...0.3</td>
<td>(\sim 2 \cdot 10^{-3})</td>
<td>0.02...0.04</td>
<td>0.03...1.3</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow \mu^- e^+ e^-)}{\text{Br}(\tau\rightarrow \mu\gamma)})</td>
<td>0.04...0.3</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>0.04...1.4</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow e^- e^+ e^-)}{\text{Br}(\tau^-\rightarrow e^- \mu^+ \mu^-)})</td>
<td>0.8...2</td>
<td>(\sim 5)</td>
<td>0.3...0.5</td>
<td>1.5...2.3</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^-\rightarrow \mu^- \mu^+ \mu^-)}{\text{Br}(\tau^-\rightarrow \mu^- e^+ e^-)})</td>
<td>0.7...1.6</td>
<td>(\sim 0.2)</td>
<td>5...10</td>
<td>1.4...1.7</td>
</tr>
<tr>
<td>(\frac{\text{R}(\mu Ti\rightarrow e Ti)}{\text{Br}(\mu\rightarrow e\gamma)})</td>
<td>(10^{-3}...10^2)</td>
<td>(\sim 5 \cdot 10^{-3})</td>
<td>0.08...0.15</td>
<td>(10^{-12}...26)</td>
</tr>
</tbody>
</table>

Disentangle the underlying dynamics of NP
3.2 $V_{us}$ determination

**From Unitarity**

- Kaon and hyperon decays
  - $K_{l3}$ decays ($+ f_+(0)$)
  - $K_{l2}/\pi_{l2}$ decays ($+ f_K/f_\pi$)
- Hyperon decays

**τ decays**

- $\tau \rightarrow s$ inclusive
- $\tau \rightarrow K\nu$ absolute ($+ f_K$)
- $\tau$ branching fraction ratio
  - $\tau \rightarrow K\nu$ / $\tau \rightarrow \pi\nu$ ($+ f_K/f_\pi$)
- Our result from Belle BR
  - $\tau \rightarrow K\pi\nu\tau$ decays ($+ f_+(0)$, FLAG)

NB: BRs measured by B factories are systematically smaller than previous measurements.

Flavianet Kaon WG’10 update by Moulson’CKM16

BaBar & Belle HFAG’17
3.2 $V_{us}$ determination

- Longstanding inconsistencies between inclusive $\tau$ and kaon decays in extraction of $V_{us}$
- Inclusive $\tau$ decays:

\[
\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}
\]

**SU(3) breaking** quantity, strong dependence in $m_s$ computed from OPE (L+T) + phenomenology

\[
\delta R_{\tau,th} = 0.0242(32)
\]

Gamiz et al’07, Maltman’11

\[
|V_{us}|^2 = \frac{R_{\tau,S}}{R_{\tau,NS} - \delta R_{\tau,th}} \left( \frac{|V_{ud}|}{|V_{us}|} \right)^2
\]

**HFAG’17**

\[
R_{\tau,S} = 0.1633(28) \\
R_{\tau,NS} = 3.4718(84) \\
|V_{ud}| = 0.97417(21)
\]

\[
|V_{us}| = 0.2186 \pm 0.0019^{\text{exp}} \pm 0.0010^{\text{th}}
\]

3.1σ away from unitarity!
3.3 \( \tau \rightarrow K\pi\nu_\tau \) CP violating asymmetry

- \( A_Q = \frac{\Gamma\left(\tau^+ \rightarrow \pi^+ K^0_S \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \rightarrow \pi^- K^0_S \nu_\tau\right)}{\Gamma\left(\tau^+ \rightarrow \pi^+ K^0_S \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \rightarrow \pi^- K^0_S \nu_\tau\right)} \)
  \[ \approx (0.36 \pm 0.01)^\% \] in the SM
  \( \text{Bigi & Sanda'05} \)
  \( \text{Grossman & Nir'11} \)

- Experimental measurement: \( \text{BaBar'11} \)
  \( A_{Q_{\text{exp}}} = \left( -0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}} \right)^\% \leftarrow 2.8\sigma \) from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM
  \( \Gamma \left( \tau^+ \rightarrow \pi^+ K^0_S \bar{\nu}_\tau \right) - \Gamma \left( \tau^- \rightarrow \pi^- K^0_S \nu_\tau \right) \)
  \( \left/ \Gamma \left( \tau^+ \rightarrow \pi^+ K^0_S \bar{\nu}_\tau \right) + \Gamma \left( \tau^- \rightarrow \pi^- K^0_S \nu_\tau \right) \right. \)
  \[ = \left( -0.54 \pm 0.14 \right)^\% \] \( \text{Belle, Babar, CLEO, FOCUS} \)

\[ |K^0_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle \]
\[ |K^0_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle \]
\[ \langle K_L|K_S\rangle = |p|^2 - |q|^2 = 2\text{Re}\left(\varepsilon_K\right) \]
3.3 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, $W_L$-$W_R$ mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha’14, Cirigliano, Crivellin, Hoferichter’17*)?

\[
\begin{array}{c}
\tau^- \\
H^- \\
\text{s[d]} \\
\text{FSI}
\end{array}
\begin{array}{c}
v \\
\uparrow \\
\uparrow
\end{array}
\quad
\begin{array}{c}
\tau^- \\
W_R \\
\text{s[d]} \\
\text{FSI}
\end{array}
\begin{array}{c}
v \\
\uparrow \\
\uparrow
\end{array}
\]

*Bigi’Tau12*

Very difficult to explain!

- Need to investigate how large can be the prediction in realistic new physics models: it looks like a tensor interaction can explain the effect but in conflict with bounds from neutron EDM and D̅D mixing

*Cirigliano, Crivellin, Hoferichter’17*

light BSM physics?
3.3 $\tau \rightarrow K\pi \nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha’14
Cirigliano, Crivellin, Hoferichter’17

- We need a tensor interaction to get some interference:

$$\mathcal{H}_T^{\text{eff}} \equiv G'(s\sigma_{\mu\nu} u)(\nu_\tau (1 + \gamma_5)\sigma^{\mu\nu} \tau)$$

with

$$G' = \frac{G_F}{\sqrt{2}} C_T, \quad C_T = |C_T| e^{i\phi_T}$$

- When integrating the interference term between vector and tensor does not vanish:

$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma^{\text{SM}}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$

$$\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2 \theta_C \frac{m^3_\tau}{32\pi^3} \left( \frac{m^2_\tau - Q^2}{m^2_\tau} \right)^2 q_1^3 \frac{Q^2}{m^2_\tau}$$

$$\times |C_T| |F_V(s)||F_T(s)| \cos \left( \delta_T(s) - \delta_V(s) + \phi_T \right)$$

In conflict with bounds from neutron EDM and $\overline{D}D$ mixing

Cirigliano, Crivellin, Hoferichter’17
3.1 Lepton Universality

- The leptonic decay width:

\[ \Gamma(\tau \to \nu_\tau l \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192 \pi^3} \left( m_l^2/m_\tau^2 \right) \left( 1 + \delta_{RC} \right) \]

Experimental inputs:
\[ \Gamma(\tau_{13}) \] Rates with well-determined treatment of radiative decays
- Branching ratios
- Tau lifetimes

- Test of \( \mu/e \) universality:

\[ \left( \frac{B_\mu}{B_e} \right)_{\text{exp}} = 0.9761 \pm 0.0028 \]

Non-BF: \( 0.9725 \pm 0.0039 \)
BaBar ‘10: \( 0.9796 \pm 0.0039 \)

\[ B_{e}^{\text{univ}} = (17.818 \pm 0.0022) \]

Inputs from theory:
\[ \delta_{RC} \] Radiative corrections

Marciano’88

\[ \begin{array}{c}
\text{HFLAV Spring 2017} \\
\text{SM lepton universality, 1\sigma}
\end{array} \]
3.5 Results

$$\text{Bound: } \sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2} \leq 0.13$$

| Process              | (BR $\times 10^8$) 90% CL | $\sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2}$ | Operator(s)               |
|----------------------|---------------------------|---------------------------------|--------------------------|
| $\tau \rightarrow \mu \gamma$ | < 4.4 [88] | < 0.016                      | Dipole                   |
| $\tau \rightarrow \mu \mu \mu$ | < 2.1 [89] | < 0.24                       | Dipole                   |
| $\tau \rightarrow \mu \pi^+\pi^-$ | < 2.1 [86] | < 0.13                       | Scalar, Gluon, Dipole    |
| $\tau \rightarrow \mu \rho$ | < 1.2 [85] | < 0.13                       | Scalar, Gluon, Dipole    |
| $\tau \rightarrow \mu \pi^0\pi^0$ | $< 1.4 \times 10^3$ [87] | < 6.3                        | Scalar, Gluon            |

Belle’08’11’12 except last from CLEO’97
3.5 What if $\tau \rightarrow \mu(e)\pi\pi$ observed?  
Reinterpreting Celis, Cirigliano, E.P’14

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!
- $Y_{u,d,s}$ poorly bounded
- For $Y_{u,d,s}$ at their SM values:

\[
\begin{align*}
Br(\tau \rightarrow \mu\pi^+\pi^-) &< 1.6 \times 10^{-11}, \quad Br(\tau \rightarrow \mu\pi^0\pi^0) < 4.6 \times 10^{-12} \\
Br(\tau \rightarrow e\pi^+\pi^-) &< 2.3 \times 10^{-10}, \quad Br(\tau \rightarrow e\pi^0\pi^0) < 6.9 \times 10^{-11}
\end{align*}
\]

- But for $Y_{u,d,s}$ at their upper bound:

\[
\begin{align*}
Br(\tau \rightarrow \mu\pi^+\pi^-) &< 3.0 \times 10^{-8}, \quad Br(\tau \rightarrow \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\
Br(\tau \rightarrow e\pi^+\pi^-) &< 4.3 \times 10^{-7}, \quad Br(\tau \rightarrow e\pi^0\pi^0) < 2.1 \times 10^{-7}
\end{align*}
\]

below present experimental limits!

- If discovered among other things upper limit on $Y_{u,d,s}$! 

Interplay between high-energy and low-energy constraints!
3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Photon mediated contribution requires the pion vector form factor:

$$\langle \pi^+ (p_{\pi^+}) \pi^- (p_{\pi^-}) \Big| \frac{1}{2} (\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \Big| 0 \rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

Gasser, Meißner’91
Guerrero, Pich’97
Oller, Oset, Palomar’01
Pich, Portolés ’08
Gómez Dumm&Roig’13

- Determined from a fit to the Belle data on $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

Celis, Cirigliano, E.P.’14

Determined from a fit to the Belle data on $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$
Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements
    \[ e^+ e^- \rightarrow \pi^+ \pi^- \quad \text{and} \quad \tau^- \rightarrow \pi^0 \pi^- \nu_\tau \] (isospin rotation)
  - Theoretically: Dispersive parametrization for $F_V(s)$

\[
F_V(s) = \exp \left[ \lambda_V^\prime \frac{s}{m_\pi^2} + \frac{1}{2} \left( \lambda_V^{\prime\prime} - \lambda_V^{\prime 2} \right) \left( \frac{s}{m_\pi^2} \right)^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s' - s - i\varepsilon)} \right]
\]

Extracted from a model including 3 resonances $\rho(770)$, $\rho'(1465)$ and $\rho''(1700)$ fitted to the data

- Subtraction polynomial + phase determined from a fit to the Belle data \[ \tau^- \rightarrow \pi^0 \pi^- \nu_\tau \]
Determination of $F_V(s)$ thanks to precise measurements from Belle!
3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Higgs exchange

\[
\Gamma_\pi(s) = \left\langle \pi^+\pi^- \left| m_u\bar{u}u + m_d\bar{d}d \right| 0 \right\rangle
\]

\[
\Delta_\pi(s) = \left\langle \pi^+\pi^- \left| m_s\bar{s}s \right| 0 \right\rangle
\]

\[
\theta_\mu = -9\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu} + \sum_{q=u,d,s} m_q\bar{q}q
\]

\[
\frac{d\Gamma(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2}{256\pi^3m_\tau^3} \frac{(|Y_{\tau\mu}|^2 + |Y_{\mu\tau}|^2)}{4m_\pi^2} \frac{M_h^4v^2}{\sqrt{s}} \left| K_{\Delta_\pi}(s) + K_{\Gamma_\pi}(s) + K_{\theta}(s) \right|^2
\]

Voloshin'85
Determination of the form factors: $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs, up to $\sqrt{s} \sim 1.4$ GeV.
  Inputs: $I=0$, S-wave $\pi\pi$ and KK data.

- Unitarity:

  $$\text{Im} F_n(s) = \sum_{m=1}^{2} T_{nm}^*(s) \sigma_m(s) F_m(s)$$
  $n = \pi\pi, K\bar{K}$

Coupled channel analysis

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13
Determination of the form factors: $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

- Inputs: $\pi\pi \rightarrow \pi\pi, KK$

- A large number of theoretical analyses Descotes-Genon et al’01, Kaminsky et al’01, Buttiker et al’03, Garcia-Martin et al’09, Colangelo et al.’11 and all agree

- 3 inputs: $\delta_\pi(s), \delta_K(s), \eta$ from B. Moussallam $\rightarrow$ reconstruct $T$ matrix
3.4.4 Determination of the form factors: $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

- General solution:
  \[
  \begin{pmatrix}
  F_\pi(s) \\
  \frac{2}{\sqrt{3}} F_K(s)
  \end{pmatrix} =
  \begin{pmatrix}
  C_1(s) & D_1(s) \\
  C_2(s) & D_2(s)
  \end{pmatrix}
  \begin{pmatrix}
  P_F(s) \\
  Q_F(s)
  \end{pmatrix}
  \]

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

- Canonical solution

\[
X(s) = C(s), D(s)
\]

**Equations:**

\[
\text{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \text{Re} \left\{ T_{nm}^{-} \sigma_{m}(s) X_m^{(N)} \right\}
\]

\[
\text{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_{\pi}^2}^{\infty} \frac{ds'}{s' - s} \text{Im} X_n^{(N+1)}
\]
Determination of the polynomial

- **General solution**

\[
\begin{pmatrix}
F_{\pi}(s) \\
\frac{2}{\sqrt{3}} F_{K}(s)
\end{pmatrix} = \begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix} \begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
\]

- **Fix the polynomial with requiring** \( F_p(s) \rightarrow 1/s \) (Brodsky & Lepage) + ChPT:

  - Feynman-Hellmann theorem:

  \[
  \Gamma_P(0) = \left( m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2 \\
  \Delta_P(0) = \left( m_s \frac{\partial}{\partial m_s} \right) M_P^2
  \]

- **At LO in ChPT:**

\[
\begin{align*}
M_{\pi^+}^2 &= (m_u + m_d) B_0 + O(m^2) \\
M_{K^+}^2 &= (m_u + m_s) B_0 + O(m^2) \\
M_{K^0}^2 &= (m_d + m_s) B_0 + O(m^2)
\end{align*}
\]

\[
\begin{align*}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\
P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\
Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left( M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots
\end{align*}
\]
Determination of the polynomial

• General solution

\[
\begin{pmatrix}
F_\pi(s) \\
\frac{2}{\sqrt{3}} F_K(s)
\end{pmatrix} =
\begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix}
\begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
\]

• At LO in ChPT:

\[
\begin{align*}
M_{\pi^+}^2 &= (m_u + m_d) B_0 + O(m^2) \\
M_{K^+}^2 &= (m_u + m_s) B_0 + O(m^2) \\
M_{K^0}^2 &= (m_d + m_s) B_0 + O(m^2)
\end{align*}
\]

\[
\begin{align*}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_K^2 + \cdots \\
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\end{align*}
\]

• Problem: large corrections in the case of the kaons!

Use lattice QCD to determine the SU(3) LECs

\[
\begin{align*}
\Gamma_K(0) &= (0.5 \pm 0.1) M_\pi^2 \\
\Delta_K(0) &= 1^{+0.15}_{-0.05} \left( M_K^2 - 1/2 M_\pi^2 \right)
\end{align*}
\]

Dreiner, Hanart, Kubis, Meissner’13
Bernard, Descotes-Genon, Toucas’12
Determination of the polynomial

- General solution

\[
\begin{pmatrix}
F_\pi(s) \\
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\end{pmatrix}
\begin{pmatrix}
P_\pi(s) \\
Q_\pi(s)
\end{pmatrix}
\]

- For \(\theta_p\) enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

  Relax the constraints and match to ChPT

\[
P_\theta(s) = 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s
\]
\[
Q_\theta(s) = \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s
\]
\[ \left\langle \pi^+ \pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s) \]

\[ \left\langle \pi^+ \pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s) \]

\[ \left\langle \pi^+ \pi^- \left| \theta^\mu_\mu \right| 0 \right\rangle \equiv \theta_\pi(s) \]
\[ \langle \pi^+ \pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \]

\[ \langle \pi^+ \pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s) \]

- Uncertainties:
  - Varying \( s_{\text{cut}} \) (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs
Comparison with ChPT

ChPT, EFT only valid at low energy.

It is not valid up to $E = !$. 

Emilie Passemar

Emilie Passemar
3.1 Lepton Universality

• What about the third family?

\[
\left| \frac{g_\tau}{g_\mu} \right|
\]

| $B_{\tau\rightarrow e}$ $\tau_\mu/\tau_\tau$ | $1.0011 \pm 0.0015$ |
| $\Gamma_{\tau\rightarrow \pi}/\Gamma_{\pi\rightarrow \mu}$ | $0.9962 \pm 0.0027$ |
| $\Gamma_{\tau\rightarrow K}/\Gamma_{K\rightarrow \mu}$ | $0.9858 \pm 0.0070$ |
| $B_{W\rightarrow \tau}/B_{W\rightarrow \mu}$ | $1.034 \pm 0.013$ |

| $B_{\tau\rightarrow \mu}$ $\tau_\mu/\tau_\tau$ | $1.0029 \pm 0.0015$ |
| $B_{W\rightarrow \tau}/B_{W\rightarrow e}$ | $1.031 \pm 0.013$ |

• Universality tested at 0.15\% level and good agreement except for
  – W decay old anomaly
  – B decays
2.2 Paths to $V_{ud}$ and $V_{us}$

- From kaon, pion, baryon and nuclear decays

<table>
<thead>
<tr>
<th>$V_{ud}$</th>
<th>$0^+ \rightarrow 0^+$</th>
<th>$\pi^\pm \rightarrow \pi^0 e\nu_e$</th>
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- From $\tau$ decays (crossed channel)

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<th>$V_{ud}$</th>
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2.2 Paths to $V_{ud}$ and $V_{us}$

- From kaon, pion, baryon and nuclear decays

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2.3 $V_{us}$ from inclusive measurement

- Tau, the only lepton heavy enough to decay into hadrons

- $m_\tau \sim 1.77 \text{GeV} > \Lambda_{QCD}$ → use perturbative tools: OPE…

- Inclusive $\tau$ decays: $\tau \to (\bar{u}d, \bar{u}s)v_\tau$ → fund. SM parameters $(\alpha_s(m_\tau), |V_{us}|, m_s)$

- We consider $\Gamma(\tau^- \to v_\tau + \text{hadrons}_{s=0})$
  $\Gamma(\tau^- \to v_\tau + \text{hadrons}_{s\neq0})$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system → huge QCD activity!

- Observable studied: $R_\tau \equiv \frac{\Gamma(\tau^- \to v_\tau + \text{hadrons})}{\Gamma(\tau^- \to v_\tau e^- \bar{v}_e)}$
2.4 Theory

\[ R_\tau \equiv \frac{\Gamma(\tau^{-} \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^{-} \rightarrow \nu_\tau \bar{e} \nu_e)} \approx N_C \]

parton model prediction

\[ R_\tau = R^{NS}_\tau + R^S_\tau \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C \]

\[ \frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R^S_\tau}{R^{NS}_\tau} \]

\[ |V_{us}| \]

Figure from M. González Alonso’13
2.4 Theory

- \( R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \nu_e)} \approx N_C \)  
  parton model prediction

- \( R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C \)

- Experimentally:
  \[ R_\tau = \frac{1 - B_\mu - B_\mu}{B_\mu} = 3.6291 \pm 0.0086 \]
2.4 Theory

- \( R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C \) parton model prediction

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- Experimentally: \( R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086 \)

- Due to QCD corrections: \( R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s) \)
2.4 Theory

- From the measurement of the spectral functions, extraction of $\alpha_S$, $|V_{us}|$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$

- Extraction of the strong coupling constant:
  $$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_S) \quad \Rightarrow \quad \alpha_S$$

- Determination of $V_{us}$:
  $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_S)$$

- Main difficulty: compute the QCD corrections with the best accuracy
2.5 Calculation of the QCD corrections

- Calculation of $R_\tau$:

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2 \frac{s}{m_\tau^2}\right) \text{Im} \Pi_{1}^{(1)}(s + i\epsilon) + \text{Im} \Pi_{1}^{(0)}(s + i\epsilon)$$

- Analyticity: $\Pi$ is analytic in the entire complex plane except for $s$ real positive

Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2 \frac{s}{m_\tau^2}\right) \Pi_{1}^{(1)}(s) + \Pi_{1}^{(0)}(s)\right]$$

- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,...} \frac{1}{(-s)^{D/2}} \sum_{\text{dim}O=D} C^{(J)}(s,\mu) \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators

$\mu$: separation scale between short and long distances

Braaten, Narison, Pich’92
2.5 Calculation of the QCD corrections

- Calculation of $R_\tau$: 

$$R_\tau (m_\tau^2) = N_C S_{EW} (1 + \delta_p + \delta_{NP})$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$  

Marciano & Sirlin’88, Braaten & Li’90, Erler’04

- Perturbative part (D=0):  

$$\delta_p = a_\tau + 5.20 \ a_\tau^2 + 26 \ a_\tau^3 + 127 \ a_\tau^4 + ... \approx 20\%$$  

$$a_\tau = \frac{\alpha_s (m_\tau)}{\pi}$$  

Baikov, Chetyrkin, Kühn’08

- D=2: quark mass corrections, neglected for $R_\tau^{NS} (\propto m_u, m_d)$ but not for $R_\tau^S (\propto m_s)$

- D ≥ 4: Non perturbative part, not known, fitted from the data
  
Use of weighted distributions
2.5 Calculation of the QCD corrections

- $D \geq 4$: Non perturbative part, not known, \textit{fitted from the data}

  Use of weighted distributions

  Exploit shape of the spectral functions to obtain additional experimental information

\[ R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left( 1 - \frac{s}{s_0} \right)^k \left( \frac{s}{s_0} \right)^\ell \frac{dR_{\tau,U}(s_0)}{ds} \]

---

Emilie Passemar

Le Diberder&Pich’92

Zhang’Tau14
2.5 Inclusive determination of $V_{us}$

- With QCD on:
  \[
  \left| V_{us} \right|^2 = \frac{R_{\tau,S}^N}{R_{\tau}^N} + O(\alpha_s)
  \]

- Use OPE:
  \[
  R_{\tau}^N(m^2) = N_C S_{EW} \left| V_{ud} \right|^2 \left( 1 + \delta_p + \delta_{ud}^{NP} \right)
  \]
  \[
  R_{\tau,S}^S(m^2) = N_C S_{EW} \left| V_{us} \right|^2 \left( 1 + \delta_p + \delta_{us}^{NP} \right)
  \]

- $\delta R_{\tau} \equiv \frac{R_{\tau,NS}}{\left| V_{ud} \right|^2} - \frac{R_{\tau,S}}{\left| V_{us} \right|^2}$

**SU(3) breaking** quantity, strong dependence in $m_s$ computed from OPE (L+T) + phenomenology

\[
\delta R_{\tau,th} = 0.0242(32) \quad \text{Gamiz et al'07, Maltman'11}
\]

HFAG’17
\[
R_{\tau,S} = 0.1633(28)
\]
\[
R_{\tau,NS} = 3.4718(84)
\]
\[
\left| V_{us} \right| = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}
\]

3.1σ away from unitarity!
NB: BRs measured by B factories are systematically smaller than previous measurements.
2.6 $V_{us}$ using info on Kaon decays and $\tau \rightarrow K\pi\nu \tau$

<table>
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<tr>
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<th>HFAG Winter 2012 fit</th>
<th>( \times 10^{-2} )</th>
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<td>$\Gamma_{10} = K^-\nu_{\tau}$</td>
<td>(0.6955 \pm 0.0096)</td>
<td>(0.713 ± 0.003)%</td>
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<td>$\Gamma_{16} = K^-\pi^0\nu_{\tau}$</td>
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<td>$\Gamma_{23} = K^-2\pi^0\nu_{\tau}$ (ex. $K^0$)</td>
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- Longstanding inconsistencies between $\tau$ and kaon decays in extraction of $V_{us}$
  - Recent studies
  
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti’17

- Crucial input:
  
  \(\tau \rightarrow K\pi\nu_{\tau}\) Br + spectrum

\[
V_{us} = 0.2229 \pm 0.0022 \text{ exp} \pm 0.0004 \text{ theo}
\]

need new data
2.6 \( V_{us} \) using info on Kaon decays and \( \tau \rightarrow K\pi \nu_\tau \)

### Table 1

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\[
|V_{us}| = 0.2229 \pm 0.0022_{\text{exp}} \pm 0.0004_{\text{theo}}
\]

- need new data
- Very good prospect from Belle II, BES?

\textit{Antonelli, Cirigliano, Lusiani, E.P. ‘13}

\textit{HFLAV Spring 2017}
4.2 Outlook

- 45 billion $\tau^+\tau^-$ pairs in full dataset from $\sigma(\tau^+\tau^-)_{E=\gamma(4S)} = 0.9$ nb @Belle II
- B2TiP initiative: define the first set of measurements to be performed at Belle II: https://confluence.desy.de/display/BI/B2TiP+WebHome
- Golden/Silver modes for the Tau, Low Multiplicity and EW working group

<table>
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<tr>
<th>Process</th>
<th>Observable</th>
<th>Theory</th>
<th>Sys. limit (Discovery) [ab$^{-1}$]</th>
<th>vs LHCb/BESIII</th>
<th>vs Belle</th>
<th>Anomaly</th>
<th>NP</th>
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<td>***</td>
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3.1 Introduction

- Tau, the only lepton heavy enough to decay into hadrons

- \( m_\tau \sim 1.77 \text{GeV} > \Lambda_{QCD} \) → use *perturbative tools: OPE*...

- Inclusive \( \tau \) decays: \( \tau \to (\bar{u}d, \bar{u}s)\nu_\tau \) → fund. SM parameters \( (\alpha_s(m_\tau), |V_{us}|, m_s) \)

- We consider

  \[
  \Gamma\left( \tau^- \to \nu_\tau + \text{hadrons}_{s=0} \right)
  \]

  \[
  \Gamma\left( \tau^- \to \nu_\tau + \text{hadrons}_{s\neq0} \right)
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- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system → huge *QCD activity*!

- Observable studied:

  \[
  R_\tau \equiv \frac{\Gamma\left( \tau^- \to \nu_\tau + \text{hadrons} \right)}{\Gamma\left( \tau^- \to \nu_\tau e^-\bar{\nu}_e \right)}
  \]
3.2 Theory

- \[ R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \]
  - parton model prediction

- \[ R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C \]

- \[ \frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} \rightarrow |V_{us}| \]

\[ d_\theta = V_{ud} d + V_{us} s \]

QCD switch

\[ (\alpha_S = 0) \]

Figure from
M. González Alonso’13

Emilie Passemard
3.2 Theory

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- From the measurement of the spectral functions, extraction of $\alpha_S, |V_{us}|$

$$R_\tau \equiv \frac{\Gamma (\tau^- \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma (\tau^- \rightarrow \nu_{\tau} e^- e^-)} \approx N_C$$

naïve QCD prediction

- Extraction of the strong coupling constant:

$$R^{NS}_\tau = |V_{ud}|^2 N_C + O(\alpha_S) \quad \Rightarrow \quad \alpha_S$$

measured

- Determination of $V_{us}$:

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R^S_\tau}{R^{NS}_\tau} + O(\alpha_S)$$

- Main difficulty: compute the QCD corrections with the best accuracy
3.3 Calculation of the QCD corrections

- Calculation of $R_\tau$: 
  \[
  R_\tau(m^2_\tau) = 12\pi S_{EW} \int_0^{m^2_\tau} \frac{ds}{s} \left(1 - \frac{s}{m^2_\tau}\right)^2 \left[\left(1 + 2 \frac{s}{m^2_\tau}\right) \text{Im} \Pi^{(1)}(s + i\epsilon) + \text{Im} \Pi^{(0)}(s + i\epsilon)\right]
  \]
  \[
  \Rightarrow R_\tau(m^2_\tau) = 6i\pi S_{EW} \oint_{|s|=m^2_\tau} \frac{ds}{s} \left(1 - \frac{s}{m^2_\tau}\right)^2 \left[\left(1 + 2 \frac{s}{m^2_\tau}\right) \Pi^{(1)}(s) + \Pi^{(0)}(s)\right]
  \]

- Analyticity: $\Pi$ is analytic in the entire complex plane except for $s$ real positive.
  Cauchy Theorem

- We are now at sufficient energy to use OPE:
  \[
  \Pi^{(j)}(s) = \sum_{D=0,2,4\ldots} \left(\frac{-s}{D}\right)^{D/2} \sum_{\text{dim}O=D} C^{(j)}(s, \mu) \langle O_D(\mu) \rangle
  \]
  Wilson coefficients \hspace{2cm} Operators

Braaten, Narison, Pich’92

$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \text{Im} \{\tau^- \hspace{0.5cm} W \hspace{0.5cm} \bar{u} \hspace{0.5cm} \nu_\tau \hspace{0.5cm} W \hspace{0.5cm} \tau^-\}$

\(\mu\): separation scale between short and long distances
3.3 Calculation of the QCD corrections

- Calculation of $R_\tau$:

$$R_\tau \left( m_\tau^2 \right) = N_C \ S_{EW} \left( 1 + \delta_p + \delta_{NP} \right)$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$  

- Perturbative part (D=0):

$$\delta_p = a_\tau + 5.20 \ a_\tau^2 + 26 \ a_\tau^3 + 127 \ a_\tau^4 + \ldots \approx 20\%$$

- $a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$  

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS} \propto m_u, m_d$ but not for $R_\tau^S \propto m_s$  

- D $\geq$ 4: Non perturbative part, not known, *fitted from the data*

  Use of weighted distributions

Braaten, Narison, Pich’92

Marciano & Sirlin’88, Braaten & Li’90, Erler’04

Baikov, Chetyrkin, Kühn’08
### 3.3 Calculation of the QCD corrections

- $D \geq 4$: Non perturbative part, not known, *fitted from the data*
  
  - Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information:

$$
R^{k\ell}_{\tau,U}(s_0) = \int_0^{s_0} ds \left( 1 - \frac{s}{s_0} \right)^k \left( \frac{s}{s_0} \right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}
$$

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**ALEPH**

- $\tau^\rightarrow (V, A) \bar{\nu}_\tau$
- QCD prediction
- parton model

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**ALEPH**

- $R^0_{\tau}$
- $R_{\tau} \equiv R^{00}_{\tau}$

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Emilie Passemar
3.4 Extraction of $\alpha_s$

**Extraction of $\alpha_s$** from hadronic $\tau$ very interesting: Moderate precision at the $\tau$ mass → very good precision at the $Z$ mass

- Beautiful test of the QCD running
3.4 Extraction of $\alpha_s$

- Several delicate points:
  - How to compute the perturbative part: CIPT vs. FOPT?
  - How to estimate the non-perturbative contribution? Where do we truncate the expansion, what is the role of higher order condensates?
  - Which weights should we use?
  - What about duality violations?

A MITP topical workshop in Mainz: March 7-12, 2016

*Determination of the fundamental parameters of QCD*

A session on Tuesday afternoon

- New data on spectral functions needed to help to answer some of these questions