Higher-order QCD in hadronic tau decays from Padé approximants

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Abstract
We use the method of Padé approximants to obtain predictions for the higher order QCD corrections to the inclusive hadronic tau decay width. Our predictions are robust and model independent.

Hadarion tau decays

- The tau hadronic width can be parametrized as
  \[ R_{\tau,\gamma/A} = \frac{N_c}{2} S_{EW} \left| V_{ud} \right|^2 \left[ 1 + \delta^{(0)} + \delta^{NP} + \delta^{EW} \right] \]

- The perturbative contribution is obtained, using analyticity, as a contour integral where the Adler function intervenes
  \[ \delta^{(0)}(s) = \frac{1}{2\pi i} \oint \frac{dZ}{Z} W(z) B_{\text{part}}(1+z)(m_{\tau}^2 Z) \]
  \[ B_{\text{part}}(z) = \sum_{a=1}^{\infty} \sum_{k=1}^{\infty} \alpha_s^k \zeta_{3(k-1)}(z) \]

- The Adler function is known to five loops \cite{1}
  \[ \tilde{B}(Q^2) = a_Q + 1.640 a_Q^2 + 6.371 a_Q^3 + 49.08 a_Q^4 + \cdots \]

- There is, however, a remaining ambiguity in \[ \delta^{(0)} \]
  \[ \text{fixed renorm.} \quad \alpha_s, \alpha_s^2 \] \quad \[ \text{running renorm.} \quad \alpha_s, \alpha_s^2 \]
  Ambiguity related to the (lack of) knowledge about higher orders

- But perturbation theory is only asymptotic (factorially divergent) and it is useful to work with the Borel transformed series defined as \cite{4}
  \[ B[\tilde{B}(t)](t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \tilde{B}(n) \]
  \[ \tilde{B}(n) = \int_{0}^{\infty} d\tau \tau^n B[\tilde{B}(t)](t) \]

Testing the method: large-\[ \beta_0 \] limit

- The results are known to all orders in the large-\[ \beta_0 \] limit of QCD.

Fadé approximants

- Padé approximants are ratios of two polynomials constructed to approximate a function \( f(z) \) whose Taylor expansion is known
  \[ f(z) = \sum_{n=0}^{\infty} a_n z^n \]
  \[ \text{efficient approximation} \quad \text{partial reconstruction of analytic properties} \]
  \[ \text{good predictions of higher orders} \quad \text{model independent results} \]

- \( f(z) = \frac{\alpha_s(x\mu^2)}{x^2} + B(x) \) approximate \( f(x) \) by Padés

D-log Padé approximants

- It is more costly (in terms of coefficients) to reproduce functions with cuts on the complex plane. Strategy - D-log Padé approximants

- One can then obtain a new approximation to the original function \cite{2}
  \[ D\log_N^M(z) = f(0) + f_1 z + f_2 z^2 + \cdots \approx f_0 + f_1 z + f_2 z^2 + \cdots \]

Results in QCD

- Having successfully tested the method, we can apply it to QCD \cite{5}

- \[ a_s, \alpha_s^2, \alpha_s^3, \alpha_s^4, \alpha_s^5 \]
  \[ \text{results equal good using Borel transform or the series in the coupling} \]
  \[ \text{completely model independent} \]

Main references and acknowledgements