# $L^- \to \ell^- \ell'^+ \ell'^-$ LFV decays in the SM with massive neutrinos

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**This work is based on arXiv:1807.06050**

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# Outline

- Motivation
- Previous calculations
  - $\mu^- \to e^- e^+ e^-$ S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977).
  - $\tau^- \to \mu^- \ell^+ \ell^-$ X. Y. Pham, Eur. Phys. J. C 8, 513 (1999).
- Our computation and results
- Conclusions

A similar study is presented in the poster session

- Revisiting $\tau \to 3\mu$ in the Standard Model and beyond, E. Passemar and P. Blackstone.

Some related works:

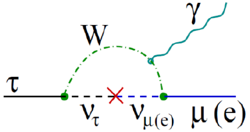
- LNU, LNV and LFV at Belle II, Ami Rostomyan.
- Leptonic LFV theory, Adrian Signer.
- LFV and neutrino mass, Ana M. Teixeira.
- Tau→3mu in Run-1 with the ATLAS detector, Matteo Bedognetti.
- The Rare and Forbidden: Testing Physics Beyond the Standard Model with Mu3e, Ann-Kathrin Perrevoor.

- LFV processes are forbidden in the original formulation of the SM (massless neutrinos).

- **However, neutrino oscillation $\Rightarrow$ LF numbers are not conserved, and claims for an extended model with tiny neutrino mass.**

- The mixing of three light neutrinos can be described through $U_{PMNS}$ matrix, which connects flavour eigenstates with mass eigenstates.

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \quad (1)$$




- The $U_{PMNS}$ matrix can also give rise, at one loop level, to cLFV

- Negligible rates are expected due to a GIM-like mechanism. ***Except for*** $\tau^\pm \to \mu^\pm \ell^\pm \ell^\mp$??????

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# Experimental limits

***So far, no evidence of cLFV!***

| Reaction | Present limit | C.L. | Experiment | Year |
|---|---|---|---|---|
| $\mu^+ \to e^+\gamma$ | $< 4.2 \times 10^{-13}$ | 90% | MEG at PSI | 2016 |
| $\mu^+ \to e^+e^-e^+$ | $< 1.0 \times 10^{-12}$ | 90% | SINDRUM | 1988 |
| $\tau \to e\gamma$ | $< 3.3 \times 10^{-8}$ | 90% | BaBar | 2010 |
| $\tau \to \mu\gamma$ | $< 4.4 \times 10^{-8}$ | 90% | BaBar | 2010 |
| $\tau \to eee$ | $< 2.7 \times 10^{-8}$ | 90% | Belle | 2010 |
| $\tau \to \mu\mu\mu$ | $< 2.1 \times 10^{-8}$ | 90% | Belle | 2010 |
| $Z \to \mu e$ | $< 7.5 \times 10^{-7}$ | 95% | LHC ATLAS | 2014 |
| $Z \to \tau e$ | $< 9.8 \times 10^{-6}$ | 95% | LEP OPAL | 1995 |
| $Z \to \tau\mu$ | $< 1.2 \times 10^{-5}$ | 95% | LEP DELPHI | 1997 |
| $h \to e\mu$ | $< 3.5 \times 10^{-4}$ | 95% | LHC CMS | 2016 |
| $h \to \tau\mu$ | $< 2.5 \times 10^{-3}$ | 95% | LHC CMS | 2017 |
| $h \to \tau e$ | $< 6.1 \times 10^{-3}$ | 95% | LHC CMS | 2017 |

- **The Mu3e experiment will search for LFV in $\mu \to 3e$ decay with a sensitivity down to $10^{-16}$.**
  - Nucl. Part. Phys. Proc. 287-288, 169 (2017).
- **Belle-II shall be able to set limits on the $\tau^- \to \ell^-\ell'^+\ell'^-$ decays at the level of O($10^{-9}$)-O($10^{-10}$) with their full data set ($50\, ab^{-1}$ ).**
  - Belle-II Physics Book, Belle-II Collaboration and B2TIP Community

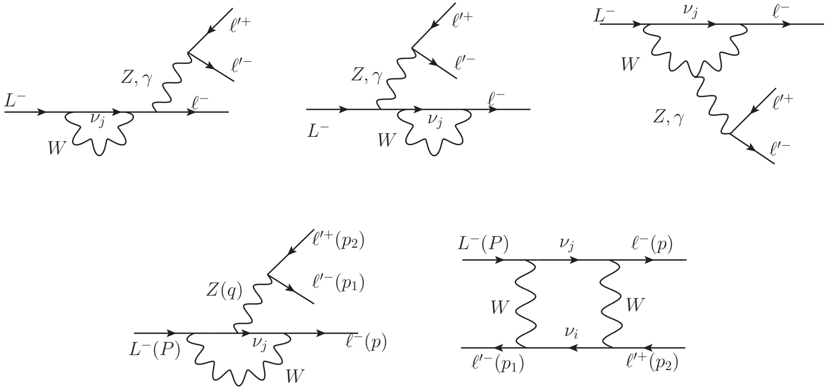
# Theoretical predictions in the SM with massive neutrinos

Several scenarios BSM predict large contributions to cLFV processes. However, we focus in the simple scenario of the SM with massive neutrinos.

- $BR(\mu \to e\gamma) \simeq \frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3}\frac{U_{\mu k}U^*_{ek}m^2_{\nu k}}{m^2_W}\right|^2 \sim 10^{-54}.$
    - T. P. Cheng and L. F. Li, Gauge Theory Of Elementary Particle Physics
- $BR(Z \to \ell'\ell) \sim 10^{-54}$
    - Phys. Rev. D 63, 053004 (2001)
- $BR(h \to \ell'\ell) \sim 10^{-55}$
    - Phys. Rev. D 71, 035011 (2005)
- $\mathrm{BR}(\mu^\pm \to e^\pm e^\pm e^\mp) \sim 10^{-53}$ (updated input)
    - S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977).
- $\mathrm{BR}(\tau^\pm \to \mu^\pm \ell^\pm \ell^\mp) > 10^{-14}$ $\qquad \mathcal{M} \sim \sum_{j=1}^{3} U^*_{\mu j}U_{\tau j}\log\left(\frac{m^2_W}{m^2_j}\right).$
    - X. Y. Pham, Eur. Phys. J. C 8, 513 (1999).

- **If the prediction in X. Y. Pham, Eur. Phys. J. C 8, 513 (1999). were right, there would be a difference of almost 40 orders of magnitude between $L^\pm \to \ell'^\pm\gamma$ and $L^\pm \to \ell'^\pm\ell^\pm\ell^\mp$.**

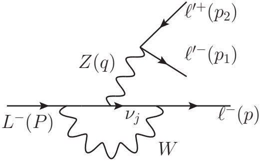
# Contributions to $L^- \to \ell^- \ell'^- \ell'^+$ LFV decays

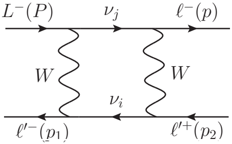



Feynman diagrams for the $L^- \to \ell^- \ell'^- \ell'^+$ decays, in the presence of lepton mixing.

# Theoretical predictions

- $\mu^{\pm} \to e^{\pm} e^{\pm} e^{\mp}$
  - ⋆ Sov. J. Nucl. Phys. 25, 340 (1977).
- Momenta and masses of the external particles are neglected from the beginning in the loop integrals for the dominant diagrams with two neutrino propagators.




- The amplitudes for these diagrams are proportional to

$$\mathcal{M} \sim \sum_{j=1}^{3} U_{ej}^{*} U_{\mu j} \frac{m_j^2}{m_W^2} \log\left(\frac{m_W^2}{m_j^2}\right).$$

- **The amplitudes vanish in the limit of massless neutrinos.**

# Theoretical predictions

## Lepton flavor changing in neutrinoless $\tau$ decays

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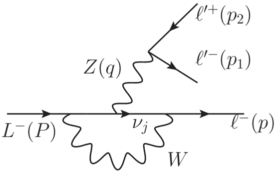
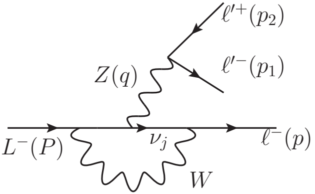



- The dominant amplitude comes from the penguin diagram with two neutrino propagators.
- In order to deal with the integrals an expansion around $q^2 = 0$ is made in Feynman parameters integrals. Then, the amplitude is proportional to

$$\mathcal{M} \sim \sum_{j=1}^{3} U^{*}_{ej} U_{\mu j} \log\left(\frac{m_W^2}{m_j^2}\right).$$

- **The amplitude won't vanish in the limit of massless neutrinos.**
- **There would be no way to cure such infrared behavior.**

# $Z$-Penguin contribution emission from internal neutrino line




The relevant part of the amplitude is the effective $ZL\ell$ vertex is defined by

$$l^{\lambda}_{L\ell} = \left(\frac{-ig}{4c_W}\right)\left(\frac{-ig}{2\sqrt{2}}\right)^2 \sum_{j=1}^{3} U^*_{\ell j} U_{Lj} \bar{u}(p) \Gamma^{\lambda}_j u(P), \tag{2}$$

where

$$\Gamma^{\lambda}_j = \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\rho(1-\gamma_5) i \left[(\not{p}+\not{k})+m_j\right] \gamma^{\lambda}(1-\gamma_5) i \left[(\not{P}+\not{k})+m_j\right] \gamma_\sigma (1-\gamma_5)(-ig^{\rho\sigma})}{\left[(p+k)^2-m_j^2\right]\left[(P+k)^2-m_j^2\right]\left[k^2-m_W^2\right]}. \tag{3}$$

- After making the loop integration

$$\begin{aligned}\Gamma^{\lambda}(q^2, m_j^2) \quad &= \quad F_a \gamma^{\lambda}(1-\gamma^5) + F_b \gamma^{\lambda}(1+\gamma^5) + F_c (P+p)^{\lambda}(1+\gamma^5) \\ &+ \quad F_d (P+p)^{\lambda}(1-\gamma^5) + F_e q^{\lambda}(1+\gamma^5) + F_f q^{\lambda}(1-\gamma^5),\end{aligned} \tag{4}$$

- **We have obtained the $F_k = F_k(q^2, m_j^2)$ $(k = a, b \ldots f)$ using both Feynman parametrization and Passarino-Veltman method.**

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# Z-Penguin contribution emission from internal neutrino line

- We reproduced the simple case where masses and momenta of the external particles are neglected S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977)

$$F_a^0 = \frac{1}{2\pi^2}\left[\frac{m_j^2}{m_W^2}\log\left(\frac{m_W^2}{m_j^2}\right) - \frac{m_j^2}{2m_W^2} + \frac{1}{2}\log\left(\frac{m_W^2}{\mu^2}\right) + \frac{1}{4} + \vartheta\left(\frac{m_j^2}{m_W^2}\right)^2\right]. \tag{5}$$

- The presence of masses and momenta of the external particles in the computation hinders the way for the derivation of analytical expressions for the loop integrals.
    - We agree with the previous expression reported in X. Y. Pham, Eur. Phys. J. C 8, 513 (1999) for the integrals in terms of the Feynman parameters.
    - However, we disagree with the expansion done around $q^2 = 0$.

- **We are studying a process where the lowest scale is the neutrino mass and $q^2$ must be non-vanishing.**
- **Taking an expansion around $q^2 = 0$ modifies substantially the behavior of the original functionsin the interesting physical region.**

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In order to deal with the integrals, we have used two approaches

- Numerical evaluation in terms of the PaVe functions
- Following the same strategy that Cheng and Li for the computation of the $\mu \to e\gamma$ decay,

In this way, we have verified that the dominant contibution comes from the $F_a$ function, which is given as follows

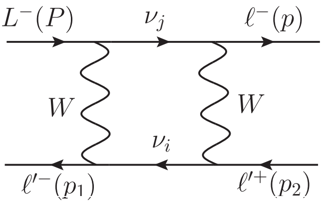
$$F_{PV_a}(q^2, m_j^2) = \frac{1}{2\pi^2}\left[Q_a + \frac{m_j^2}{m_W^2} R_a + \vartheta\left(\frac{m_j^4}{m_W^4}\right)\right], \tag{6}$$

where

$$\begin{aligned} R_a &= -m_W^2 \lambda(m^2, M^2, q^2)^{-1}\left[f_{R_{a_1}} C_0(m^2, M^2, q^2, 0, m_W^2, 0) + f_{R_{a_2}} \log\left(\frac{m_W^2}{m_W^2 - m^2}\right)\right. \\ &+ \left. f_{R_{a_3}} \log\left(\frac{m_W^2}{m_W^2 - M^2}\right) + f_{R_{a_4}} \log\left(\frac{m_W^2}{q^2}\right) + f_{R_{a_5}}\right], \end{aligned} \tag{7}$$

where $\lambda$ is the Kallen function.

# Box contributions




$$\mathcal{M} = (\frac{-ig}{2\sqrt{2}})^4 \sum_j \sum_i U_{Lj} U^*_{lj} U_{\ell' i} U^*_{\ell' i} T_{\sigma\sigma'} I^{\sigma\sigma'}$$

where

$$T_{\sigma\sigma'} = 4\,\bar{u}(p)\gamma_\mu\gamma_\sigma\gamma_\nu(1-\gamma_5)u(P)\times \bar{u}(p_1)\gamma_\nu\gamma'_\sigma\gamma_\mu(1-\gamma_5)v(p_2)$$

and the relevant loop integral is

$$I^{\sigma\sigma'} = \int \frac{d^4k}{(2\pi)^4}\frac{(P+k)^\sigma(k+p_1)^{\sigma'}}{(k^2-m_W^2)((p_1+p_2+k)^2-m_W^2)((P+k)^2-m_j^2)((k+p_1)^2-m_i^2)}.$$

After making the loop integration

$$I^{\sigma\sigma'} = i\Big(g^{\sigma\sigma'}H_a + P^\sigma P^{\sigma'}H_b + P^\sigma p_1^{\sigma'}H_c + P^\sigma p_2^{\sigma'}H_d + p_1^\sigma P^{\sigma'}H_e + p_1^\sigma p_1^{\sigma'}H_f + p_1^\sigma p_2^{\sigma'}H_g + p_2^\sigma P^{\sigma'}H_h + p_2^\sigma p_1^{\sigma'}H_i + p_2^\sigma p_2^{\sigma'}H_j\Big). \quad (8)$$

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- **Analogously to the penguin diagram, we have obtained the $H_k = H_k(s_{12}, s_{13}, m_j^2, m_i^2)$ using both Feynman parametrization and Passarino-Veltman method.**
    - In the simple case where masses and momenta of the external particles are neglected the only non-zero function is

$$
\begin{aligned}
H_a^0(m_j^2, m_i^2) &= \frac{1}{64\pi^2 m_W^4}\left[\left(m_i^2+m_j^2\right)\left(\log\left(\frac{m_W^2}{m_j^2}\right)-1\right)\right. \\
&+ \left.\frac{m_i^2 m_j^2}{m_W^2}\left(2\log\left(\frac{m_W^2}{m_j^2}\right)-1\right) - m_W^2 + \vartheta\left(\frac{m_i^4}{m_W^2}\right) + \vartheta\left(\frac{m_j^4}{m_W^2}\right)\right].
\end{aligned}
\tag{9}
$$

- As far as the general case is concerned, the dominant contributions comes from the $H_a$ function associated with a $(V-A)\times(V-A)$ operator.
    - We estimate the relevant dependence on the neutrino mass for the $H_a$ function fitting the curve in the physical region evaluated in terms of the PaVe functions considering fixed values for the other parameters.

$$
H_a = \frac{1}{16\pi^2}\left(Q_{H_a} + \frac{m_j^2}{m_W^4} R_{H_a}\right), \tag{10}
$$

where $R_{H_a} \approx 1.5 + i0.007$, for all different $\tau$ channels, whereas $R_{H_a} \approx 1.5$ for the $\mu \to 3e$ channel.

# Results

| Decay channel | Our Result | Petcov's Result* | Our Result | Petcov's Result* |
|---|---|---|---|---|
| $\mu^- \to e^- e^+ e^-$ | $9{,}5 \cdot 10^{-55}$ | $1{,}0 \cdot 10^{-53}$ | $2{,}1 \cdot 10^{-56}$ | $2{,}6 \cdot 10^{-53}$ |
| $\tau^- \to e^- e^+ e^-$ | $5{,}0 \cdot 10^{-56}$ | $1{,}8 \cdot 10^{-54}$ | $3{,}6 \cdot 10^{-57}$ | $4{,}5 \cdot 10^{-54}$ |
| $\tau^- \to \mu^- \mu^+ \mu^-$ | $1{,}0 \cdot 10^{-54}$ | $3{,}7 \cdot 10^{-53}$ | $7{,}6 \cdot 10^{-56}$ | $9{,}7 \cdot 10^{-53}$ |
| $\tau^- \to e^- \mu^+ \mu^-$ | $2{,}9 \cdot 10^{-56}$ | $1{,}0 \cdot 10^{-54}$ | $1{,}7 \cdot 10^{-57}$ | $2{,}2 \cdot 10^{-54}$ |
| $\tau^- \to \mu^- e^+ e^-$ | $7{,}3 \cdot 10^{-55}$ | $2{,}5 \cdot 10^{-53}$ | $4{,}0 \cdot 10^{-56}$ | $5{,}0 \cdot 10^{-53}$ |

| Decay channel | Our Result | Petcov's Result* |
|---|---|---|
| $\mu^- \to e^- e^+ e^-$ | $7{,}4 \cdot 10^{-55}$ | $8{,}5 \cdot 10^{-54}$ |
| $\tau^- \to e^- e^+ e^-$ | $3{,}2 \cdot 10^{-56}$ | $1{,}4 \cdot 10^{-54}$ |
| $\tau^- \to \mu^- \mu^+ \mu^-$ | $6{,}4 \cdot 10^{-55}$ | $3{,}2 \cdot 10^{-53}$ |
| $\tau^- \to e^- \mu^+ \mu^-$ | $2{,}1 \cdot 10^{-56}$ | $9{,}4 \cdot 10^{-55}$ |
| $\tau^- \to \mu^- e^+ e^-$ | $5{,}2 \cdot 10^{-55}$ | $2{,}1 \cdot 10^{-53}$ |

- Individual penguin contributions
- Box contributions
- Total contributions

* We considered the state of the art best fit values of the three neutrino oscillation parameters.

# Conclusions

- We have re-evaluated $L^- \to \ell^- \ell'^- \ell'^+$ using Feynman-parametrization and Passarino-Veltman functions methods keeping finite masses and momenta of the external particles.
- We find Branching ratios even smaller than using the approximation in Ref. S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977) (vanishing masses and momenta).
- Large room for effects of new physics.

Thank you!