# Isospin Breaking in $\tau$ input for $(g-2)_\mu$ from Lattice QCD


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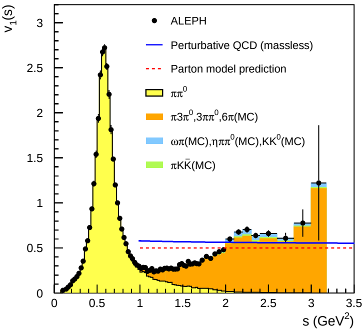
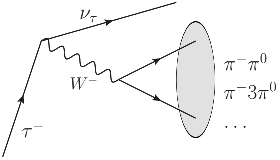
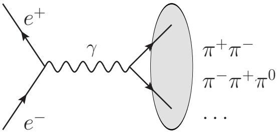
# Motivations




$V - A$ current

Final states $I = 1$ charged




EM current

Final states $I = 0, 1$ neutral

$\tau$ data can improve $a_\mu[\pi\pi]$
$\rightarrow 72\%$ of total Hadronic LO

or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]
[talks by Lopez Castro, Gonzalez-Alonso]

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# Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\rm EW}}$$

Isospin correction $v_0 = R_{\rm IB} v_-$ $\quad R_{\rm IB} = \frac{\rm FSR}{G_{\rm EM}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

**0.** $S_{\rm EW}$ electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

**1.** Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

**2.** $G_{\rm EM}$ (long distance) radiative corrections in $\tau$ decays
   Chiral Resonance Theory [Cirigliano et al. '01, '02]
   Meson Dominance [Flores-Talpa et al. '06, '07]

**3.** Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

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# Contribution to $a_\mu$

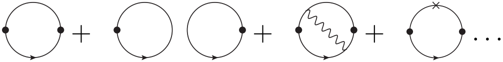
Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

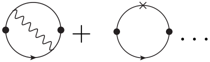
Isospin decomposition of $u, d$ current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$



$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$



$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

# Neutral vs Charged

$$\tfrac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),\ \begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \to \quad j_\mu^{(1,-)} = \tfrac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right),\ \begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^{W} = \frac{1}{3}\sum_k \int d\vec{x}\ \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

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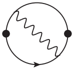
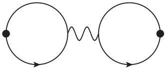
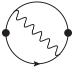
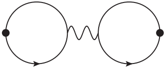
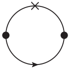
$$\delta G^{(1,1)} \equiv G_{11}^{\gamma} - G_{11}^{W}$$

$$= Z_V^4 (4\pi\alpha) \frac{(Q_{\rm u} - Q_{\rm d})^4}{4} \Bigg[$$  $$+$$  $$\Bigg]$$

$$G_{01}^{\gamma} = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Bigg[$$  $$+\ 2\times$$  $$+$$  $$+\dots\Bigg]$$

$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Bigg[2\times$$  $$+\dots\Bigg]$$

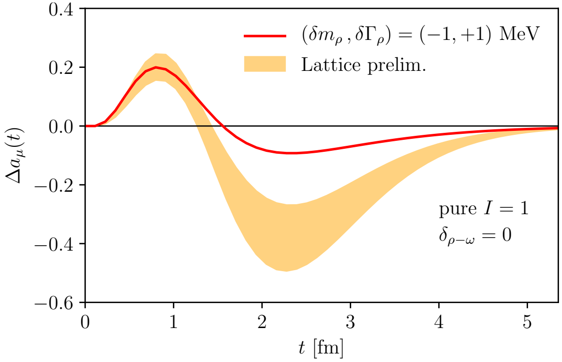
$\dots$ = subleading diagrams currently not included

# Lattice: Preliminary results

Study integrand in euclidean time → as important as integral

direct comparison
Lattice vs. EFT+Pheno

1. validate previous estimates of $R_{\mathrm{IB}}$
2. study neutral/charged $\rho$ and $\omega$ properties




$a_\mu^\tau - a_\mu^e$ sensitive to new physics [Cirigliano et al. '18] [Miranda-Roig '18]

work in progress:

✓ finite vol. errs

✓ better stat. errs

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# Conclusions

For precise prediction:

study systematic errors $\rightarrow$ ongoing finite volume study

improvement of errs $\rightarrow$ high stat. data set from HLbL

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Outlook:

**1.** full lattice calculation of $\Delta a_\mu[\tau]$ on the way

**2.** tests/checks previous calculations

comparing $v_-$ with experiment requires $\mathrm{FSR}$, $S_{\mathrm{EW}}$ and $G_{\mathrm{EM}}$
$\rightarrow$ test of long distance QED corrections
$\rightarrow$ direct computation

study $G_{01}^{\gamma}$ alone $\rightarrow$ $\rho\omega$ mixing; $\delta G^{(1,1)}$ alone $\rightarrow$ $\rho^0$ vs $\rho^-$

**3.** possibly sensitive to new physics

**Thanks for your attention**

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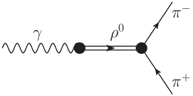
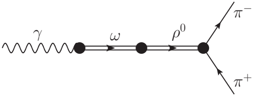
# Pion form factors

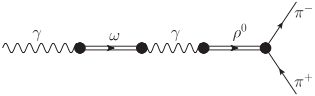
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$

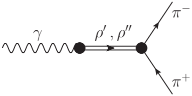



$$\times \left[1 + \delta_{\rho\omega} \frac{s}{D_\omega(s)}\right]$$




$$+ \frac{m_X^2}{D_X(s)} \qquad X = \rho' \,, \rho''$$

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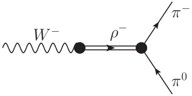
$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \,, \rho'')$$




Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$ , $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$ , $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_u - m_d) + O(e^2)$

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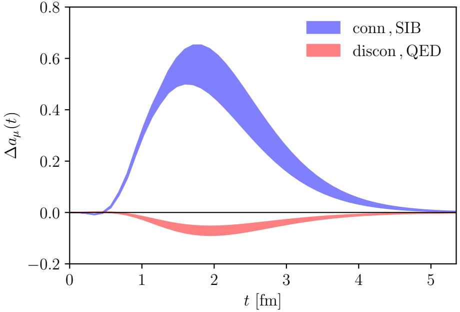
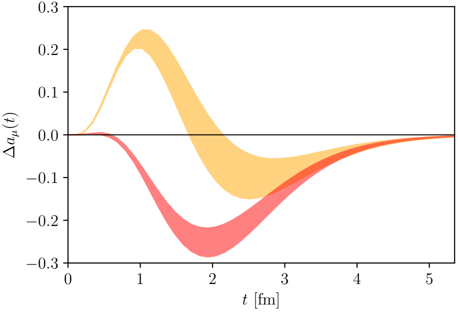
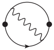
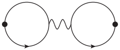
# Lattice: Preliminary results

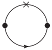
$\Delta a_\mu$ from $G_{01}^{\gamma}$ (QED and SIB):




Pure $I = 1$ only $O(\alpha)$ terms:




$V =$  $F =$  $S =$ 

$M =$  $\rightarrow$ dominates noise

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