Combined Analysis of $B \to D^{(*)} l \nu$

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Helicity Amplitude Module
for Matrix Element Reweighting

Based on: 1610.02045 (Z. Ligeti, M. Papucci & DR)

1703.05330 (F. Bernlochner, Z. Ligeti, M. Papucci & DR)

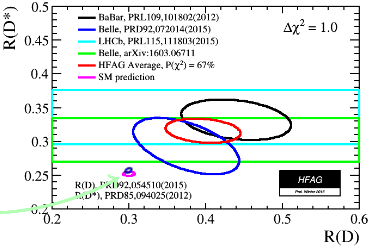
170x.xxxxx (F. Bernlochner, Z. Ligeti, M. Papucci & DR)

170x.xxxxx (`Hammer`: F. Bernlochner, S. Duell, Z. Ligeti, M. Papucci & DR)

Overview

- Newly published, unfolded Belle data [1702.01521]
- Measurement of $B \to D^{(*)} l\nu$ distributions constrain all $B \to D^{(*)}$ FFs, by constraining leading and subleading Isgur-Wise functions in HQET.
- Precise predictions for $B \to D^{(*)}\tau\nu_\tau$ rates in SM and all NP.

New prediction: **combined** $D^{(*)}$ fit with HQET beyond LO to unfolded Belle data



HQET

HQET to $\mathcal{O}(\varepsilon_{c,b} = \overline{\Lambda}/2m_{c,b})$ plus α_s perturbative corrections.

- Trace formalism: to order $\varepsilon_{c,b} = \overline{\Lambda}/2m_{c,b}$

Leading IW function
HQET: $\xi(1) = 1$

$$\frac{\langle D^{(*)} | \bar{c}\,\Gamma\, b | \bar{B}\rangle}{\sqrt{m_{D^{(*)}} m_B}} = -\xi(w)\Big\{\mathrm{Tr}\big[\overline{H}^{(c)}_{v'}\,\Gamma\, H^{(b)}_v\big] + \varepsilon_c\,\mathrm{Tr}\big[\overline{H}^{(c,1)}_{v',v}\,\Gamma\, H^{(b)}_v\big] + \varepsilon_b\,\mathrm{Tr}\big[\overline{H}^{(c)}_{v'}\,\Gamma\, H^{(b,1)}_{v,v'}\big]\Big\}\,,$$

- Perturbative α_s corrections are extracted from one-loop matching of QCD onto HQET: All long-known (Neubert '92).
- We use $\Upsilon(1S)$ short distance scheme to control renormalon ambiguities.

FF expressions

For $\bar{B} \to D$

SM FFs:

$$\hat{h}_+ = 1 + \hat{\alpha}_s\Big[C_{V_1} + \frac{w+1}{2}\,(C_{V_2} + C_{V_3})\Big] + (\varepsilon_c + \varepsilon_b)\,\hat{L}_1\,,$$

$$\hat{h}_- = \hat{\alpha}_s\,\frac{w+1}{2}\,(C_{V_2} - C_{V_3}) + (\varepsilon_c - \varepsilon_b)\,\hat{L}_4\,,$$

$$\hat{h}_S = 1 + \hat{\alpha}_s\,C_S + (\varepsilon_c + \varepsilon_b)\Big(\hat{L}_1 - \hat{L}_4\,\frac{w-1}{w+1}\Big)\,,$$

Not in literature!

$$\hat{h}_T = 1 + \hat{\alpha}_s(C_{T_1} - C_{T_2} + C_{T_3}) + (\varepsilon_c + \varepsilon_b)(\hat{L}_1 - \hat{L}_4)\,.$$

For $\bar{B} \to D^*$

SM FFs:

$$\hat{h}_V = 1 + \hat{\alpha}_s\,C_{V_1} + \varepsilon_c(\hat{L}_2 - \hat{L}_5) + \varepsilon_b(\hat{L}_1 - \hat{L}_4)\,,$$

$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s\,C_{A_1} + \varepsilon_c\Big(\hat{L}_2 - \hat{L}_5\,\frac{w-1}{w+1}\Big) + \varepsilon_b\Big(\hat{L}_1 - \hat{L}_4\,\frac{w-1}{w+1}\Big)\,,$$

$$\hat{h}_{A_2} = \hat{\alpha}_s\,C_{A_2} + \varepsilon_c(\hat{L}_3 + \hat{L}_6)\,,$$

$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s(C_{A_1} + C_{A_3}) + \varepsilon_c(\hat{L}_2 - \hat{L}_3 + \hat{L}_6 - \hat{L}_5) + \varepsilon_b(\hat{L}_1 - \hat{L}_4)\,,$$

$$\hat{h}_P = 1 + \hat{\alpha}_s\,C_P + \varepsilon_c\big[\hat{L}_2 + \hat{L}_3(w-1) + \hat{L}_5 - \hat{L}_6(w+1)\big] + \varepsilon_b(\hat{L}_1 - \hat{L}_4)\,,$$

Not in literature!

$$\hat{h}_{T_1} = 1 + \hat{\alpha}_s\Big[C_{T_1} + \frac{w-1}{2}\,(C_{T_2} - C_{T_3})\Big] + \varepsilon_c\hat{L}_2 + \varepsilon_b\hat{L}_1\,,$$

$$\hat{h}_{T_2} = \hat{\alpha}_s\,\frac{w+1}{2}\,(C_{T_2} + C_{T_3}) + \varepsilon_c\hat{L}_5 - \varepsilon_b\hat{L}_4\,,$$

$$\hat{h}_{T_3} = \hat{\alpha}_s\,C_{T_2} + \varepsilon_c(\hat{L}_6 - \hat{L}_3)\,.$$

The $\hat{L}$ functions contain three subleading Isgur-Wise functions

$$\hat{L}_1 = -4(w-1)\hat{\chi}_2 + 12\hat{\chi}_3\,, \qquad \hat{L}_2 = -4\hat{\chi}_3\,, \qquad \hat{L}_3 = 4\hat{\chi}_2\,,$$

$$\hat{L}_4 = 2\eta - 1\,, \qquad \hat{L}_5 = -1\,, \qquad \hat{L}_6 = -2(1+\eta)/(w+1)\,.$$

All FFs are specified by $1 + 3$ IW functions!

Consistent HQET+SR

- QCDSR results available for zero recoil subleading IW.
- Keep only contributions to $\mathcal{O}(\varepsilon_{c,b}(w-1))$ for subleading IW functions $\chi(w) = \chi(1) + \chi'(1)(w-1)$
- Treat $\widehat{\chi}_2(1)$, $\widehat{\chi}'_{2,3}(1)$, $\eta(1)$ and $\eta'(1)$ as either free or QCDSR constrained

QCDSR central values lead to,

$$R_1(w) = 1.268 - 0.114(w-1) + \ldots$$
$$R_2(w) = 0.760 + 0.136(w-1) + \ldots$$

Each coeff is fixed! Compare with literature

$$R_1(w) = R_1(1) - 0.12\,(w-1) + \ldots$$
$$R_2(w) = R_2(1) + 0.11\,(w-1) + \ldots$$

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Literature: "$R_1(1)$ and $R_2(1)$ are left as fitting parameters"

$$R_1(w) = R_1(1) - 0.12\,(w-1) + \ldots$$
$$R_2(w) = R_2(1) + 0.11\,(w-1) + \ldots$$

Literature: Uses HQET+SR predictions here and elsewhere

World Av: $R_1(1) \simeq 1.40 \pm 0.03$
Not consistent with $R_1(1) = 1.34 - 0.12\eta(1)$ and $\eta(1) \simeq 0.6$

Fit strategies

Unitarity/dispersive constraints on $B \to D l \nu$ FF, $\mathcal{G}(w)$

- HQET

$$\frac{\mathcal{G}(w)}{\mathcal{G}(w_0)} \times \frac{\xi(w)}{\mathcal{G}(w)} \times \frac{\mathcal{G}(w_0)}{\xi(w_0)} \to \frac{\xi(w)}{\xi(w_0)}$$

$[\widehat{h}_+ - \widehat{h}_-(1-r_D)/(1+r_D)]^{-1}$

- Also $\xi(1) = 1 \Rightarrow \xi(w_0) \Rightarrow \xi(w)$

In principle, HQET+parametrization+data fix $|V_{cb}|$ without other theory information! However,

- lattice data† is expected to include higher order effects in normalization, that could be large.
- Also, calibration of tagging effs $\to$ large systematics in normalization. Want to examine 'shape only' information.

†We use FNAL/MILC

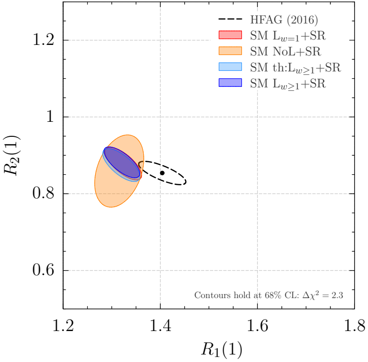
Fit summary

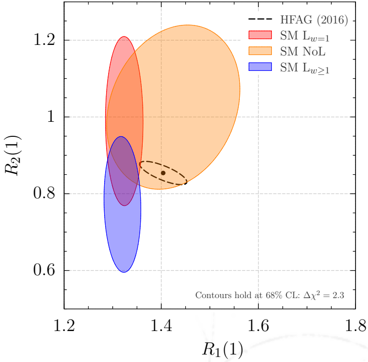
Fit	QCDSR	Lattice QCD			Belle Data
		$\mathcal{F}(1)$	$f_{+,0}(1)$	$f_{+,0}(w>1)$	
$\mathrm{L}_{w=1}$	—	✓	✓	—	✓
$\mathrm{L}_{w=1}$+SR	✓	✓	✓	—	✓
NoL	—	—	—	—	✓
NoL+SR	✓	—	—	—	✓
$\mathrm{L}_{w\geq1}$	—	✓	✓	✓	✓
$\mathrm{L}_{w\geq1}$+SR	✓	✓	✓	✓	✓
th:$\mathrm{L}_{w\geq1}$+SR	✓	✓	✓	✓	—

For each fit: Also turn on the QCDSR constraints. Can also perform a 'pure theory' fit without Belle data

Fits introduce extra HQET breaking in normalizations, but preserve subleading HQET structure

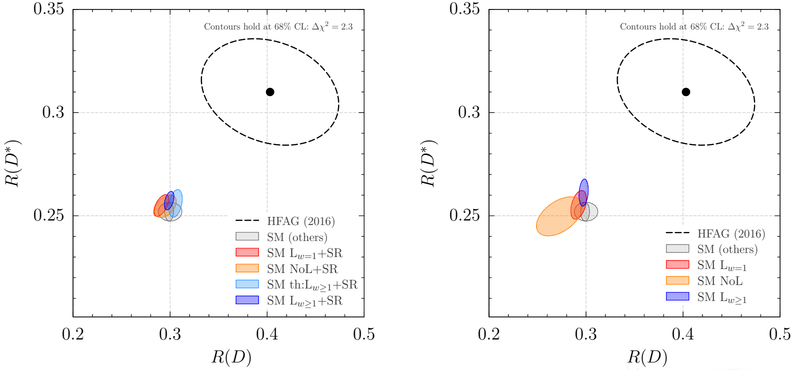
FF Ratios





- Tension with HQET-inconsistent approach at $\sim 1\sigma$ level
- No inconsistencies between the data, lattice QCD results, and QCD sum rule predictions

$R(D^{(*)})$



- Most precise prediction ($L_{w\geq1}$+SR):

$$R(D) = 0.299 \pm 0.003\,, \qquad R(D^*) = 0.257 \pm 0.003\,,$$

and 44% correlation.

- No inconsistencies between the data, lattice QCD results, and QCD sum rule predictions

Fit Quality and $|V_{cb}|$

	$L_{w=1}$	$L_{w=1}$+SR	NoL	NoL+SR	$L_{w\geq1}$	$L_{w\geq1}$+SR	th:$L_{w\geq1}$+SR
χ^2	40.2	44.0	38.7	43.1	49.0	53.8	7.4
dof	44	48	43	47	48	52	4
$\|V_{cb}\| \times 10^3$	38.8 ± 1.2	38.5 ± 1.1	—	—	39.1 ± 1.1	39.3 ± 1.0	—

Compare: $|V_{cb}| \simeq (42.2 \pm 0.8) \times 10^{-3}$ from $B \to X_c l \nu$

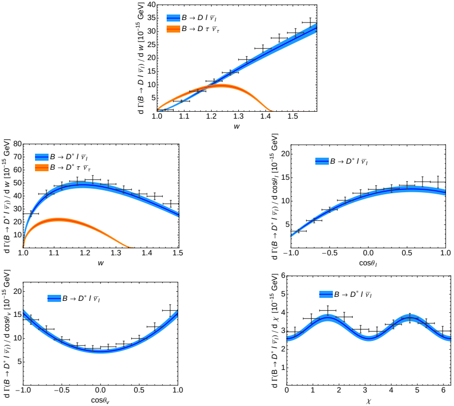
And $|V_{cb}| \simeq (37.4 \pm 1.3) \times 10^{-3}$ from Belle

And $|V_{cb}| \simeq (39.3 \pm 0.7) \times 10^{-3}$ and $|V_{cb}| \simeq (40.9 \pm 1.0) \times 10^{-3}$ from FLAG

Preliminary: $|V_{cb}| \simeq (39.0 \pm 1.0) \times 10^{-3}$ from BGL-f adapted parametrization

Distributions

For $\mathrm{L}_{w\geq 1}$+SR fit:



Uses all theory inputs

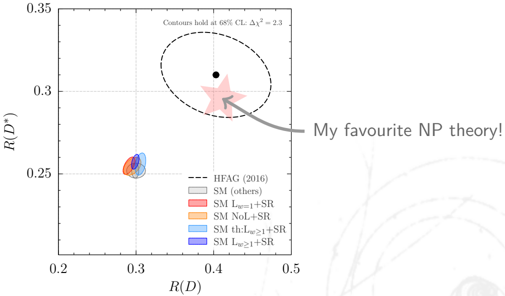
$\chi^2/\mathrm{dof} = 53.8/52$

$|V_{cb}| = (39.3 \pm 1.0) \times 10^{-3}$

hammer advertisement

- Non-trivial τ and D^* interference effects
- Non-trivial phase space cuts
- τ frame not (yet) reconstructible
- Simultaneous BG+signal float: model dependent template!

Should be done with caution and caveats:



Black ellipse tells us confidence to reject SM, not confidence to accept NP.

hammer will allow Belle II (and LHCb?) to efficiently reweight fully simulated samples

Ongoing Puzzles

- V_{cb}
  - How much do FF parametrizations matter?
  - BGL/BCL have more leading order parameters; larger $|V_{cb}|$ [Bigi, Gambino, Schacht '17, and Grinstein, Kobach '17]
  - Subleading HQET analysis relaxes tension somewhat.

- HQET
  - $\alpha_s/m_{c,b}$ or $1/(2m_c)^2$ effects: Beyond current scope, many subleading IW functions.
  - How important is the $w-1$ subleading expansion?

Thanks!

Adapted CLN parametrization

Unitarity/dispersive constraints on $B \to Dl\nu$ FF, $\mathcal{G}(w)$

- Transform to optimized conformal parameter z_*: parametric range is minimized

$$z_* = \frac{\sqrt{w+1} - \sqrt{2}\,a}{\sqrt{w+1} + \sqrt{2}a}$$

$$a = \left(\frac{1 + r_D}{2\sqrt{r_D}}\right)^{1/2}$$

- Obtain slope–curvature constraint

$$\frac{\mathcal{G}(w)}{\mathcal{G}(w_0)} \simeq 1 - 8a^2\rho_*^2 z_* + \left(V_{21}\rho_*^2 - V_{20}\right) z_*^2\,.$$

$V_{21} \simeq 57.$, $V_{20} \simeq 7.5$ and $w_0 : z_*(w_0) = 0$. For $B \to Dl\nu$, $|z_*| < 0.032$.

- HQET: We know FF ratios!

$$\frac{\mathcal{G}(w)}{\mathcal{G}(w_0)} \times \frac{\xi(w)}{\mathcal{G}(w)} \times \frac{\mathcal{G}(w_0)}{\xi(w_0)} \to \frac{\xi(w)}{\xi(w_0)}$$

$$[\widehat{h}_+ - \widehat{h}_-(1 - r_D)/(1 + r_D)]^{-1}$$

- Also $\xi(1) = 1 \Rightarrow \xi(w_0) \Rightarrow \xi(w)$