On the QED effects on $R_K$ and $R_{K^*}$

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$R_K$ and $R_{K^*}$ within the SM

Which are the sources of LFUV contributions to $R_{K(\ast)}$ in the SM?

- kinematics and form factor effects $\sim \frac{m_l^2}{q^2}$
- naive estimation of QED corrections $\sim \frac{\alpha}{\pi} \log^2 \left( \frac{m^2_l}{q^2} \right)$
- interplay between kinematic effects and QED corrections

Can we trust the $\mathcal{O}(10^{-3}/10^{-4})$ uncertainties that are quoted in the literature?

semi-analytic calculation of radiative corrections
Calculation setup for the region $q^2 \in [1, 6] \text{ GeV}^2$

- $B \to K^* \ell \ell(\gamma)$ and $B \to K \ell \ell(\gamma)$ decays can be treated in complete analogy (NB: real+virtual QED effects $\Rightarrow$ IR safe observables)

- limit $m_\ell^2 \ll q^2$

- interested to extract log-enhanced terms $\sim \frac{\alpha}{\pi} \log \left( \frac{m_\ell^2}{q^2} \right)$ and $\sim \frac{\alpha}{\pi} \log^2 \left( \frac{m_\ell^2}{q^2} \right)$
  - since they depend on $m_\ell$ they can be the only terms responsible of LFU violation
  - can be extracted from term associated with collinear and soft divergences due to the photon emission

- neglect $\mathcal{O}(\alpha/\pi)$ finite corrections ($\sim 0.2\%$)

- radiation from meson leg is negligible (not log-enhanced)
Calculation setup for the region $q^2 \in [1, 6] \text{ GeV}^2$

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**Main purpose**: understand if QED effects are correctly taken in account in the experimental analysis + estimate residual th. error due to unknown finite corrections
Radiator function

\( \omega(x, x_\ell) \): probability density function that a dilepton system retains a fraction \( \sqrt{x} \) of its original invariant mass \( q_0^2 \) after bremsstrahlung

\[
\omega(x, x_\ell, \theta_K) = \omega_1(x, x_\ell, \theta_K) \theta(1 - x - x_*) + \omega_2(x, x_\ell, x_*) \delta(1 - x)
\]

- \( \omega_1 \): real emission
- \( x = q^2/q_0^2 \)
- \( x_\ell = m_\ell^2/q_0^2 \)
- \( x_* \): IR regulator
- \( \theta_K \): angle between \( K^{(*)} \) and the photon
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- \( \omega_1 \): real emission
- \( \omega_2 \): soft emission and virtual corrections, obtained from
  \[
  \int_{-1}^{1} d\cos \theta_K \int_{2x_\ell}^{1} dx \, \omega(x, x_\ell, \theta_K) = 1 + \mathcal{O}\left( \frac{\alpha}{\pi} \right)
  \]
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Implementation of the radiator into the non radiative spectrum

Double-differential decay width

\[ \frac{d^2 \Gamma}{dq_0^2 dx} (B \to K\ell\ell(\gamma)) = \mathcal{F}^{(0)}_K(q_0^2) \omega(x, x_\ell, \theta_K) \]

\( \mathcal{F}^{(0)}_K(q^2) \): non radiative spectrum of the decay \( B \to K\ell\ell \)

To obtain the radiative-spectrum we need to perform the following convolution

\[ \mathcal{F}^{\ell}_K(q^2) = \int_{q^2}^{q_0^2, \text{max}} dq_0^2 \frac{dq_0^2}{q_0^2} \mathcal{F}^{(0)}_K(q_0^2) \omega \left( \frac{q^2}{q_0^2}, \frac{2m_\ell^2}{q_0^2}, \theta_K \right) \]

where the kinematical region of integration depends on experimental cuts, namely \( m_B^{\text{rec}} \)
Modelling the $J/\Psi$

Non-perturbative spectrum

$$C_9(q^2) = C_9^{\text{pert}} + \kappa_\Psi \frac{q^2}{q^2 - m_\Psi^2 + im_\Psi \Gamma_\Psi}$$

- $C_9^{\text{pert}}$ ensures the behaviour at low $q^2$ region
- BW reproduces the presence of $J/\Psi$, $\kappa_\Psi$ normalised to $\mathcal{B}(B \to K^{(*)} J/\Psi)$
- relative phase between $C_9^{\text{pert}}$ and BW doesn’t affect the result
- we do not claim this is the "true" shape of the resonance, but still it is suitable toy to study the behaviour around the $J/\Psi$
- interference not included in PHOTOS-based models
$J/\Psi$ tail

$m^\text{rec}_B$: reconstructed mass of the $B$ meson from charged tracks

- **Key-variable**: $m^\text{rec}_B$, that determines the size of the effect of radiation we need to take in account.
$J/\Psi$ tail

$m_{B}^{\text{rec}}$: reconstructed mass of the $B$ meson from charged tracks

- **Key-variable**: $m_{B}^{\text{rec}}$, that determines the size of the effect of radiation we need to take in account

- even with the looser cut $m_{B}^{\text{rec}} = 4.880$ GeV the tail is safely above the interesting region $[1, 6]$ GeV$^2$
\( B \rightarrow K\ell\ell(\gamma) \) for \( 1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2 \)

\[
\begin{array}{c|cc}
\hline
\text{\( m_B^{\text{rec}} \)} & \ell = e & \ell = \mu \\
\hline
4.880 \text{ GeV} & -7.60\% & -1.8\% \\
5.175 \text{ GeV} & -16.9\% & -4.6\% \\
\hline
\end{array}
\]

- radiative correction can be sizable
$B \rightarrow K\ell\ell(\gamma)$ for $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$

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- due to the cuts applied in the analysis the overall effect is less important
$B \to K\ell\ell(\gamma)$ for $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$

- The estimate effect on $R_K$ is: $\Delta R_K = 3\%$
- The effect is the same for $R_{K^*}$
- Most important, we are in agreement at the $\%$ level with PHOTOS-based signal model
- The shift has already been taken into account in the analysis
- We associate a conservative error of $\pm 0.01$

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- Radiative correction can be sizable
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$q^2 \leq 1 \text{ GeV}^2$

1. Kinematic effect are non universal for electron and muons and they may cause distortion
   - the radiator for QED corrections must include the complete mass dependence
   - the error due to the form factors is not negligible

2. Light-quark resonances ($\eta, f_0, \ldots$) provide non-bremsstrahlung terms not included in PHOTOS-based signal model
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Main results:
- The theory uncertainty on \( R_{K^*} \) increases to a few %
- \( \mathcal{O}(1\%) \) negative shift in \( R_{K^*} \)
Non-bremsstrahlung effects

- The radiative tails is modified by the non-bremsstrahlung (direct emission) terms
- The effect is enhanced for the electron, due to kinematic effects and the cuts on $m_{B}^{\text{rec}}$
- Potentially sizeable effects due to

$$\mathcal{B}(B \rightarrow K^* \eta(\rightarrow e^+ e^- \gamma)) \sim 30\% \mathcal{B}(B \rightarrow K^* e^+ e^-, q^2 < 0.1 \text{ GeV}^2)$$
The $\eta$ case

Given the cuts on $q^2$ we estimate a shift of $\Delta R_{K^*} \sim -0.017$ to which we assign an error of $\mathcal{O}(100\%)$

$$R_{K^*}[0.045, 1.1]^{SM} = 0.906 \pm 0.020_{\text{QED}} \pm 0.020_{\text{FF}}$$
Non-bremsstrahlung effects

- The non-bremsstrahlung effects lead to a non-vanishing contribution to $R_{K^*}$ only near the threshold.
- If we look at the region $q^2 > 0.1 \text{ GeV}^2$ all these effects are negligible, and

$$R_{K^*}[0.1, 1.1]^{SM} = 0.983 \pm 0.010_{\text{QED}} \pm 0.010_{\text{FF}}$$

We recommend for future analysis to take in account the $q^2 \in [0.1, 1.1] \text{ GeV}^2$.
Summary

- In the central region $q^2 \in [1.1, 6]$ GeV$^2$
  \[ R_{K^*} = 1.00 \pm 0.01 \]
  where the uncertainty is driven by QED corrections

- In the $q^2 \in [0.045, 1.1]$ GeV$^2$ region
  \[ R_{K^*} = 0.91 \pm 0.03 \]
  where the central value is shifted by the presence of non-bremsstrahlung contribution due to light resonances and the uncertainty is due to QED and non vanishing form factor effects