Knowns & Unknowns in LFUV fits

based on arXiv: 1704.05447

in collaboration with

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ALSO SUPPORTED BY

NPFlavour
The Flavour of New Physics

erc
European Research Council
Funding for top researchers from anywhere in the world

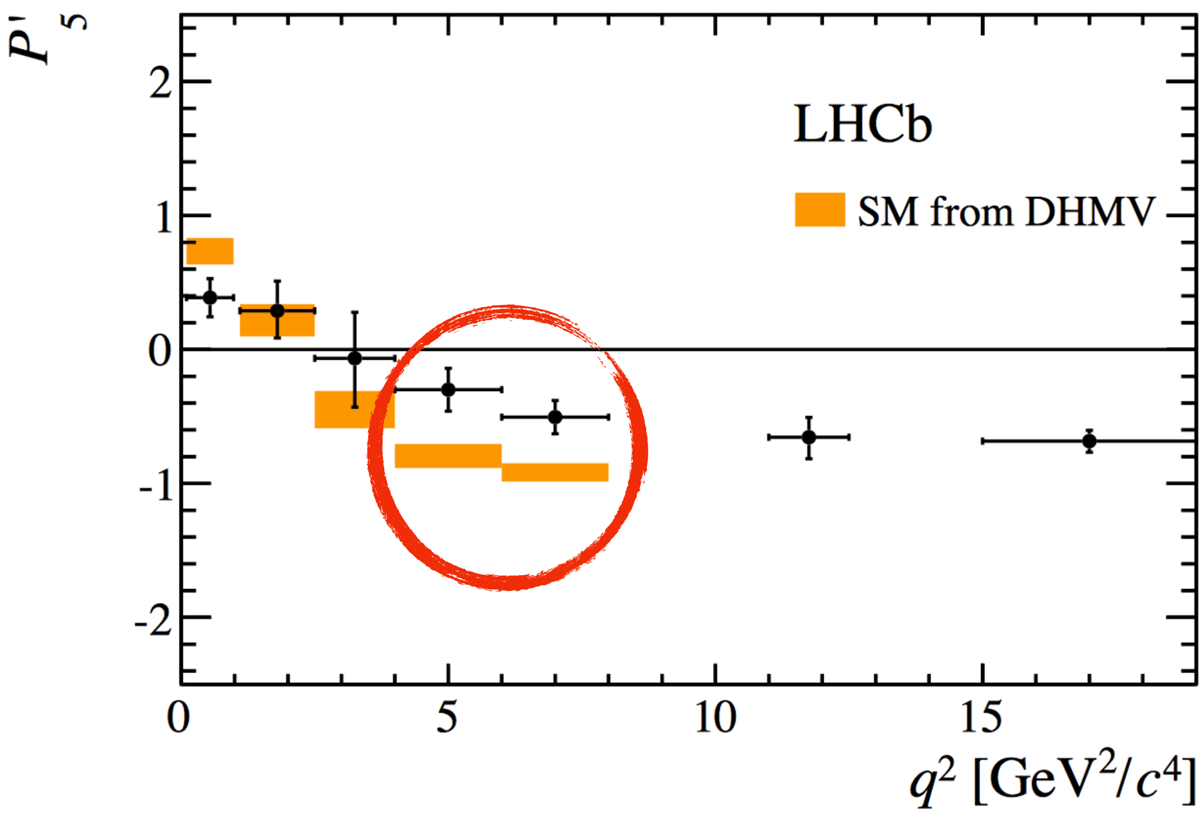
NOT SO LONG TIME BACK …

Talking about *B —> K*$\mu\mu$ channel & the P'$_5$ anomaly:*

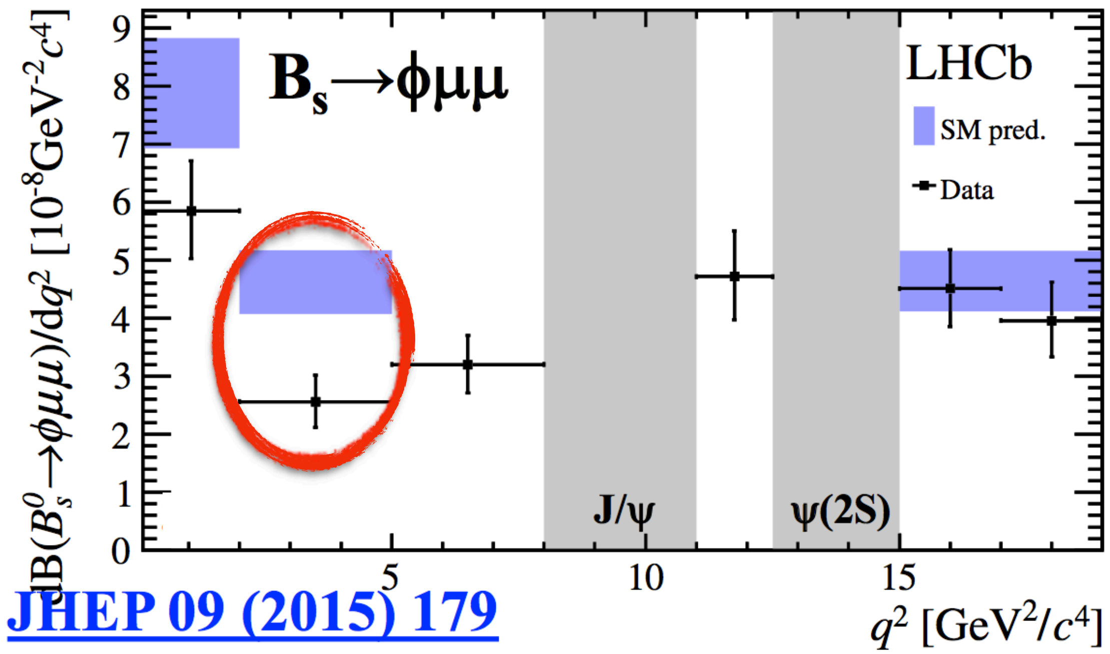
" Unless any other statistically significant anomaly shows up, from this single decay mode we cannot learn much about New Physics! "

In our analysis of this exclusive decay, anomalies disappear when one conservatively (gu)e(s)stimates non-factorizable QCD power corrections.

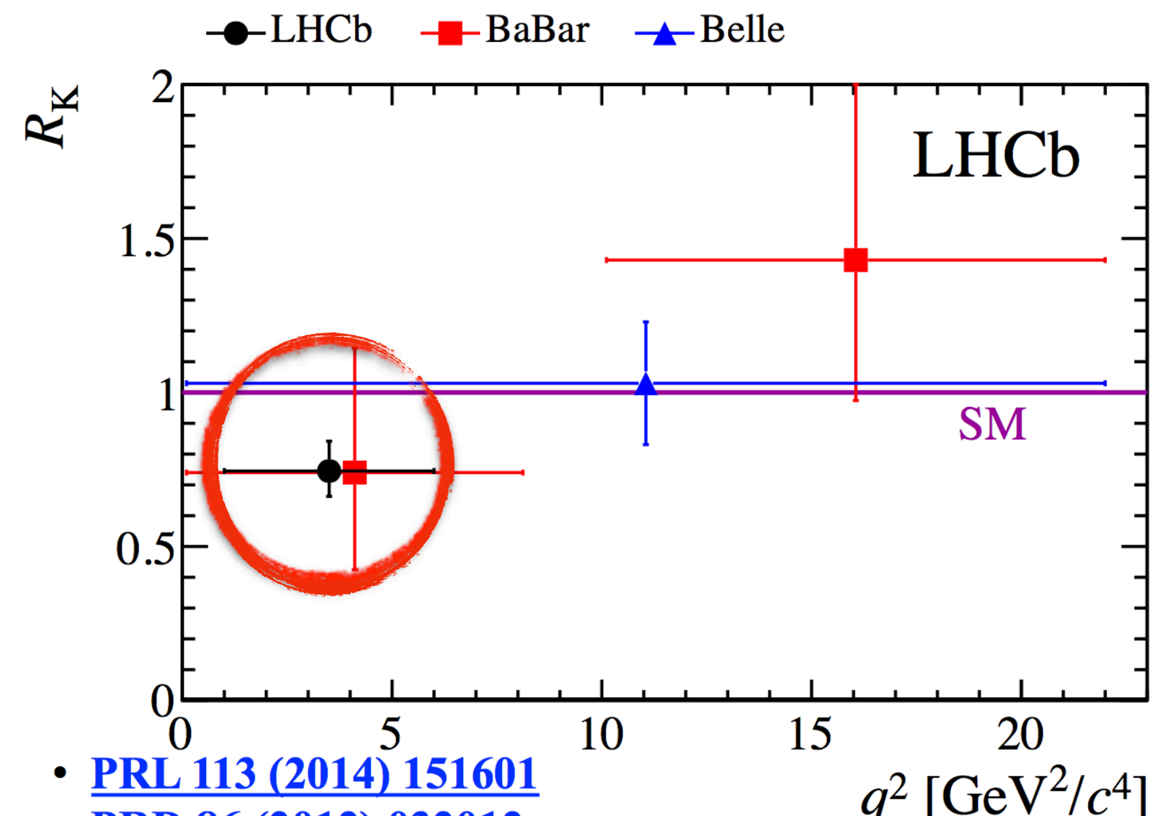
Ciuchini et al. , *JHEP 1606 (2016) 116*



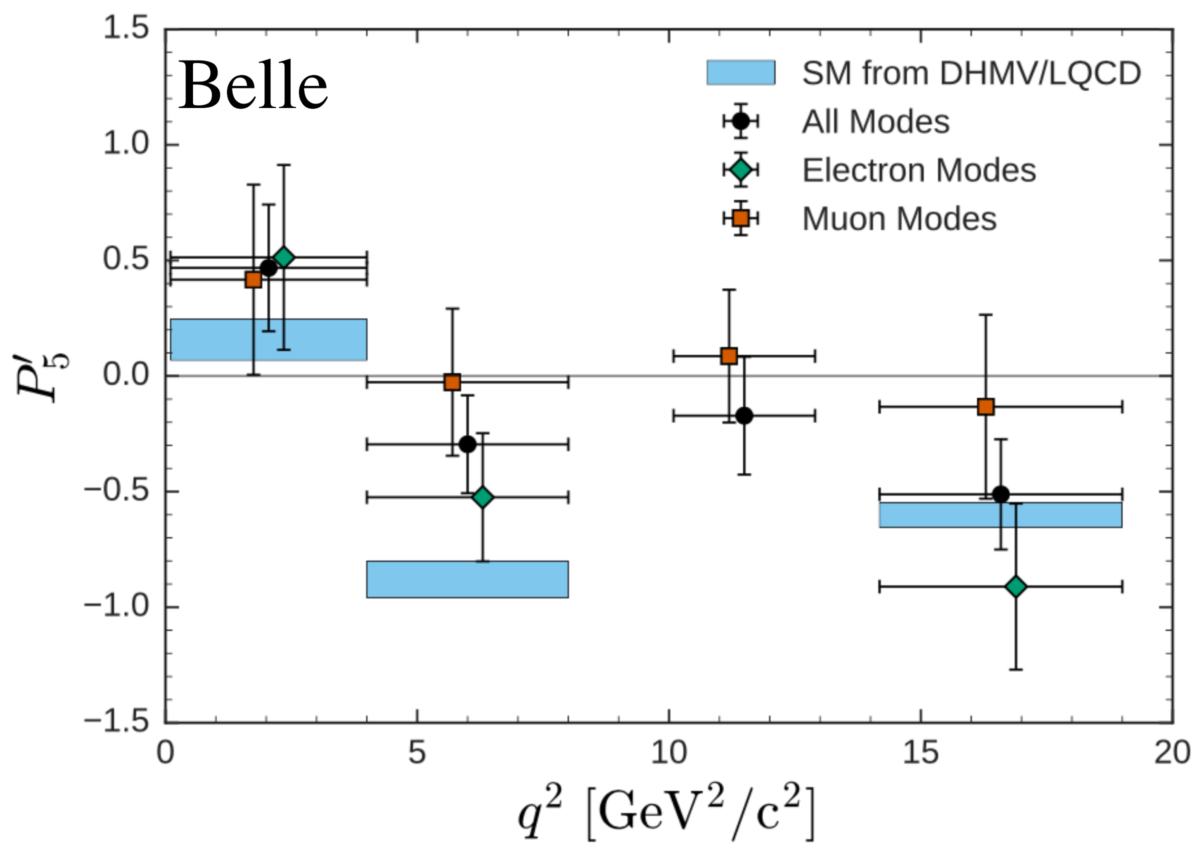
JHEP 1602 (2016) 104



JHEP 09 (2015) 179



- PRL 113 (2014) 151601
- PRD 86 (2012) 032012
- PRL 103 (2009) 171801



PRL 118 (2017) 111801

SEE NOW: 1705.05802 !

LHCb Preliminary	low-q^2	central-q^2
$\mathcal{R}_{K^{*0}}$	$0.660\ ^{+\ 0.110}_{-\ 0.070} \pm 0.024$	$0.685\ ^{+\ 0.113}_{-\ 0.069} \pm 0.047$
95% CL	[0.517–0.891]	[0.530–0.935]
99.7% CL	[0.454–1.042]	[0.462–1.100]

UPDATE: CURRENT SITUATION MAY LOOK LIKE THIS

i.e., my collaborators & I harmlessly lying on the ground … OR MAYBE NOT?

$$R^{\mathrm{LHCb}}_{K^{(*)},[1(.1),6]} < 1$$

$$Q^{\mathrm{Belle}}_{5,[4,8]} > 0$$

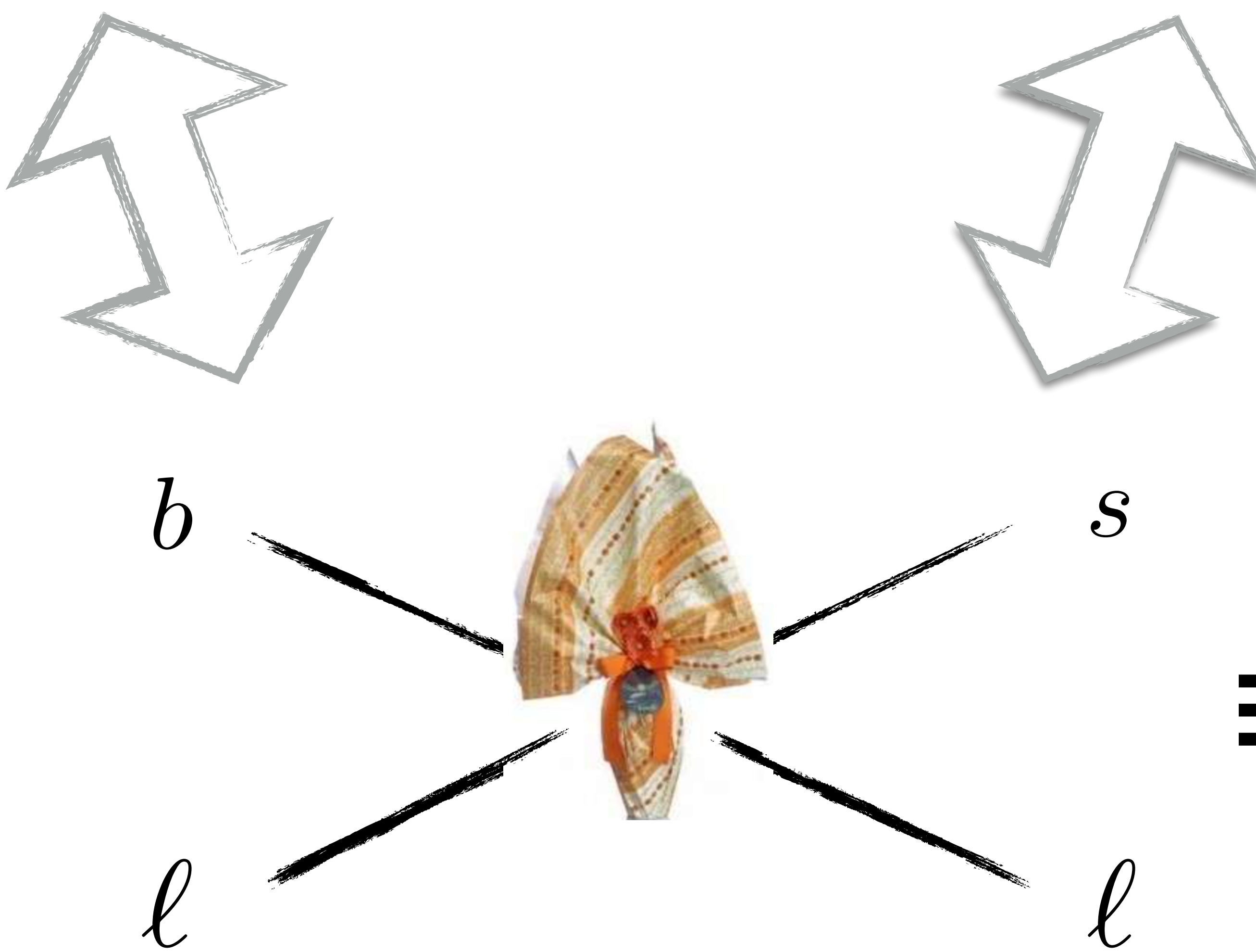
Barcelona $C_{9\mu}$

… INDEED:

Is evidence for New Physics (NP) in Q_{9V} operator indisputable?

What is its significance @ present?

Main content of this ~10 min contribution

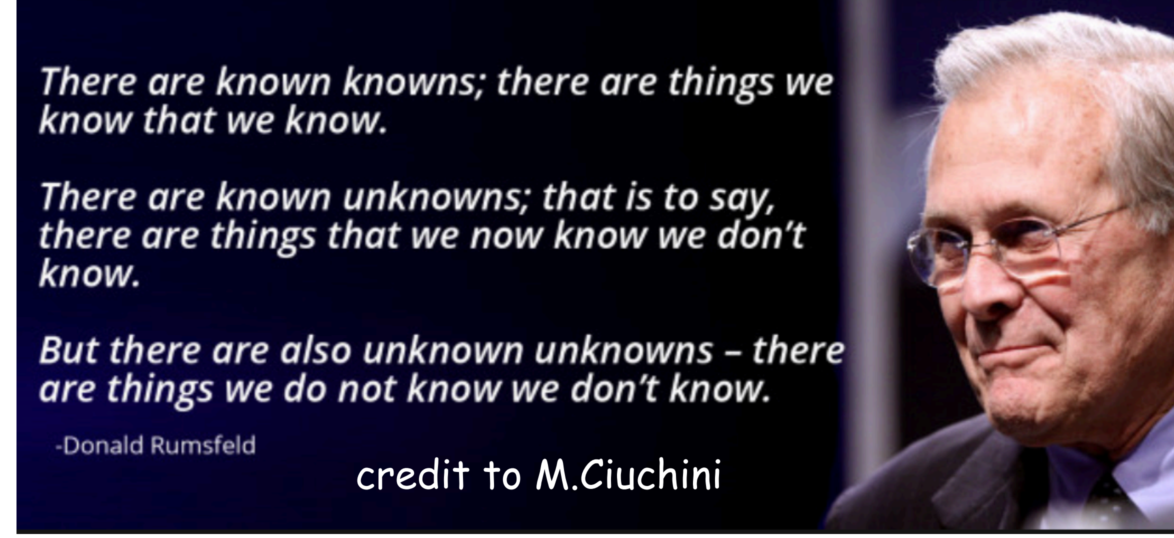


$\equiv$ " Flavourful Easter eggs "

Helicity (λ) amplitudes relevant in this study:

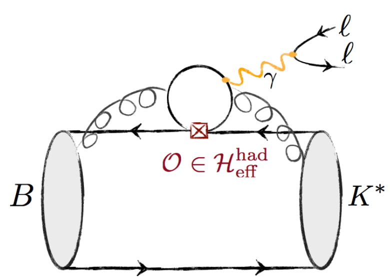
$$H_V(\lambda) \propto C_9 \tilde{V}_\lambda + \frac{2 m_b m_B}{q^2} C_7 \tilde{T}_\lambda - \frac{16\pi^2 m_B^2}{q^2} h_\lambda ,$$

$$H_A(\lambda) \propto C_{10} \tilde{V}_\lambda , \; H_P \propto \frac{2 m_\ell m_B}{q^2} C_{10} \left(1 + \frac{m_s}{m_B} \right) \tilde{S} .$$



The known knowns: (form factors) QCD sum rules on light-cone (LCSR) at large recoil in agreement with extrapolated Lattice results at low recoil.

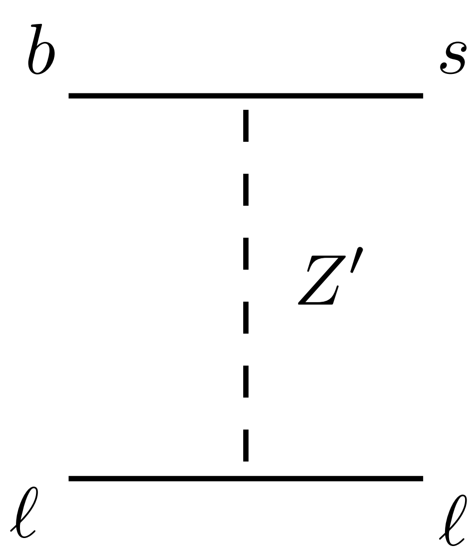
The known unknowns: (long-distance effects) non-factorizable power corrections to the amplitude of exclusive decay modes; estimates from LCSR may suggest small effects in B to K amplitude, but not in B to K* one!

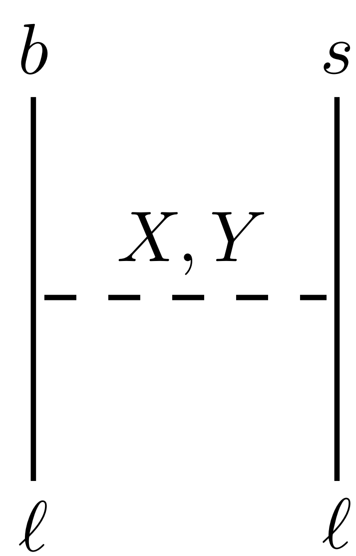


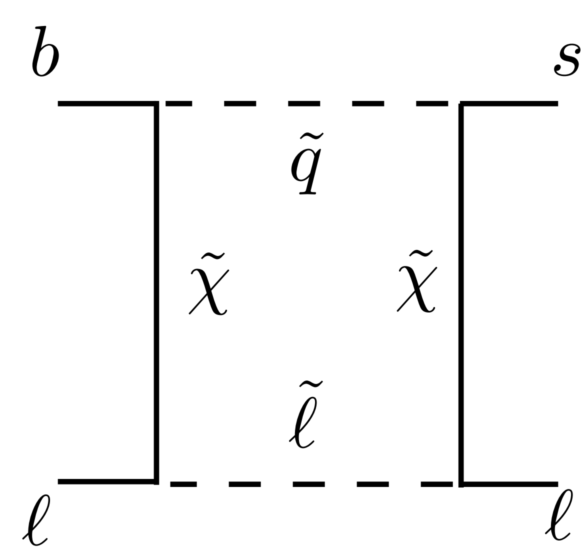
—> possible degeneracy in H_V with NP in $Q_{7\gamma}$ & Q_{9V}

See Jäger & Virto's talks for further details …

The unknowns (not so) unknown …

 = 

, 

, 

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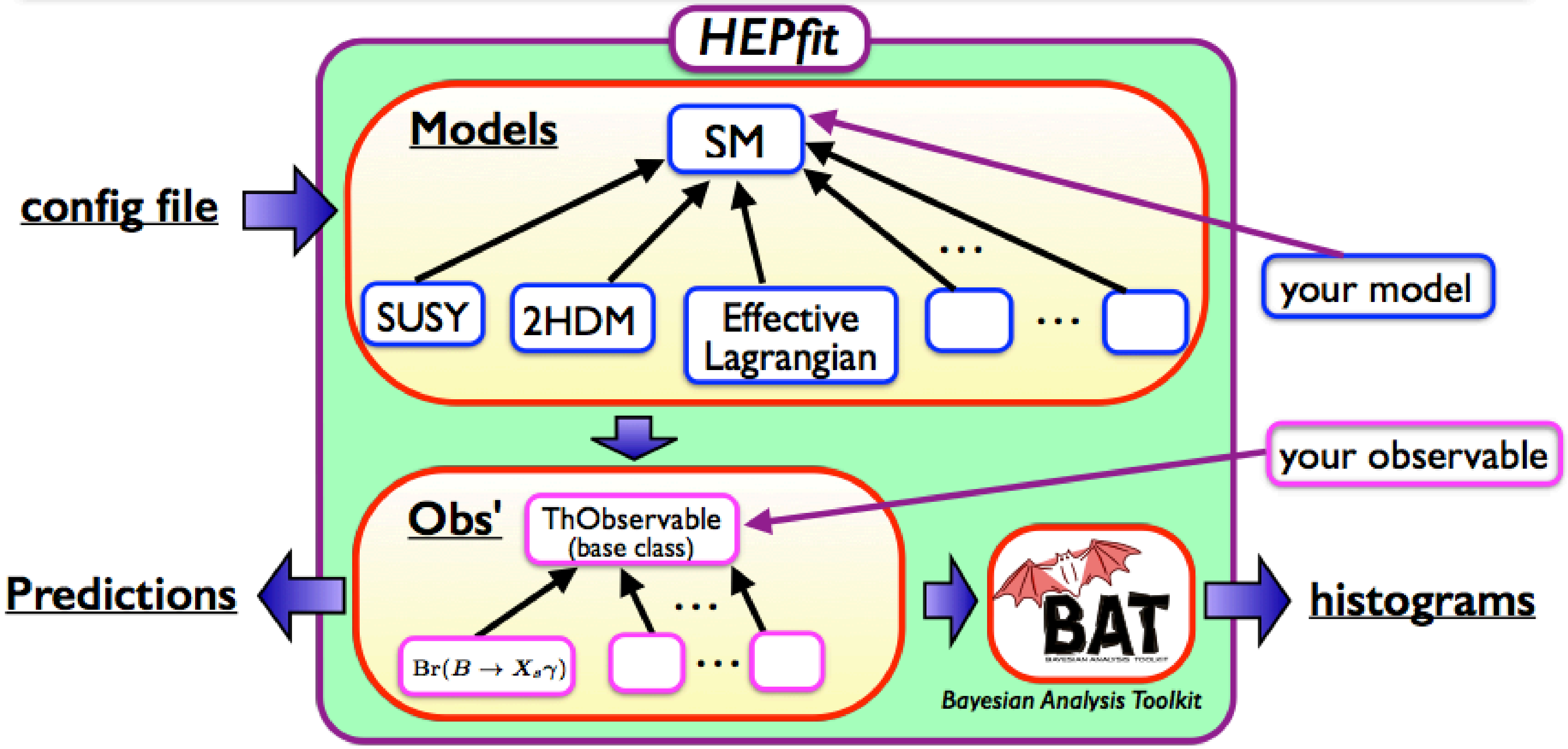


public@http://hepfit.roma1.infn.it

Higgs & Electroweak Precision Tests

PoS ICHEP2016 (2017) 690

Nucl.Part.Phys.Proc. 273-275 (2016)

JHEP 1612 (2016) 135

JHEP 1308 (2013) 106

Flavour Physics

Nucl.Part.Phys.Proc. 285-286 (2017)

PoS ICHEP2016 (2016) 584

JHEP 1606 (2016) 116

Our global analysis is carried out by means of Bayesian inference.

(see e.g. *10.5170/CERN-99-03 — CERN Yellow Reports* — G.D'Agostini)

We use the Information Criterion for Predictive Bayesian Model Selection:

$$IC = -2\overline{\log L} + 4\sigma^2_{\log L}$$

T. Ando, "Predictive Bayesian Model Selection", AJMMS 31 (2011)

1st term —> *how well model fits data*

2nd term —> *penalty on model complexity*

$\Delta IC \simeq \ln \frac{\mathcal{P}(\text{data}\|M_1)}{\mathcal{P}(\text{data}\|M_2)}$	Evidence against higher $IC\,(M_1)$
0 to 2	about bare mention
2 to 6	positive/substantial
6 to 10	strong
> 10	very strong/decisive

Kass & Raftery, "Bayes Factors", JASS 90 (1995) 430

MCMC with **total of 93 SM parameters** with Gaussian or flat prior distribution.

<u>DATA INCLUDED</u>: MEASUREMENTS RELATED TO $K^{(*)}$ & PHI IN LARGE RECOIL REGION ONLY + B_S TO MU MU & B TO X_S GAMMA (**LHCB, BELLE, ATLAS, CMS**, SEE BACKUP!).

About known knowns ...

Form factors from **LCSR &/or Lattice**, with correlations.

JHEP 1608 (2016) 098
Bharucha et al.
PRD 93 (2016) 025026
Bailey et al.

... and known unknowns

Non-factorizable part of *B* to *K*ll* amplitude parametrized as follows:

Ciuchini et al., *JHEP 1606 (2016) 116*

$$h_\lambda(q^2) = h_\lambda^{(0)} + \frac{q^2}{1\,\mathrm{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\,\mathrm{GeV}^4} h_\lambda^{(2)}$$

$$\left|h_{0,\pm}^{(0,1,2)}\right| \in [0, 2\cdot 10^{-3}]$$

$$Arg\left(h_{0,\pm}^{(0,1,2)}\right) \in [0, 2\pi)$$

also used as a proxy also for $B_s \longrightarrow \phi\,\mu\,\mu$ mode.

i.e. flat priors with large ranges

Unknown unknowns in ΔB=1 EFT

2 different approaches implemented

$$C_7^{NP}, C_{9,e}^{NP}, C_{9,\mu}^{NP}, C_{10,e}^{NP}, C_{10,\mu}^{NP}$$

<u>PMD approach</u>, LCSR on cc-loop + dispersion relation in whole large recoil region
PHENOMENOLOGICAL MODEL DRIVEN

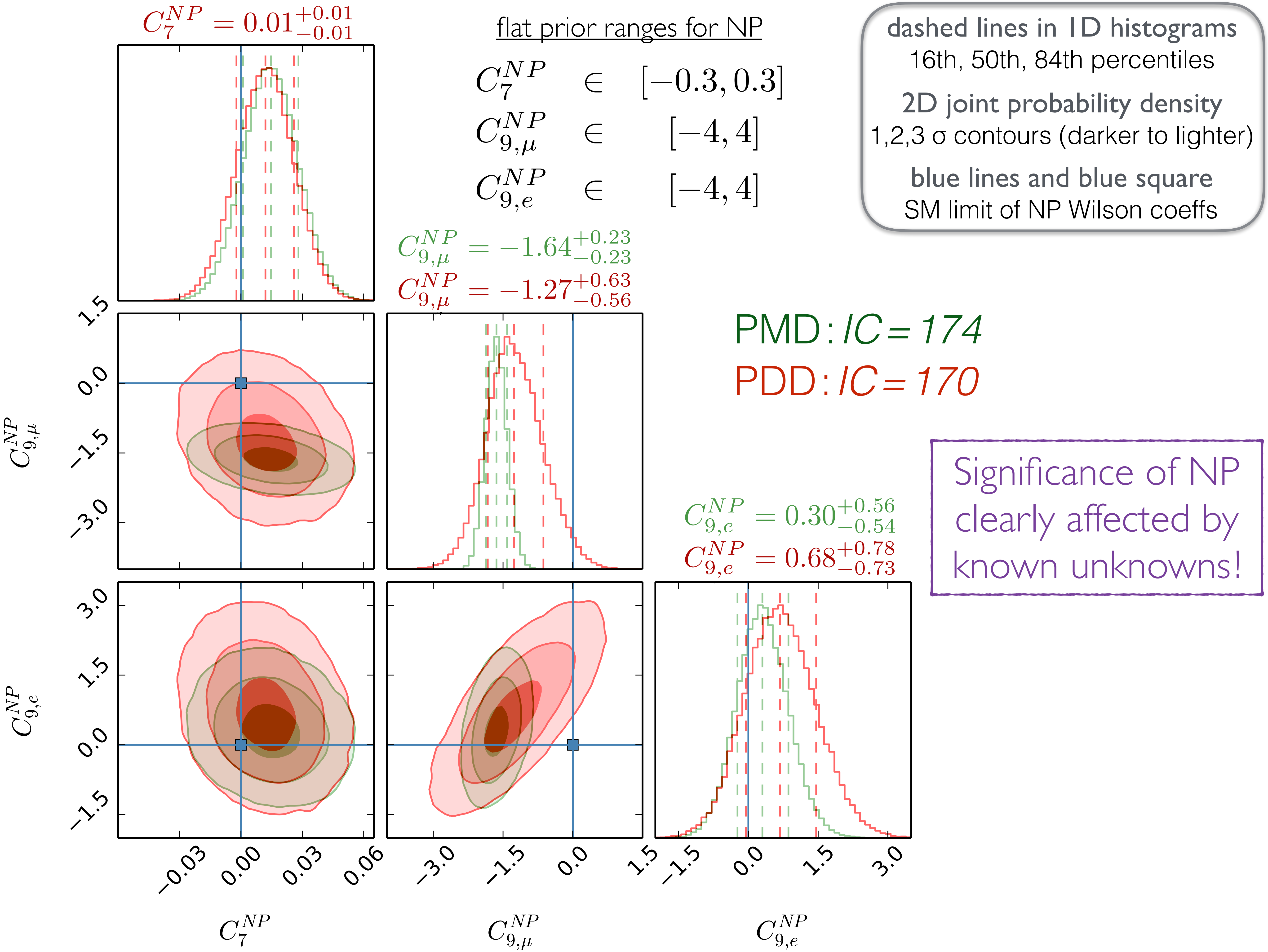
<u>PDD approach</u>, LCSR result enforced only nearby the light-cone, i.e. $q^2 \leq 1$ GeV^2
PHENOMENOLOGICAL DATA DRIVEN

Khodjamirian et al., *JHEP 1009 (2010) 089*

NP results — Part I —

"Chasing well-known Easter eggs!"

Q : What is the significance of $C_{9,\mu}^{NP} \neq 0$?

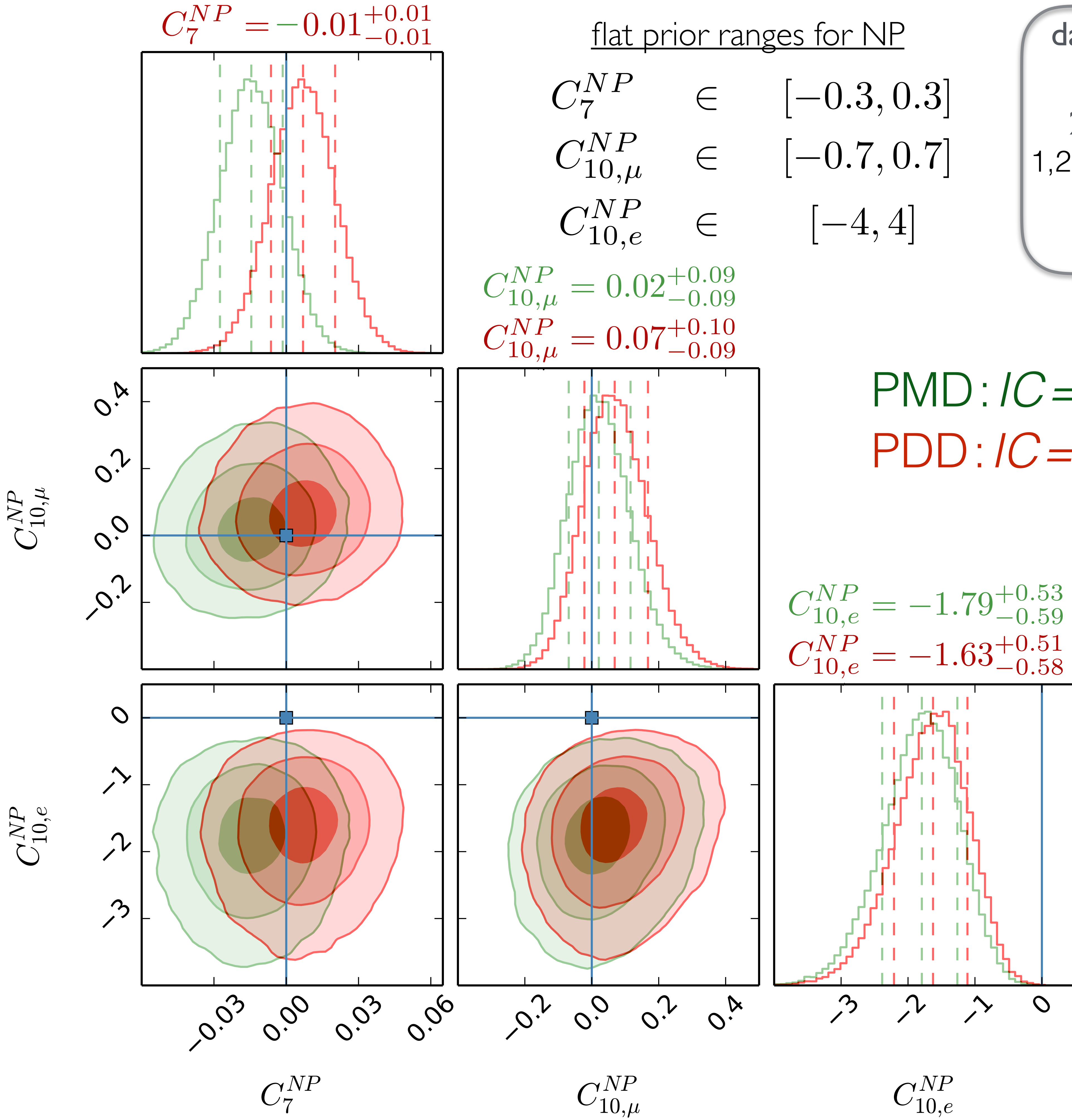


$C_7^{NP} = 0.01^{+0.01}_{-0.01}$
flat prior ranges for NP
$C_7^{NP} \in [-0.3, 0.3]$
$C_{9,\mu}^{NP} \in [-4, 4]$
$C_{9,e}^{NP} \in [-4, 4]$
dashed lines in 1D histograms
16th, 50th, 84th percentiles
2D joint probability density
1,2,3 σ contours (darker to lighter)
blue lines and blue square
SM limit of NP Wilson coeffs
$C_{9,\mu}^{NP} = -1.64^{+0.23}_{-0.23}$
$C_{9,\mu}^{NP} = -1.27^{+0.63}_{-0.56}$
PMD : IC = 174
PDD : IC = 170
Significance of NP clearly affected by known unknowns!
$C_{9,e}^{NP} = 0.30^{+0.56}_{-0.54}$
$C_{9,e}^{NP} = 0.68^{+0.78}_{-0.73}$
$C_{9,\mu}^{NP}$
$C_{9,e}^{NP}$
1.5
0.0
−1.5
−3.0
3.0
−0.03
0.00
0.03
0.06
C_7^{NP}
$C_{9,\mu}^{NP}$
$C_{9,e}^{NP}$

NP results — Part II —

"Digging in ~no man's land …"

Q : Is NP in $Q_{9V,\mu}$ the « only way to go » ?



flat prior ranges for NP

$$C_7^{NP} \in [-0.3, 0.3]$$
$$C_{10,\mu}^{NP} \in [-0.7, 0.7]$$
$$C_{10,e}^{NP} \in [-4, 4]$$

dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs

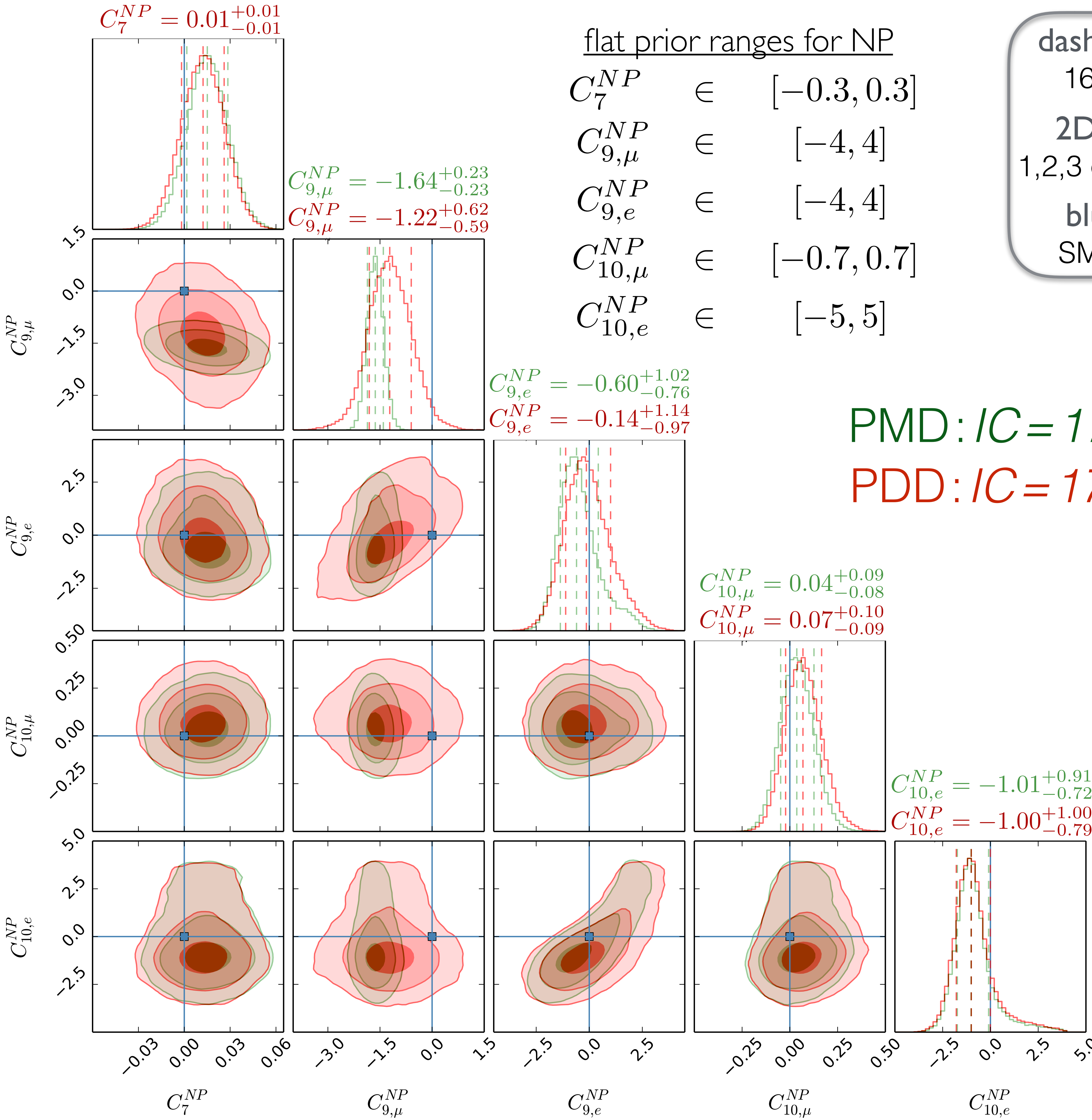
PMD : *IC = 221*

PDD : *IC = 172*

"Disfavoured" NP scenarios allowed by a more careful treatment of non-factorizable QCD power corrections.

NP results — Part III —

*** The Bayesian State Of Mind ***



flat prior ranges for NP

C_7^{NP}	$\in$	$[-0.3, 0.3]$
$C_{9,\mu}^{NP}$	$\in$	$[-4, 4]$
$C_{9,e}^{NP}$	$\in$	$[-4, 4]$
$C_{10,\mu}^{NP}$	$\in$	$[-0.7, 0.7]$
$C_{10,e}^{NP}$	$\in$	$[-5, 5]$

dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs

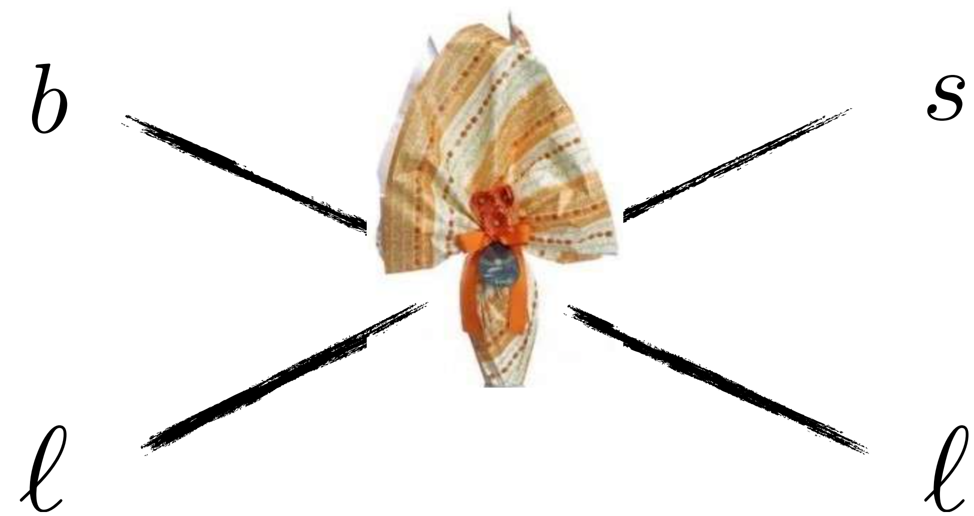
PMD: *IC = 178*

PDD: *IC = 174*

NP significance boils down by the interplay among **$C_{9\mu}$, C_{9e} & C_{10e}** when we leave room for more generous hadronic contributions.

Summary & Conclusions

Size of arrows proportional to money I may bet on …



Size of arrows proportional to amount of money some of my collaborators might put on …

NOVEL NP SCENARIOS ARISE WHEN WE TAKE A MORE CONSERVATIVE APPROACH FOR POORLY ESTIMATED HADRONIC UNCERTAINTIES.

E.g.: *Electronic Axial NP models*

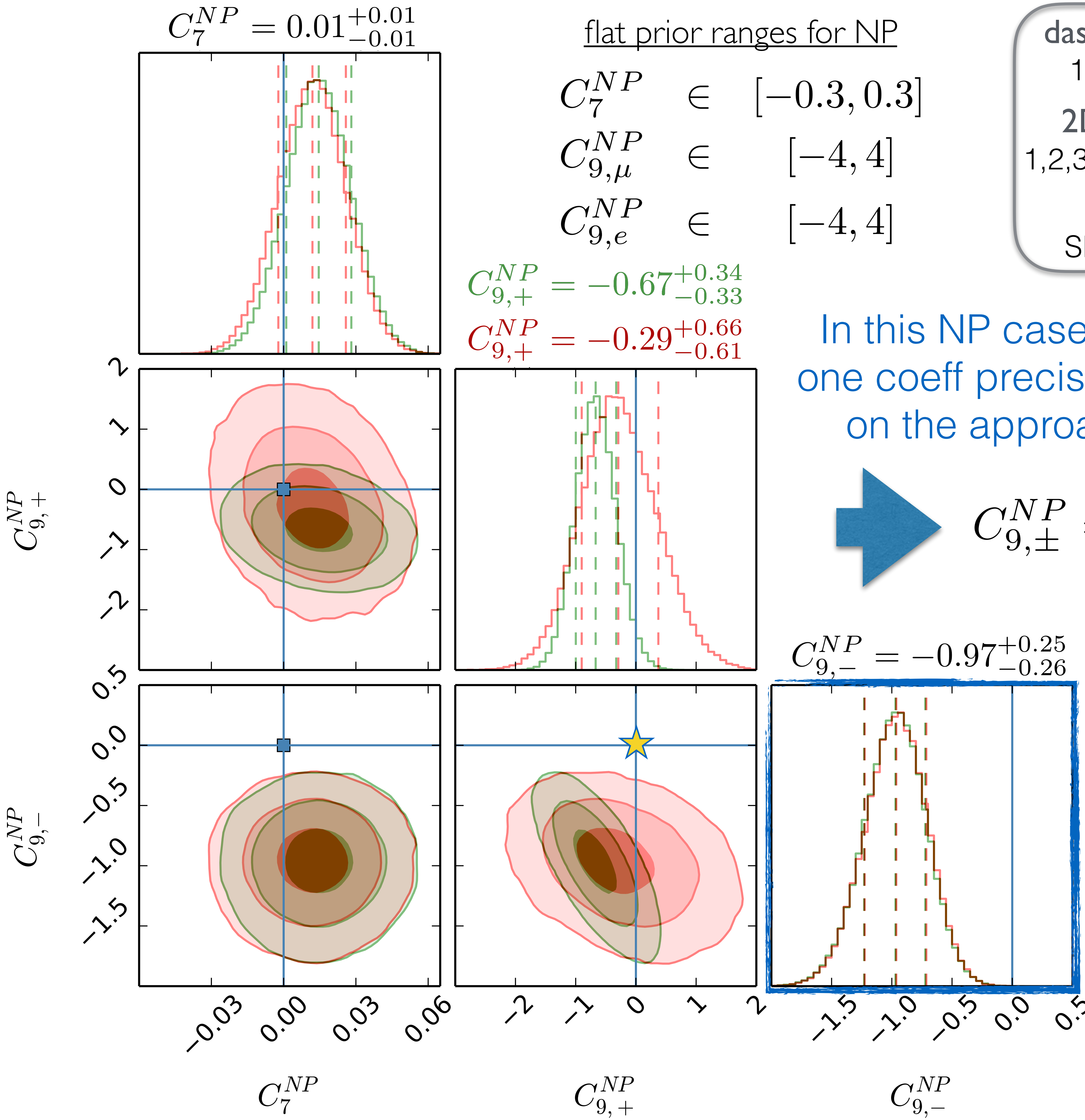
SIGNIFICANCE (~2σ TO ~7σ) OF NP IN THE MUONIC VECTOR CURRENT DEPENDS ON ESTIMATED HADRONIC UNCERTAINTIES.

On general grounds it is the NP scenario preferred by current data.

WE MAY BE SITTING IN FRONT OF ~2σ-3σ NP EVIDENCE …

I.E., AS OF NOW, THE SM PREDICTION IS COMPATIBLE AT ~2.5σ LEVEL WITH R_K & R_{K^*}.

Backups



flat prior ranges for NP

$$C_7^{NP} \in [-0.3, 0.3]$$
$$C_{9,\mu}^{NP} \in [-4, 4]$$
$$C_{9,e}^{NP} \in [-4, 4]$$

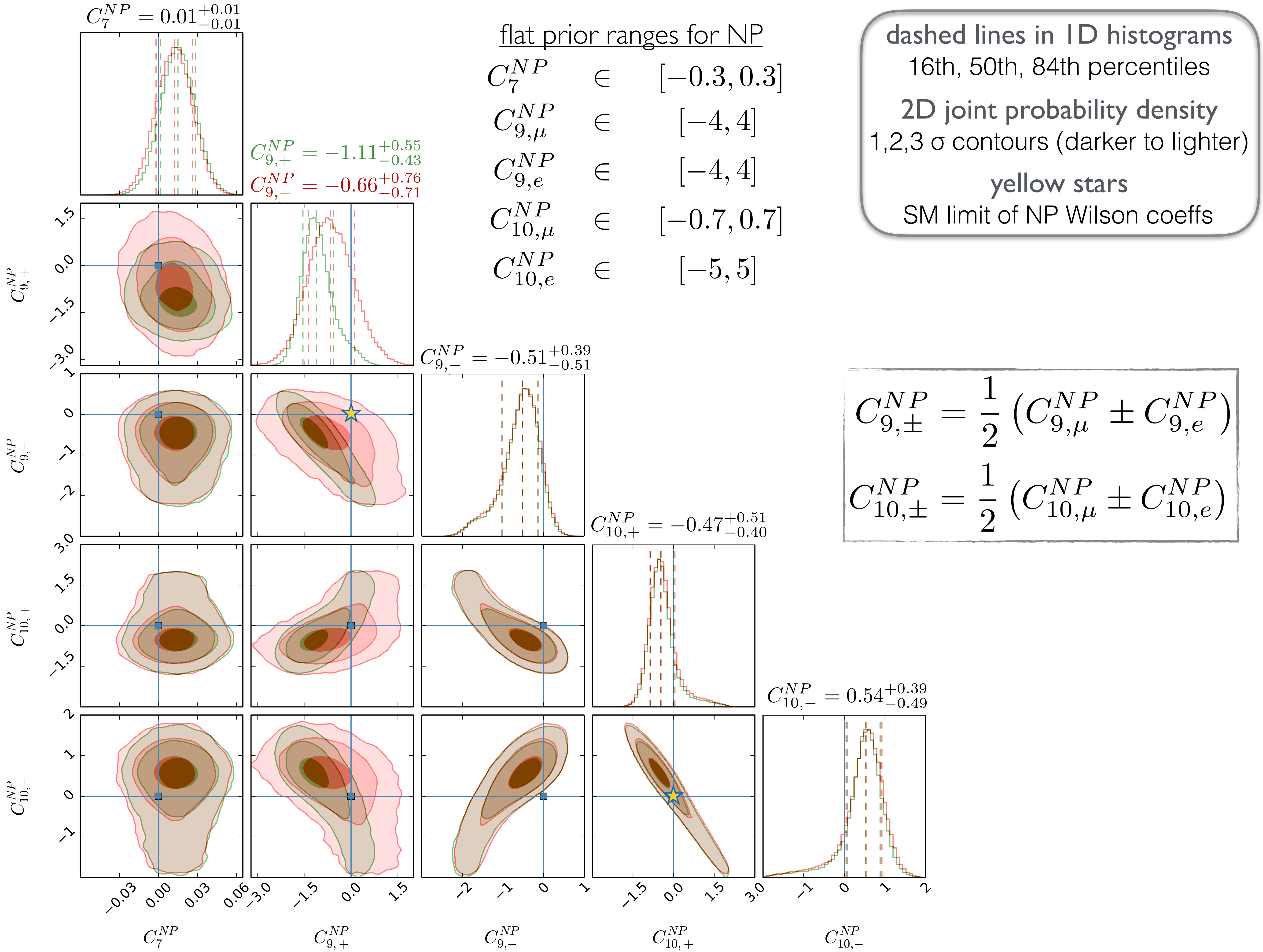
dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

yellow star
SM limit of NP Wilson coeffs

In this NP case we are constraining one coeff precisely, i.e. independently on the approach adopted for h_λ!

$$C_{9,\pm}^{NP} = \frac{1}{2}\left(C_{9,\mu}^{NP} \pm C_{9,e}^{NP}\right)$$



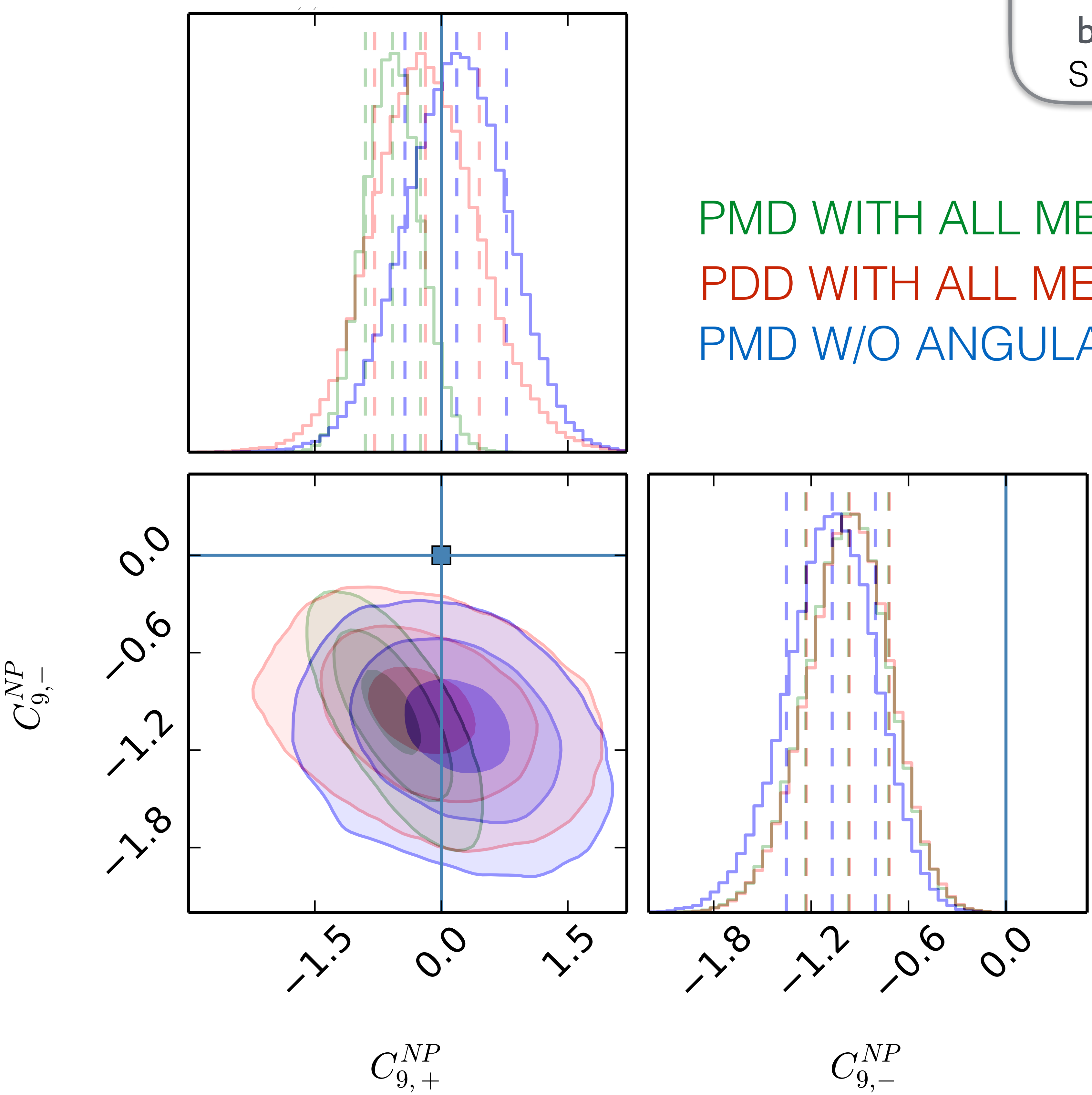
$C_7^{NP} = 0.01^{+0.01}_{-0.01}$
$C_{9,+}^{NP} = -1.11^{+0.55}_{-0.43}$
$C_{9,+}^{NP} = -0.66^{+0.76}_{-0.71}$
$C_{9,-}^{NP} = -0.51^{+0.39}_{-0.51}$
$C_{10,+}^{NP} = -0.47^{+0.51}_{-0.40}$
$C_{10,-}^{NP} = 0.54^{+0.39}_{-0.49}$
C_7^{NP}
$C_{9,+}^{NP}$
$C_{9,-}^{NP}$
$C_{10,+}^{NP}$
$C_{10,-}^{NP}$
flat prior ranges for NP
$C_7^{NP} \in [-0.3, 0.3]$
$C_{9,\mu}^{NP} \in [-4, 4]$
$C_{9,e}^{NP} \in [-4, 4]$
$C_{10,\mu}^{NP} \in [-0.7, 0.7]$
$C_{10,e}^{NP} \in [-5, 5]$
dashed lines in 1D histograms
16th, 50th, 84th percentiles
2D joint probability density
1,2,3 σ contours (darker to lighter)
yellow stars
SM limit of NP Wilson coeffs
$C_{9,\pm}^{NP} = \frac{1}{2}\left(C_{9,\mu}^{NP} \pm C_{9,e}^{NP}\right)$
$C_{10,\pm}^{NP} = \frac{1}{2}\left(C_{10,\mu}^{NP} \pm C_{10,e}^{NP}\right)$

Fitting w/o any of the Angular Observables.

dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs



PMD WITH ALL MEASUREMENTS

PDD WITH ALL MEASUREMENTS

PMD W/O ANGULAR OBS DATA

Exploring NP effects only in the muon channel.

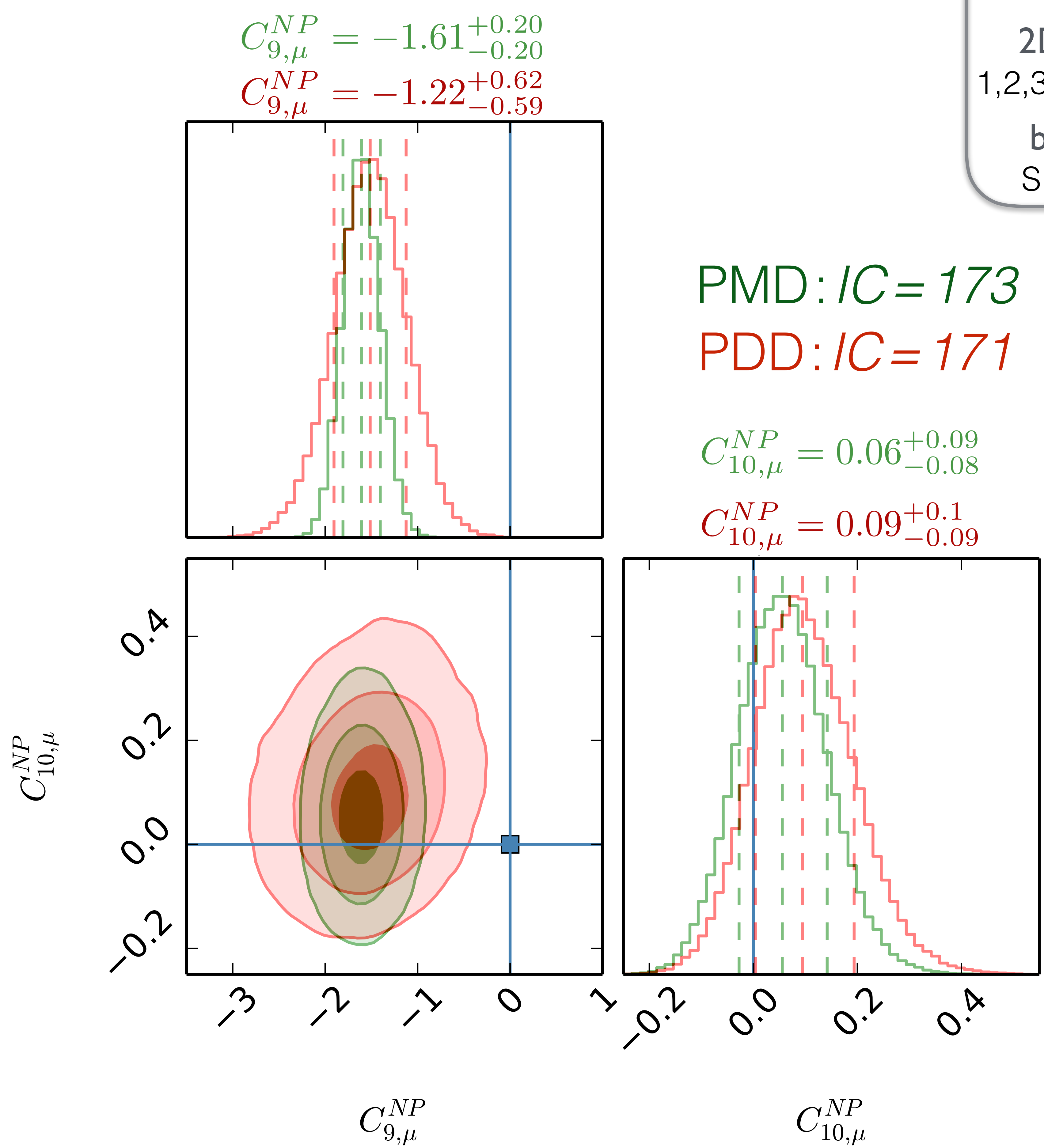
dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs

PMD: $IC = 173$

PDD: $IC = 171$

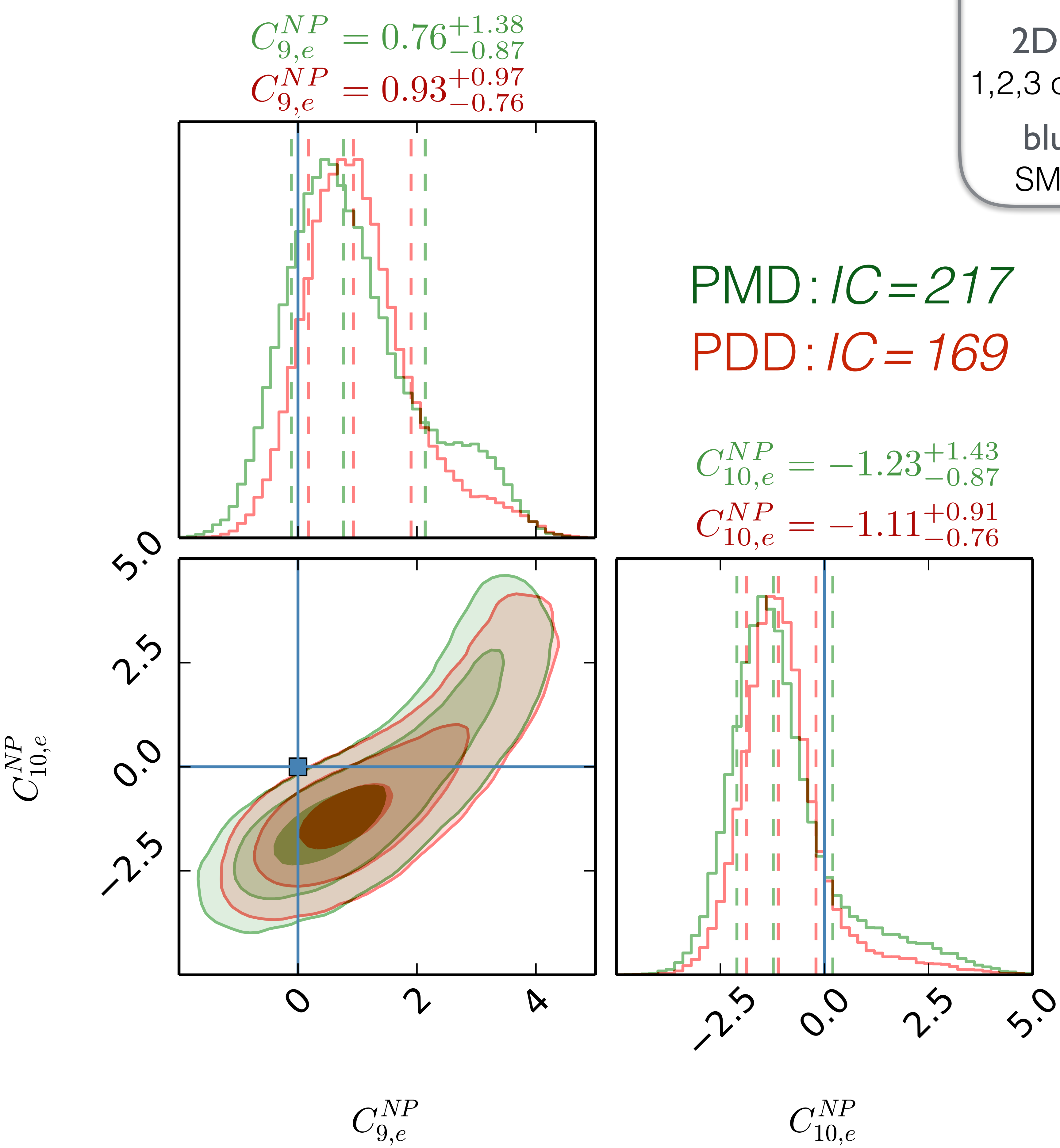


Exploring NP effects only in the electron channel.

dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs

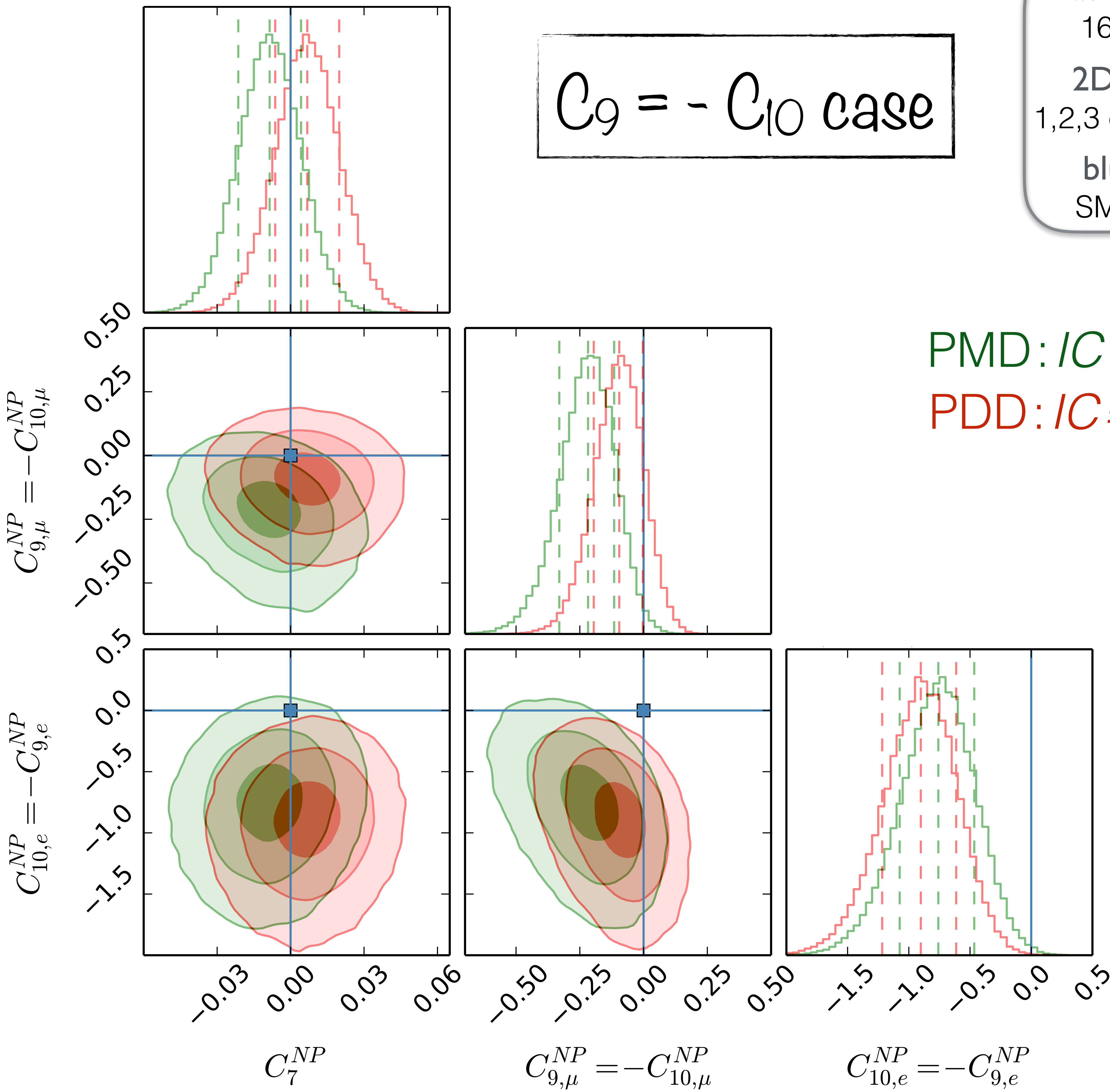


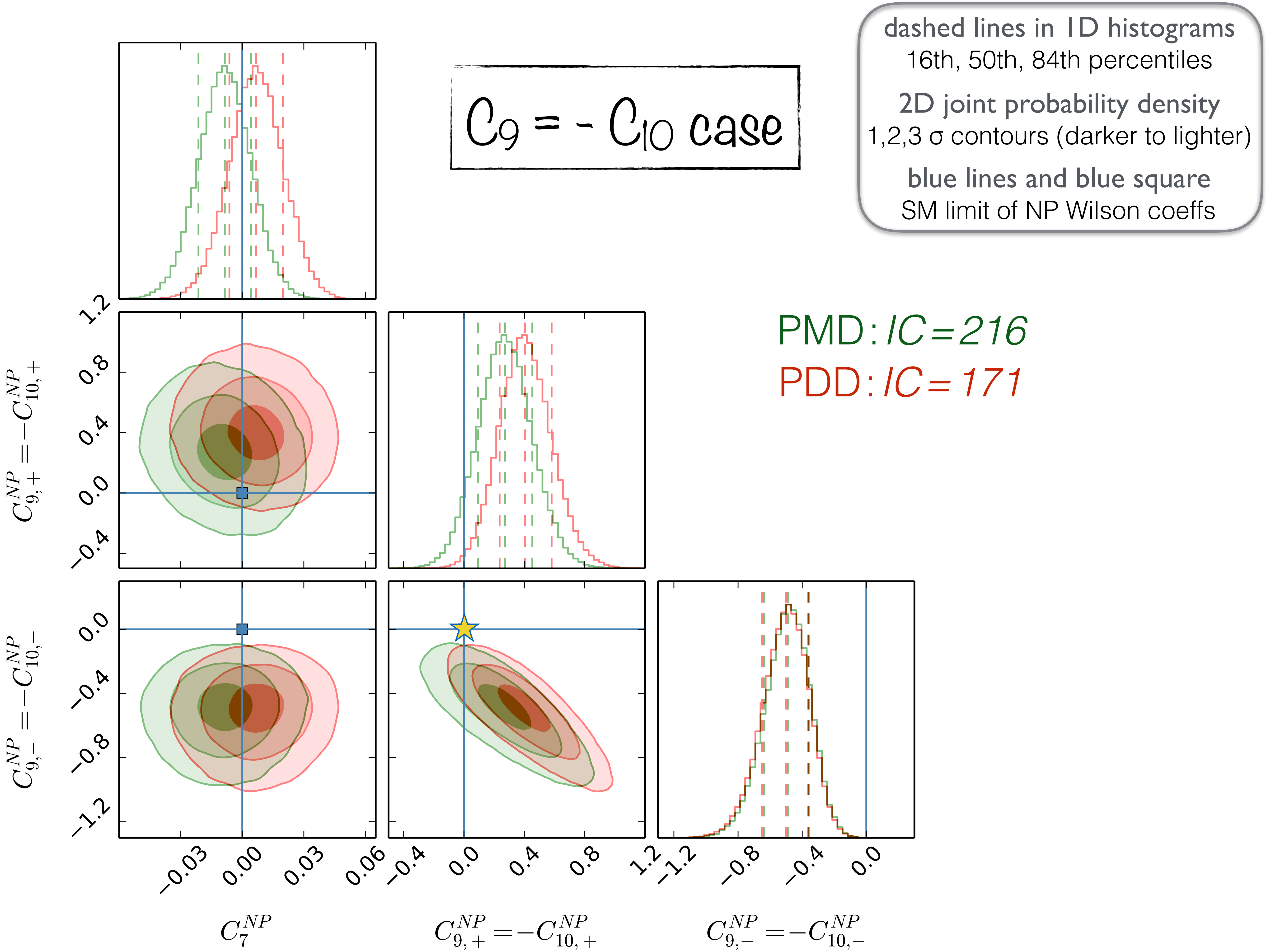
C_9 = - C_{10} case

dashed lines in 1D histograms
16th, 50th, 84th percentiles

2D joint probability density
1,2,3 σ contours (darker to lighter)

blue lines and blue square
SM limit of NP Wilson coeffs




dashed lines in 1D histograms
16th, 50th, 84th percentiles
2D joint probability density
1,2,3 σ contours (darker to lighter)
blue lines and blue square
SM limit of NP Wilson coeffs
C9 = - C10 case
PMD: IC=216
PDD: IC=171
$C_{9,+}^{NP} = -C_{10,+}^{NP}$
$C_{9,-}^{NP} = -C_{10,-}^{NP}$
C_7^{NP}
$C_{9,+}^{NP} = -C_{10,+}^{NP}$
$C_{9,-}^{NP} = -C_{10,-}^{NP}$

Vector-like + C_7 NP case

PMD approach

$$\overline{\log L} = -70.9\,,\ \sigma^2_{\log L} = 8.1\,,\ IC \simeq 174$$

PDD approach

$$\overline{\log L} = -68.4\,,\ \sigma^2_{\log L} = 8.3\,,\ IC \simeq 170$$

(IC less than ~2 units w/o C_7)

Axial-like + C_7 NP case

PMD approach

$$\overline{\log L} = -87.0\,,\ \sigma^2_{\log L} = 11.8\,,\ IC \simeq 221$$

PDD approach

$$\overline{\log L} = -69.1\,,\ \sigma^2_{\log L} = 8.6\,,\ IC \simeq 172$$

(IC less than ~2 units w/o C_7)

Bayesian NP case

PMD approach

$$\overline{\log L} = -70.8\,,\ \sigma^2_{\log L} = 9.0\,,\ IC \simeq 178$$

PDD approach

$$\overline{\log L} = -68.4\,,\ \sigma^2_{\log L} = 9.2\,,\ IC \simeq 174$$

Purely Muonic NP case

PMD approach

$$\overline{\log L} = -70.7\,,\ \sigma^2_{\log L} = 7.9\,,\ IC \simeq 173$$

PDD approach

$$\overline{\log L} = -68.6\,,\ \sigma^2_{\log L} = 8.5\,,\ IC \simeq 171$$

Purely Electronic NP case

PMD approach

$$\overline{\log L} = -86.7\,,\ \sigma^2_{\log L} = 10.8\,,\ IC \simeq 217$$

PDD approach

$$\overline{\log L} = -68.2\,,\ \sigma^2_{\log L} = 8.2\,,\ IC \simeq 169$$

~$SU(2)_L$ NP case

PMD approach

$$\overline{\log L} = -85.8\,,\ \sigma^2_{\log L} = 11.1\,,\ IC \simeq 216$$

PDD approach

$$\overline{\log L} = -68.3\,,\ \sigma^2_{\log L} = 8.7\,,\ IC \simeq 171$$

Set of measurements included in the present analysis

LHCb	$F_L, A_{FB}, S_{3,4,5,7,8,9}$ i.e. available angular info for $K^{(*)}, \phi$ modes $\mathcal{B}(B \to K^{(*)}\ell\ell, \gamma)$ $\mathcal{B}(B_s \to \phi\,\mu\mu, \gamma)$ $R_{K,[1,6]}, R_{K^*,[0.045,1.1],[1.1,6]}$	JHEP 1611 (2016) 047 JHEP 1602 (2016) 104 JHEP 1509 (2015) 179 JHEP 1504 (2015) 064 Nucl.Phys. B867 (2013) 1-18 PRL 113 (2014) 151601 indico.cern.ch/event/580620/
ATLAS	$F_L, A_{FB}, S_{3,4,5,7,8}$	ATLAS-CONF-2017-023
CMS	$P_1, P_5', F_L, A_{FB}, \mathcal{B}(B \to K^*\mu\mu)$	CMS-PAS-BPH-15-008 twiki.cern/…/CMSPublic/…
Belle	$P'_{5\,(\mu,e)}$	PRL 118 (2017) 111801

We use data in the large recoil region only, i.e. where anomalies show up.

We take into account correlation matrices when experimentally provided.

LHCb, HFAG	$\mathcal{B}(B_s \to \mu\mu), \mathcal{B}(B \to X_s\gamma)$	LHCB-PAPER-2017-001 FERMILAB-PUB-16-611-ND

Effective Field Theory of Weak Interactions for b —> s l l transitions

$$Q_1^{q=u,c} = (\bar{s}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L)$$

$$Q_2^{q=u,c} = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L)$$

$$P_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q)$$

$$P_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q)$$

$$P_5 = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} b_L) \sum_q (\bar{q} \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} q)$$

$$P_6 = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} T^a b_L) \sum_q (\bar{q} \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} T^a q)$$

$$Q_{8g} = \frac{g_s}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} P_R G^{\mu\nu} b$$

$$Q_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b$$

$$Q_{9V} = \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell)$$

$$Q_{10A} = \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma^5 \ell)$$

$$\mathcal{H}_{\text{eff}}^{\Delta B=1} \sim \mathcal{H}_{\text{eff}}^{\text{had}} + \mathcal{H}_{\text{eff}}^{\text{sl}+\gamma}$$

Within Standard Model (SM), quantum running from M_W down to low scale ~ 5 GeV

$C_1 \sim -0.25$, $C_2 \sim 1.0$, $C_8 \sim -0.2$

$C_7 \sim -0.3$, $C_9 \sim 4.2$, $C_{10} \sim -4.1$

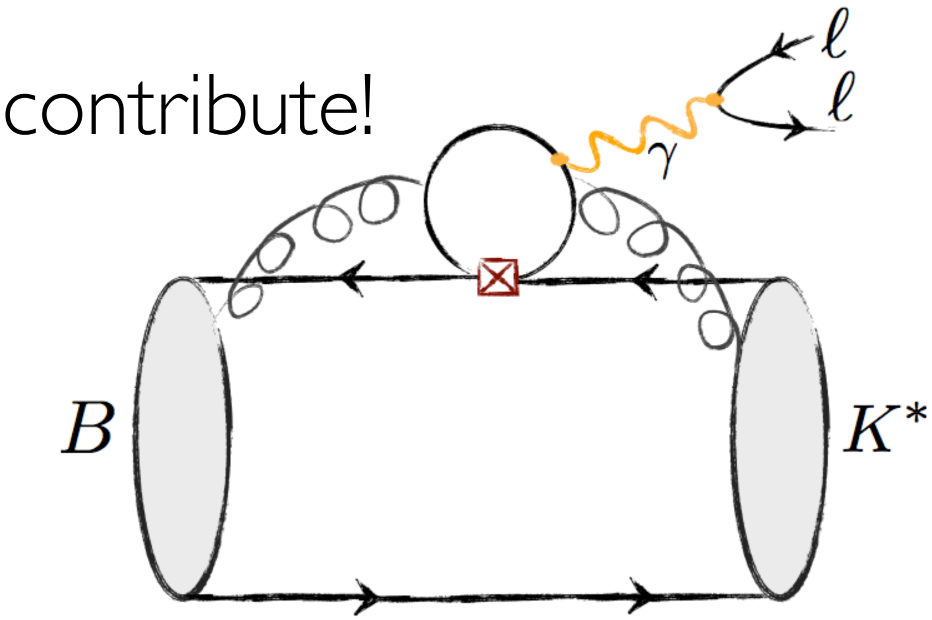
FOCUS OF PRESENT ANALYSIS

CP-conserving New Physics (NP) effects, phenomenologically seen as shifts of SM Wilson coefficients at the low scale:

$$C_7^{NP}, C_{9,e}^{NP}, C_{9,\mu}^{NP}, C_{10,e}^{NP}, C_{10,\mu}^{NP}$$

$$\mathcal{A}^{(\mathrm{had})}(\bar{B} \to \bar{K}^* \ell\ell) \sim \frac{e^2}{q^2} \langle \ell^- \ell^+ | \,\bar{\ell}\, \gamma_\mu \,\ell\, | 0 \rangle \int \underline{d^4x\, e^{i\,qx} \langle \bar{K}^* | \,T\Big\{ \bar{q}(x) \gamma^\mu q(x)\, \mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0) \Big\} | \bar{B} \rangle}$$

i.e. @ first order in $\boldsymbol{\alpha}_{\mathrm{em}}$ the hadronic piece can contribute!



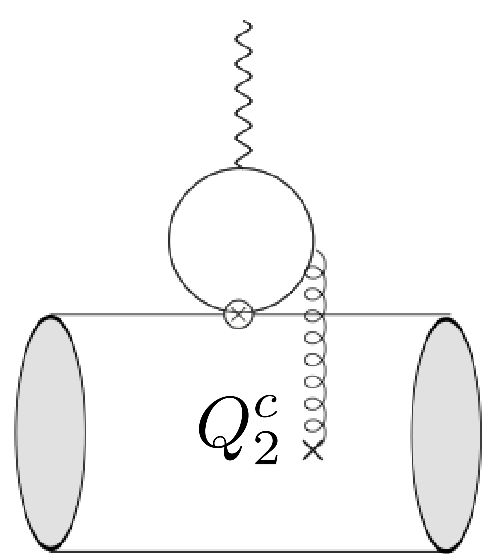
The above correlator is the weakest part of the theoretical prediction.

OBS.

Single soft gluon emission from charm-loop estimated with LCSRs.

A.Khodjamirian, T.Mannel, A.A. Pivovarov and Y.-M.Wang JHEP 1009 (2010) 089

arXiv:1006.4945



DRAWBACKS ON PHENO APPLICATIONS !

- Correlator expanded on the light-cone: maybe a **right estimate**, but **for small q^2**.
- Multiple soft gluon emission is likely relevant: negligible when **q^2 << 4m^2_c**.

$$h_\lambda \equiv \frac{\epsilon_\mu(\lambda)}{m_B^2} \int d^4x\, e^{iqx} \langle \bar{K}^* | T\{j^\mu_{\rm em}(x) \mathcal{H}_{\rm eff}^{\rm had}(0)\} | \bar{B} \rangle$$

non-factorizable hadronic part of B to K*ll amplitude

In the K* helicity basis, $\langle \bar{K}^* \ell^- \ell^+ | \mathcal{H}_{\rm eff}^{\Delta B=1} | \bar{B} \rangle$ can be decomposed as:

$$H_V(\lambda) \propto C_9 \tilde{V}_\lambda + \frac{2 m_b m_B}{q^2} C_7 \tilde{T}_\lambda - \frac{16\pi^2 m_B^2}{q^2} h_\lambda \,, \qquad (\lambda = 0, \pm)$$

$$H_A(\lambda) \propto C_{10} \tilde{V}_\lambda \,, \quad H_P \propto \frac{2 m_\ell m_B}{q^2} C_{10} \left(1 + \frac{m_s}{m_B}\right) \widetilde{S} \,,$$

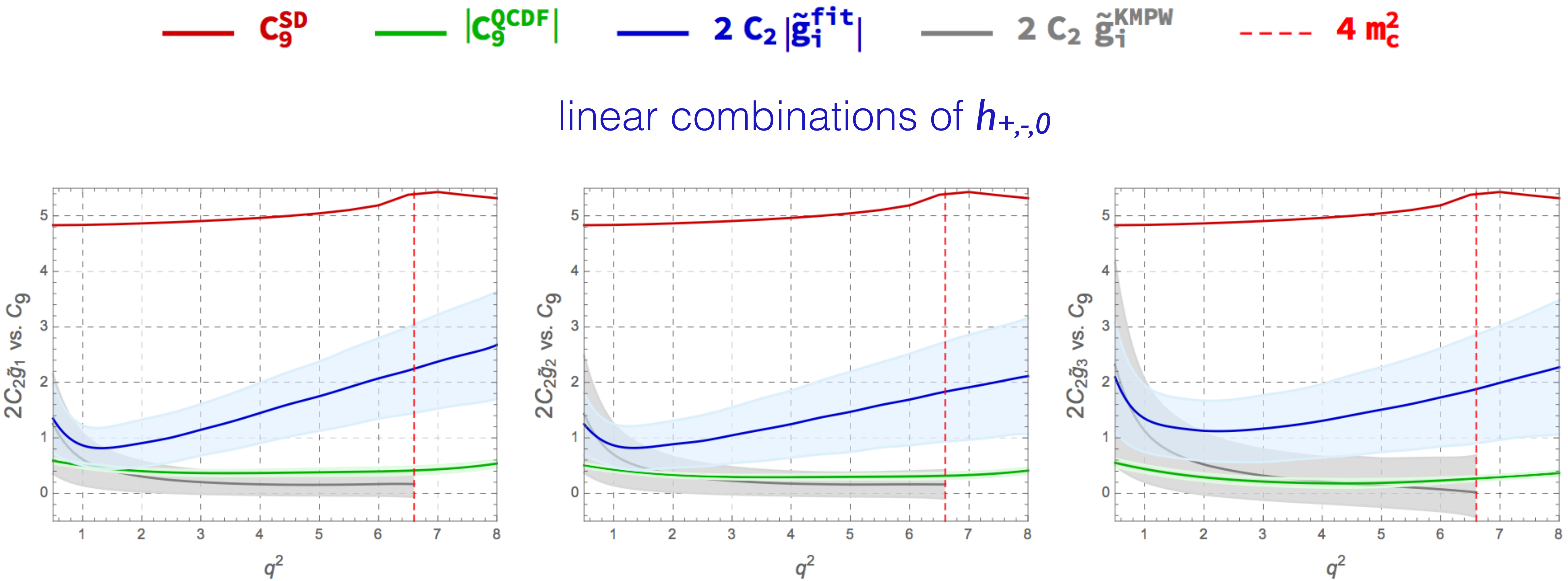
1) All the observables introduced so far are functions of $H_{V,A,P}$.

In the approximation of ~const form factors in q^2:

2) The 0th order and 1st order power correction in q^2 is degenerate with NP effects in Q_7 and Q_9 respectively .

3) Higher order power-corrections in q^2 instead likely associated to genuinely Standard hadronic physics .

short distance (SD) physics   long distance (LD) cc-loop from **JHEP 1009 (2010) 089**



1) Light-blue band identifiable with LD cc-loop read from the fit

IMPORTANT DEPARTURE FROM THEORETICAL ESTIMATES BASED ON LCSR + SINGLE SOFT GLUON APPROX VALID FOR $q^2 << 4\,m_c^2$

2) NP contributing to Q_{9V} should be independent of dilepton mass

q^2 DEPENDENCE SHOWN IN LIGHT-BLUE BAND DISFAVORS NEW SD EFFECTS, BUT POINTS TO UNDERESTIMATED HADRONIC PHYSICS!

$\|h_0^{(0)}\| \cdot 10^4$	1.74 ± 0.99	1.83 ± 0.99	1.8 ± 1.0	1.3 ± 1.1	1.2 ± 1.0	1.7 ± 1.0
$\|h_+^{(0)}\| \cdot 10^4$	0.068 ± 0.054	0.068 ± 0.053	0.067 ± 0.053	0.064 ± 0.051	0.067 ± 0.053	0.068 ± 0.053
$\|h_-^{(0)}\| \cdot 10^4$	0.49 ± 0.12	0.49 ± 0.12	0.49 ± 0.12	0.47 ± 0.12	0.49 ± 0.12	0.49 ± 0.12
$\|h_0^{(1)}\| \cdot 10^4$	0.70 ± 0.65	0.70 ± 0.65	0.73 ± 0.68	0.80 ± 0.67	0.76 ± 0.66	0.69 ± 0.67
$\|h_+^{(1)}\| \cdot 10^4$	0.107 ± 0.093	0.108 ± 0.093	0.111 ± 0.096	0.096 ± 0.089	0.107 ± 0.098	0.112 ± 0.097
$\|h_-^{(1)}\| \cdot 10^4$	0.14 ± 0.12	0.14 ± 0.12	0.15 ± 0.12	0.18 ± 0.13	0.18 ± 0.13	0.15 ± 0.13
$\|h_0^{(2)}\| \cdot 10^4$	0.11 ± 0.10	0.11 ± 0.10	0.11 ± 0.11	0.12 ± 0.10	0.11 ± 0.10	0.11 ± 0.10
$\|h_+^{(2)}\| \cdot 10^4$	0.023 ± 0.020	0.023 ± 0.019	0.023 ± 0.020	0.019 ± 0.018	0.021 ± 0.020	0.024 ± 0.020
$\|h_-^{(2)}\| \cdot 10^4$	0.026 ± 0.022	0.026 ± 0.022	0.026 ± 0.022	0.033 ± 0.022	0.032 ± 0.023	0.026 ± 0.022

PMD h_λ

Ciuchini et al. '16

Parameter	Absolute value
$h_0^{(0)}$	$(5.7 \pm 2.0) \cdot 10^{-4}$
$h_0^{(1)}$	$(2.3 \pm 1.6) \cdot 10^{-4}$
$h_0^{(2)}$	$(2.8 \pm 2.1) \cdot 10^{-5}$
$h_+^{(0)}$	$(7.9 \pm 6.9) \cdot 10^{-6}$
$h_+^{(1)}$	$(3.8 \pm 2.8) \cdot 10^{-5}$
$h_+^{(2)}$	$(1.4 \pm 1.0) \cdot 10^{-5}$
$h_-^{(0)}$	$(5.4 \pm 2.2) \cdot 10^{-5}$
$h_-^{(1)}$	$(5.2 \pm 3.8) \cdot 10^{-5}$
$h_-^{(2)}$	$(2.5 \pm 1.0) \cdot 10^{-5}$

Columns identify NP cases (I) - (VI) in 1704.05447.

PDD h_λ

$\|h_0^{(0)}\| \cdot 10^4$	1.8 ± 1.1	2.0 ± 1.1	1.8 ± 1.1	1.5 ± 1.2	1.4 ± 1.1	1.8 ± 1.1
$\|h_+^{(0)}\| \cdot 10^4$	0.078 ± 0.067	0.078 ± 0.068	0.077 ± 0.067	0.084 ± 0.073	0.087 ± 0.075	0.077 ± 0.067
$\|h_-^{(0)}\| \cdot 10^4$	0.53 ± 0.21	0.53 ± 0.21	0.52 ± 0.21	0.58 ± 0.23	0.58 ± 0.23	0.53 ± 0.21
$\|h_0^{(1)}\| \cdot 10^4$	1.21 ± 0.99	1.26 ± 1.00	1.17 ± 0.94	1.4 ± 1.0	1.5 ± 1.1	1.22 ± 0.99
$\|h_+^{(1)}\| \cdot 10^4$	0.40 ± 0.29	0.41 ± 0.29	0.40 ± 0.29	0.39 ± 0.29	0.40 ± 0.29	0.42 ± 0.30
$\|h_-^{(1)}\| \cdot 10^4$	0.51 ± 0.37	0.44 ± 0.33	0.50 ± 0.37	0.74 ± 0.45	0.74 ± 0.44	0.51 ± 0.38
$\|h_0^{(2)}\| \cdot 10^4$	0.19 ± 0.17	0.19 ± 0.17	0.18 ± 0.16	0.19 ± 0.15	0.19 ± 0.15	0.18 ± 0.16
$\|h_+^{(2)}\| \cdot 10^4$	0.132 ± 0.089	0.143 ± 0.093	0.126 ± 0.086	0.124 ± 0.087	0.128 ± 0.089	0.134 ± 0.090
$\|h_-^{(2)}\| \cdot 10^4$	0.123 ± 0.090	0.112 ± 0.089	0.122 ± 0.088	0.192 ± 0.095	0.185 ± 0.095	0.127 ± 0.091