Standard Model Prediction of $R(D^*)$

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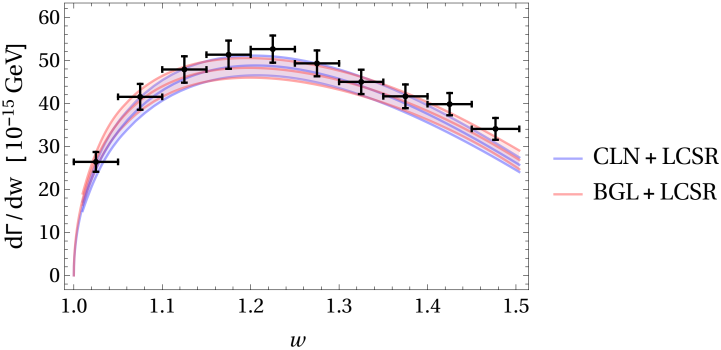
based on work in progress with Dante Bigi and Paolo Gambino
and Phys.Lett. B769 (2017) 441-445 [1703.06124]

Belle has new (preliminary) data

- First time w and angular deconvoluted distributions independent of parametrization.
  ➥ Possible to use different parametrizations.



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}, \; q^2 = (p_B - p_{D^*})^2$$

Form factor parametrizations

Boyd Grinstein Lebed: Unitarity, crossing symmetry, analyticity

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n, \quad z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}, \quad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}.$$

- Theory input: unitarity constraints $\sum_{n=0}^{\infty} \left(a_n^i\right)^2 < 1$.
- $0 < z < 0.056$ for $B \to D^* l\nu \Rightarrow$ truncation at $N = 2$ enough, $z^3 \sim 10^{-4}$.
- $P_i(z)$: "Blaschke factor": removes poles, $\phi_i(z)$: phase space factors.

Caprini Lellouch Neubert: Use HQET relations between form factors

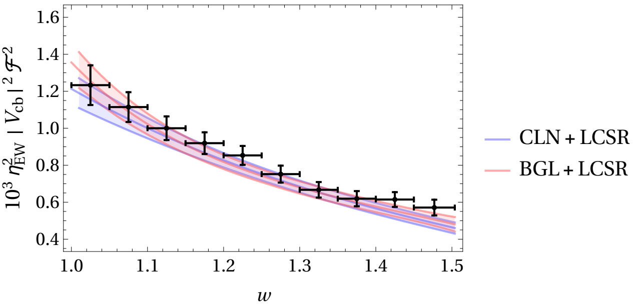
- Less parameters, slope of form factor ratios R_i fixed.

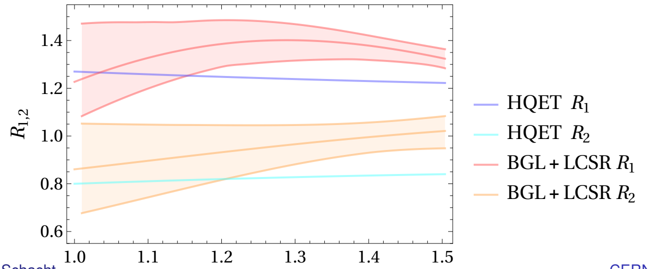
$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right),$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

Fit results for $B \to D^* l \nu$





Different results for V_{cb}

[Bigi Gambino Schacht, 1703.06124, agreeing with Grinstein Kobach, 1703.08170]

BGL Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	$27.9/32$	$31.4/35$
$\lvert V_{cb}\rvert$	$0.0417\left({}^{+20}_{-21}\right)$	$0.0404\left({}^{+16}_{-17}\right)$

CLN Fit:	Data + lattice	Data + lattice + LCSR
χ^2/dof	$34.3/36$	$34.8/39$
$\lvert V_{cb}\rvert$	$0.0382\,(15)$	$0.0382\,(14)$

- $|V_{cb}|$ central values deviate by 9% and 6% (with LCSR).
- LCSR: Light Cone Sum Rule results [Faller, Khodjamirian, Klein, Mannel 0809.0222]
  $h_{A_1}(w_{\text{max}}) = 0.65(18), \quad R_1(w_{\text{max}}) = 1.32(4), \quad R_2(w_{\text{max}}) = 0.91(17).$
- Lattice: $h_{A_1}(1) = 0.906 \pm 0.013.$ [FNAL/MILC 1403.0635]

Anatomy of $R(D^*) \equiv \frac{\int_1^{w_{\tau,\max}} dw\, d\Gamma_\tau/dw}{\int_1^{w_{\max}} dw\, d\Gamma/dw}$

Differential decay rate for $B \to D^* \tau \nu_\tau$

[BGL, hep-ph/9705252]

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw}$$

$$\frac{d\Gamma_{\tau,1}}{dw} = \left(1 - m_\tau^2/q^2\right)^2 \left(1 + m_\tau^2/(2q^2)\right) \frac{d\Gamma}{dw}$$

$$\frac{d\Gamma_{\tau,2}}{dw} = |V_{cb}|^2 m_\tau^2 \times \text{kinematics} \times \mathcal{F}_2(z)^2$$

- $d\Gamma/dw$: Measured differential decay rate of $B \to D^* l \nu$ with $m_l = 0$, depends on axial vector form factors f, $\mathcal{F}_1$ and vector form factor g.
- $\mathcal{F}_2$: Additional unconstrained pseudoscalar form factor.
- $d\Gamma_{\tau,2}/dw$ contributes $\sim 10\%$ to $R(D^*)$.
- Common normalization/notation:

$$R_0 = \frac{P_1}{A_1} = m_{D^*} \left(\frac{1+w}{1+r}\right) \frac{\mathcal{F}_2}{f}, \qquad r = m_{D^*}/m_B$$

Calculating $R_0(w)$

Heavy quark limit

[BGL, hep-ph/9705252]

$$R_0(w) = 1 \quad \forall\, w.$$

HQET at $\mathcal{O}(\Lambda/m_{c,b})$

[Bernlochner Ligeti Papucci Robinson (BLPR), 1703.05330, Neubert, Phys. Lett. B264 (1991) 455; hep-ph/9408290, hep-ph/9306320]

$$R_0(w) = R_0(1) + R_0'(1)(w-1)\,,$$

$$R_0(1) = 1.09 + 0.25\eta(1)\,,$$

$$R_0'(1) \equiv \frac{d}{dw}R_0(w)\Big|_{w=1} = -0.18 + 0.87\hat{\chi}_2(1) + 0.06\eta(1) + 0.25\eta'(1)\,,$$

Sum rule parameters

$$\eta(1) = 0.62 \pm 0.02 \qquad \eta'(1) = 0 \pm 0.2 \qquad \hat{\chi}_2(1) = -0.06 \pm 0.02$$

How large could higher order corrections to $R_0(w)$ beyond $\mathcal{O}(\Lambda/m_{c,b},\ \alpha_s)$ be?

Rough dimensional analysis of higher order corrections

$$\Lambda^2/m_c^2 \sim (0.3)^2 \simeq 10\%$$

$$\alpha_s(m_c)^2 \sim (0.4)^2 \simeq 16\%$$

$$\alpha_s(m_c) \times \Lambda/m_c \sim 0.3 \times 0.4 \simeq 12\%$$

Direct Comparison of HQET and Lattice QCD Results

A_1/V_1 at $w = 1$: Central values deviate by up to 12%.

Lattice QCD: $\left.\frac{A_1(w=1)}{V_1(w=1)}\right|_{\text{FNAL/MILC}} = 0.859(14)$ [obtained from 1403.0635, 1503.07237]

HQET: $\left.\frac{A_1(w=1)}{V_1(w=1)}\right|_{\text{CLN}} = 0.948$ [hep-ph/9712417]

HQET: $\left.\frac{A_1(w=1)}{V_1(w=1)}\right|_{\text{BLPR}} = 0.966(30)$ [obtained from 1703.05330]

f_0/f_+ at $w = 1$: Central values deviate by 3%.

Lattice QCD: $\left.\frac{f_0(w=1)}{f_+(w=1)}\right|_{\text{FNAL/MILC}} = 0.753(3)$ [obtained from 1503.07237]

HQET: $\left.\frac{f_0(w=1)}{f_+(w=1)}\right|_{\text{CLN}} = 0.775$ [obtained from hep-ph/9712417]

HPQCD: Less precise but generally consistent results for the form factors.

A_1/V_1: only marginally consistent with FNAL, but even lower result.

Direct Comparison of HQET and Lattice QCD Results

Slope of f_0/f_+ at $w = 1$: Central values deviate by 20%

Lattice QCD: $$\frac{d}{dw}\left(\frac{f_0}{f_+}\right)\Bigg|_{w=1,\text{FNAL/MILC}} = 0.457(35)$$ [obtained from 1503.07237]

HQET: $$\frac{d}{dw}\left(\frac{f_0}{f_+}\right)\Bigg|_{w=1,\text{CLN}} = 0.382$$ [obtained from hep-ph/9712417]

Implications of Dimensional Analysis and Comparison HQET ⇔ Lattice QCD results

Possible size of higher order corrections of HQET results

- Corrections could modify form factor ratios by $\sim 12\%$.

  ➥ For prediction of $R(D^*)$ vary $R_0(w)$ in a band of 12%.

Take this into account by variation of additional parameter:

$$R_0(w, E) = E\left(R_0(1) + R_0'(1))(w-1)\right)$$

$$\text{vary} \quad E = 1.0 \pm 0.12$$

Overview on Sources of Uncertainty

preliminary results

- Our analysis leads to a central value $R(D^*) = 0.258$.
  Very good agreement to [BLPR, 1703.05330].

Error due to experimental error of measurement of $B \to D^* l\nu$.

$$\delta R(D^*) = 0.005$$

Theory error due to sum rule parameters.

- Scan: $\delta R(D^*) = 0.003$
- Gaussian: $\delta R(D^*) = 0.002$

Theory error due to higher order effects.

- Scan/Gaussian: $\delta R(D^*) = {}^{+0.007}_{-0.006}$

Total Uncertainties for $R(D^*)$

preliminary results

BGL fit

Higher orders	Sum rule parameters	Prediction for $R(D^*)$
Scan	Scan	0.258^{+15}_{-13}
Scan	Gaussian	0.258^{+12}_{-11}
Gaussian	Gaussian	0.258^{+9}_{-8}

CLN fit

Higher orders	Sum rule parameters	Prediction for $R(D^*)$
Scan	Scan	0.257^{+15}_{-13}
Scan	Gaussian	0.257^{+12}_{-11}
Gaussian	Gaussian	0.257^{+9}_{-8}

Experiment: $0.310 \pm 0.015 \pm 0.008$ (HFAG average [1612.07233])

Conclusions

- Belle has new data: Deconvoluted, independent of parametrization.
- Different parametrizations give different results for $|V_{cb}|$.
- $R(D^*)$ depends to an amount of $\sim 10\%$ on the unconstrained form factor $\mathcal{F}_2$, which has to be estimated by theory.
  ➡ No lattice calculation available, use HQET input from BLPR.
- Our central value 0.258 agrees well with the literature.
- We find a larger uncertainty, coming from three sources:
  - Experimental error in $B \to D^* l\nu$: 0.005 .
  - Sum rule parameters : 0.003 (scan) ,
    0.002 (gaussian) .
  - Higher order HQET corrections : ${}^{+0.007}_{-0.006}$ (scan/gaussian).
- The anomaly is persistent.

[numbers: preliminary results]