Flavor Physics and CP violation

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II. The CKM matrix and neutral meson mixing

III. Rare B and K decays

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I. Introduction to Flavor Physics and CP violation

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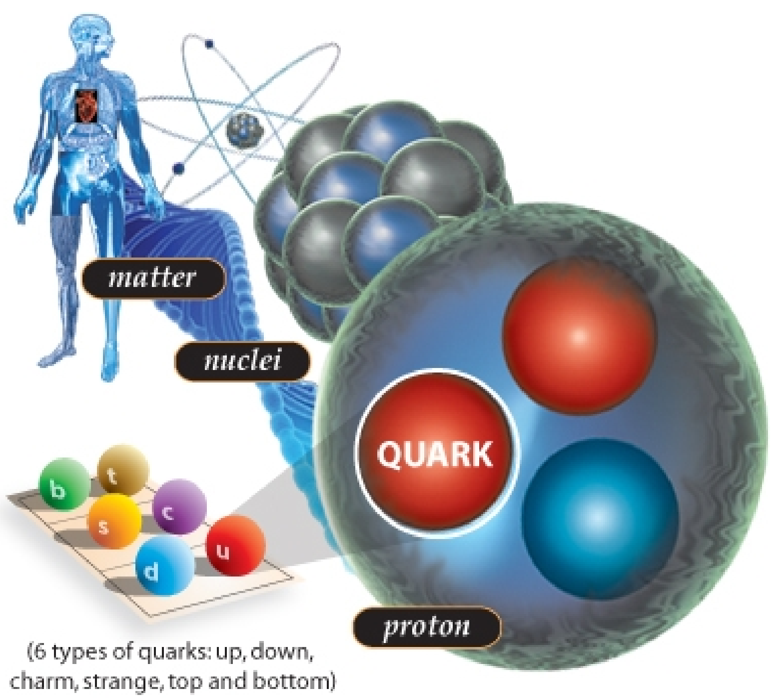
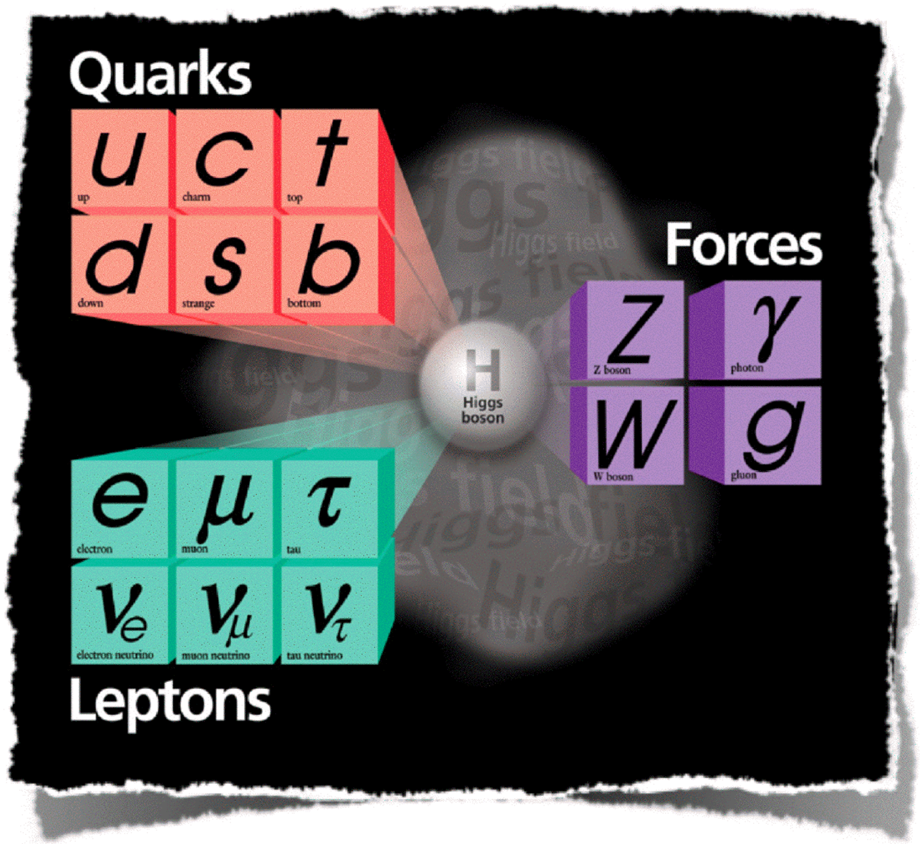
  - *Flavor and CP*
- The CKM matrix

Introduction

In last 40 years a highly successful Theory has emerged in the study of fundamental interactions: the so-called Standard Model.

The Standard Model describes with success both *nature* and *interactions* of matter constituents. It is a Theory valid over a huge range of energies: from the few eV of atomic bounding energies up to (at least) the few TeV energy reached in LHC collisions.

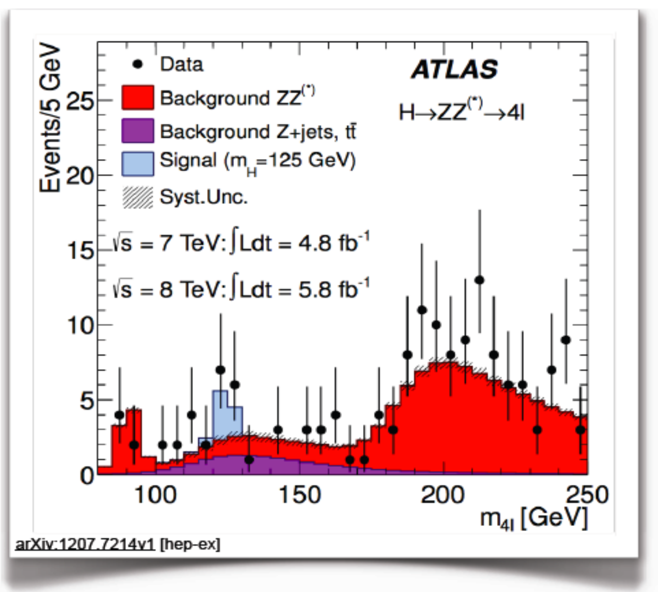


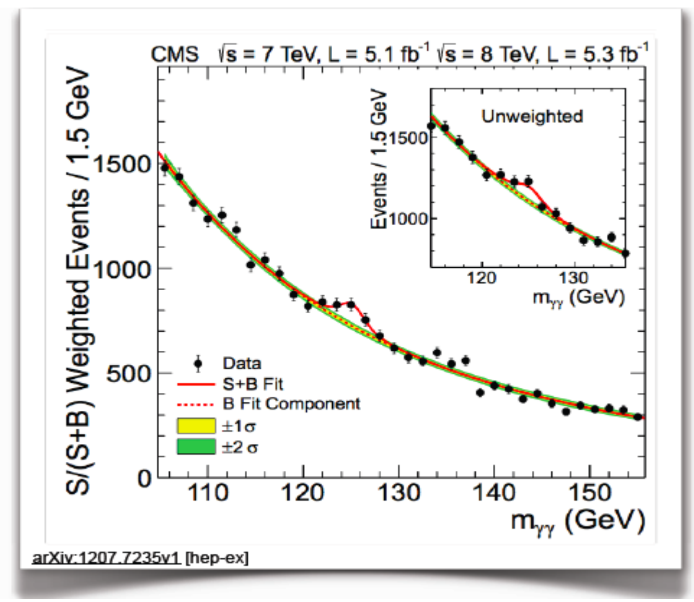


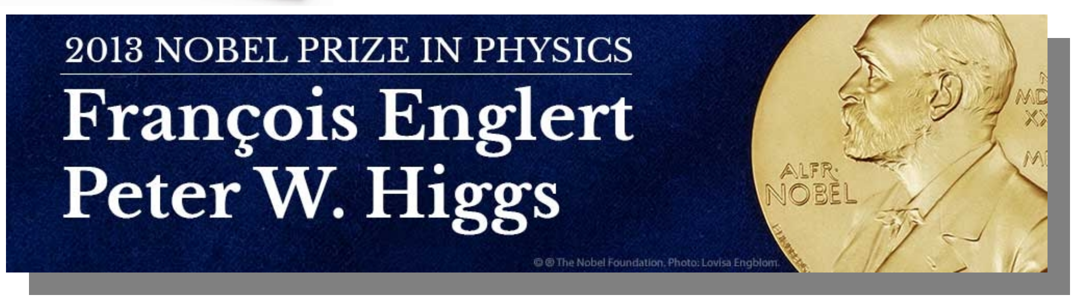
Introduction

It has been called "model" since, till recently, many of us were not convinced this was the right Theory of Nature...

...but the situation has changed after the discovery of the Higgs boson at the LHC in 2012: this "model" is now the reference Theory describing strong, weak, and electromagnetic forces, consistently with the principles of quantum mechanics and special relativity.







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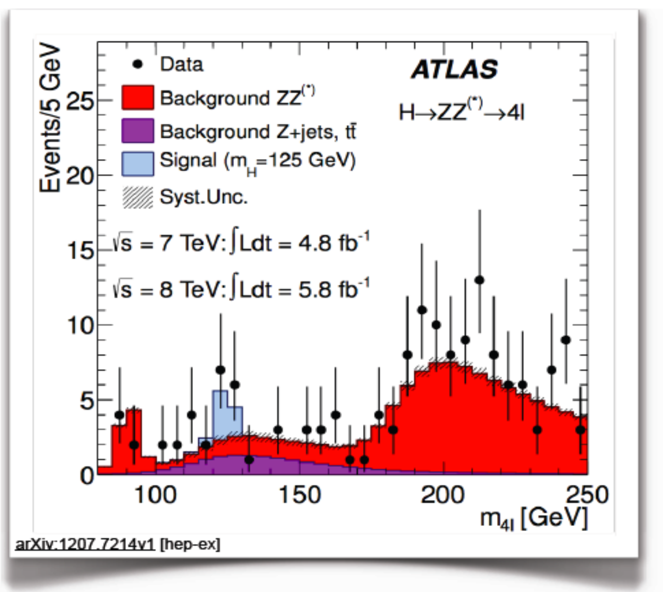

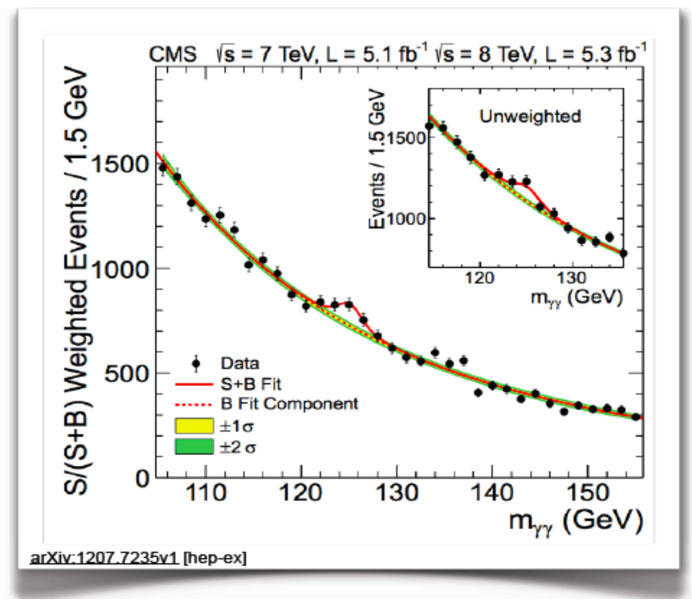

Given we identified this successful Theory,
shall we stop investigating fundamental physics?

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Physical Theories at the end of the 19th century:

Electromagnetism (Maxwell's Theory)	Gravity (Newton's Theory)

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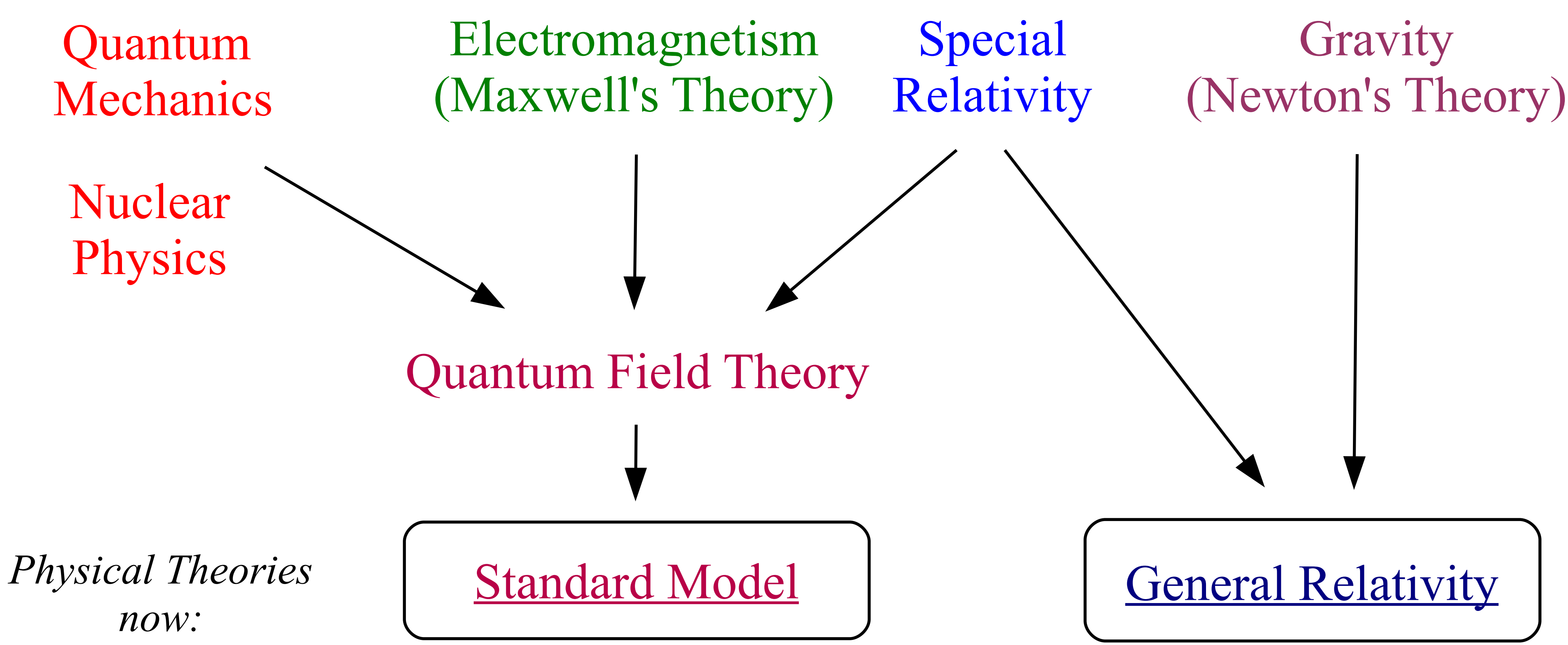


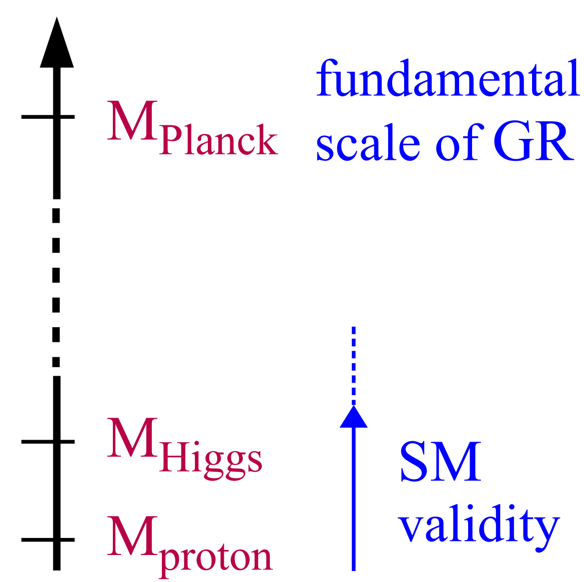
Introduction

The discovery of the Higgs boson is certainly a great triumph for the SM.
But there are a few important questions that are still open.
One of the most pressing one is the so-called "(mass) hierarchy" problem:

The Higgs boson mass (non predicted within the model) is $M_{Higgs} \sim 125$ GeV.
This is the only fundamental scale of energy within the SM.
This energy scale is higher compared to the proton mass, but is still well below

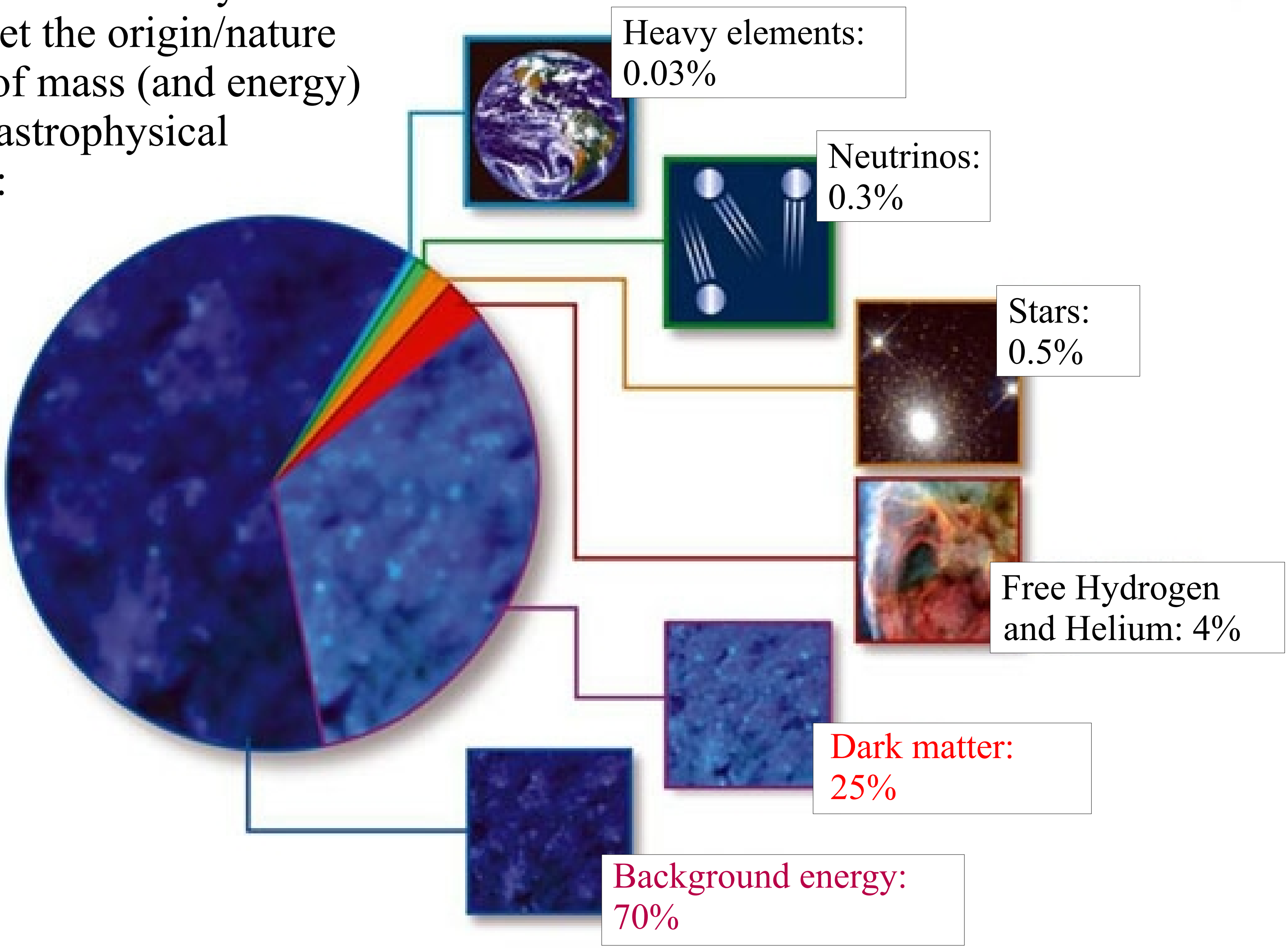
$M_{Planck} = (\hbar c/G_N)^{-1/2} \sim 10^{19}$ GeV (*universal energy scale associated to Gravity*)

- Why $M_{Higgs} \ll M_{Planck}$?
- Is there something in between M_{Higgs} & M_{planck} ?
- What determines the coupling of the Higgs boson to the various particles?
- ...



Introduction

A clear clue we don't fully understand yet the origin/nature of all forms of mass (and energy) comes from astrophysical observations:



Introduction

Physical Theories now:

Standard Model	General Relativity

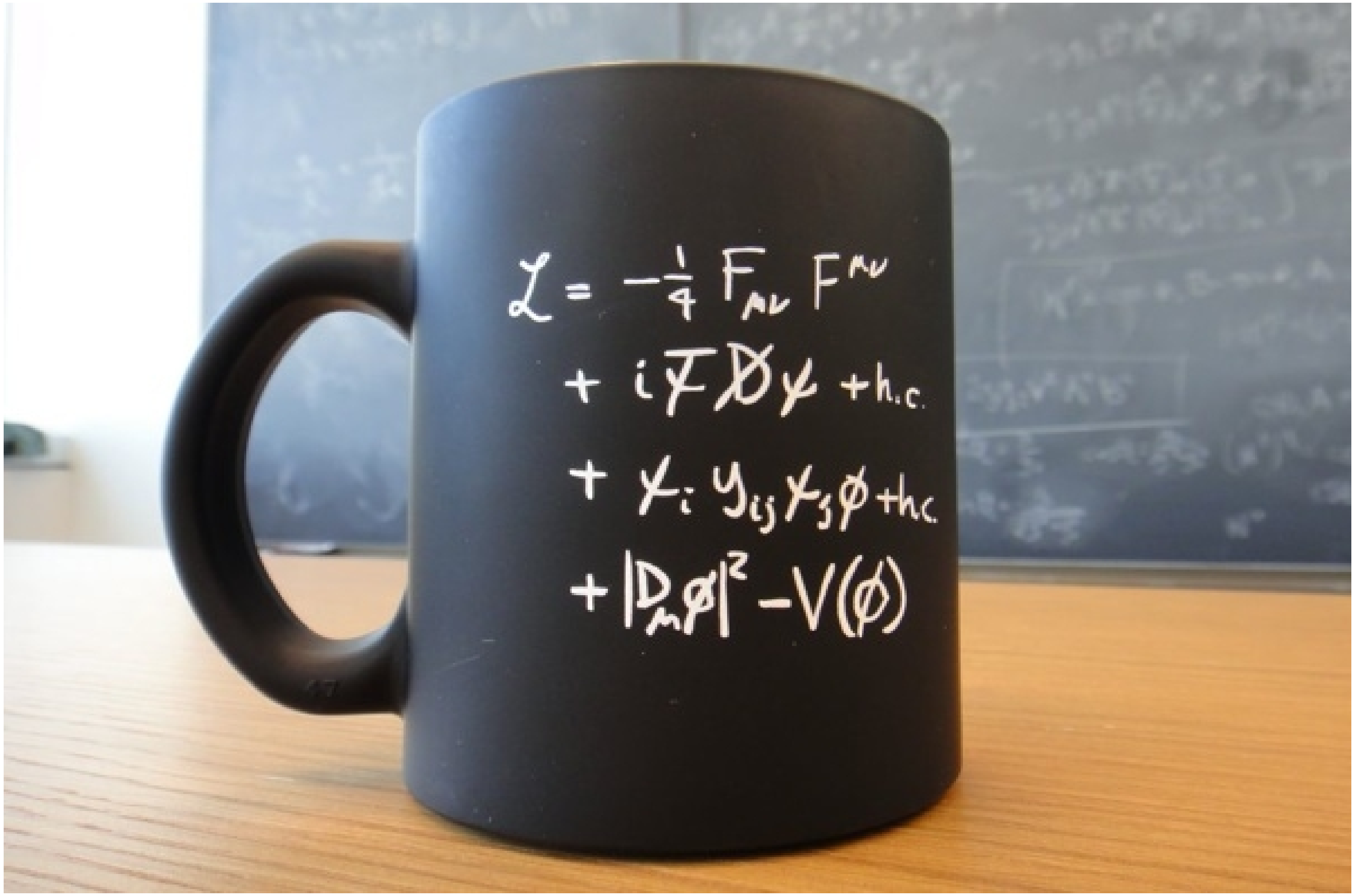
We are certainly in a special time in particle physics, where "near-by" discoveries are not anymore guaranteed: the Standard Model is a very successful theory *with no intrinsic energy limitations*.

However, the SM cannot be the end of the story: on the one hand, it cannot be completely reconciled with the principles of General Relativity; on the other hand, it leaves unanswered some intriguing fundamental questions.

The problems related to "*Flavor Physics and CP violation*", that we will discuss in these lectures, are among these intriguing questions.

As I will try to illustrate, deeper theoretical and experimental studies of these problems may reveal clues on how to go beyond the SM.

A closer look to the Standard Model



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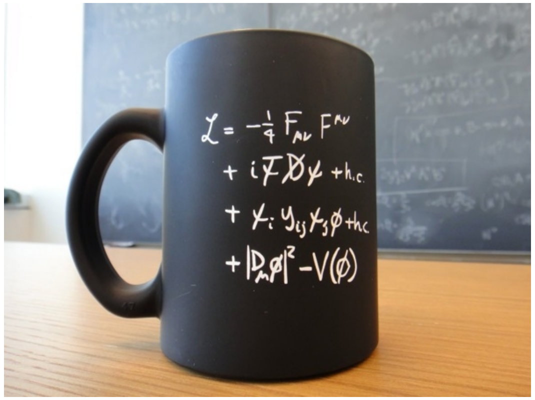

The SM is a relativistic Quantum Field Theory based on

- Two fundamental (gauge) symmetries:
  - the color symmetry ($\rightarrow$ *strong interactions*)
  - the electro-weak symmetry
- Three sets of Fundamental Constituents:
  - 3 generations (flavors) of quarks & leptons
- A peculiar symmetry-breaking sector
  non-trivial Higgs vacuum expectation value

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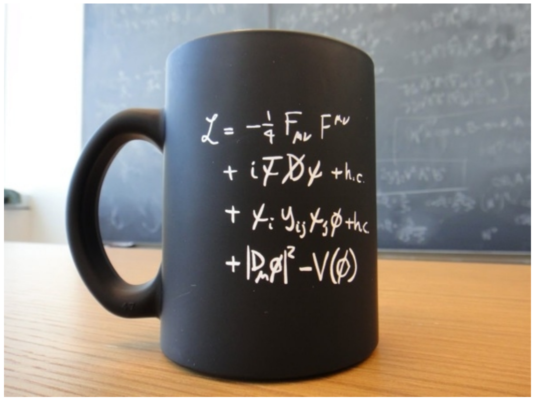

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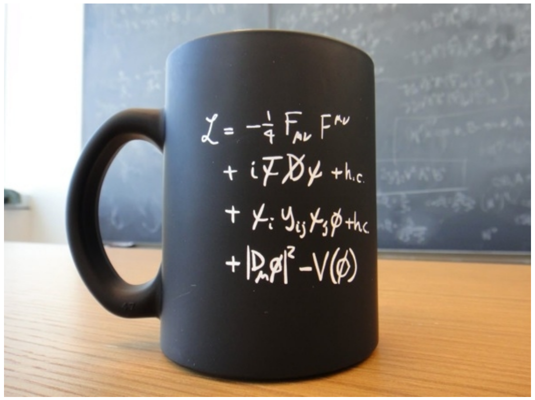
A team game played with a ball...

...the ball is spherical and can be touched only by feet, the filed is square...

...each team has 11 players...

A closer look to the Standard Model



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A team game played with a ball...

...the ball is spherical and can be touched only by feet, the filed is square...

...each team has 11 players...

the field is covered by grass, the players are "humans" & are a bit different one from the other...

A closer look to the Standard Model

The two main pillars on which *quantum field theory* is based are the two "revolutionary" theories developed at the beginning of last century:

Quantum Mechanics
[Uncertainty principle $\Delta E \, \Delta t > \hbar$]

Quantum Field Theory

Special Relativity
[Equivalence of mass & energy $E = m\, c^2$]

QFT generalizes and combines these two theories: it is the most advanced theoretical tool we have to describe natural phenomena...

To achieve this goal, the last classical concept that has to be abandoned is the idea that the number of the matter constituents is conserved: all elementary particles (including the electron) are described by *excitations* of specific *fields*.

All particles can be created and destroyed transforming mass in energy and viceversa (*they are somehow like "waves"*) → resolution of the particle/wave dualism of non-relativistic quantum mechanics.

A closer look to the Standard Model

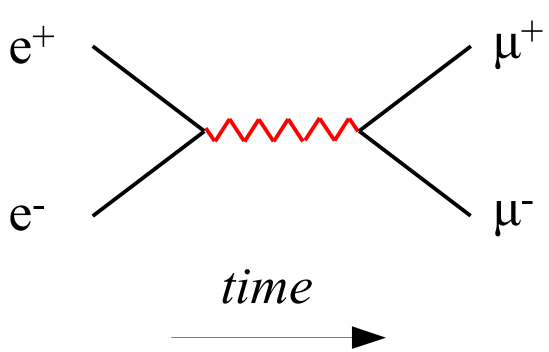
Quantum Mechanics
[Uncertainty principle $\Delta E\ \Delta t > \hbar$]

Quantum Field Theory

Special Relativity
[Equivalence of mass & energy $E = m\ c^2$]

All elementary particles are described by *excitations* of specific *fields* and can be *created* & *destroyed* transforming mass in energy and viceversa

This is possible because to each "particle" is necessarily associated a corresponding "anti-particle", with same mass & spin but opposite quantum numbers (*generalized charges*)



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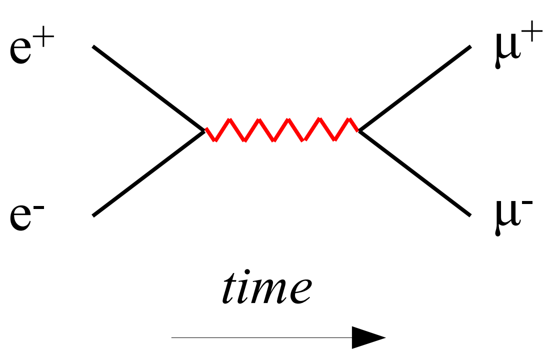

CP (= *Charge-conjugation* × *Parity*)
is the symmetry that connects
and particles and anti-particles

CP-violation is a necessary ingredient to explain the asymmetry between matter and anti-matter that we observe in our Universe (→ *more later...*)

A closer look to the Standard Model

To specify the nature of the SM, within the context of QFT, we need to specify the type of fields and their interactions, and we do this via the Lagrangian:

$$\mathscr{L}_{\mathrm{SM}} = \mathscr{L}_{\mathrm{gauge}}(A_a, \psi_i) + \mathscr{L}_{\mathrm{Higgs}}(\phi, A_a, \psi_i)$$

- Two fundamental (gauge) symmetries:
  - the color symmetry (→ *strong interactions*)
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- Three sets of Fundamental Constituents:
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- A peculiar symmetry-breaking sector

  non-trivial Higgs vacuum expectation value

The SM gauge sector

$$\mathscr{L}_{\text{gauge}} = \Sigma_a - \frac{1}{4g_a^{\ 2}} (F_{\mu\nu}^{\ \ a})^2 + \Sigma_\psi \Sigma_i \overline{\psi}_i i \not{D} \psi_i$$

The requirement of local-gauge-invariance (symmetry principle) severely restricts the form of the gauge Lagrangian

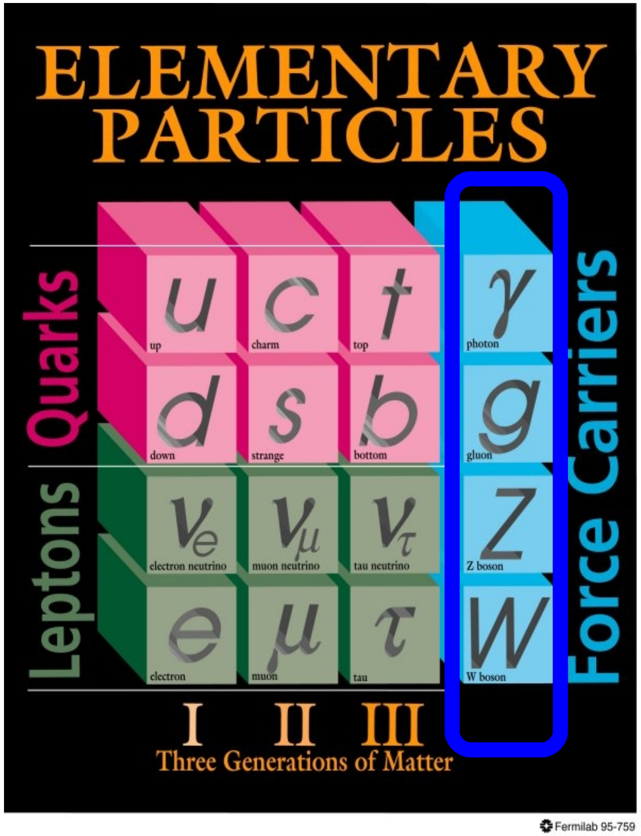
$$\psi(x) \longrightarrow e^{\,i\alpha^a(x)T^a} \psi(x) \quad \longleftrightarrow \quad A^a_{\ \mu}(x)$$

The number and the self-interactions on the gauge fields are completely specified by the choice of the gauge group:

$$SU(3)_c \times SU(2)_L \times U(1)_Y \qquad \text{Unbroken symmetry (above e.w. scale)}$$
$$[T,T_3] \quad [Y]$$

$$\downarrow$$

$$SU(3)_c \times U(1)_Q \qquad \text{low-energy symmetry}$$
$$[Q=T_3+Y]$$



The SM gauge sector

$$\mathscr{L}_{\rm gauge} = \Sigma_a - \frac{1}{4g_a^2}(F_{\mu\nu}{}^a)^2 + \Sigma_\psi \Sigma_i \overline{\psi}_i i\not{D}\,\psi_i$$

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$SU(3)_c \times U(1)_Q$ low-energy symmetry

$[Q = T_3 + Y]$

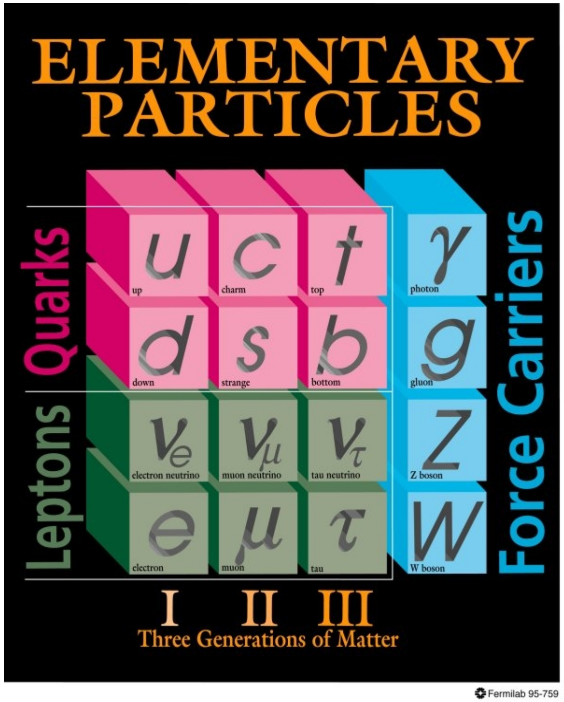
The "naturalness" of this part of the SM Lagrangian is somehow indicated by the values of the 3 gauge couplings above the weak scale:

	E ~ 200 GeV
g_3	~ 1.2
g_2	~ 0.6
g_1	~ 0.3

The SM gauge sector

$$\mathscr{L}_{\rm gauge} = \Sigma_a - \frac{1}{4g_a^{\,2}} (F_{\mu\nu}^{\ a})^2 + \Sigma_\psi \Sigma_i \,\bar{\psi}_i\, i\not{D}\, \psi_i$$

The fermion part of the Lagrangian has more "freedom": number of fermions & quantum numbers not specified



[Q=T$_3$+Y]

basic matter family [3 replica]	$SU(3)_c$	$SU(2)_L$	Y	Q
$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$	3	2	+1/6	+2/3 -1/3
u_R	3	1	+2/3	+2/3
d_R	3	1	-1/3	-1/3
$L_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$	1	2	-1/2	0 -1
e_R	1	1	-1	-1

The SM gauge sector

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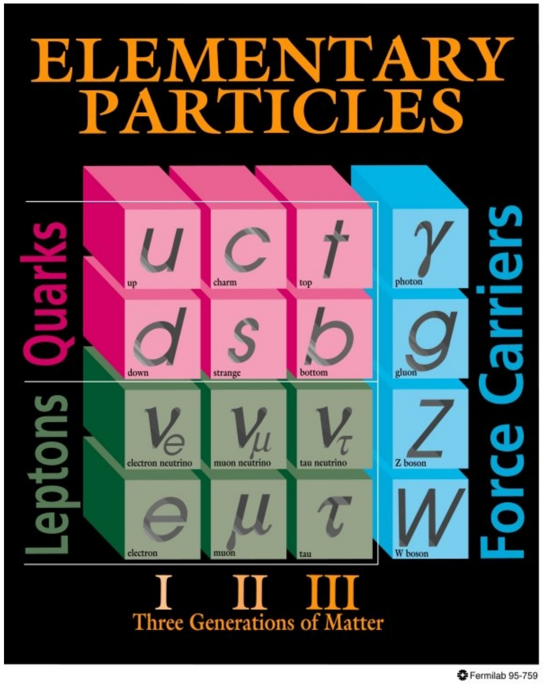
$$e_R$$

The fermion part of the Lagrangian has more "freedom": number of fermions & quantum numbers not specified

Everything is "natural" also here but for two points:

I. Why we have 3 (almost) identical replica?



The 1st family is composed by up & down quarks (↔ *protons & neutrons*), the electron & the electron neutrino (*copiously produced in the sun*): all the forms of matter we observe around us are composed by these basic constituents

The 2nd & 3rd families are <u>identical copies</u> but for <u>different</u> (heavier) <u>masses</u>

The SM gauge sector

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Everything is "natural" also here but for two points:

I. Why we have 3 (almost) identical replica?

II. Why the U(1) charge is quantized?

→ possible indication of a deeper layer with unified gauge group

$[Q=T_3+Y]$

basic matter family [3 replica]	$SU(3)_c$	$SU(2)_L$	Y	Q
$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$	3	2	+1/6	+2/3 -1/3
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The fermion part of the Lagrangian has more “freedom”: number of fermions & quantum numbers not specified

N.B.: a RH neutrino is something we do not include in the SM since it would be completely “neutral” under the gauge symmetry

[$Q=T_3+Y$]

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e_R	1	1	-1	-1
ν_R	1	1	0	0

The Higgs sector

The symmetry structure the SM implies that neither (chiral) fermions nor gauge bosons can have a mass
[→ *the corresponding excitations should propagate at the speed of light*].

In particular, the $SU(2)_L$ gauge symmetry forbid mass terms for the fermions:

$$\psi_L(x) \longrightarrow e^{i\,\alpha_L(x)}\,\psi_L(x)$$

$$\psi_R(x) \longrightarrow e^{i\,\alpha_R(x)}\,\psi_R(x)$$

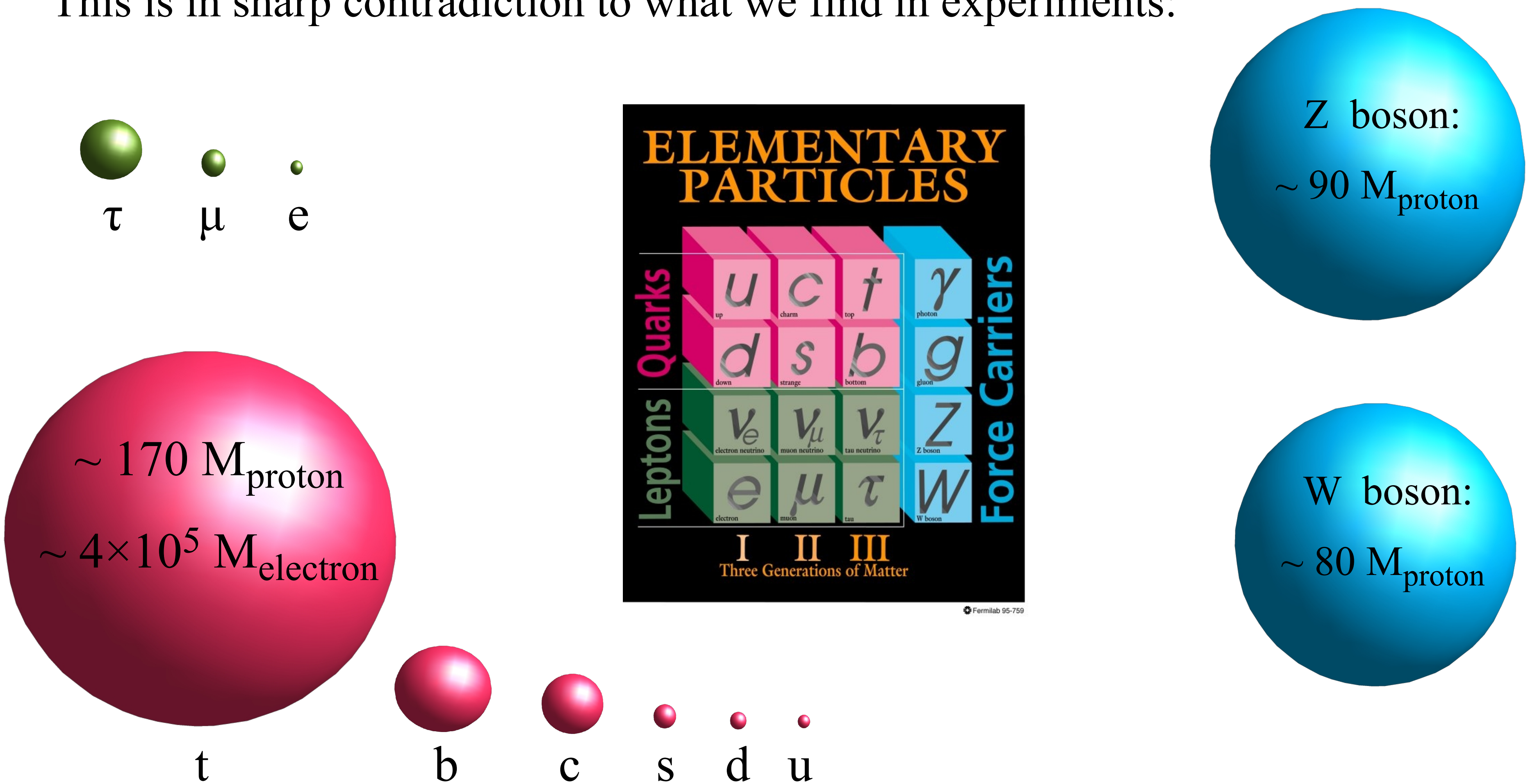
$$\cancel{m\,\overline{\psi}_L\,\psi_R}$$

not invariant

The Higgs sector

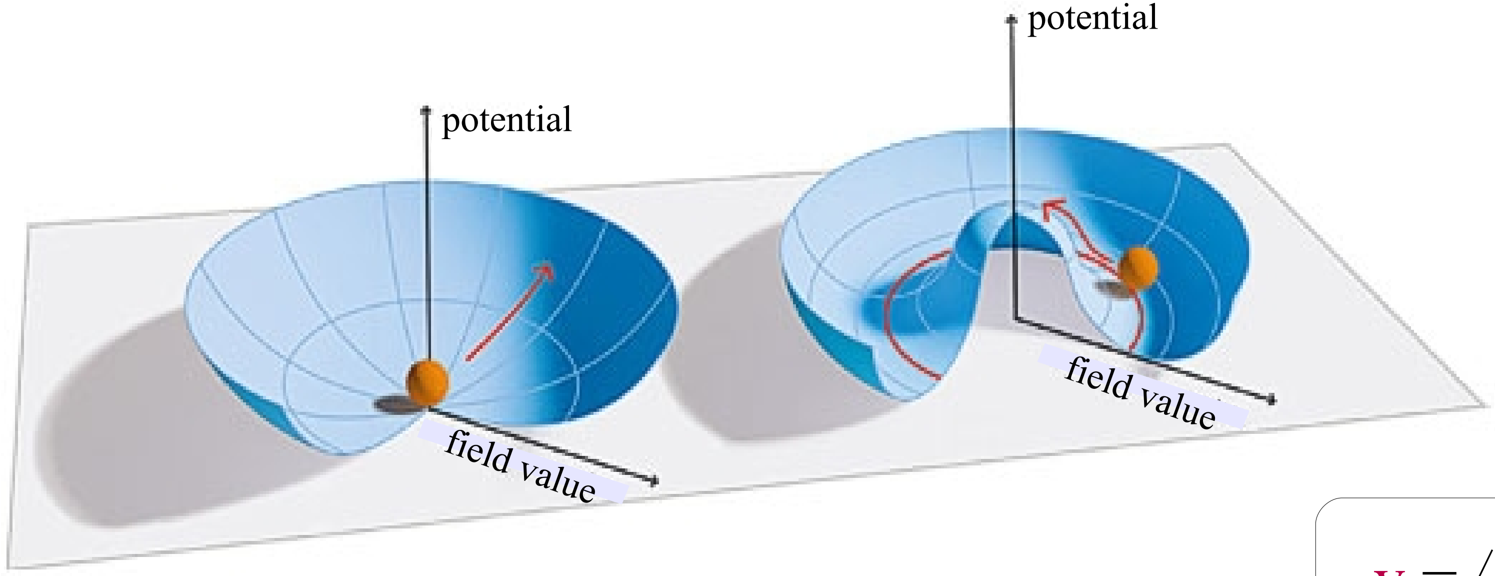
The symmetry structure the SM implies that neither (chiral) fermions nor gauge bosons can have a mass
[→ *the corresponding excitations should propagate at the speed of light*].

This is in sharp contradiction to what we find in experiments:



The Higgs sector

The introduction of an elementary $SU(2)_L$ scalar doublet, with ϕ^4 potential, is the most economical & simple choice to achieve the spontaneous symmetry breaking of both gauge [$SU(2)_L \times U(1)_Y \to U(1)_Q$] and flavor symmetries (*i.e. the degeneracy among the various fermions families*) that we observe in Nature.



$$v = \langle \phi \rangle \sim 246 \text{ GeV}$$

$$\mathscr{L}_{\rm higgs}(\phi, A_a, \psi_i) = D\phi^+ D\phi - V(\phi)$$

[$m_W = \frac{1}{2} g\, v$]

$$V(\phi) = -\mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2 + Y^{ij}\, \overline{\psi}_L{}^i\, \psi_R{}^j\, \phi$$

The Higgs sector

“Effective” elementary particle masses are the result of the interaction of the various elementary fields with the background value of the Higgs field.

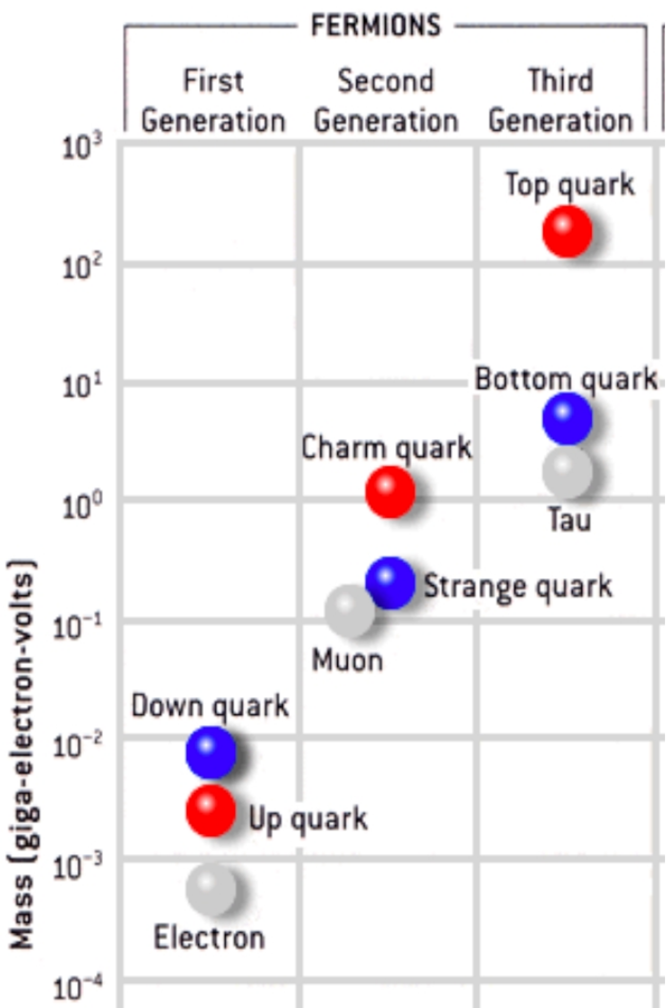
$$Y^{ij}\, \psi_L^{\,i}\, \psi_R^{\,j}\, \phi \longrightarrow Y^{ij}\, \psi_L^{\,i}\, \psi_R^{\,j}\, \langle\phi\rangle = (m_{eff})^{ij}\, \psi_L^{\,i}\, \psi_R^{\,j}$$

Although this solution works from the technical point of view, it is not very satisfactory:

- The Higgs field is essentially a new interaction. However, contrary to the four “standard forces”, it is not based on a symmetry principle.
- The Higgs mechanism does not solve the problem of why each particle has a different mass (*it does not allow us to predict/compute particle masses*) and this is why we suspect it is only an *effective description* of something more fundamental.

Flavor physics and CP violation

$$Y^{ij}\, \psi_L^{\ i}\, \psi_R^{\ j}\, \phi \longrightarrow Y^{ij}\, \psi_L^{\ i}\, \psi_R^{\ j}\, \langle\phi\rangle = (m_{\rm eff})^{ij}\, \psi_L^{\ i}\, \psi_R^{\ j}$$



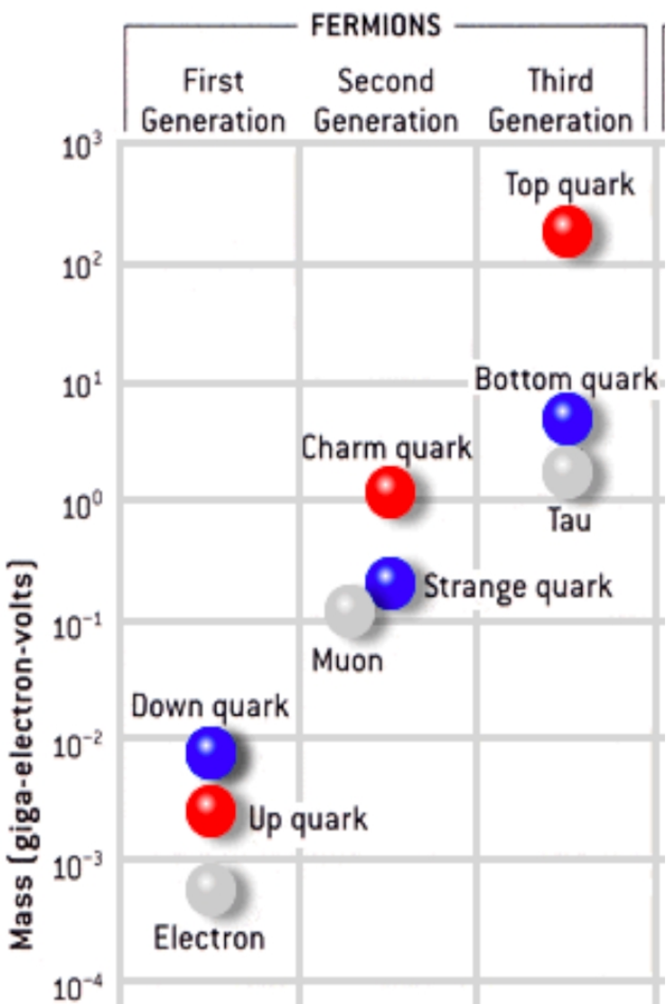
We denote by *"Flavor Physics"* all the phenomena related to *interactions differentiating the various fermion families*.

Within the SM, Flavor Physics is completely controlled by the Yukawa couplings that, in turn, are fixed by the fermion masses. But in general (i.e. when going beyond the SM), this might be no longer true.

Interestingly enough, within the SM the complex phases present in the Yukawa couplings are also the only sources of CV violation

Flavor physics and CP violation

$$Y^{ij}\,\psi_L^{\,i}\,\psi_R^{\,j}\,\phi \longrightarrow Y^{ij}\,\psi_L^{\,i}\,\psi_R^{\,j}\,\langle\phi\rangle = (m_{eff})^{ij}\,\psi_L^{\,i}\,\psi_R^{\,j}$$



We denote by *"Flavor Physics"* all the phenomena related to *interactions differentiating the various fermion families*.

Twofold role of Flavor physics, related to the following two key open questions:

- *Are there other sources of flavor (and CP) symmetry breaking, beside the SM Yukawa couplings?*
- *What determines the observed pattern of quark & lepton mass matrices?*

Answering this questions is likely to shed light on physics beyond the SM...

The CKM matrix

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$\mathscr{L}_{\rm SM} = \mathscr{L}_{\rm gauge}(A_a, \psi_i) + \mathscr{L}_{\rm Higgs}(\phi, A_a, \psi_i)$$

3 identical replica of the basic fermion family
$[\psi = Q_L, u_R, d_R, L_L, e_R] \Rightarrow$ huge flavor-degeneracy

$$\Sigma_{\psi = Q_L, u_R, d_R, L_L, e_R} \Sigma_{i=1..3} \bar{\psi}_i i \not{D} \psi_i$$

The gauge Lagrangian is invariant under <u>5 independent U(3) global rotations</u> for each of the 5 independent fermion fields

E.g.: $Q_L^i \rightarrow U^{ij} Q_L^j$

$$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}, \quad u_R, \quad d_R, \quad L_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}, \quad e_R$$

U(1) flavor-independent phase
+
SU(3) flavor-dependent mixing matrix

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Within the SM the flavor-degeneracy is broken only by the Yukawa interaction:

in the quark sector:

$$\begin{cases} \bar{Q}_L^{\,i}\, Y_D^{ik}\, d_R^{\,k}\, \phi + h.c. \longrightarrow \bar{d}_L^{\,i}\, M_D^{ik}\, d_R^{\,k} + \dots \\ \bar{Q}_L^{\,i}\, Y_U^{ik}\, u_R^{\,k}\, \phi_c + h.c. \longrightarrow \bar{u}_L^{\,i}\, M_U^{ik}\, u_R^{\,k} + \dots \end{cases}$$

$$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix} \qquad \langle\phi\rangle = \begin{bmatrix} 0 \\ v \end{bmatrix} \qquad \langle\phi_c\rangle = \begin{bmatrix} v \\ 0 \end{bmatrix}$$

$$\mathscr{L}_{\rm SM} = \mathscr{L}_{\rm gauge}(A_a, \psi_i) + \mathscr{L}_{\rm Higgs}(\phi, A_a, \psi_i)$$

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$[\psi = Q_L, u_R, d_R, L_L, e_R] \Rightarrow$ huge flavor-degeneracy.

Within the SM the flavor-degeneracy is broken only by the Yukawa interaction:

in the quark sector:

$$\bar{Q}_L{}^i Y_D{}^{ik} d_R{}^k \phi + h.c. \rightarrow \bar{d}_L{}^i M_D{}^{ik} d_R{}^k + \ldots$$

$$\bar{Q}_L{}^i Y_U{}^{ik} u_R{}^k \phi_c + h.c. \rightarrow \bar{u}_L{}^i M_U{}^{ik} u_R{}^k + \ldots$$

The Y are not hermitian $\rightarrow$ diagonalized by bi-unitary transformations:

$$V_D{}^+ Y_D U_D = D_D = \mathrm{diag}(y_b, y_s, y_d)$$
$$V_U{}^+ Y_U U_U = D_U = \mathrm{diag}(y_t, y_c, y_u)$$

$$y_i = \frac{2^{1/2}\, \mathrm{m}_{\mathrm{q_i}}}{\langle \phi \rangle} \approx \frac{\mathrm{m}_{\mathrm{q_i}}}{174 \text{ GeV}}$$

$$\mathscr{L}_{\rm SM} = \mathscr{L}_{\rm gauge}(A_a, \psi_i) + \mathscr{L}_{\rm Higgs}(\phi, A_a, \psi_i)$$

3 identical replica of the basic fermion family
[$\psi = Q_L, u_R, d_R, L_L, e_R$] $\Rightarrow$ huge flavor-degeneracy.

Within the SM the flavor-degeneracy is broken only by the Yukawa interaction:

in the quark sector:

$$\bar{Q}_L{}^i\, Y_D{}^{ik}\, d_R{}^k\, \phi + h.c. \;\longrightarrow\; \bar{d}_L{}^i\, M_D{}^{ik}\, d_R{}^k + \ldots$$

$$\bar{Q}_L{}^i\, Y_U{}^{ik}\, u_R{}^k\, \phi_c + h.c. \;\longrightarrow\; \bar{u}_L{}^i\, M_U{}^{ik}\, u_R{}^k + \ldots$$

The residual flavor symmetry let us to choose a (gauge-invariant) flavor basis where <u>only one</u> of the two Yukawas is diagonal:

$$Y_D = V_D\, D_D\, U_D{}^+ \;\rightarrow\; D_D$$

$$Y_U = V_U\, D_U\, U_U{}^+ \;\rightarrow\; (V_D{}^+ V_U)\; D_U$$

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The residual flavor symmetry let us to choose a (gauge-invariant) flavor basis where <u>only one</u> of the two Yukawas is diagonal:

$$Y_D = V_D D_D U_D^{+} \rightarrow D_D \qquad\qquad \rightarrow (V_U^{+} V_D)\, D_D$$

or

$$Y_U = V_U D_U U_U^{+} \rightarrow (V_D^{+} V_U)\, D_U \qquad\qquad \rightarrow D_U$$

$(V_U^{+} V_D) = \mathrm{V} =$ unitary matrix

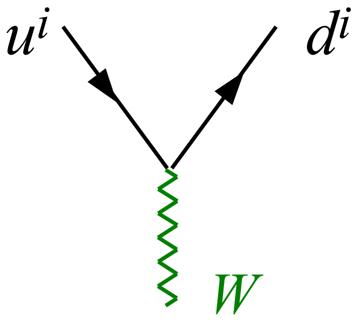
Diagonalizing the down-quark Yukawa coupling, we get:

$$M_D = \mathrm{diag}(m_d, m_s, m_b) \qquad M_U = V^{+} \times \mathrm{diag}(m_u, m_c, m_t)$$

To diagonalize also the second mass matrix we need to rotate separately u_L & d_L (non gauge-invariant basis)

$$\mathscr{L}_{gauge} \rightarrow \frac{g}{\sqrt{2}} W_\mu J_W^\mu$$

$$J_W^\mu = \bar{u}_L^i \gamma^\mu d_L^i$$



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To diagonalize also the second mass matrix we need to rotate separately u_L & d_L (non gauge-invariant basis) → V appears in charged-current gauge interactions:

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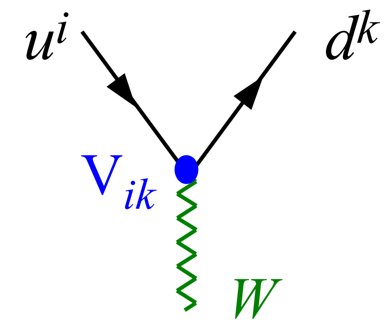

$$J_W^\mu = \bar{u}_L^{\,i} \gamma^\mu d_L^{\,i} \rightarrow \bar{u}_L^{\,i} V_{ik} \gamma^\mu d_L^{\,k}$$

<u>Cabibbo-Kobayashi-Maskawa</u>
(CKM) mixing matrix

...however, it must be clear that this non-trivial mixing originates only from the Higgs sector: $V_{ij} \rightarrow \delta_{ij}$ if we *switch-off* Yukawa interactions !

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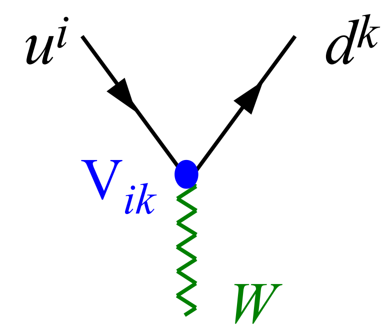

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Several equivalent parameterizations [unobservable phases] in terms of

- 3 real parameters (rotational angles)

  +

- 1 complex phase (source of CP violation)

Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \qquad V V^{+} = \mathrm{I}$$

Diagonalizing the down-quark Yukawa coupling, we get:

$$M_D = \mathrm{diag}(m_d, m_s, m_b) \qquad M_U = V^{+} \times \mathrm{diag}(m_u, m_c, m_t) \rightarrow \mathrm{diag}(m_u, m_c, m_t)$$

To diagonalize also the second mass matrix we need to rotate separately u_L & d_L (non gauge-invariant basis) → V appears in charged-current gauge interactions:

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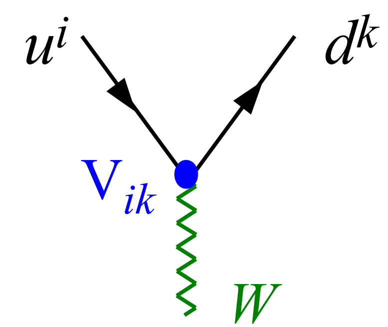

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Several equivalent parameterizations [unobservable phases] in terms of

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The SM quark flavor sector is described by 10 observable parameters [6 quark masses, 3+1 CKM parameters]

N.B.:

- *The rotations on the right-handed sector are not observable*
- *Neutral currents remain flavor diagonal*