Flavor Physics and CP violation

Gino Isidori
[University of Zürich]

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II. The CKM matrix and neutral meson mixing

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- Properties of the CKM matrix & CKM fits
  - Some properties of the CKM matrix
  - How to measure the CKM elements
  - Status of CKM fits
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  - CP symmetry and neutral meson mixing
  - The DF=2 amplitude
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Brief summary of the previous lecture

\[ \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} (A_a, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi, A_a, \psi_i) \]

- Two fundamental (gauge) symmetries:
  - the color symmetry \((\rightarrow \text{strong interactions})\)
  - the electro-weak symmetry

- Three sets of Fundamental Constituents:
  - 3 generations (flavors) of quarks & leptons

- A peculiar symmetry-breaking sector
  non-trivial Higgs vacuum expectation value
Brief summary of the previous lecture

\[ \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}}(A_a, \psi_i) + \mathcal{L}_{\text{Higgs}}(\phi, A_a, \psi_i) \]

3 identical replica of the basic fermion family
\[ [\psi = Q_L, u_R, d_R, L_L, e_R] \Rightarrow \text{large flavor-degeneracy.} \]

\[ \sum_{\psi} \sum_{i=1..3} \bar{\psi}_i i\mathcal{D}_\psi \psi_i \]
Brief summary of the previous lecture

\[ \mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} (A_a, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi, A_a, \psi_i) \]

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\[ [\psi = Q_L, u_R, d_R, L_L, e_R] \Rightarrow \text{large flavor-degeneracy.} \]

Within the SM the flavor-degeneracy is broken only by the Yukawa interaction.
In the quark sector this is of the form
\[ \bar{Q}_L^i Y_D^{ik} d_R^k \phi + h.c. \rightarrow \bar{d}_L^i M_D^{ik} d_R^k + ... \]
\[ \bar{Q}_L^i Y_U^{ik} u_R^k \phi_c + h.c. \rightarrow \bar{u}_L^i M_U^{ik} u_R^k + ... \]
**Brief summary of the previous lecture**

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We can diagonalize only one the two mass matrices without “disturbing” weak interactions:
\[ M_D = \text{diag}(m_d, m_s, m_b) \quad M_U = V^+ \times \text{diag}(m_u, m_c, m_t) \]
Brief summary of the previous lecture

$$M_D = \text{diag}(m_d, m_s, m_b) \quad M_U = V^+ \times \text{diag}(m_u, m_c, m_t) \to \text{diag}(m_u, m_c, m_t)$$

To diagonalize also the second mass matrix we need to rotate separately $u_L$ & $d_L$ (non gauge-invariant basis) $\to$ $V$ appears in charged-current gauge interactions:

$$\mathcal{L}_{\text{gauge}} \rightarrow \frac{g}{\sqrt{2}} W^\mu J^\mu_w$$

$$J^\mu_w = \overline{u}^i_L \gamma^\mu d^i_L \to \overline{u}^i_L V_{ik} \gamma^\mu d^k_L$$

Several equivalent parameterizations [unobservable phases] in terms of

- 3 real parameters (rotational angles)
- 1 complex phase (source of CP violation)

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \quad V V^+ = I$$

Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix
Brief summary of the previous lecture

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M_D = \text{diag}(m_d, m_s, m_b) \quad M_U = V^+ \times \text{diag}(m_u, m_c, m_t) \rightarrow \text{diag}(m_u, m_c, m_t)
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\mathcal{L}_{\text{gauge}} \rightarrow \frac{g}{\sqrt{2}} \ W_\mu \ J^\mu_w \\
J^\mu_w = \bar{u}_L^i \gamma^\mu d_L^i \rightarrow \bar{u}_L^i V_{ik} \gamma^\mu d_L^k
\]

Several equivalent parameterizations [unobservable phases] in terms of

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The SM quark flavor sector is described by 10 observable parameters [6 quark masses, 3+1 CKM parameters]

- The rotations on the right-handed sector are not observable
- Neutral currents remain flavor diagonal

N.B.:
Properties of the CKM matrix & CKM fits
Some properties of the CKM matrix

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$V_{CKM}^+ V_{CKM} = I$$

6 “triangular” relations:

$$(V^+ V)_{i \neq j} = (V^+)_i^1 V_{1j} + (V^+)_i^2 V_{2j} + (V^+)_i^3 V_{3j} = 0$$

- 3 real parameters (rotational angles)
- 1 complex phase (source of CP violation)

N.B.: The elements of the CKM matrix are complex.

Many phases are not observables since they can be eliminated by phase-redefinitions of the fields (e.g. $u \rightarrow e^{i \alpha} u$), but one (physical) phase survive $\rightarrow$ CP violation
Some properties of the CKM matrix

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6 “triangular” relations:

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(V^+V)_{i \neq j} = (V^+)_{i1} V_{1j} + (V^+)_{i2} V_{2j} + (V^+)_{i3} V_{3j} = 0
\]

E.g. for i=b and j=d we get

\[
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0
\]
Some properties of the CKM matrix

\[ V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \]

- 3 real parameters (rotational angles)
  - +
- 1 complex phase (source of CP violation)

\[ V^+_{CKM} V_{CKM} = I \]

6 “triangular” relations:

\[(V^+V)_{i \neq j} = (V^+)_{i1}V_{1j} + (V^+)_{i2}V_{2j} + (V^+)_{i3}V_{3j} = 0\]

the area of these triangles is:

- always the same
- phase-convention independent
- zero in absence of CP violation
Some properties of the CKM matrix

\[ V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \]

Experimental indication of a strongly hierarchical structure:

\[
\begin{bmatrix} 1-\lambda^2/2 & \lambda & 0 \\ -\lambda & 1-\lambda^2/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} \lambda^2 \end{bmatrix}
\]

\[ V^+_{CKM} V_{CKM} = I \]

6 “triangular” relations:

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Wolfenstein, '83

\[ \lambda = 0.22 \]
Some properties of the CKM matrix

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\[ \lambda = 0.22 \]

\[ \Lambda, \quad |\rho+i\eta| = O(1) \]

Mixing

- 1-2 → O(\lambda)
- 2-3 → O(\lambda^2)
- 1-3 → O(\lambda^3)
Some properties of the CKM matrix

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Experimental indication of a strongly hierarchical structure:

\[ \lambda = 0.22 \quad \Lambda, \quad |\rho + i\eta| = O(1) \]

Only the 3-1 triangles have all sizes of the same order in \( \lambda \)

\[ V_{CKM}^+ V_{CKM} = I \]

The \( b \rightarrow d \) triangle:

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

Wolfenstein, '83
Some properties of the CKM matrix

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Experimental indication of a strongly hierarchical structure:

\[ \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \]

\[ V_{\text{CKM}}^+ V_{\text{CKM}} = I \]

The \( b \to d \) triangle:

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]
How to measure the CKM elements

The simplest way to determine the various elements of the CKM matrix is by means of processes mediated by charged-current amplitudes

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\]

Actually we never observe free quarks, but we are able to compute semi-leptonic weak decays (β decays) of the hadrons.

\[
\mathcal{L}_{\text{eff}} = \frac{g^2}{2M_W^2} \ V_{us} \ \bar{u}_L \ \gamma^\mu \ s_L \ \bar{e}_L \ \gamma^\mu \ \nu_L
\]
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\mathcal{L}_{\text{eff}} = \frac{g^2}{2M_W^2} V_{us} \bar{u}_L \gamma^\mu s_L \bar{e}_L \gamma_\mu \nu_L
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The simplest way to determine the various elements of the CKM matrix is by means of processes mediated by charged-current amplitudes.

The effective Lagrangian is given by:

$$\mathcal{L}_{\text{eff}} = \frac{g^2}{2M_W^2} V_{us} \bar{u}_L \gamma^\mu s_L \bar{e}_L \gamma_\mu \nu_L$$

Nuclear $\beta$ decay

$$V_{CKM} = \begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
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\end{bmatrix}$$

Excellent determination (error $\sim 0.1\%$)
Very good determination (error $\sim 0.5\%$)
The simplest way to determine the various elements of the CKM matrix is by means of processes mediated by charged-current amplitudes. Excellent determination (error ~ 0.1%) Very good determination (error ~ 0.5%) Good determination (error ~ 2%) Sizable error (5-15 %) Not competitive with unitarity constraints
The simplest way to determine the various elements of the CKM matrix is by means of processes mediated by charged-current amplitudes.

Also the phase $\gamma = \arg(V_{ub})$ can be obtained by (quasi-) tree-level processes, such as

$B \rightarrow D$ ($\bar{D}$) + $K \rightarrow f + K$:
How to measure the CKM elements

The only two CKM elements we cannot access via (tree-level) charged-current processes are $V_{ts}$ & $V_{td}$

Loop-induced amplitudes:
(neutral-meson mixing)

As we will see in the second part of today's lecture, these amplitudes are dominated by the top-quark contribution: $[ A \sim m_t^2 V_{tq}^* V_{tb} ]$
Present status of CKM fits

At present, measurements of sides and angels of the CKM unitarity triangle show a remarkable success of the SM:

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V_{CKM} = \begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
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\end{bmatrix}
\]

\[
\begin{bmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
- \lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{bmatrix}
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**Present status of CKM fits**

At present, measurements of sides and angles of the CKM unitarity triangle show a remarkable success of the SM *(redundant and consistent determination of various CKM elements)*.

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\end{bmatrix}$$
CP violation and neutral meson mixing
**CP symmetry and neutral meson mixing**

The lightest bound states (mesons) composed by a *quark-antiquark pair* of same charge but *different flavor* form a very interesting systems: a pair of pseudo-scalar mesons with

- tiny mass difference (*due to 2nd order weak interactions*)
- mass eigenstates different from flavor eigenstates

Four systems of this type: $K^0 = |s d>$, $B_d = |b d>$, $B_s = |b s>$, $D = |c s>$

The interesting time-evolution of these systems has allowed to discover the phenomenon of CP violation in fundamental interactions (so far observed both in the $K^0$-$K^0$ and in the $B_d$-$B_d$ systems).

\[ M_{B_d} = 5.279 \text{ GeV} \]

\[ \Delta M_{B_d} = 3.4 \times 10^{-13} \text{ GeV} \]
CP symmetry and neutral meson mixing

The effective Hamiltonian describing the ground state (i.e. the mass matrix) of these systems has a relatively simple structure:

\[
i \frac{d}{dt} \begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix} = \begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix} \begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix}
\]

- A general theorem of QFT (the CPT theorem) implies \( M_{11} = M_{22} = M_0 = \text{real} \)

\[
M_0 = M_{B_d} = 5.279 \text{ GeV} \quad \quad |M_{12}| = \Delta M_{B_d} = 3.4 \times 10^{-13} \text{ GeV}
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\]

- A general theorem of QFT (*the CPT theorem*) implies \( M_{11} = M_{22} = M_0 = \text{real} \)
- *If CP is a good symmetry* \[ A(\text{matter} \rightarrow \text{antimatter}) = A(\text{antimatter} \rightarrow \text{matter}) \] then \( M_{12} = M_{21} \rightarrow M_{12} = \text{real} \)

The complex CKM (↔ complex Yukawa couplings) induce CPV

\[
M_0 = M_{B_d} = 5.279 \text{ GeV}
\]

\[
|M_{12}| = \Delta M_{B_d} = 3.4 \times 10^{-13} \text{ GeV}
\]
The $\Delta F=2$ amplitude

- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy
- *Calculable with good accuracy* $\leftrightarrow$ top-quark dominance
- *Measurable with good accuracy* $\leftrightarrow$ time evolution of the neutral meson system

Highly-suppressed amplitude potentially very sensitive to contributions beyond the SM
The reason why the amplitude is dominated by the top-quark contribution (hence is calculable with high precision) can be somehow understood even without performing a complete QFT calculation:

\[ A_{\Delta F=2} = \sum_{q,q'=u,c,t} (V_{qb}^* V_{qd}) (V_{q'b}^* V_{q'd}) A_{q'q} \]

\[ A_{qq'} \sim \frac{g^4}{16\pi^2 m_W^2} \left[ \text{Const.} + \frac{m_q m_{q'}}{m_W^2} + \ldots \right] \]
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\]

\[
V_{ub}^* V_{ud} = - V_{tb}^* V_{td} - V_{cb}^* V_{cd}
\]  
[CKM unitarity]

\[
A_{\Delta F=2} = \sum_{q=u,c,t} (V_{qb}^* V_{qd}) \left[ V_{tb}^* V_{td} (A_{tq} - A_{uq}) + V_{cb}^* V_{cd} (A_{cq} - A_{uq}) \right]
\]

\[
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[CKM unitarity]

\[
A_{\Delta F=2} = \sum_{q=u,c,t} (V_{qb}^* V_{qd}) \left[ V_{tb}^* V_{td} (A_{tq} - A_{uq}) + V_{cb}^* V_{cd} (A_{cq} - A_{uq}) \right]
\]

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A_{qq'} \sim \frac{g^4}{16\pi^2m_W^2} \left[ \text{Const.} + \frac{m_q m_{q'}}{m_W^2} + \ldots \right]
\]

since \( m_t \gg m_{u,c} \)

\[
A_{\Delta F=2} \sim (V_{tb}^* V_{td})^2 \frac{g^4m_t^2}{16\pi^2m_W^4}
\]
Mass eigenstates:

\[ |B_L\rangle = p |B^0\rangle + q |\bar{B}^0\rangle \quad |B_H\rangle = p |\bar{B}^0\rangle + q |B^0\rangle \]

Taking into account that the heavy quarks inside the mesons decay, we can describe the time evolution of the neutral mesons in full generality by means of a non-Hermitian Hamiltonian.
Time evolution and time-dependent CP asymmetries

\[
\frac{i}{\hbar} \frac{d}{dt} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = [M - i \Gamma/2] \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}
\]

Taking into account that the heavy quarks inside the mesons decay, we can describe the time evolution of the neutral mesons in full generality by means of a non-Hermitian Hamiltonian

\[
\begin{pmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{pmatrix}
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Mass eigenstates:

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|B_L\rangle = p |B^0\rangle + q |\bar{B}^0\rangle \quad |B_H\rangle = p |\bar{B}^0\rangle + q |B^0\rangle
\]

\[
\frac{q}{p} = \arg(M_{12}) = \frac{(V_{tb}^* V_{td})^2}{\left| (V_{tb} V_{td}^*)^2 \right|} = e^{-2i\beta}
\]

Large CPV phase
(in the standard CKM phase convention)
The study of time-dependent decays of neutral B into CP eigenstates is a marvelous tool to extract CPV phases in a clean way:

\[
\begin{align*}
[ t=0 ] & \quad B^0 & \xrightarrow{q/p} & \quad \bar{B}^0 & \xrightarrow{A_f} & \quad f \quad [ t ] \\
\lambda_f = \frac{q}{p} \bar{A}_f & \quad \text{Phase-convention independent quantity} & \quad [ \rightarrow \text{observable}] \\
\end{align*}
\]

\[
\Gamma(B^0(t)\to f) \propto e^{-\Gamma_B t} \left[ 1 - \eta_f \text{Im}(\lambda_f) \sin(\Delta m_B t) \right] \\
\Gamma(\bar{B}^0(t)\to f) \propto e^{-\Gamma_B t} \left[ 1 + \eta_f \text{Im}(\lambda_f) \sin(\Delta m_B t) \right]
\]

If \(|\lambda_f| = 1\) (i.e. if \(A_f\) is dominated by a single weak phase) then:

\[\text{Im}(\lambda_f) \neq 0 \iff \text{CP}\]
The study of time-dependent decays of neutral $B$ into CP eigenstates is a marvelous tool to extract CPV phases in a clean way:

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\begin{align*}
[t=0] & \quad B^0 & \xrightarrow{q/p} & \quad B^0 & \xrightarrow{A_f} & \quad f & \quad [t]
\end{align*}
\]

Phase-convention independent quantity [ $\rightarrow$ observable]

\[
\lambda_f = \frac{q}{p} \frac{\overline{A}_f}{A_f}
\]

If $|\lambda_f| = 1$ (i.e. if $A_f$ is dominated by a single weak phase) then:

\[
\Gamma(B^0(t) \rightarrow f) \propto e^{-\frac{\Gamma_B}{t}} \left[ 1 - \eta_f \operatorname{Im}(\lambda_f) \sin(\Delta m_B t) \right]
\]

$\operatorname{Im}(\lambda_f) \neq 0 \iff \text{CP}$

Key points to successfully use this method:

- [EXP]: flavor tagging and time-dependent resolution are essential ingredients
- [TH]: identify final states such that $A_f$ is dominated by a single weak phase
A few words about flavor tagging: B factories vs. hadron colliders

B factories:

\[ e^+ + e^- \rightarrow \Psi(4S) \rightarrow \bar{B} \ B \]

- clean environment [ \( \sigma(B) / \sigma(bkg) \sim 0.3 \) ]
- coherent quantum state for neutral B
- “easy” and clean flavor tag from the decay of the opposite meson (e.g. \( b \rightarrow c e^- \nu \))

Hadron colliders:

- dirty environment [ \( \sigma(B) / \sigma(bkg) < 0.01 \) ]
- incoherent quantum state

- high stat. [ \( \sim 10^{12} \) B pairs / 1 fb\(^{-1}\) ]
- all hadrons with b-quarks produced
A few words about flavor tagging: B factories vs. hadron colliders
When is $A_f$ dominated by a single weak phase?

$|B_d \rangle \rightarrow | \psi K_s \rangle$

$[b + d \rightarrow c\bar{c}s + d]$

\[ \text{real } O(\lambda^2) \quad \text{real } O(\alpha_s \lambda^2) \quad \text{O}(\alpha_s \lambda^5) \]

\[ \text{dominant} \quad \text{amplitude} \quad \text{pollution } \lesssim 1\% \]

\[ \text{Im}(\lambda_f) = \sin(2\beta) \quad \text{(from the mixing)} \]

extremely precise constraint in the $\rho$--$\eta$ plane

\[ \text{Golden channel for B factories} \]
extremely precise constraint in the $\rho-\eta$ plane:
Today's summary

At present, measurements of sides and angels of the CKM unitarity triangle show a remarkable success of the SM (*redundant and consistent determination of various CKM elements*):