Flavor Physics and CP violation

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I. Introduction

II. The CKM matrix and neutral meson mixing

III. Rare B and K decays

IV. Flavor Physics beyond the SM

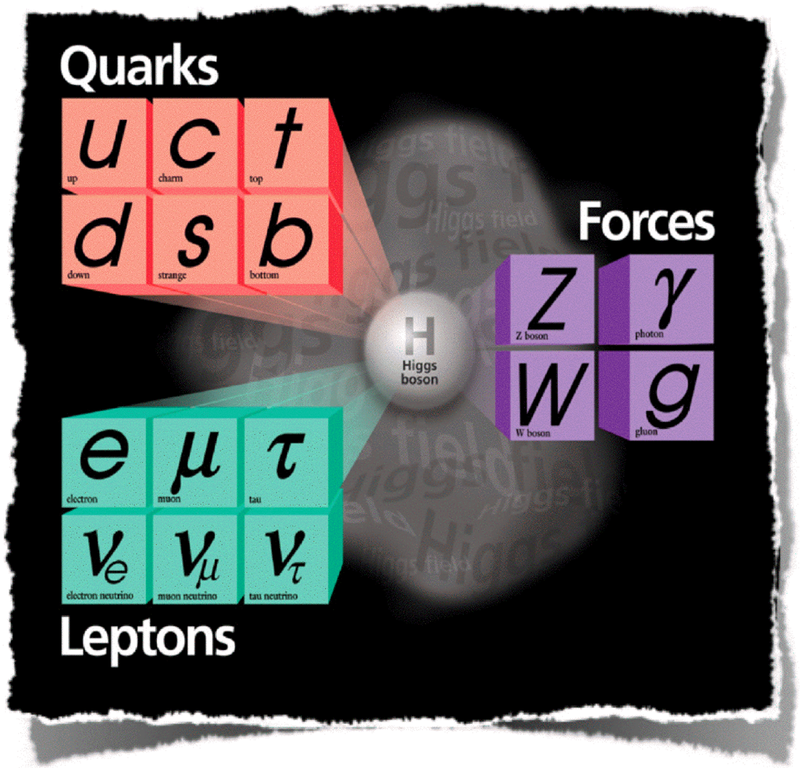
II. The CKM matrix and neutral meson mixing

- Brief summary of the previous lecture
- Properties of the CKM matrix & CKM fits
  - *Some properties of the CKM matrix*
  - *How to measure the CKM elements*
  - *Status of CKM fits*
- CP violation and neutral meson mixing
  - *CP symmetry and neutral meson mixing*
  - *The DF=2 amplitude*
  - *Time evolution and time-dependent asymmetries*

Brief summary of the previous lecture

$$\mathscr{L}_{SM} = \mathscr{L}_{gauge}(A_a, \psi_i) + \mathscr{L}_{Higgs}(\phi, A_a, \psi_i)$$

- Two fundamental (gauge) symmetries:
  - the color symmetry (→ *strong interactions*)
  - the electro-weak symmetry
- Three sets of Fundamental Constituents:
  - 3 generations (flavors) of quarks & leptons
- A peculiar symmetry-breaking sector
  non-trivial Higgs vacuum expectation value



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↑

3 identical replica of the basic fermion family
$[\psi = Q_L, u_R, d_R, L_L, e_R] \Rightarrow$ large flavor-degeneracy.

$$\left[\sum_{\psi} \sum_{i=1..3} \bar{\psi}_i \, i\not{D} \, \psi_i \right]$$

▶ *Brief summary of the previous lecture*

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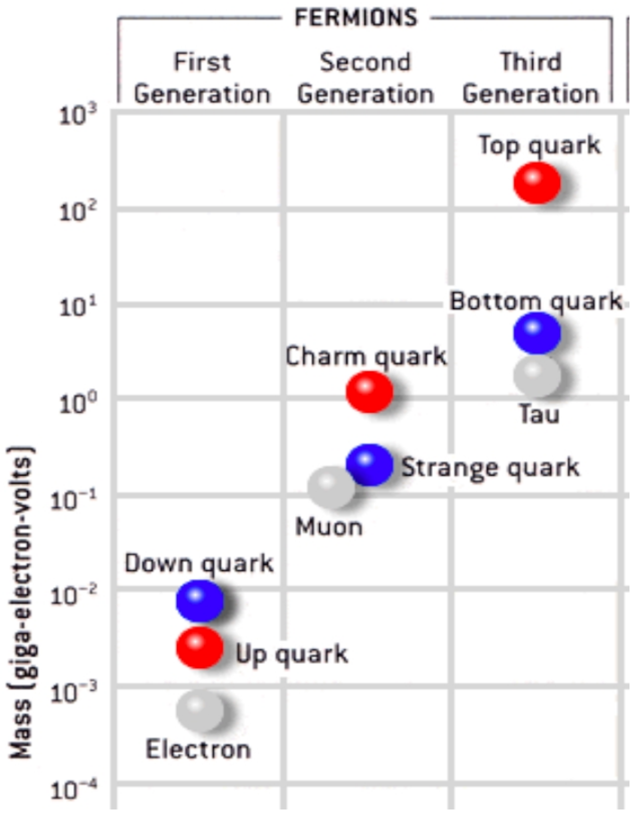
Within the SM the flavor-degeneracy is broken only by the Yukawa interaction.
In the quark sector this is of the form

$$\bar{Q}_L^{\,i}\, Y_D^{ik}\, d_R^{\,k}\, \phi + h.c. \rightarrow \bar{d}_L^{\,i}\, M_D^{ik}\, d_R^{\,k} + \ldots$$

$$\bar{Q}_L^{\,i}\, Y_U^{ik}\, u_R^{\,k}\, \phi_c + h.c. \rightarrow \bar{u}_L^{\,i}\, M_U^{ik}\, u_R^{\,k} + \ldots$$



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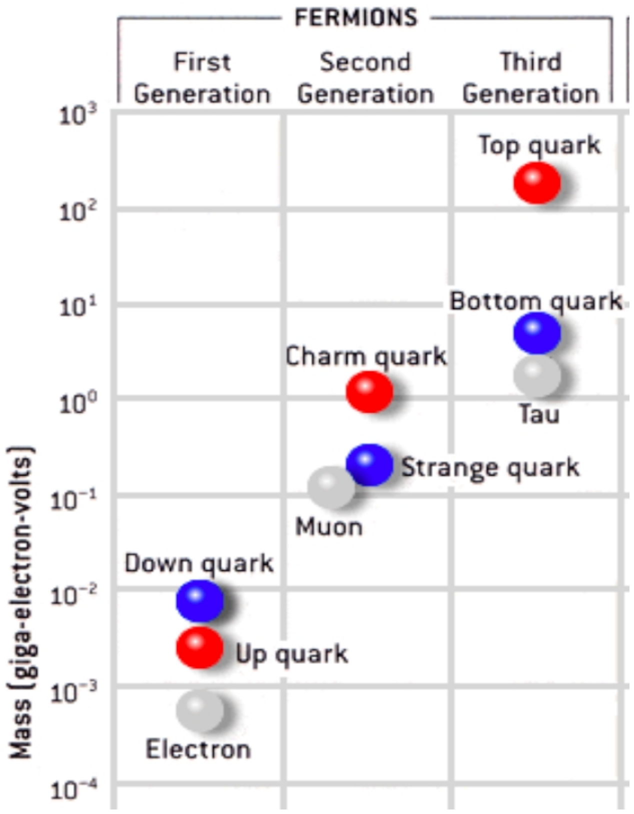
$$\bar{Q}_L^i Y_U^{ik} u_R^k \phi_c + h.c. \rightarrow \bar{u}_L^i M_U^{ik} u_R^k + \ldots$$

We can diagonalize only one the two mass matrices without "disturbing" weak interactions:

$$M_D = {\rm diag}(m_d, m_s, m_b) \qquad M_U = V^+ \times {\rm diag}(m_u, m_c, m_t)$$



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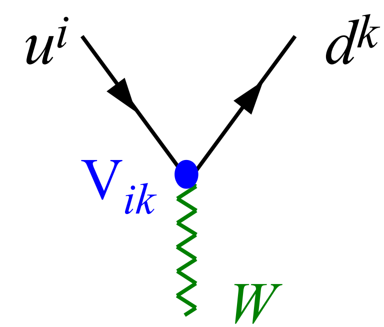
To diagonalize also the second mass matrix we need to rotate separately u_L & d_L (non gauge-invariant basis) → V appears in charged-current gauge interactions:

$$\mathscr{L}_{gauge} \rightarrow \frac{g}{\sqrt{2}} W_\mu J_W^\mu$$



$$J_W^\mu = \bar{u}_L^i \gamma^\mu d_L^i \rightarrow \bar{u}_L^i V_{ik} \gamma^\mu d_L^k$$

→ Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix

Several equivalent parameterizations [unobservable phases] in terms of

- 3 real parameters (rotational angles)

  +

- 1 complex phase (source of CP violation)

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \qquad V V^{+} = I$$

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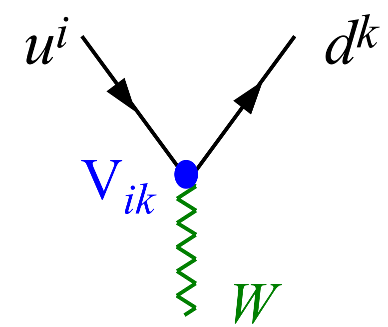

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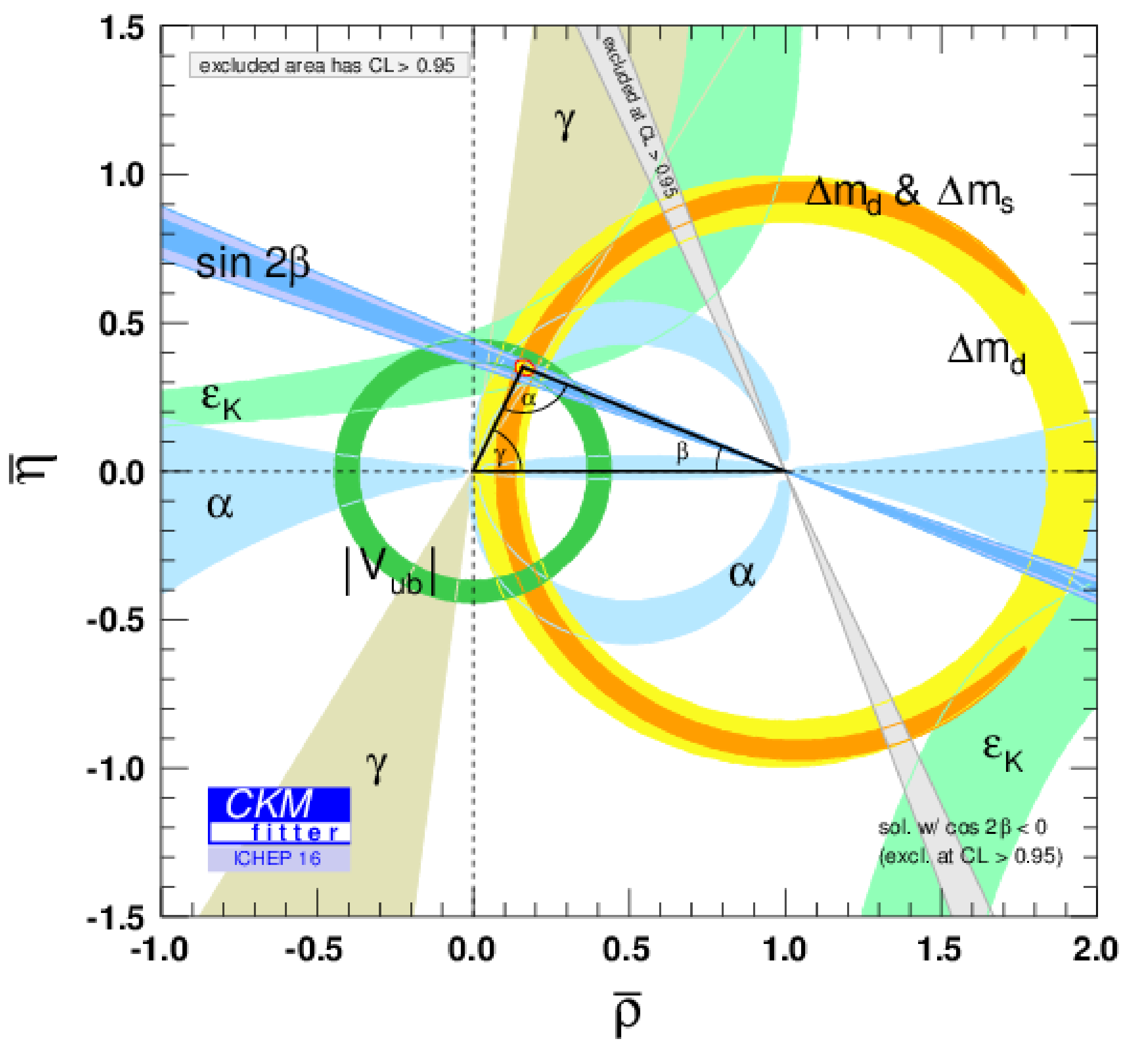
The SM quark flavor sector is described by 10 observable parameters [6 quark masses, 3+1 CKM parameters]

N.B.:

- *The rotations on the right-handed sector are not observable*
- *Neutral currents remain flavor diagonal*

Properties of the CKM matrix & CKM fits



Some properties of the CKM matrix

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$V^{+}_{CKM} V_{CKM} = I$$

↓

6 "triangular" relations:

$$(V^{+}V)_{i\neq j} = (V^{+})_{i1} V_{1j} + (V^{+})_{i2} V_{2j} + (V^{+})_{i3} V_{3j} = 0$$

- 3 real parameters (rotational angles)

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- 1 complex phase (source of CP violation)

N.B.: The elements of the CKM matrix are complex.

Many phases are not observables since they can be eliminated by phase-redefinitions of the fields (e.g. $u \to e^{i\alpha} u$), but one (physical) phase survive → CP violation

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E.g. for i=b and j=d we get

$$V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0$$

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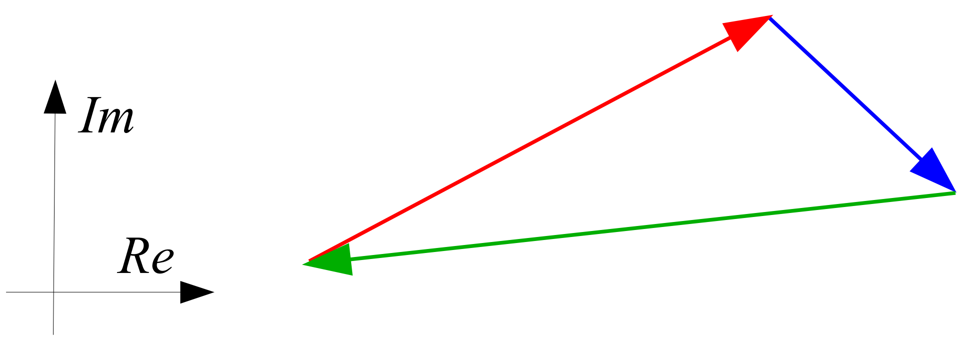

the area of these triangles is:

- always the same
- phase-convention independent
- zero in absence of CP violation

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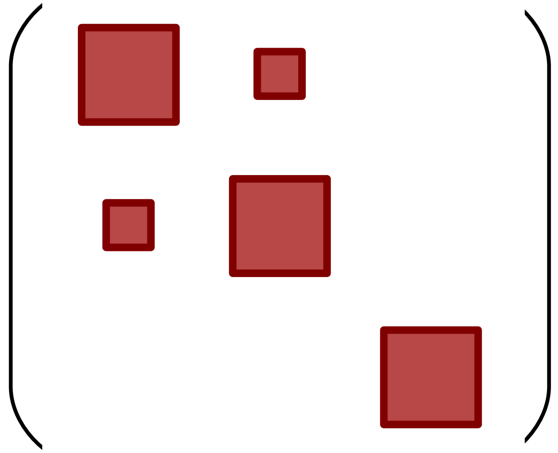
Experimental indication of a strongly hierarchical structure:

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & 0 \\ -\lambda & 1-\lambda^2/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + O(\lambda^2)$$

Wolfenstein, '83

$\lambda = 0.22$

$V_{CKM} \sim$ 

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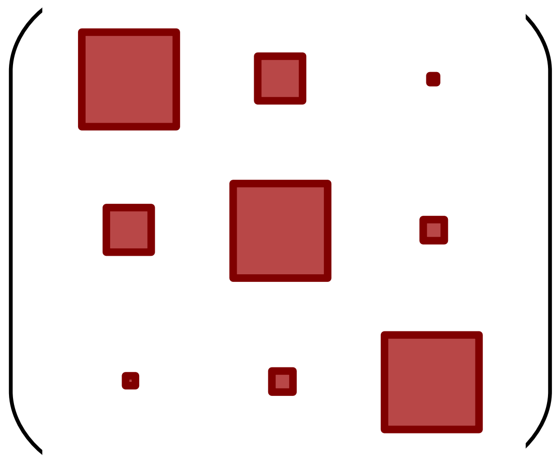
Wolfenstein, '83

$V_{CKM} \sim$ 

$\lambda = 0.22$

A, $|\rho + i\eta| = O(1)$

mixing 1-2 $\rightarrow O(\lambda)$
mixing 2-3 $\rightarrow O(\lambda^2)$
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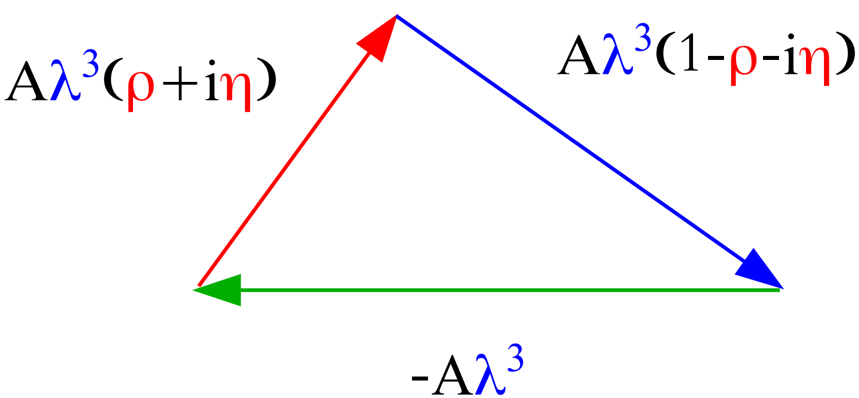
The b → d triangle:

$$V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0$$



only the 3-1 triangles have all sizes of the same order in λ

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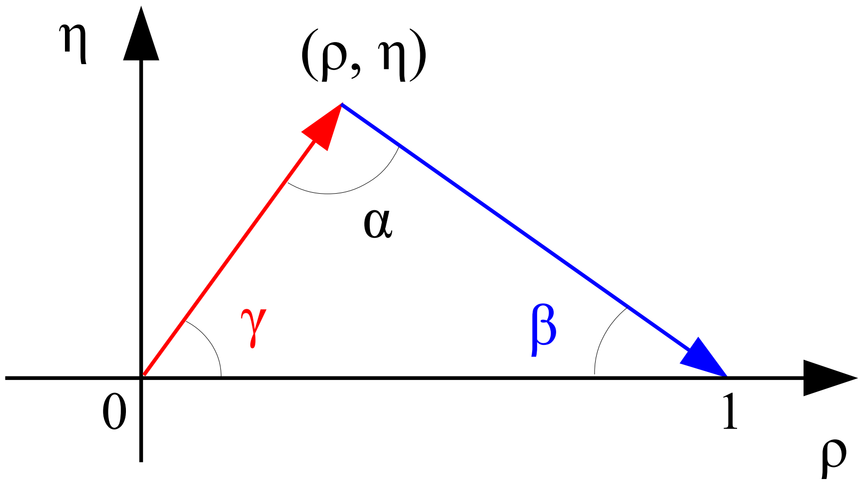
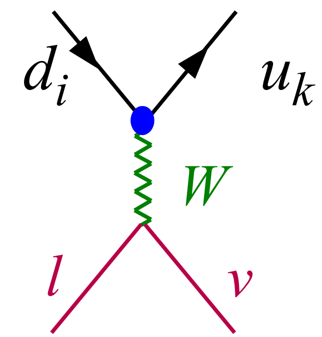


How to measure the CKM elements

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

The simplest way to determine the various elements of the CKM matrix is by means of processes mediated by charged-current amplitudes

$$\mathscr{L}_{gauge} \to \frac{g}{\sqrt{2}} W_\mu \bar{u}_L^i V_{ik} \gamma^\mu d_L^k + h.c.$$



$\Rightarrow V_{ik}$

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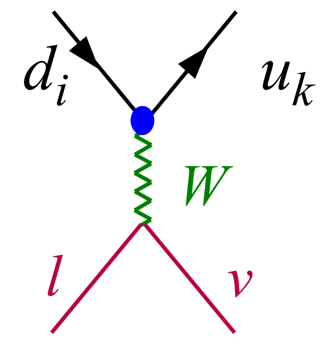
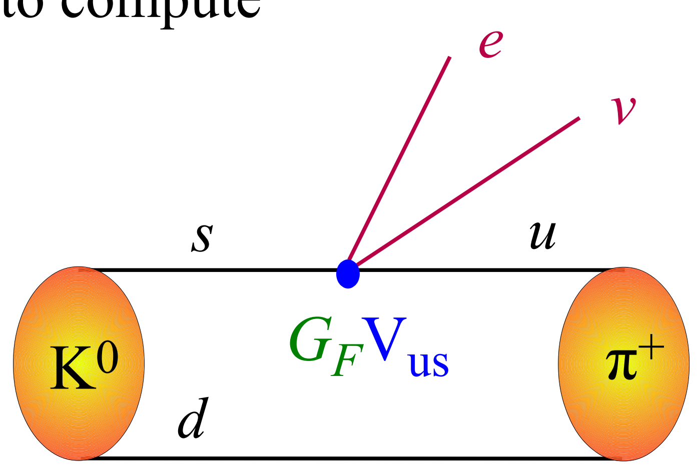


$\Rightarrow V_{ik}$

Actually we never observe free quarks, but we are able to compute semi-leptonic weak decays (β decays) of the hadrons

$$\mathscr{L}_{eff} = \frac{g^2}{2M_W^2} V_{us} \, \bar{u}_L \gamma^\mu s_L \; \bar{e}_L \gamma_\mu \nu_L$$

$\frac{g^2}{2M_W^2} \leftrightarrow G_F$



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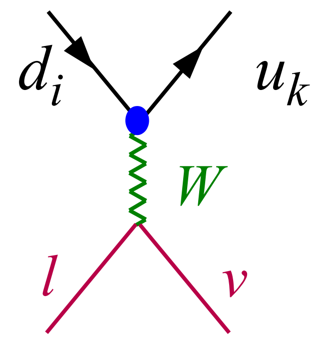

$\rightarrow$ V_{ik}

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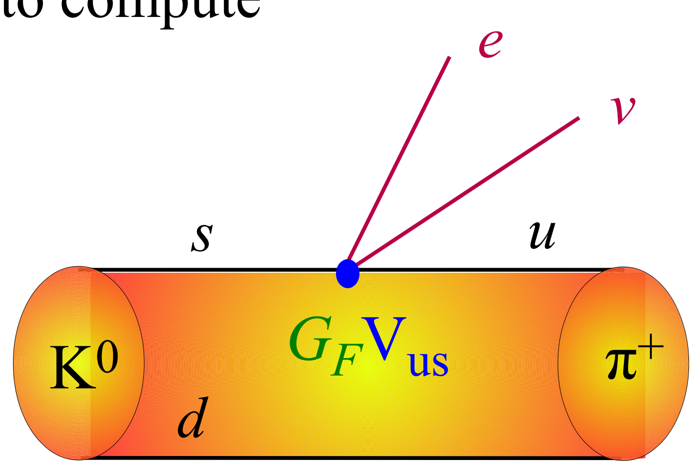

Nuclear β decay (V_{ud}) $K \to \pi l \nu$ (V_{us})

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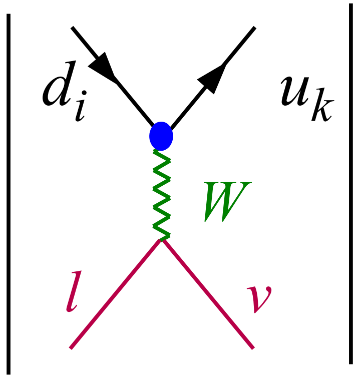

→ $|V_{ik}|$

Excellent determination (error ~ 0.1%)
Very good determination (error ~ 0.5%)

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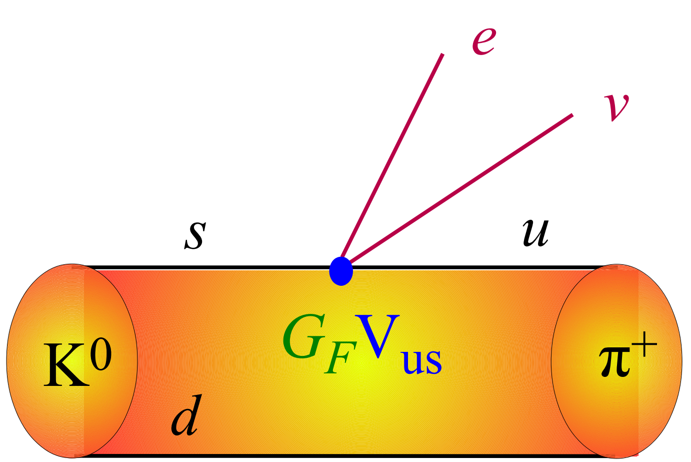

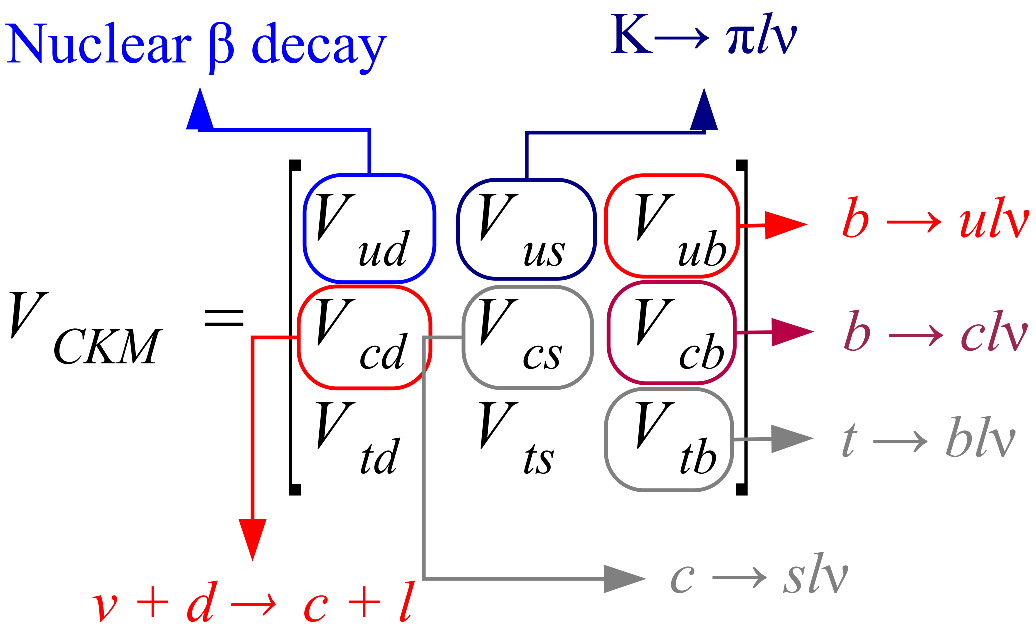

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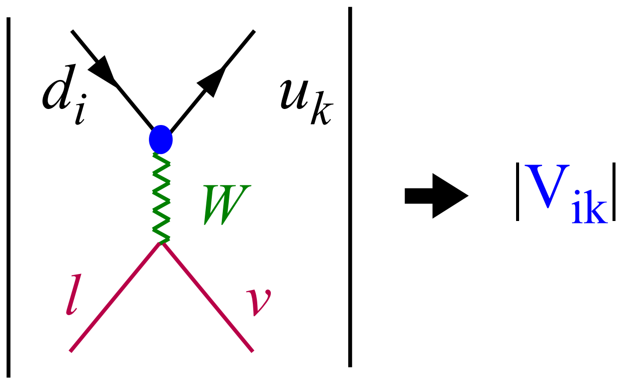
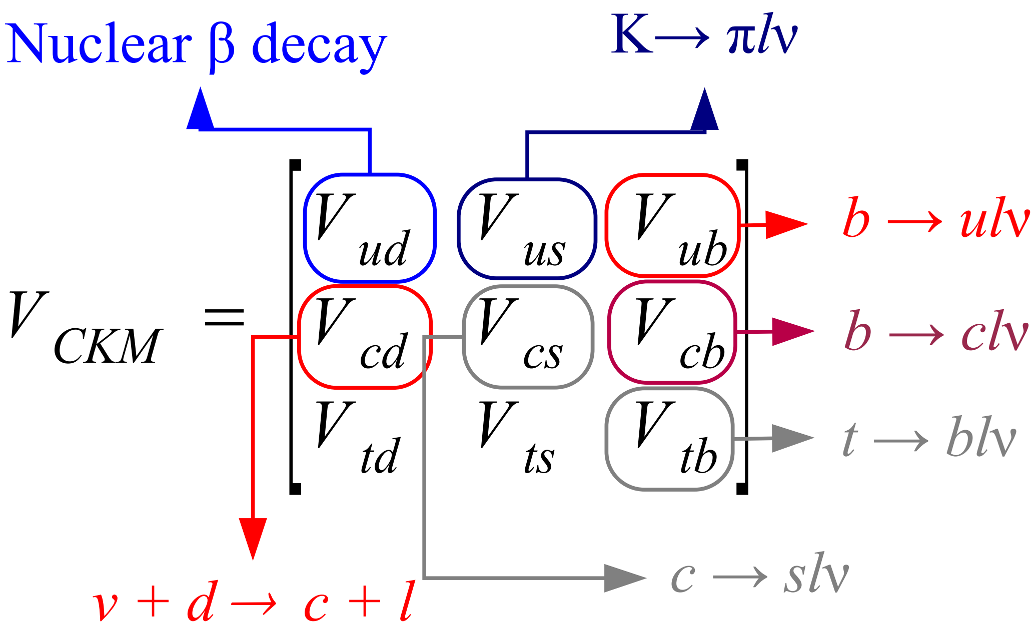


Excellent determination (error ~ 0.1%)
Very good determination (error ~ 0.5%)
Good determination (error ~ 2 %)
Sizable error (5-15 %)
Not competitive with unitarity constraints



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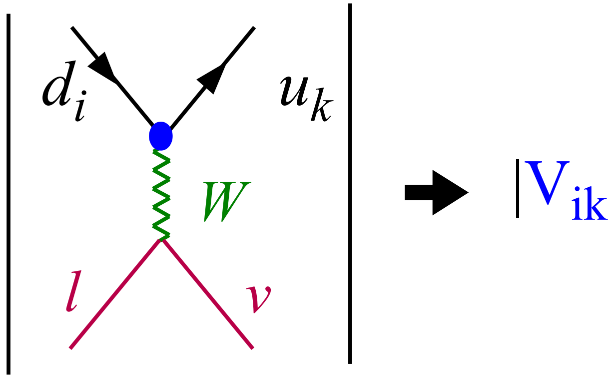

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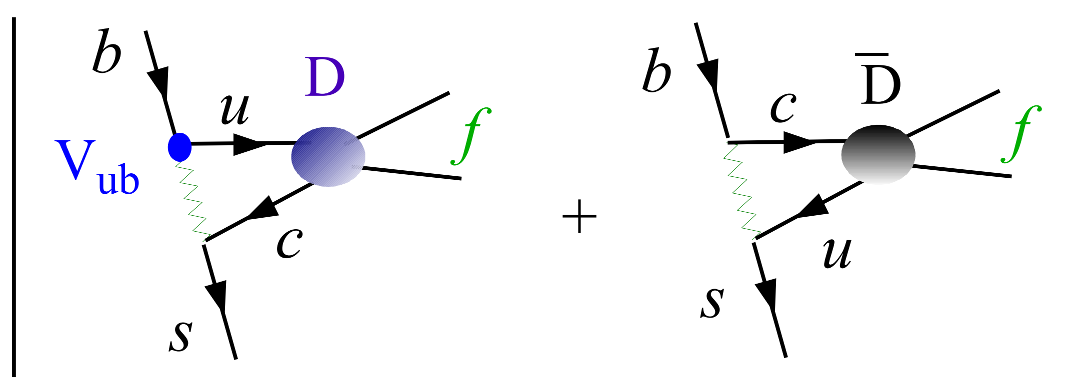
Not competitive with unitarity constraints

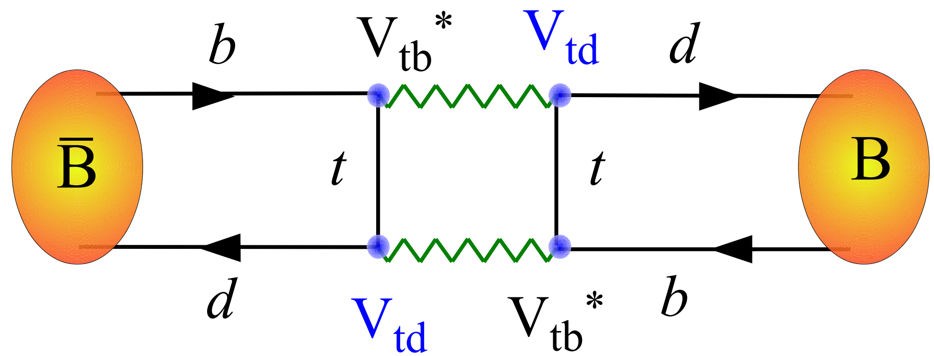
Also the phase $\gamma = \arg(V_{ub})$ can be obtained by (quasi-) tree-level processes, such as

$B \to D\,(\bar{D}) + K \to f + K$:

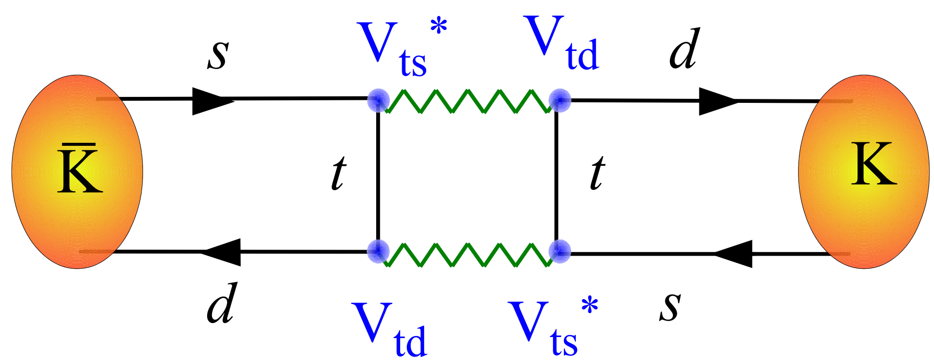


How to measure the CKM elements

The only two CKM elements we cannot access via (tree-level) charged-current processes are V_{ts} & V_{td}



Loop-induced amplitudes:
(neutral-meson mixing)



As we will see in the second part of today's lecture, these amplitudes are dominated by the top-quark contribution: $[\, A \sim m_t^2\ V_{tq}^*\, V_{tb}\,]$

Present status of CKM fits

At present, measurements of sides and angels of the CKM unitarity triangle show a remarkable success of the SM:

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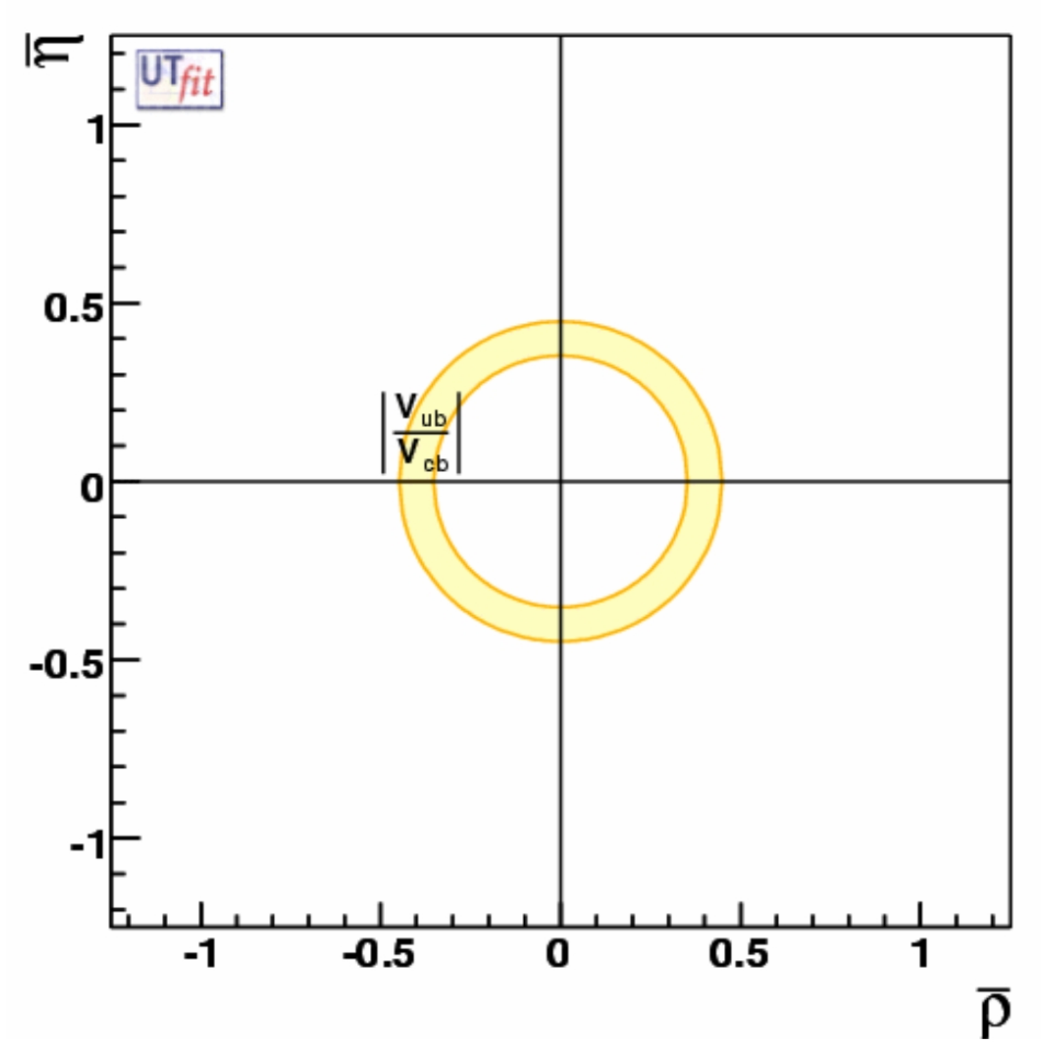

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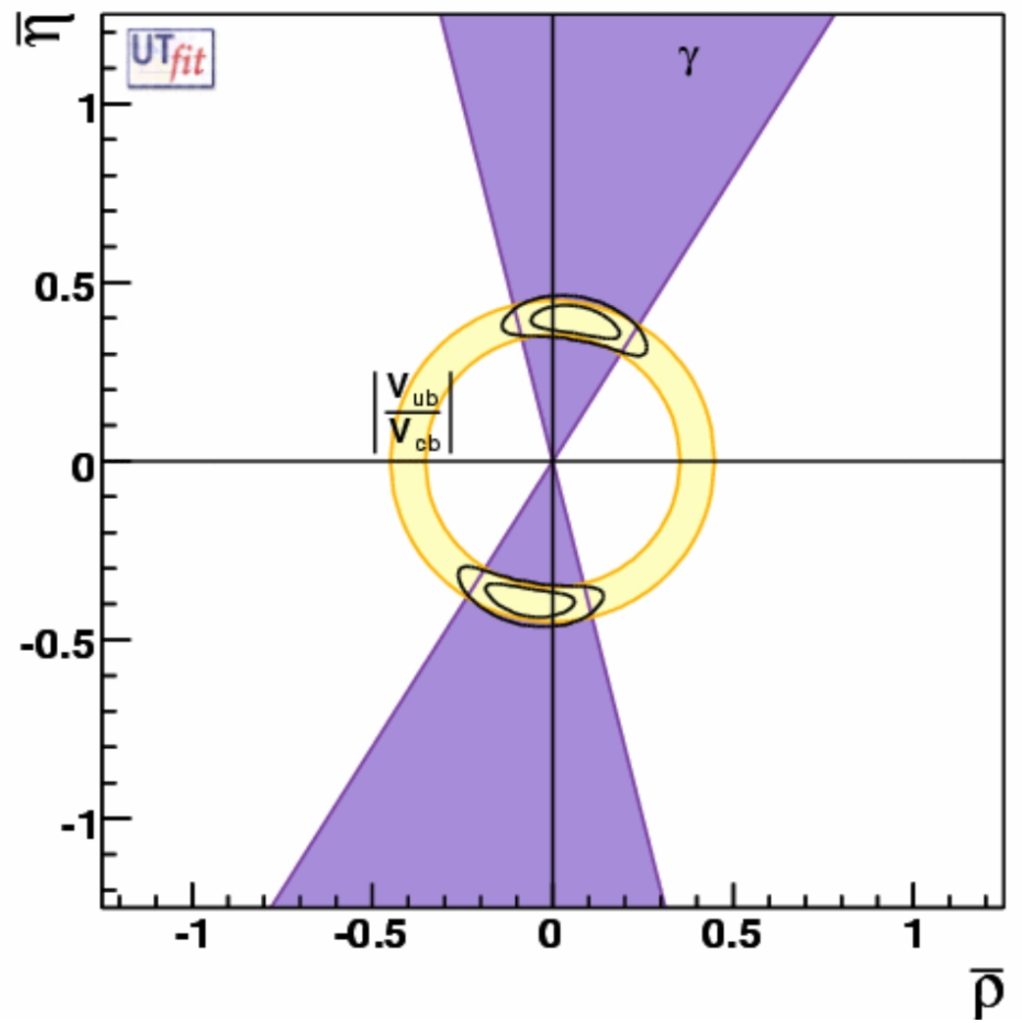

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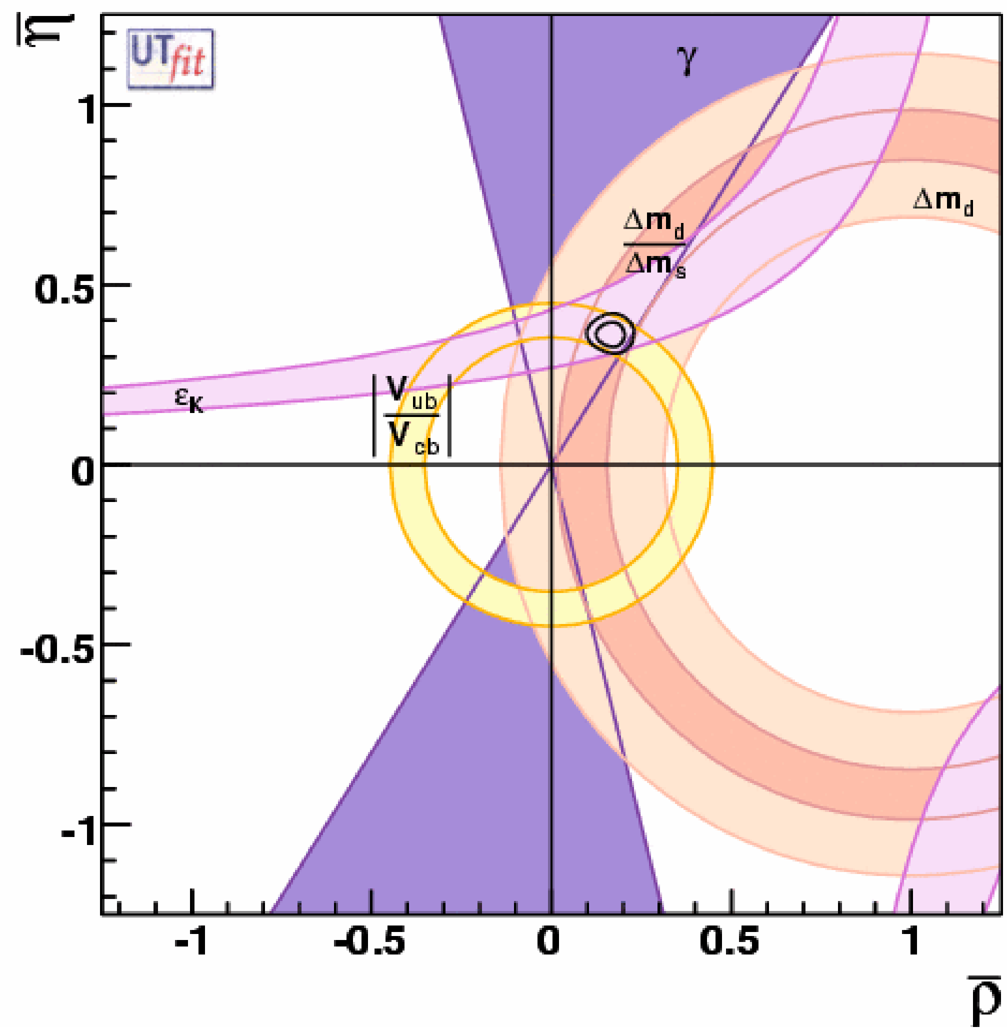
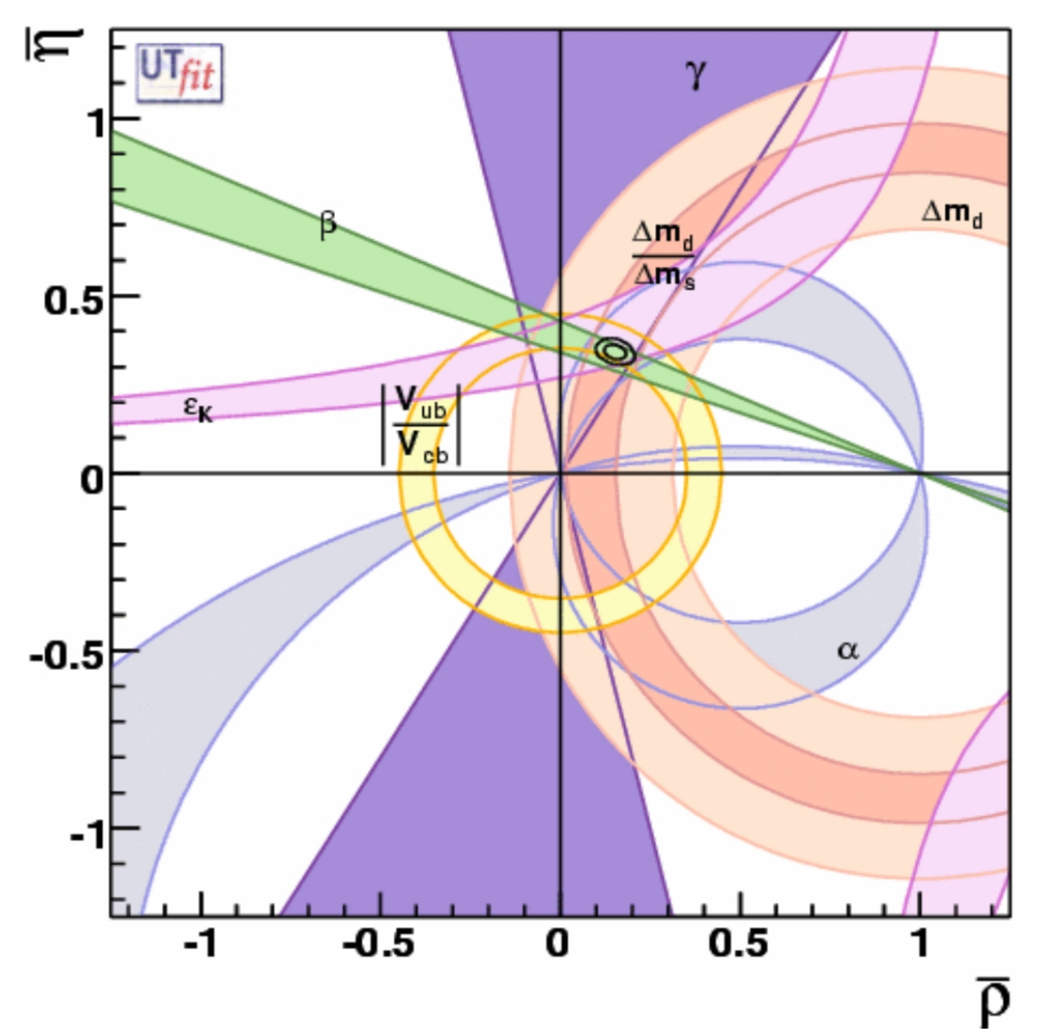


Present status of CKM fits

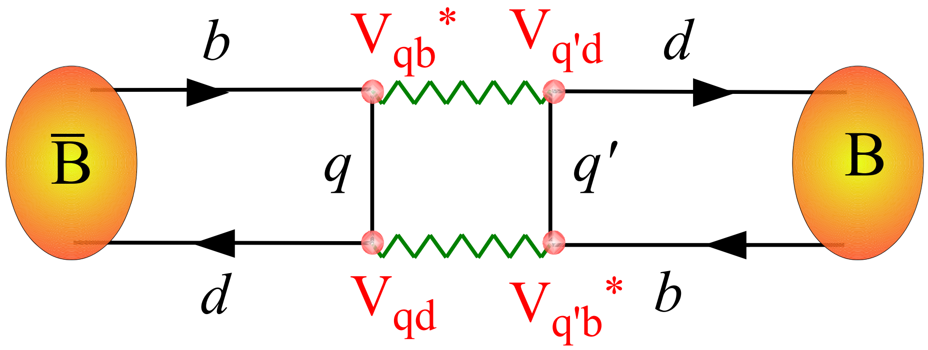
At present, measurements of sides and angels of the CKM unitarity triangle show a remarkable success of the SM (*redundant and consistent determination of various CKM elements*).

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$



CP violation and neutral meson mixing



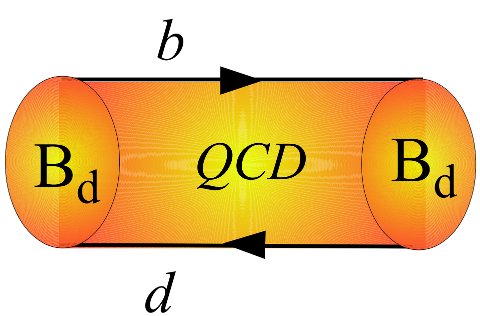
CP symmetry and neutral meson mixing

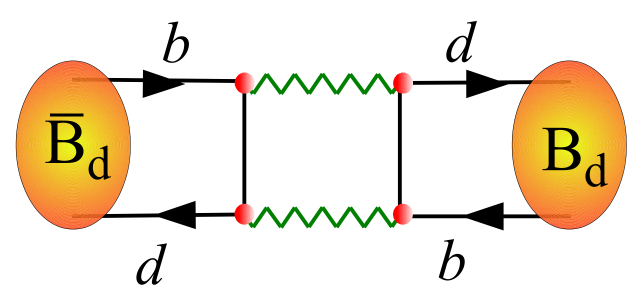
The lightest bound states (mesons) composed by a *quark-antiquark pair* of same charge but *different flavor* form a very interesting systems: a pair of pseudo-scalar mesons with

- tiny mass difference (*due to 2nd order weak interactions*)
- mass eigenstates different from flavor eigenstates

Four systems of this type: $K^0 = |\underline{s}d\rangle$, $B_d = |\underline{b}d\rangle$, $B_s = |\underline{b}s\rangle$, $D = |\underline{c}s\rangle$

The interesting time-evolution of these systems has allowed to discover the phenomenon of CP violation in fundamental interactions (so far observed both in the K^0-K^0 and in the B_d-B_d systems).





$M_{Bd} = 5.279$ GeV

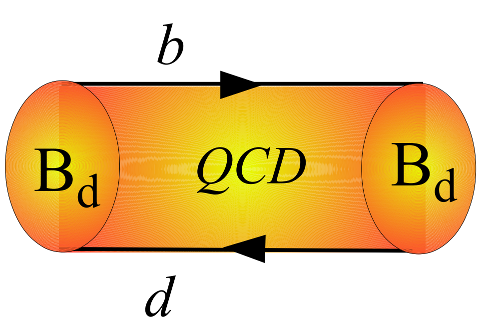
$\Delta M_{Bd} = 3.4\times 10^{-13}$ GeV

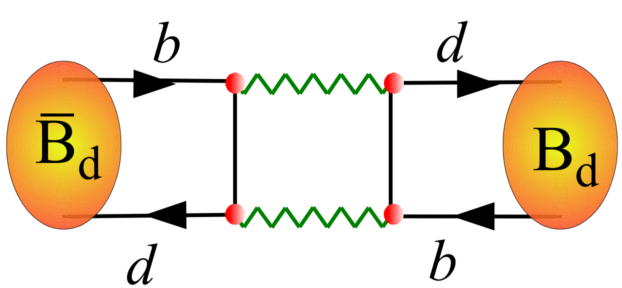
CP symmetry and neutral meson mixing

The effective Hamiltonian describing the ground state (i.e. the mass matrix) of these systems has a relatively simple structure:

$$i \frac{d}{dt} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = \begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}$$

- A general theorem of QFT (*the CPT theorem*) implies $M_{11} = M_{22} = M_0 = \textit{real}$





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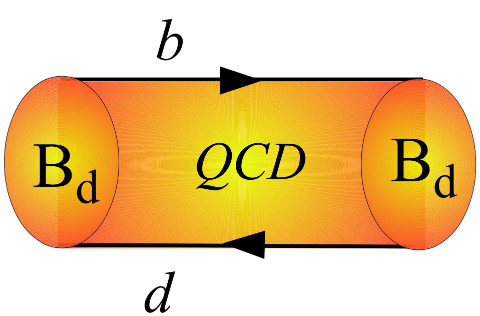
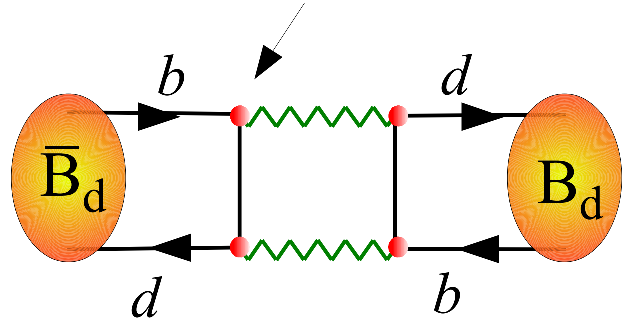
- *If CP is a good symmetry* [*A(matter→antimatter) = A(antimatter→matter)*] then $M_{12} = M_{21} \rightarrow M_{12} = real$

The complex CKM (↔ *complex Yukawa couplings*) *induce CPV*





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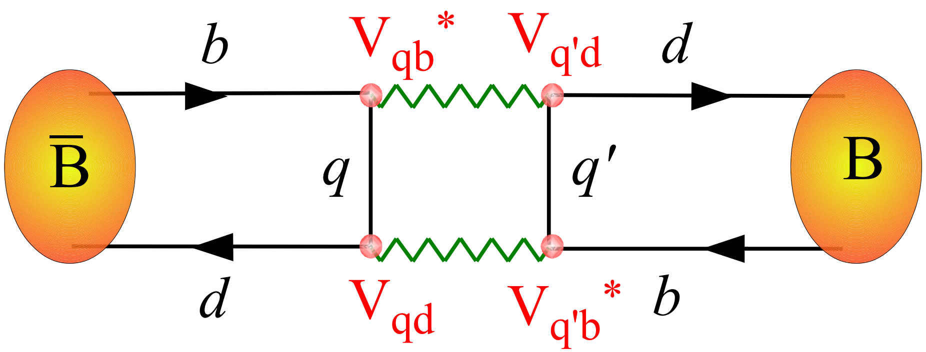
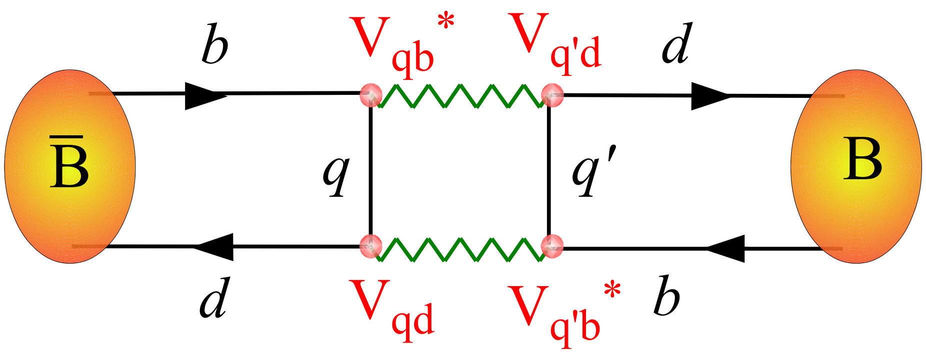
The ΔF=2 amplitude



Highly-suppressed amplitude potentially very sensitive to contributions beyond the SM

- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy
- *Calculable with good accuracy* [↔ top-quark dominance]
- *Measurable with good accuracy* [↔ time evolution of the neutral meson system]

The ΔF=2 amplitude



The reason why the amplitude is dominated by the top-quark contribution (*hence is calculable with high precision*) can be somehow understood even without performing a complete QFT calculation:

$$A_{\Delta F=2} = \Sigma_{q,q'=u,c,t} (V_{qb}^* V_{qd}) (V_{q'b}^* V_{q'd}) \, A_{q'q}$$

$$A_{qq'} \sim \frac{g^4}{16\pi^2 m_W^2} \left[\text{Const.} + \frac{m_q \, m_{q'}}{m_W^2} + \ldots \right]$$

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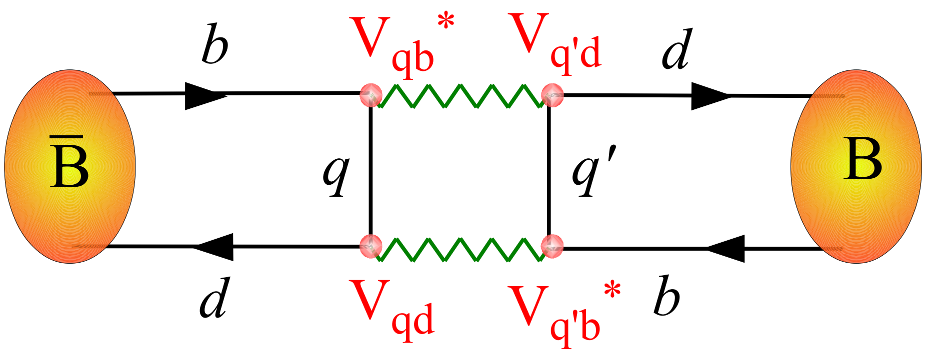

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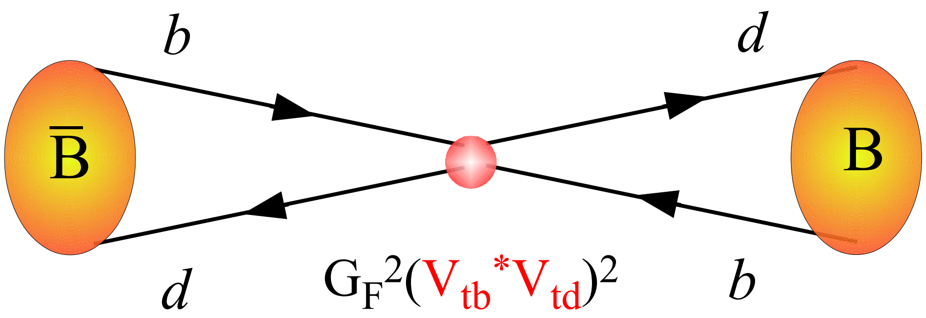

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since $m_t >> m_{u,c}$

$$A_{\Delta F=2} \sim (V_{tb}^* V_{td})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4}$$

Time evolution and time-dependent CP asymmetries

$$i \frac{d}{dt} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = [\, M - i\,\Gamma/2 \,] \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}$$

Taking into account that the heavy quarks inside the mesons decay, we can describe the time evolution of the neutral mesons in full generality by means of a non-Hermitian Hamiltonian

$$\begin{bmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{bmatrix}$$

Mass eigenstates:

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \qquad |B_H\rangle = p|\bar{B}^0\rangle + q|B^0\rangle$$

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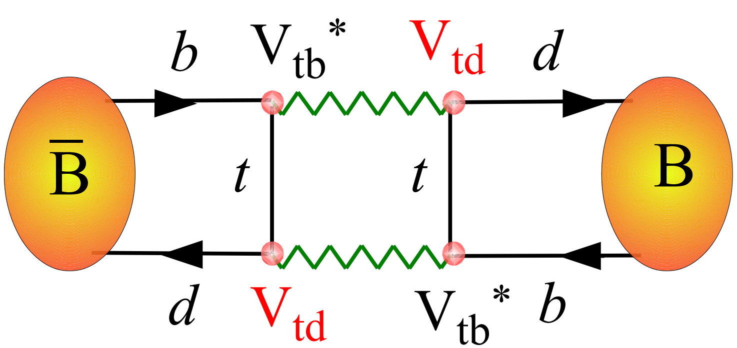
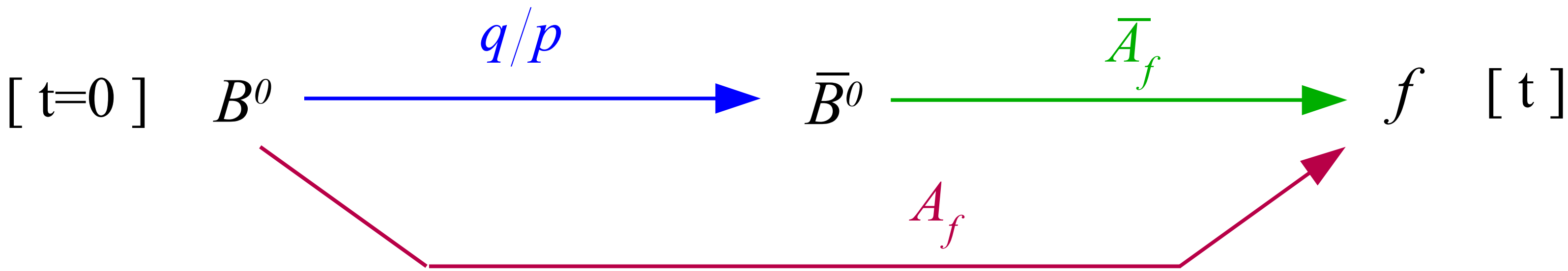


$$
\frac{q}{p} = \arg(M_{12}) = \frac{(V_{tb}^* V_{td})^2}{|(V_{tb} V_{td}^*)^2|} = e^{-2i\beta}
$$

Large CPV phase

(*in the standard CKM phase convention*)

The study of time-dependent decays of neutral B into CP eigenstates is a a marvelous tool to extract CPV phases in a clean way:



$$CP|f\rangle = \eta_f |f\rangle$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

Phase-convention independent quantity [→ observable]

If $|\lambda_f| = 1$ (i.e. if A_f is dominated by a single weak phase) then :

$$\Gamma(B^0(t)\to f) \propto e^{-\Gamma_B t}\left[1-\eta_f \,\text{Im}(\lambda_f)\,\sin(\Delta m_B t)\right]$$

$$\Gamma(\bar{B}^0(t)\to f) \propto e^{-\Gamma_B t}\left[1+\eta_f \,\text{Im}(\lambda_f)\,\sin(\Delta m_B t)\right]$$

$\text{Im}(\lambda_f) \neq 0 \leftrightarrow$ ~~CP~~

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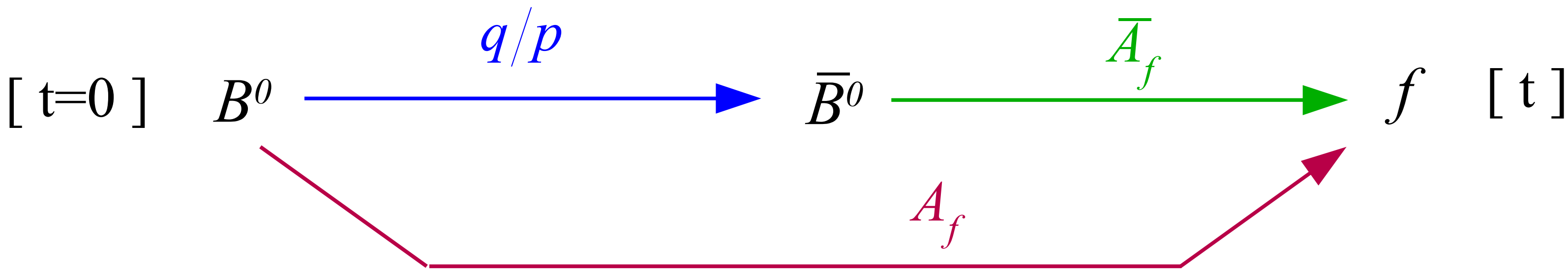

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Key points to successfully use this method:

- [EXP]: flavor tagging and time-dependent resolution are essential ingredients
- [TH]: identify final states such that A_f is dominated by a single weak phase

A few words about flavor tagging: B factories vs. hadron colliders

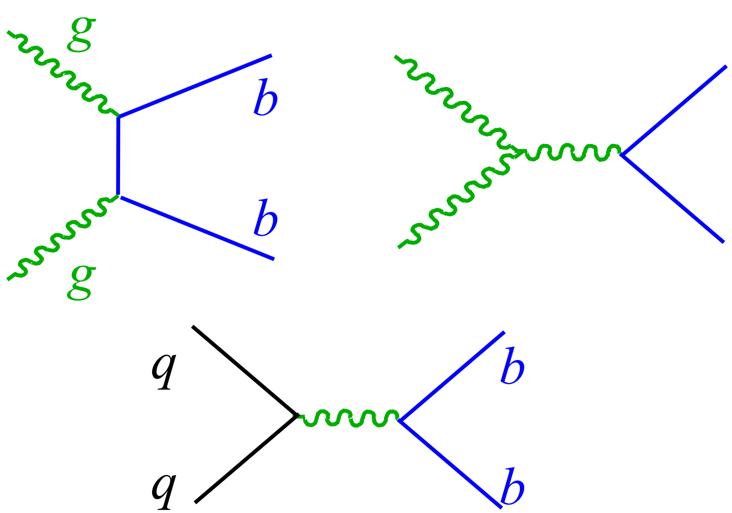
B factories:

$$e^+ + e^- \rightarrow \Psi(4S) \rightarrow \bar{B}\ B$$

- clean environment [σ(B) /σ(bkg) ~ 0.3]
- coherent quantum state for neutral B
- "easy" and clean flavor tag from the decay of the opposite meson (e.g. $b \rightarrow c\, e^- \nu$)

- low stat. [~ 10^8 B pairs / 100 fb^{-1}]

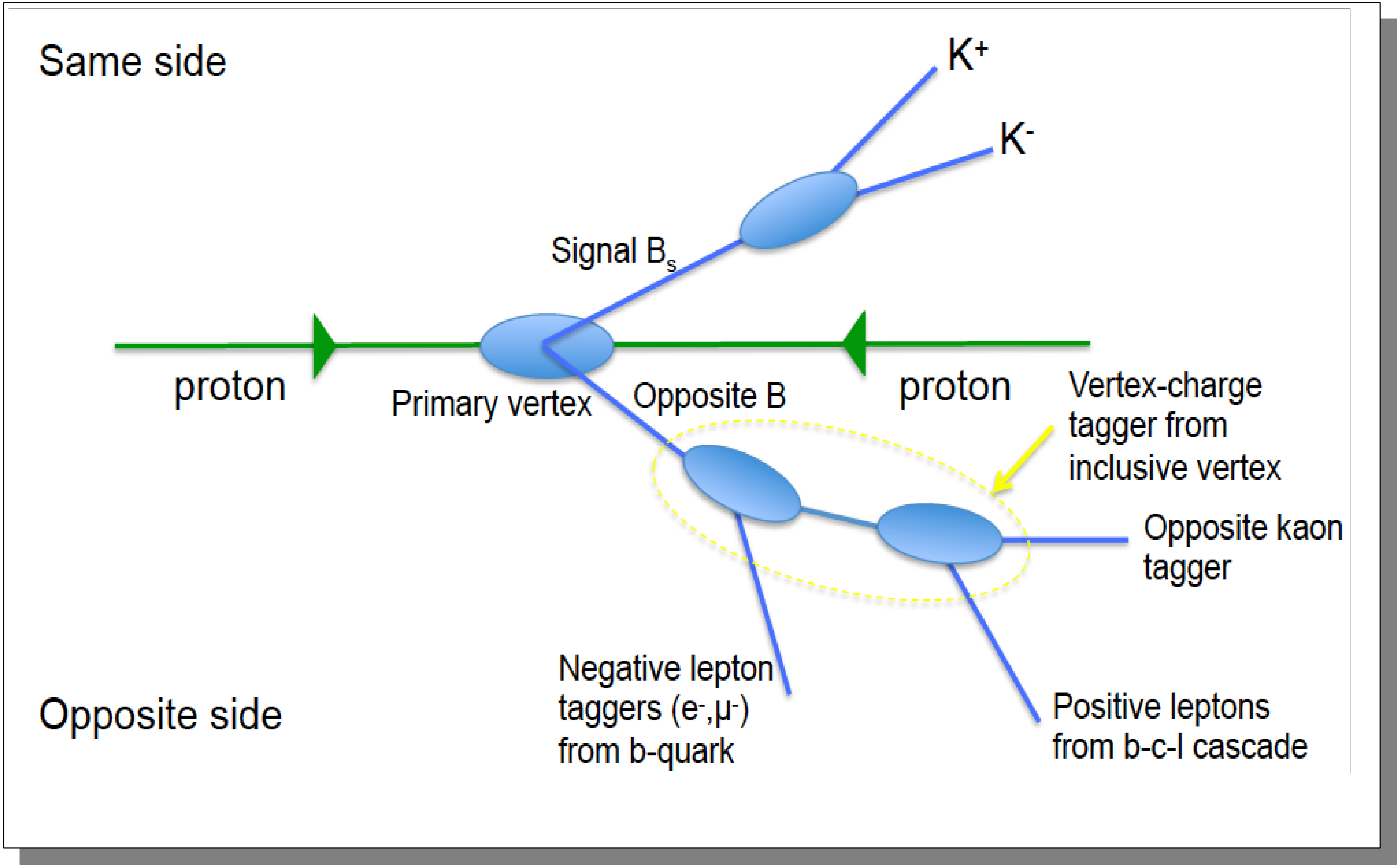
Hadron colliders:



- dirty environment [σ(B) /σ(bkg) < 0.01]
- incoherent quantum state

- high stat. [~ 10^{12} B pairs / 1 fb^{-1}]
- all hadrons with b-quarks produced

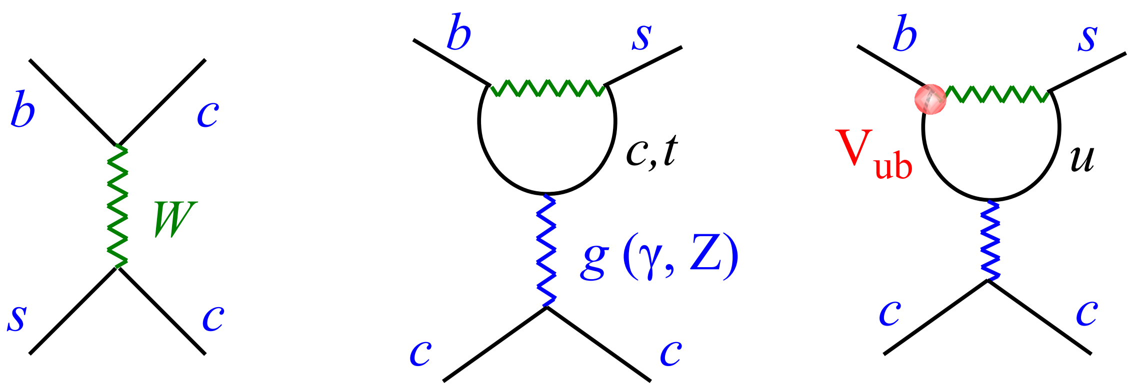
A few words about flavor tagging: B factories vs. hadron colliders



S. Benson, 2013

When is A_f dominated by a single weak phase?

$|\,B_d\rangle \to |\,\psi K_S\rangle$ [b+d → $c\bar{c}s$+d]



real $O(\lambda^2)$ real $O(\alpha_S\,\lambda^2)$ $O(\alpha_S\,\lambda^5)$

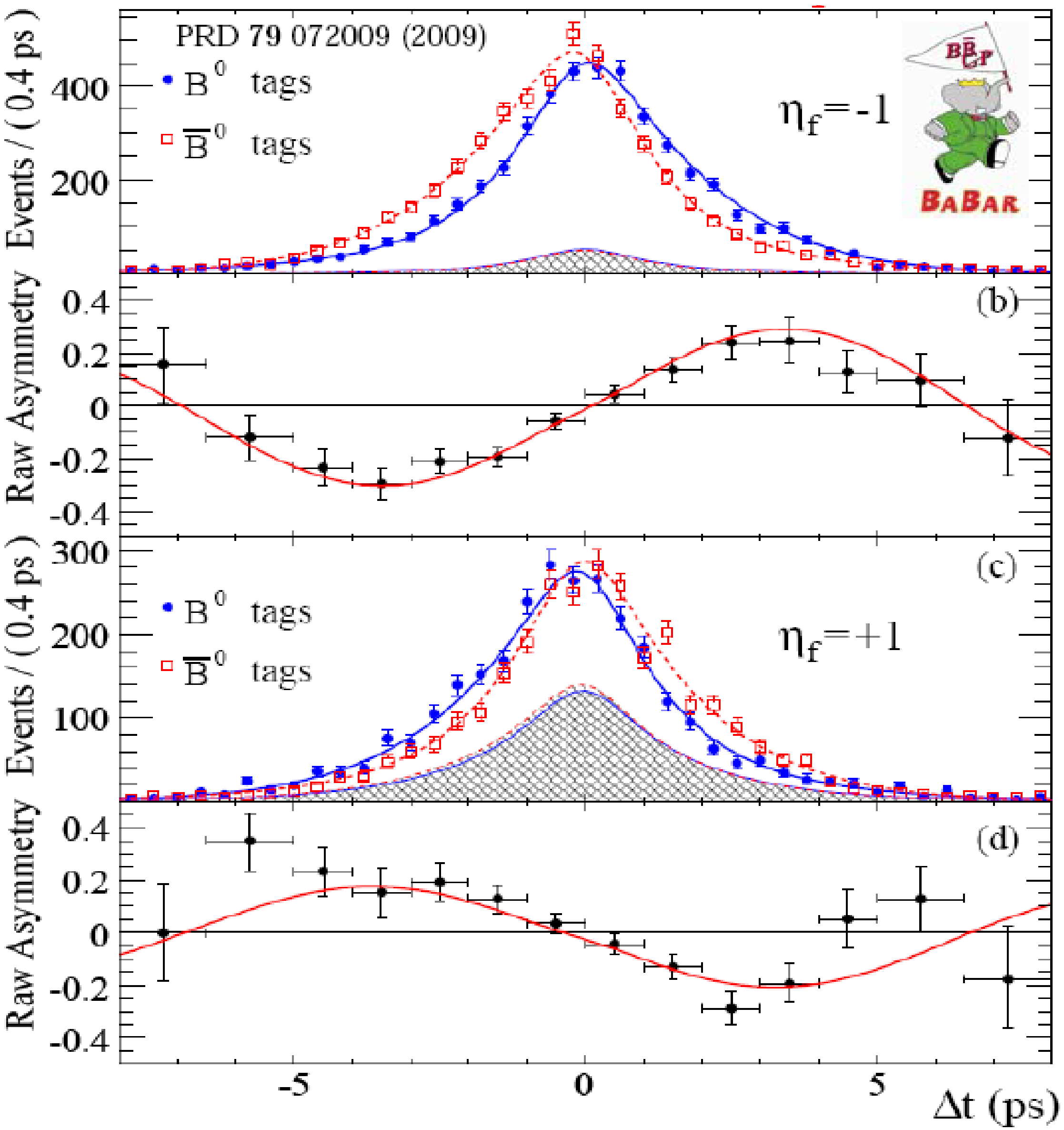
dominant amplitude pollution $\lesssim 1$ %

$\mathrm{Im}(\lambda_f) = \sin(2\beta)$

(from the mixing)

extremely precise constraint in the ρ−η plane ➡ *Golden channel* for B factories

BaBar's final result



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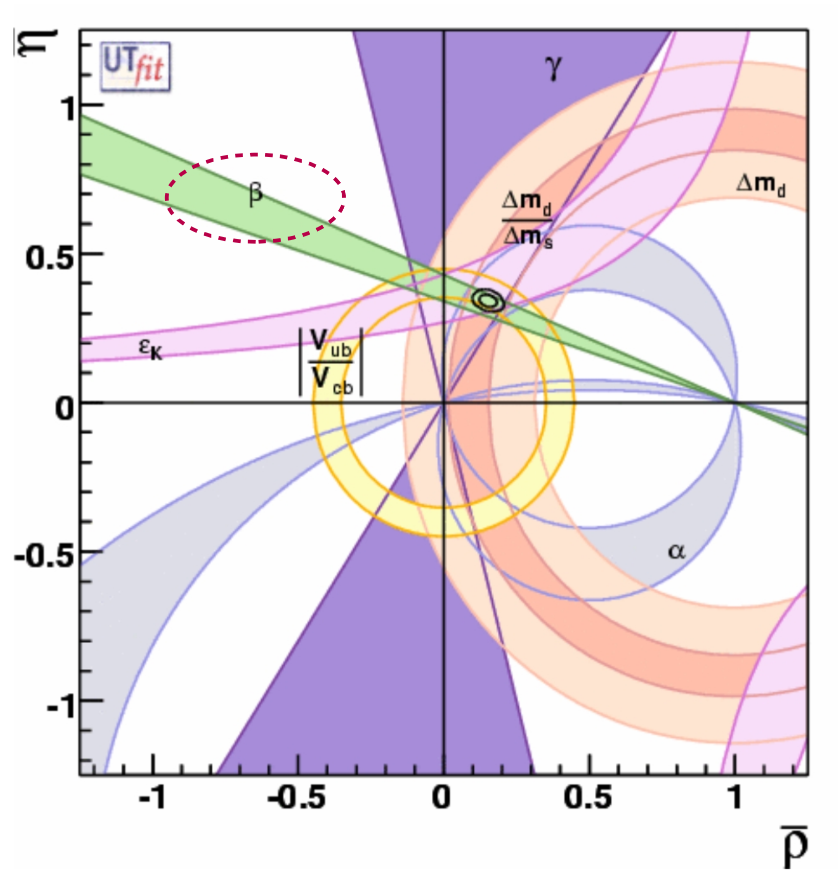

Today's summary

At present, measurements of sides and angels of the CKM unitarity triangle show a remarkable success of the SM (*redundant and consistent determination of various CKM elements*):