Introduction to Heavy-Ion Physics
Part I

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Summer Student Lectures 2017
What is Heavy-Ion Physics?

• A way to study QCD
  … without confinement
  … with quarks at their bare masses

• A way to study matter
  … at energy densities like $10 \, \mu s$ after the Big Bang
  … at temperatures $10^5$ times larger than in the sun core
Most searched for signals at the LHC are rare

Triggers select very small fraction of all collision events

Today we discuss about the rest – the bulk of all LHC collisions
The bulk is…

… soft (small momentum transfer)

… governed by strong interaction

… in the non-perturbative regime

This lecture discusses how heavy-ion physics helps the understanding of QCD in the non-perturbative regime
Strong-Interaction Physics

• Strong interaction
  – binds quarks into hadrons
  – binds protons and neutrons into nuclei

• QCD is a very successful theory…
  e.g. for jet production at high $p_T$ and heavy-flavour production
  … with some open puzzles

<table>
<thead>
<tr>
<th>Confinement</th>
<th>Hadron Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impossible to find an isolated quark or gluon</td>
<td>Proton consists of 2 u and 1 d quark</td>
</tr>
<tr>
<td>Why?</td>
<td>$m_p = 938$ MeV $\neq \sim 10$ MeV $= m_{uud}$</td>
</tr>
<tr>
<td></td>
<td>Where is the extra mass generated?</td>
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</table>

in a regime where perturbative methods are not applicable … unfortunately!
Yang–Mills theory

From Wikipedia, the free encyclopedia

Yang–Mills theory is a gauge theory based on the SU(N) group, or more generally any compact, semi-simple Lie group. Yang–Mills theory seeks to describe the behavior of elementary particles using these non-Abelian Lie groups and is at the core of the unification of the electromagnetic and weak forces (i.e. U(1) × SU(2)) as well as quantum chromodynamics, the theory of the strong force (based on SU(3)). Thus it forms the basis of our understanding of particle physics, the Standard Model.

List of unsolved problems in physics

Yang–Mills theory in the non-perturbative regime: The equations of Yang–Mills remain unsolved at energy scales relevant for describing atomic nuclei. How does Yang–Mills theory give rise to the physics of nuclei and nuclear constituents?

https://en.wikipedia.org/wiki/Yang%E2%80%93Mills_theory
Fundamental Questions (2)

• How do “free” quarks and gluons behave?
• How do quarks and gluons behave when chiral symmetry is restored?
• What generates the constituent masses?

• In the early universe a phase with free quarks and gluons and restored chiral symmetry has existed
  – *Quark-gluon plasma* (QGP)
  – Recreate in the laboratory with heavy-ion collisions

• How does matter behave at very large densities and temperatures?
Big Bang

Big Bang → WE symmetry breaking → Quark-gluon plasma → t \sim 10^6 \mu s → Protons, neutrons → Atomic nuclei
Basic Concepts
Confinement

• QCD vacuum
  – Gluon-gluon self-interaction (non-abelian)
  – QCD field lines compressed in flux tube
• Potential grows linearly with distance
  \[ V(r) = -\frac{\alpha}{r} + \sigma r \]
• Pulled apart, energy in string increases
• New q-qbar pair is created once energy is above production threshold

No free quark can be obtained \(\rightarrow\) confinement
Phenomenology of Confinement

- QCD vacuum can be seen as liquid of gluon-gluon pairs
- Why does this create confinement?
- *MIT bag model*: hadrons are confined in bubbles of perturbative (= empty) vacuum
  - Surrounded by QCD vacuum exerting pressure

**PRD9, 3471 (1974)**
Bag Model

- Quarks in bubble $\rightarrow$ kinetic pressure
- QCD vacuum $\rightarrow$ bag pressure
- Bag pressure = phenomenological quantity for non-perturbative effects of QCD
- Massless fermions in spherical cavity
  \[ E = \frac{2.04N}{R} + \frac{4\pi}{3} R^3 B \]
  - N quarks
  - R radius
  - B bag pressure
- Equilibrium defines bag radius
- Proton radius ($\sim 0.8$ fm)
  $\rightarrow$ $B^{1/4} \sim 206$ MeV

If kinetic pressure exceeds bag pressure $\rightarrow$ deconfinement

\[ \text{PRD9, 3471 (1974)} \]
Bag Model (2)

- If kinetic pressure exceeds bag pressure $\rightarrow$ deconfinement
- Relativistic massless quark gas

$$p = \left( g_B + \frac{7}{8} g_F \right) \frac{\pi^2 T^4}{90}$$

$$g_B = 16 \quad g_F = 24$$

8 gluons x 2 spins
2 quarks x 2 spins x 3 colors + antiquarks

- Pressure exceeds bag pressure ($p > B$) at $T_C \sim 144$ MeV
  - Quark-gluon plasma above $T_C$

More thorough estimate of the phase transition temperature can be done with [lattice QCD] $\rightarrow T_C \sim 156$ MeV
Chiral Symmetry

• QCD Lagrangian symmetric under SU(2)$_L$ x SU(2)$_R$

• Light quarks have finite (small) bare masses
  – Explicit chiral symmetry breaking

• Creation of coherent q-qbar pairs in QCD vacuum (compare to cooper pairs in superconductivity)
  – Has a chiral charge
  – Not symmetric under SU(2)$_L$ x SU(2)$_R$
  → Spontaneous symmetry breaking (pseudo-goldstone bosons: pions)

• Quarks acquire ~350 MeV additional mass
  – Constituent mass
  – Relevant only for u, d, s

Σ$p = 0$  Σ$L = 0$  → chirality $\neq 0$
Spontaneous Breaking of Chiral Symmetry

- Consequences
  - $m(u) \approx m(d) \rightarrow \text{isospin symmetry}$

  Isospin symmetry is not based on a fundamental relation, but an ‘accident’ because acquired masses are much larger than bare masses

  - $m(p) \gg m(\text{bare } 2u+1d)$
    - $938 \text{ MeV} \gg 10 \text{ MeV}$

- In the QGP, spontaneous chiral symmetry breaking is expected to be restored (partial restoration)

X. Zhu et al., PLB 647 (2007) 366
Two Phase Transitions

• Spontaneous breaking of chiral symmetry
  – Present below $T_{SSB}$ (~170 MeV, lattice QCD)
  – Quark masses enhanced to constituent masses

• Confinement/deconfinement transition
  – Confinement scale depends on quark masses

\[ T < T_{SSB} \]
Quarks: constituent masses
\[ \rightarrow \text{Confinement} \]

\[ T > T_{SSB} \]
Quarks: bare masses
\[ \rightarrow \text{Deconfinement} \]

Both phase transition occur at the same $T$
(again an accident – not linked from first principles)
QCD Phase Diagram

- Early universe
- Particle accelerators
- Phase transition
- Quark-gluon plasma
  - Deconfined
  - Chiral symmetry
- Color superconductivity (several phases)
- Neutron stars
- Confined
- Hadron phase
- Chiral symmetry broken
  - \( \approx \) baryons - antibaryons

Adapted from hep-ph/0503184
Phases Heavy-Ion Collision

- Pre-equilibrium collision: $t = 0 \text{ fm/c}$
- Hydrodynamic evolution: $t = \sim 1 \text{ fm/c}$
- Chemical freeze-out: $t = 10 \text{ fm/c}$
- Kinetic freeze-out: $t = 4 \text{ cm/c}$
- Particle detection: $1 \text{ fm/c} = 3 \cdot 10^{-24} \text{ s}$

Real example...
Outline of the Lecture

- How to use a particle detector to learn about the QGP?
- This lecture will focus on the main topics
  
  1. Jet quenching, energy loss & quarkonia
  2. Particle yields & Statistical model
  3. Collective flow & hydrodynamics
  4. Collectivity in small systems

- And the currently *hottest* topic
Outline of the Lecture

what I have no time to cover...

- Direct Photons
- Nuclear Parton Distribution Functions
- Collective flow & hydrodynamics
- Jet quenching, energy loss & quarkonia
- Particle yields & Statistical model
- Femtoscopy
- Global Event properties
- Jet Reconstruction
- Fragmentation Functions
- Higher Moments
- Ultra peripheral collisions
- Dileptons
- Heavy-Ion Experiments
- Creating Heavy-Ion Beams
- Diffraction
- Collectivity in small systems
- Jet Structure
- Collective flow & hydrodynamics
Accelerators

<table>
<thead>
<tr>
<th></th>
<th>SPS</th>
<th>RHIC</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s_{NN}}$ (GeV)</td>
<td>17</td>
<td>200</td>
<td>5020 (5500)</td>
</tr>
<tr>
<td>Volume at freeze-out (fm$^3$)</td>
<td>1200</td>
<td>2300</td>
<td>5000</td>
</tr>
<tr>
<td>Energy density (GeV/fm$^3$)</td>
<td>3-4</td>
<td>4-7</td>
<td>10</td>
</tr>
<tr>
<td>Life time (fm/c)</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

Heavy-ion collisions:
$\sqrt{s}$ given per nucleon pair ($\sqrt{s_{NN}}$)
$\sqrt{s_{NN}} = 5$ TeV $\rightarrow \sqrt{s_{Pb-Pb}} = 1040$ TeV

NA57 (SPS) ~3m
PHENIX (RHIC) ~8m
ALICE (LHC) ~16m
Literature

- **Lectures**
  - J. Stachel, K. Reygers (2011)
  - Quark Matter Student Day (2014)
    [https://indico.cern.ch/event/219436/timetable/#20140518.detailed](https://indico.cern.ch/event/219436/timetable/#20140518.detailed)
  - Quark Matter Student Day (2017)
    [https://indico.cern.ch/event/433345/timetable/#20170205.detailed](https://indico.cern.ch/event/433345/timetable/#20170205.detailed)

- **Books**
  - C.Y. Wong, Introduction to High-Energy Heavy-Ion Collisions, World Scientific, 1994
    [http://books.google.de/books?id=Fnxvrdj2NOQC&printsec=frontcover](http://books.google.de/books?id=Fnxvrdj2NOQC&printsec=frontcover)
  - L. P. Csernai, Introduction to Relativistic Heavy-Ion Collisions, 1994 *(free as pdf)*
  - E. Shuryak, The QCD vacuum, hadrons, and superdense matter, World Scientific, 2004
    [http://books.google.de/books?id=rbcQMK6a6ekC&printsec=frontcover](http://books.google.de/books?id=rbcQMK6a6ekC&printsec=frontcover)
  - Yagi, Hatsuda, Miake, Quark-Gluon Plasma, Cambridge University Press, 2005
    [http://books.google.de/books?id=C2bpwxUXJngC&printsec=frontcover](http://books.google.de/books?id=C2bpwxUXJngC&printsec=frontcover)
  - R. Vogt, Ultrarelativistic Heavy-ion Collisions, Elsevier, 2007
    [http://books.google.de/books?id=F1P8WMESgkMC&printsec=frontcover](http://books.google.de/books?id=F1P8WMESgkMC&printsec=frontcover)
  - W. Florkowski, Phenomenology of Ultra-Relativistic Heavy-Ion Collisions, World Scientific, 2010
    [http://books.google.de/books?id=4glp05n9lz4C&printsec=frontcover](http://books.google.de/books?id=4glp05n9lz4C&printsec=frontcover)
Jet Quenching & Energy Loss

How does a quark-gluon plasma affect particles traversing it?
A Back-to-Back Jet

One jet disappears in the QGP → “Jet quenching”

ATLAS, PRL105:252303, 2010
Drawing: A. Mischke
Dijet Asymmetry

• How often do jets lose lot of energy?
• Quantify by dijet asymmetry
• 2 highest energy jets with $\Delta \phi > 2\pi/3$

\[ A_J = \frac{|p_{T1} - p_{T2}|}{p_{T1} + p_{T2}} \]

- $p_{T1} = p_{T2} \rightarrow A_J = 0$
- $1/3 \ p_{T1} = p_{T2} \rightarrow A_J = 0.5$

• Peripheral collisions: Pb-Pb ~ Pythia
• Central collisions: Significant difference

**Central / peripheral will be introduced soon**

PRC 84 (2011) 024906
PRL105:252303,2010
Jets lose up to two thirds of their energy!

Something significant happening in heavy-ion collisions!
Hard Probes

• Ideally: a Rutherford experiment

• But
  – QGP exists in the lab only for $\sim 10^{-23}$ s
  – No free color charges as probes

• Instead
  – Use probes generated in the heavy-ion collision itself
    \( \rightarrow \) "self-generated" probes
Self-Generated Probes

• Produced early, before the plasma forms
  \[ t \sim \frac{\hbar}{Q} \quad Q > 2 \text{ GeV/c} \rightarrow t < 0.1 \text{ fm/c} \]

• Production rate “known”
  – Ideally calculable perturbatively
  – Not produced in the medium

• Interact with dense medium (QGP)

• Large cross-section

... as usual there is no such thing as a free lunch ...

<table>
<thead>
<tr>
<th>Per central LHC collision</th>
<th>LHC Run 1 (~ 150/ub)</th>
<th>Rec. efficiency, branching ratios factors ~ 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 D mesons (&gt; 2 GeV/c)</td>
<td>10^8 D mesons (&gt; 2 GeV/c)</td>
<td></td>
</tr>
<tr>
<td>0.2 B mesons (&gt; 10 GeV/c)</td>
<td>10^7 B mesons (&gt; 10 GeV/c)</td>
<td></td>
</tr>
<tr>
<td>10^{-3} jets above 100 GeV</td>
<td>10^5 jets above 100 GeV</td>
<td></td>
</tr>
<tr>
<td>10^{-6} jets above 400 GeV</td>
<td>120 jets above 400 GeV</td>
<td></td>
</tr>
</tbody>
</table>
Heavy-Ion Environment

- Measurements in an environment with $dN_{ch}/d\eta$ up to 1600 ($\sqrt{s}_{NN} = 2.76$ TeV)
  - $= 400$ pp MB collisions = 1 event with 399 pile-up events
    (ATLAS/CMS reconstruct up to 100)
- In one collision, there are in the tracker acceptances
  - 3200 tracks in ALICE | 8000 tracks in CMS/ATLAS

for comparison
pp : $dN_{ch}/d\eta \sim 4$
Probes Traverse the QGP

Quark-Gluon Plasma

Quarks and gluons

Initial state

Heavy quarkonia

Electro-weak probes

Final state

Detector

Nuclear-Modification Factor

- Hard processes occur in nucleon-nucleon (NN) collisions
- Heavy-ion collision: many NN collisions
  - Hard process is independent of number of NN collisions
- Without QGP, HI collision is superposition of NN collisions with incoherent fragmentation

\[
\frac{dN_{AA}}{dp_T} = \left\langle N_{coll} \right\rangle \frac{dN_{pp}}{dp_T}
\]

- Let’s turn this into an observable

\[
R_{AA} = \frac{\frac{dN_{AA}}{dp_T}}{\left\langle N_{coll} \right\rangle \frac{dN_{pp}}{dp_T}}
\]

- \(R_{AA} = 1 \rightarrow \text{no modification}\)
- \(R_{AA} \neq 1 \rightarrow \text{medium effects}\)
Nuclear-Modification Factor (2)

- How do we measure this quantity?

\[ R_{AA} = \frac{dN_{AA} / dp_T}{\langle N_{coll} \rangle dN_{pp} / dp_T} \]

For example:
- \( p_T \) distribution in AA collisions
  - Select events
  - Select & count tracks
  - Correct for detector effects
  - Estimate systematic uncertainties

- \( p_T \) distribution in pp collisions
- Number of binary collisions \( N_{coll} \)
  - Glauber modelling (see next slides)
  - Centrality (see next next slides)
How to Measure $N_{\text{coll}}$?

- Each nucleon (Pb-Pb: 2x208) has momentum and energy
- Calculating the number of collisions is in principle a $2x(208+208+1) = 834$-dimensional integral

Some simplification seems to be needed…
Glauber Monte Carlo

- Nucleons travel on straight lines
- Collisions do not alter their trajectory (energy of nucleons large enough)
- No quantum-mechanical interference
- Interaction probability for two nucleons is nucleon-nucleon cross-section

“Blue” nucleon has suffered 5 NN collisions

Need to repeat for all other nucleons in A

Strongly dependent on impact parameter \( b \)

More details in nucl-ex/0701025
Realistic Example

**Transverse view**

![Transverse view of Au+Au collision with b = 6 fm. Light nucleons are red and have not participated (spectators). Dark nucleons are blue and have participated.](image)

**Along the beam axis**

![Along the beam axis view of Au+Au collision with b = 6 fm. Light nucleons are red and have not participated (spectators). Dark nucleons are blue and have participated.](image)

- **light nucleons**: have not participated (spectators)
- **dark nucleons**: have participated

Figure: nucl-ex/0701025
Input to Glauber MC

• Distribution of nucleons in nuclei
  – Based on nuclear density
  – Typically Woods-Saxon distribution

\[ \rho(r) = \rho_0 \frac{1}{1 + \exp \left( \frac{r-R}{a} \right)} \]

  Density in the center
  Nuclear radius R
  Skin depth a

• Nucleon-nucleon cross-section
  – From pp measurements / extrapolations

Figure: nucl-ex/0701025

\( \rho(r)/\rho(0) \) vs. \( r \)
Glauber MC Output

- Number of spectators
  - Nucleons which did not collide
- Participant/wounded nucleons
  - Collided at least once
  - Called $N_{\text{part}}$
  - Scale with $2A$ ($A = \text{number of nucleons}$)
- Number of binary collisions
  - Called $N_{\text{coll}}$
  - Scales with $A^{4/3}$
- Rule of thumb
  - Soft (low $p_T$) observables scale with $N_{\text{part}}$
  - Hard (high $p_T$) observables scale with $N_{\text{coll}}$

$$N_{\text{part}} \sim A + A$$

$$N_{\text{coll}} \sim A \cdot L = A^{4/3}$$

$L = A^{1/3}$
• 10% most central at RHIC (Au-Au, 200 GeV)
  – \( N_{\text{coll}} \sim 1200 \)
  – \( N_{\text{part}} \sim 380 \)

• 5% most central collisions at LHC (Pb-Pb, 5 TeV)
  – \( N_{\text{coll}} \sim 1770 \)
  – \( N_{\text{part}} \sim 384 \)

• Difference mainly due to cross-section increase

Can also be calculated analytically: Optical Glauber (see backup)
Recap

• We are trying to understand heavy-ion collisions
• For that, we are trying to measure the difference between AA and pp collisions, expressed as $R_{AA}$

$$R_{AA} = \frac{dN_{AA} / dp_T}{\langle N_{coll} \rangle dN_{pp} / dp_T}$$

• For that we need to estimate the number of nucleon-nucleon collisions $N_{coll}$
• Using the Glauber Monte Carlo, for a given impact parameter $b$, we are now able to estimate $N_{coll}$

How do we measure $b$?
Centrality

- How do measure the impact parameter $b$?

Lower multiplicity  $b > 0$  
Upper multiplicity  $b = 0$

Striking relation between $b$ and multiplicity
Centrality (2)

- Multiplicity anti-proportional to $b$
  - Glauber MC + particle production model calculates multiplicity
- Multiplicity correlated in different phase space (e.g. forward and mid rapidity) regions in HI collisions

Instead of multiplicity, calorimeter energy can also be used (e.g. ATLAS/CMS)

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Introduction to Heavy-Ion Physics – Jan Fiete Grosse-Oetringhaus
Centrality (3)

- Use multiplicity to split events into classes
- Called 0-5%, 5-10%, … 100% (“0%” = most central)
- Glauber MC calculates $N_{\text{part}}$ and $N_{\text{coll}}$ per class

Number of events vs. multiplicity
Recap

- We are trying to measure $R_{AA}$

$$R_{AA} = \frac{\frac{dN_{AA}}{dp_T}}{\frac{\langle N_{coll}\rangle dN_{pp}}{dp_T}}$$

- We will do this in different event classes based on the event multiplicity
- For each class we can estimate the number of nucleon-nucleon collisions $N_{coll}$ using the Glauber Monte Carlo

So... let’s go!
\[ R_{AA} \]

\[ R_{AA} = \frac{dN_{AA} / dp_T}{\langle N_{coll}\rangle dN_{pp} / dp_T} \]

- \( R_{AA} = 1 \) → no modification
- 70-80% (peripheral) → \( R_{AA} \approx 0.7 \)
- 0-5% (central) → \( R_{AA} \) drops to 0.14

Soft particle production does not scale with \( N_{coll} \)

ALICE, PLB696 (2011) 30
$R_{AA}$ at High $p_T$

$R_{AA}$ vs. $p_T$

- 60-80% (peripheral) $\rightarrow R_{AA}$ increases up to 0.9
- 0-5% (central) $\rightarrow R_{AA}$ increases up to 0.6

$R_{AA}$ reaches asymptotic value for $p_T > 50$ GeV/c

ATLAS, JHEP09(2015)050
Recent $R_{AA}$

• If you were wondering how to compare the plots on the previous slides…

• … but all consistent 😊
$R_{AA}$ for Color-Neutral Probes

No suppression for color-neutral probes

→ No interaction with QGP
→ Experimental check on $N_{\text{coll}}$ calculation (and nuclear PDFs)
Recap

• Peripheral collisions
  – \( R_{AA} \approx 0.8 \) – 0.9 for colored probes

• Central collisions
  – \( R_{AA} \approx 0.14 \) at \( p_T \approx 6\)–7 GeV/c
  – \( R_{AA} \approx 0.6 \) at high \( p_T \)

• \( R_{AA} \approx 1 \) for color-neutral probes

• Interpretation
  – \( R_{AA} \approx 0.14 \approx 1/7 \) → naïve conclusion: only 1 out of 7 particles escape the QGP?

⚠️ We are looking at a ratio and the particle spectrum is shifted by energy loss

Let’s try to understand this in more detail…
\( R_{AA} \) Interpretation

- \( \p_T \) spectrum is power law
  \[
  \frac{1}{\p_T} \frac{dN}{dp_T} \sim \p_T^{-n}
  \]
  \( \Delta E = 3 \text{ GeV} \)
  \( \Delta E/E = 0.23 \)
  RHIC (200 GeV): \( n = 7.2 \)
  LHC (2.76 TeV): \( n = 5.4 \)

- Constant energy loss
  \[
  R_{AA} \approx \left(1 - \frac{\Delta E}{\p_T}\right)^n
  \]
  rising with \( \p_T \)

- Constant fractional energy loss \( \Delta E/E \)
  \[
  R_{AA} \approx \left(1 - \frac{\Delta E}{E}\right)^{n-1}
  \]
  \( \p_T \) independent

- Steepness of spectra influences \( R_{AA} \)
- Similar \( R_{AA} \) does not imply similar energy loss

Medium energy loss: a few GeV

\[ R_{AA} \text{ vs. } \p_T \text{ (RHIC)} \]
\[ \Delta E = 3 \text{ GeV} \]
\[ \Delta E/E = 0.23 \]

\[ R_{AA} \text{ vs. } \p_T \text{ (LHC)} \]
\[ \Delta E = 5 \text{ GeV} \]
\[ \Delta E/E = 0.23 \]

LHC
Energy Loss

- Particle production in central collisions is strongly suppressed

How does the medium achieve this suppression?
Energy Loss in the QGP

- QGP: high density of quarks and gluons / color sources
- Traversing quark / gluon feels color fields
- Collisional energy loss
  - Elastic scatterings
  - Dominates at low momentum
- Radiative energy loss
  - Inelastic scatterings
  - Dominates at high momentum
  - Gluon bremsstrahlung

\[ \Delta E = \Delta E_{\text{coll}} + \Delta E_{\text{rad}} \]
Lattice QCD

• More thorough estimate of the phase transition temperature can be done with lattice QCD
• Approach to solve non-perturbative QCD
• Discretize the QCD Lagrangian on a space-time grid
• Limited to chemical potential $\mu_B = 0$ (some workarounds exist)
• Calculate $T$ dependence of
  – energy density
  – pressure
• Steep rise = change in number of degrees of freedom → phase transition

![Graph showing $\varepsilon/T^4$, $3p/T^4$ vs. $T$]

$156 \text{ MeV} \approx 2 \times 10^{12} \text{ K}$
(Sun core: $1.5 \times 10^7 \text{ K}$)

Transition temperature $T_C \sim 156 \text{ MeV}$ (consistent with bag model estimate)
Optical Glauber

- Probability to find a specific nucleon at s
  \[ T_A(s) \quad T_B(s) \]

- Overlap function
  \[ T_{AB}(b) = \int T_A(s-b)T_B(s) \, ds \]
  - Effective overlap area for which a specific nucleon in A can interact with a given nucleon in B

- Probability for interaction
  \[ T_{AB}(b) \sigma_{NN} \]

Figure: nucl-ex/0701025
Optical Glauber (2)

- Probability for \( n \) interactions

\[
P(n, b) = {AB \choose n} \left[ \hat{T}_{AB}(b) \sigma_{\text{incl}}^{NN} \right]^n \left[ 1 - \hat{T}_{AB}(b) \sigma_{\text{incl}}^{NN} \right]^{AB-n}
\]

- Number of collisions

\[
N_{\text{coll}}(b) = \sum_{n=1}^{AB} nP(n, b) = AB \hat{T}_{AB}(b) \sigma_{\text{inel}}^{NN}
\]

- Number of participants

\[
N_{\text{part}}(b) = A \int \hat{T}_A(s) \left\{ 1 - \left[ 1 - \hat{T}_B(s - b) \sigma_{\text{inel}}^{NN} \right]^B \right\} d^2s + B \int \hat{T}_B(s - b) \left\{ 1 - \left[ 1 - \hat{T}_A(s) \sigma_{\text{inel}}^{NN} \right]^A \right\} d^2s
\]

A,B number of nucleons
probability for not a single nucleon in a specific place

nucl-ex/0701025
Optical Glauber (3)

- Overlap function $T_{AA}$ allows to rewrite nuclear-modification factor in terms of pp cross-section

\[
R_{AA} = \frac{d N_{AA} / dp_T}{\langle N_{coll} \rangle d N_{pp} / dp_T}
\]

Identical nuclei: $AB \rightarrow A^2$

\[
N_{coll}(b) = A^2 T_{AA}(b) \sigma^{NN}
\]

- Reduces uncertainties if cross-section measurement is available

\[
R_{AA} = \frac{d N_{AA} / dp_T}{A^2 \langle T_{AA} \rangle d \sigma_{pp} / dp_T}
\]

sometimes factor $A^2$ included in $\langle T_{AA} \rangle$
Optical vs. MC Glauber

• Optical Glauber calculates the average $N_{\text{coll}} / N_{\text{part}}$ analytically
  – Exact

• MC Glauber arrives within MC approach at same values

• Advantage: Initial state fluctuations can be included
  (random distributions of nucleons in nuclei)
  $\rightarrow$ needed to describe many observables

Figure: nucl-ex/0701025