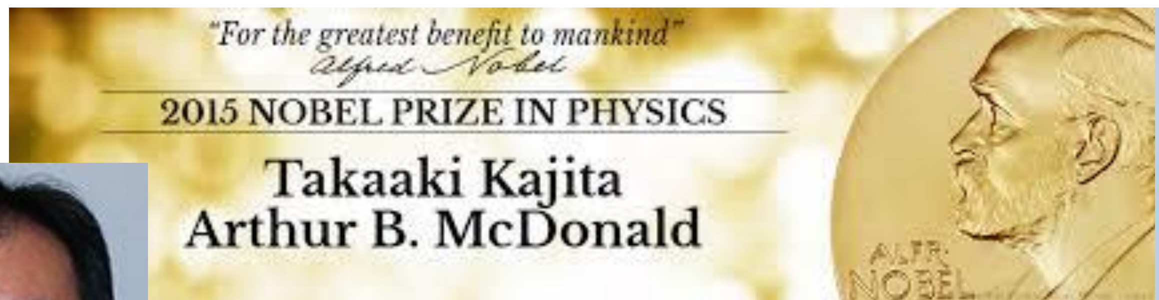
PLAN

- **Lecture I:** Neutrinos in the SM
  Neutrino masses and mixing: Majorana vs Dirac
- **Lecture II:** Neutrino oscillations and the discovery of neutrino masses and mixings
- **Lecture III:** The quest for leptonic CP violation
  A neutrino look at BSM and the history of the Universe

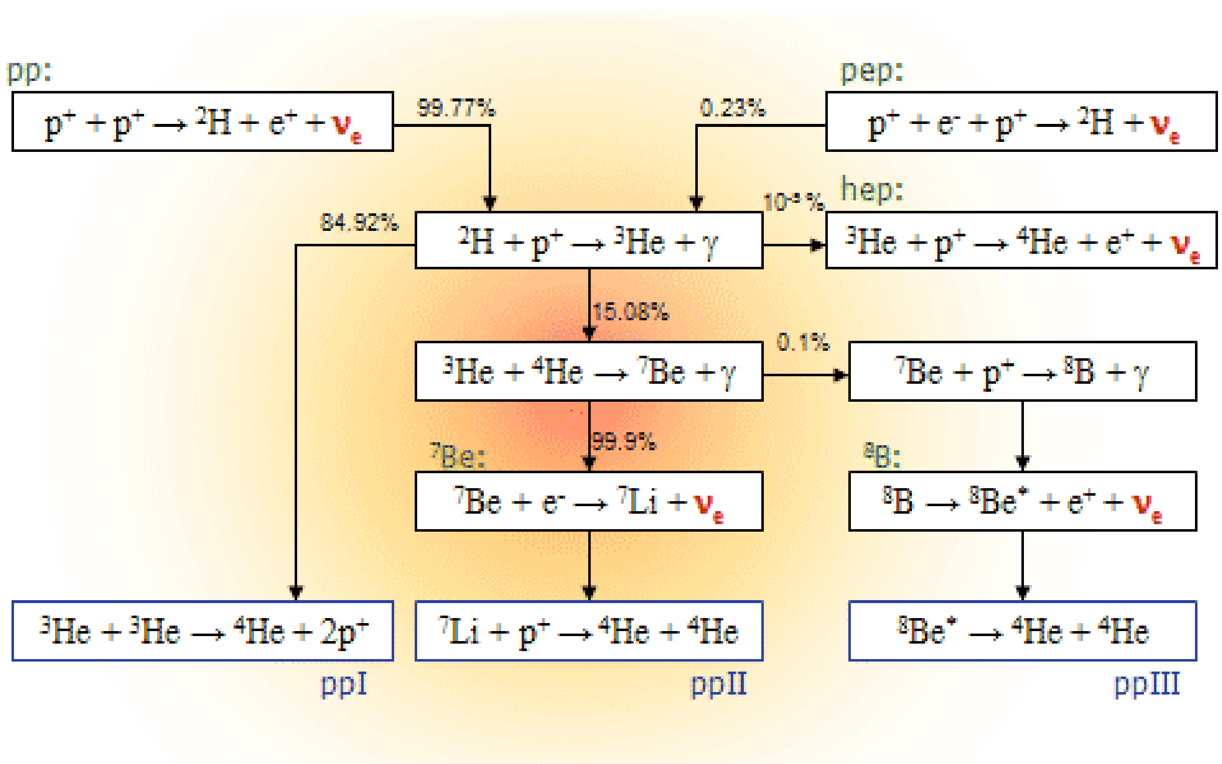
"For the discovery of neutrino oscillations,
which shows that neutrinos have mass"



Stars shine neutrinos

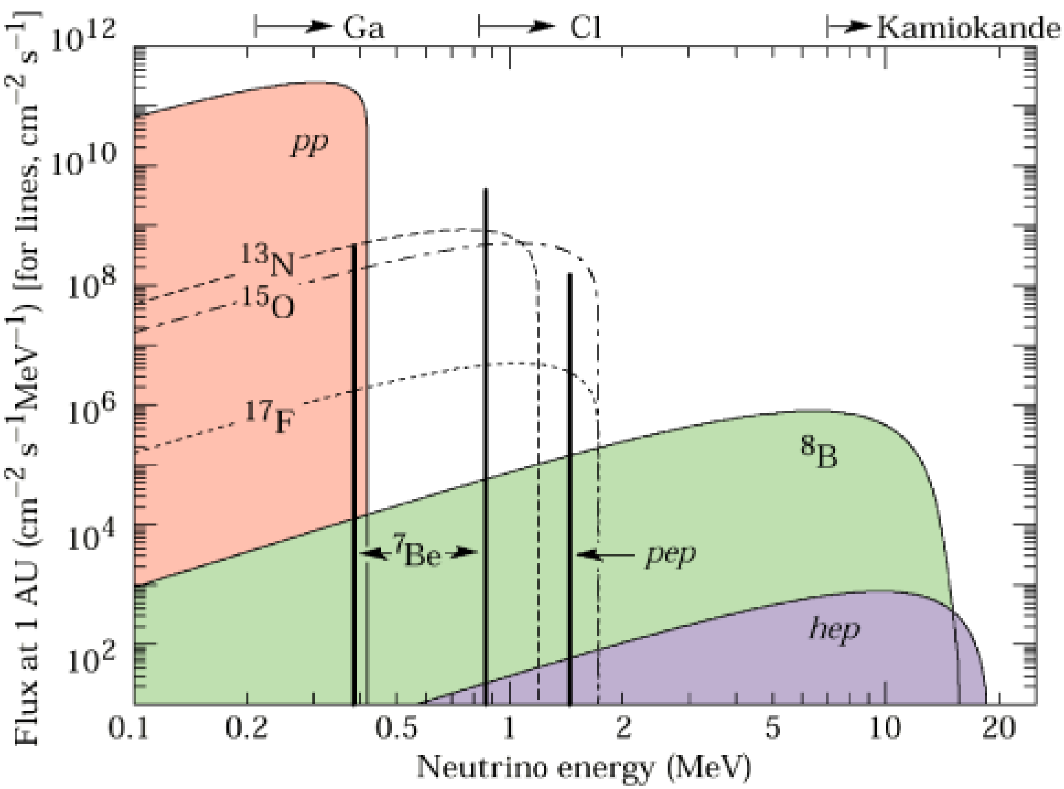
1939 Bethe

Stablishes the theory of stelar nucleosynthesis



Nobel 1967

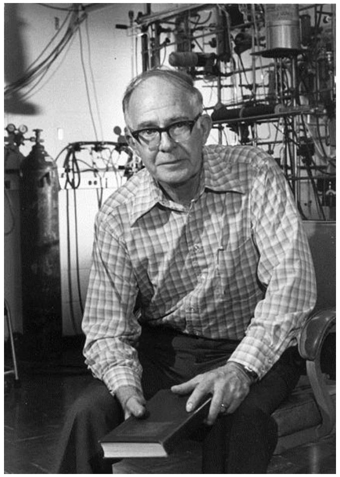
¿How many neutrinos from the Sun ?



Bahcall

The hero of the caves

1966 he detects for the first time
solar neutrinos in a tank of
400000 liters 1280m underground
(Homestake mine)



R. Davis
Nobel 2002

$$^{37}\mathrm{Cl} + \nu_e \to ^{37}\mathrm{Ar} + e^-$$

Did not convince because he saw 0.4 of the expected....

Problem in detector ? In solar model ? In neutrinos ?

Neutrino masses & lepton mixing (Dirac)

Couplings to the Higgs give rise to fermion masses and connect families!

$$\mathcal{L}^{\text{free}}_{\text{lepton}} = \sum_\alpha \bar{\nu}_\alpha i \not{\partial} \nu_\alpha + \sum_\alpha \bar{l}_\alpha i \not{\partial} l_\alpha - \left(\sum_{\alpha\beta} \bar{\nu}_{L\alpha} \underbrace{Y_{\nu\alpha\beta} \frac{v}{\sqrt{2}}}_{M_{\nu\alpha\beta}} \nu_{R\beta} + \sum_{\alpha\beta} \bar{l}_{L\alpha} \underbrace{Y_{l\alpha\beta} \frac{v}{\sqrt{2}}}_{M_{l\alpha\beta}} l_{R\beta} + h.c. \right)$$

$$M_\nu = U_\nu^\dagger \ \mathrm{Diag}(m_1, m_2, m_3) \ V_\nu, \ \ M_l = U_l^\dagger \ \mathrm{Diag}(m_e, m_\mu, m_\tau) \ V_l$$

In the mass eigenbasis

$$\nu'_{Li} \equiv \sum_\alpha (U_\nu)_{i\alpha} \nu_{L\alpha}, \quad \nu'_{Ri} = \sum_\alpha (V_\nu)_{i\alpha} \nu_{R\alpha},$$

$$l'_{Li} \equiv \sum_\alpha (U_l)_{i\alpha} l_{L\alpha}, \quad l'_{Ri} = \sum_\alpha (V_l)_{i\alpha} l_{R\alpha},$$

Neutrino masses & lepton mixing (Dirac)

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$$M_\nu = U_\nu^\dagger \ \mathrm{Diag}(m_1, m_2, m_3) \ V_\nu, \ \ M_l = U_l^\dagger \ \mathrm{Diag}(m_e, m_\mu, m_\tau) \ V_l$$

In the mass eigenbasis

$$\mathcal{L}^{\mathrm{free}}_{\mathrm{lepton}} = \sum_{i=1,2,3} \bar{\nu}'_i (i \not{\partial} - m_i) \nu'_i + \sum_{l'=e,\mu,\tau} \bar{l}'(i \not{\partial} - m_{l'}) l'$$

Three massive neutrinos and three massive leptons

Neutrino masses & lepton mixing (Dirac)

$$\mathcal{L}^{\text{Weak}}_{\text{lepton}} = -\frac{g}{\sqrt{2}} \sum_{\alpha} \bar{\nu}_{L\alpha} \gamma_\mu l_{L\alpha} W^+_\mu - \frac{g}{2\cos\theta_W} \sum_{\alpha} \bar{\nu}_{L\alpha} \gamma_\mu \nu_{L\alpha} Z_\mu + h.c.$$

In the new basis:

$$\mathcal{L}^{\text{Weak}}_{\text{lepton}} = -\frac{g}{\sqrt{2}} \sum_{i} \bar{\nu}'_{Li} \underbrace{(U_\nu U_l^\dagger)_{ij}}_{U^\dagger_{\text{PMNS}}} \gamma_\mu l'_{Lj} W^+_\mu - \frac{g}{2\cos\theta_W} \sum_{\alpha} \bar{\nu}'_{Li} \gamma_\mu \nu'_{Lj} Z_\mu + h.c.$$

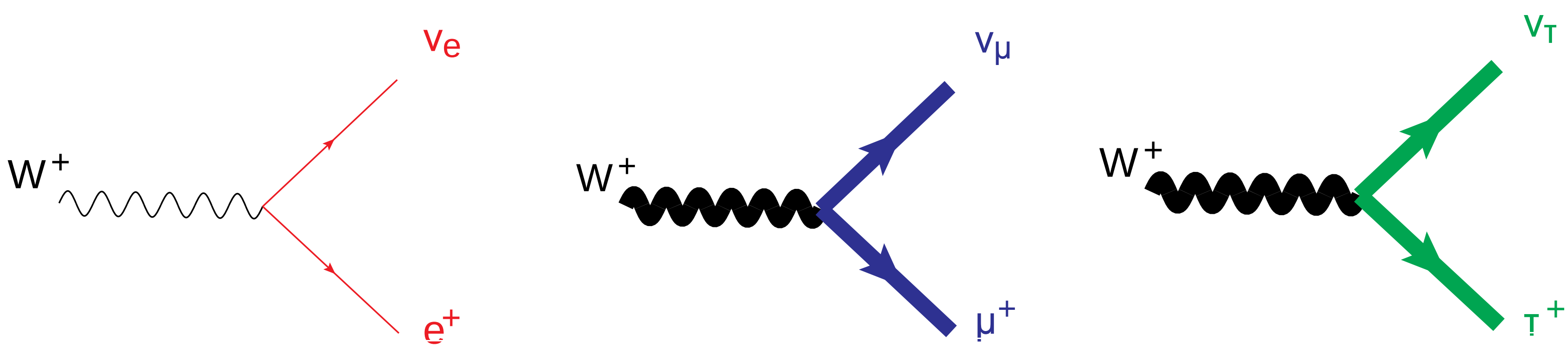
Pontecorvo-Maki-Nakagawa-Sakata Matrix

$U_{\text{PMNS}}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ unitary matrix analogous to CKM

Lepton mixing

$$\mathcal{L}_{\text{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix} W_\mu^- \gamma_\mu P_L \, U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + h.c.$$

The neutrino flavour basis:



States produced in a CC interaction in combination with e, μ, τ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

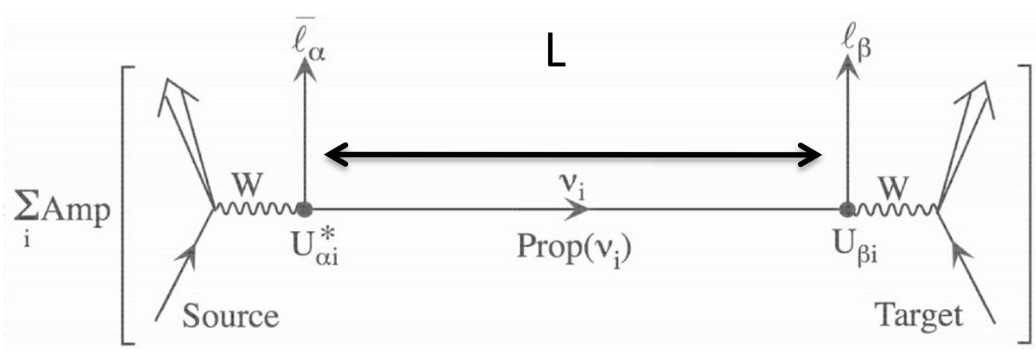
Eigenstates of the free Hamiltonian

Neutrino oscillations

Neutrinos are produced and detected via weak interactions as flavour states:

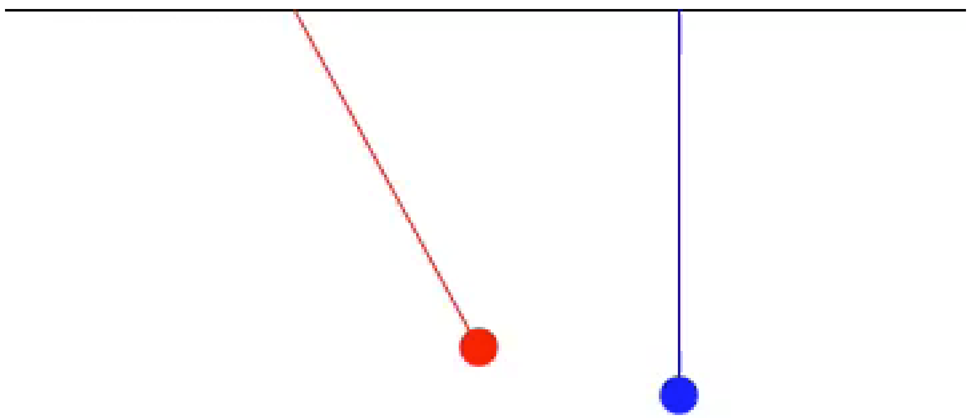
$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle, \quad \alpha = e, \mu, \tau$$

A neutrino experiment is an interferometer in flavour space, because neutrinos are so weakly interacting that can keep coherence over very long distances !



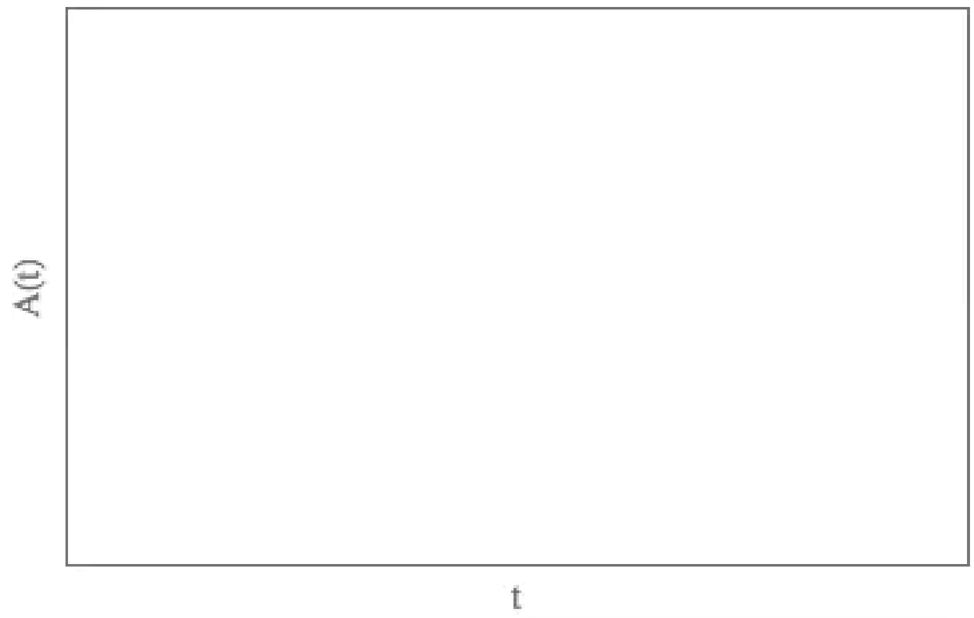
Classical analogy I: no flavour mixing

A ν_e is produced and stays a ν_e ...



ν_e

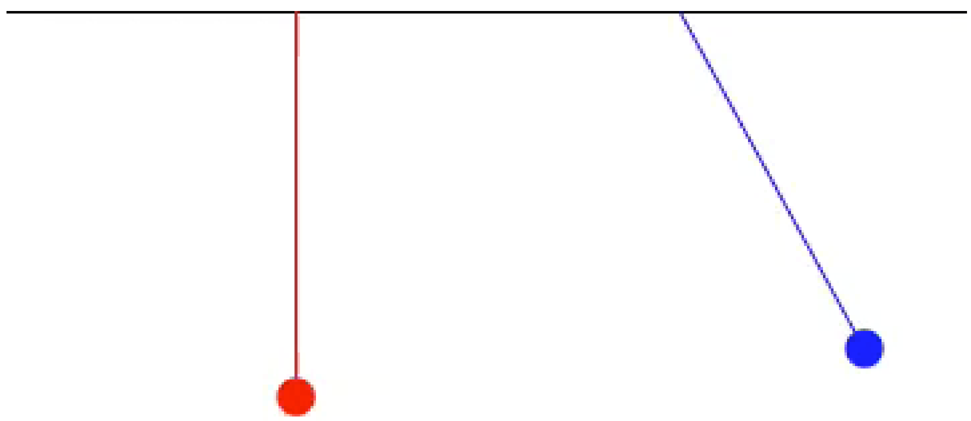
$$\mathrm{Prob}(\nu_e)(t) = |\mathrm{Amplitude}(t)|^2, \;\; \mathrm{Prob}(\nu_e)(t_0) = 1$$



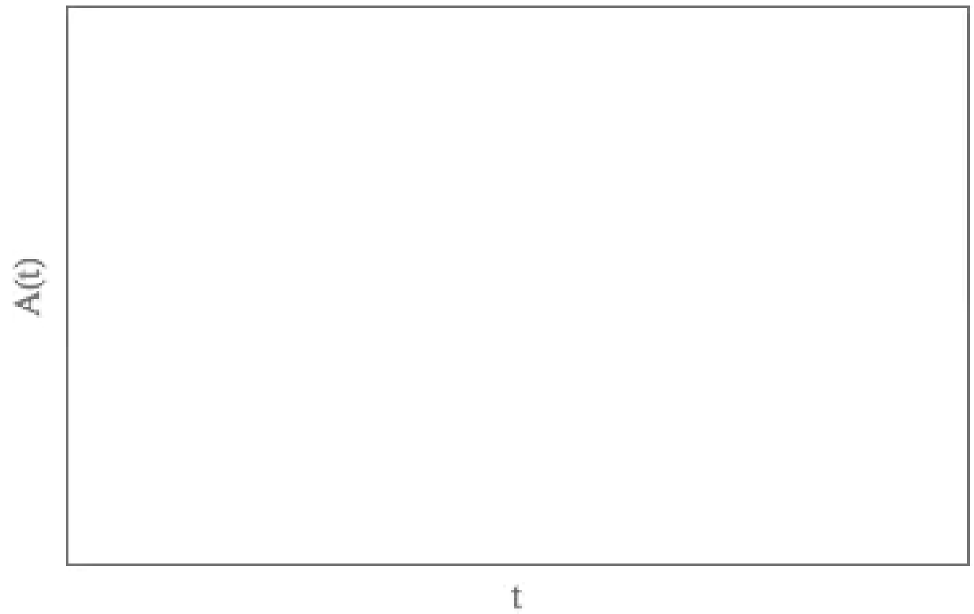
The probability to find a ν_e at any time is the same, but the probability to find a ν_μ is zero.

No flavour mixing

A ν_μ is produced and stays a ν_μ …

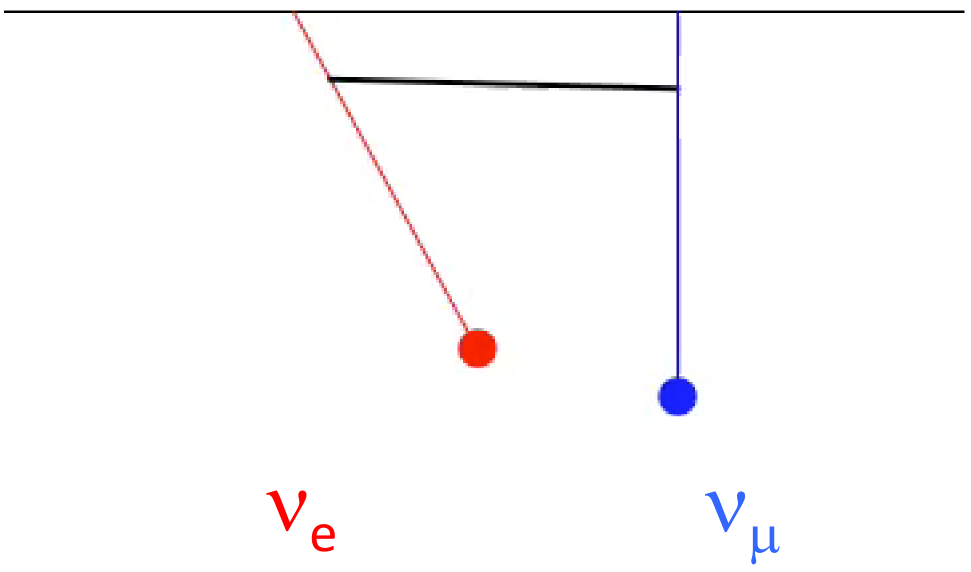


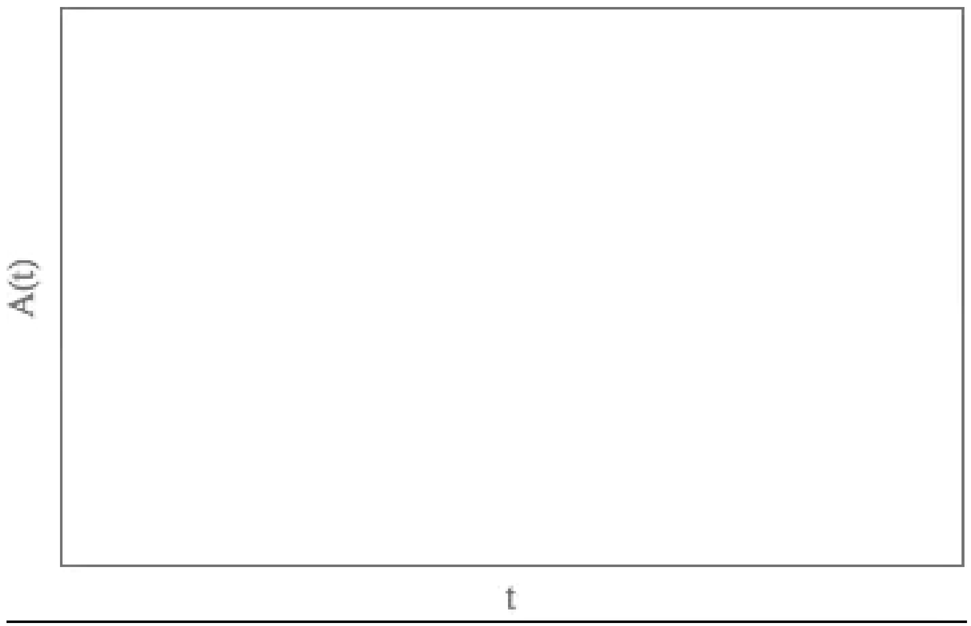
ν_μ



The probability to find a ν_μ at any time is the same, but the probability to find a ν_e is zero.

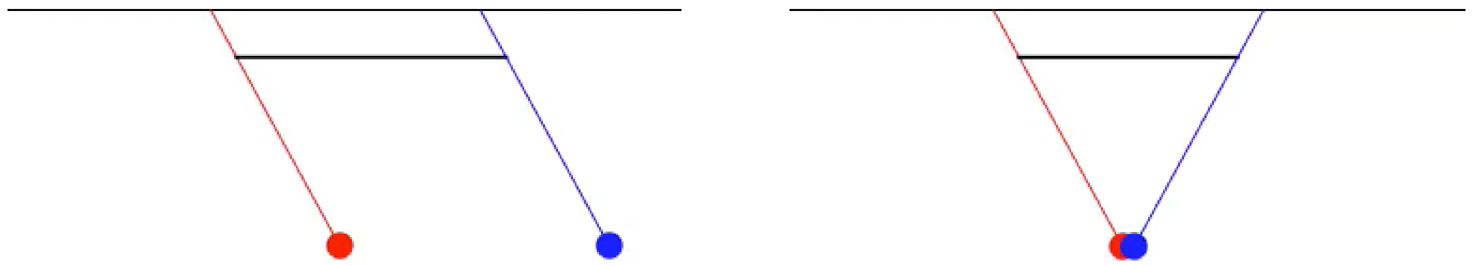
Classical analogy II: Maximal flavour mixing

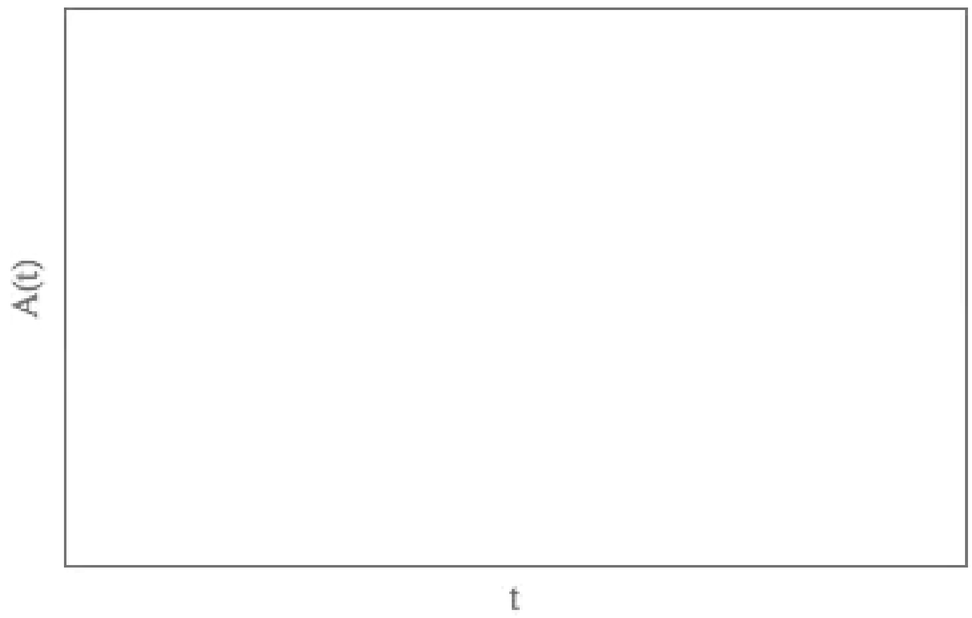




The probability to find a ν_e oscillates with time and so does that of ν_μ

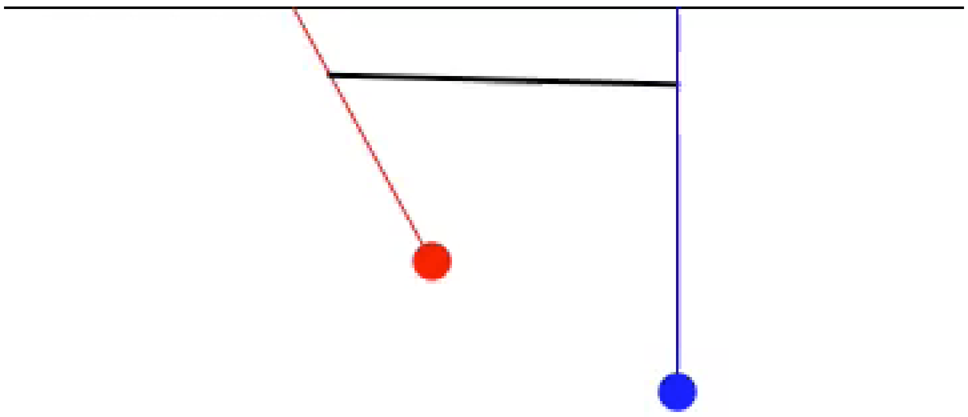
Mass eigenstates=normal modes

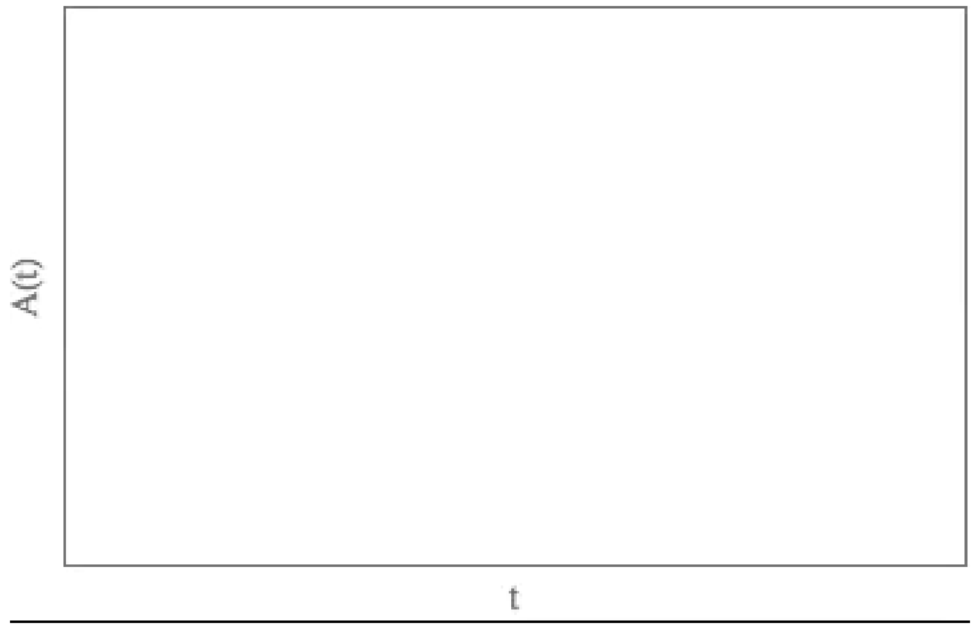




The probability to find a ν_e o ν_μ does not change with time

Classical analogy III: small flavour mixing





The probability to find a ν_e oscillates with time and so does that of ν_μ

Neutrino oscillations in vacuum

We start at time t_o with a neutrino state of flavour α:

$$|\nu_\alpha(t_0)\rangle = \sum_i U^*_{\alpha i}|\nu_i(\mathbf{p})\rangle, \qquad \hat{H}|\nu_i(\mathbf{p})\rangle = E_i(\mathbf{p})|\nu_i(\mathbf{p})\rangle, \quad \mathbf{p}^2 + m_i^2 = E_i^2(\mathbf{p})$$

$$\downarrow \text{time evolution} \qquad \equiv e^{-i\hat{H}(t-t_0)}$$

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i} e^{-iE_i(\mathbf{p})(t-t_0)}|\nu_i(\mathbf{p})\rangle$$

We want to know the probability that at time t a measurement reveals that the state is one with flavour β:

$$|\nu_\beta\rangle = \sum_i U^*_{\beta i}|\nu_i(\mathbf{p})\rangle$$

Neutrino oscillations in vacuum

$$P(\nu_\alpha \to \nu_\beta)(t) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_i U_{\beta i} U^*_{\alpha i} e^{-iE_i(t-t_0)} \right|^2$$

$$= \sum_{i,j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{-i(E_i - E_j)(t-t_0)}$$

$$E_i(\mathbf{p}) - E_j(\mathbf{p}) \simeq \frac{1}{2} \frac{m_i^2 - m_j^2}{|\mathbf{p}|} + \mathcal{O}(m^4) \qquad L \simeq t - t_0, v_i \simeq c$$

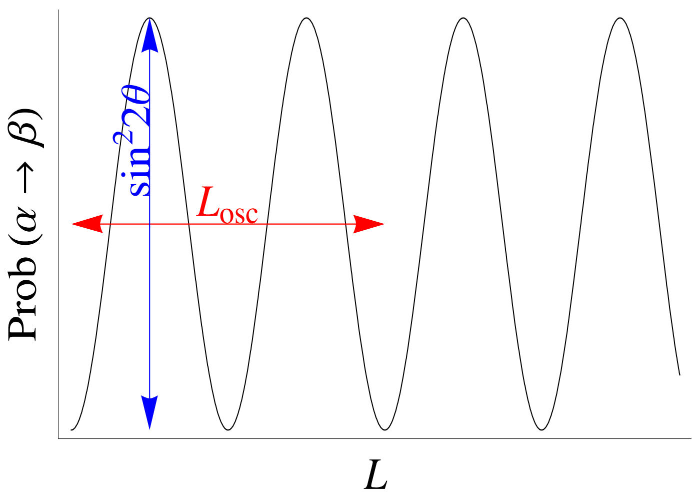
$$\boxed{P(\nu_\alpha \to \nu_\beta)(L) \simeq \sum_{i,j} e^{i \frac{\Delta m^2_{ji} L}{2E}} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}}$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2(eV^2)L(km)}{E(GeV)}\right)$$

(appearance probability)



$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

(disappearance or survival probability)

$$L_{osc}(km) = \frac{\pi}{1.27}\frac{E(GeV)}{\Delta m^2(eV^2)}$$

Optimal experiment: $\frac{E}{L} \sim \Delta m^2$

$\frac{E}{L} \gg \Delta m^2$ Oscillation suppressed

$$P(\nu_\alpha \to \nu_\beta) \propto \sin^2 2\theta \left(\Delta m^2\right)^2$$

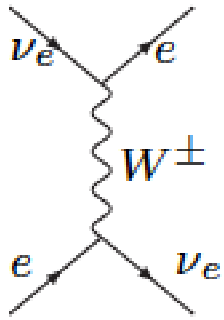
$\frac{E}{L} \ll \Delta m^2$ Fast oscillation regime

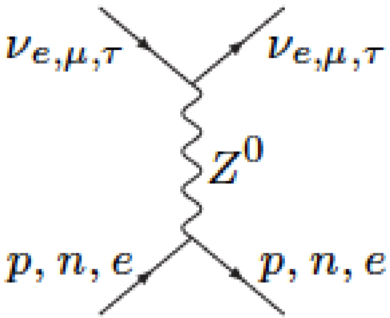
$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2 2\theta \, \langle \sin^2 \frac{\Delta m^2 L}{4E} \rangle \simeq \frac{1}{2} \sin^2 2\theta = |U^*_{\alpha 1} U_{\beta 1}|^2 + |U^*_{\alpha 2} U_{\beta 2}|^2$$

Equivalent to incoherent propagation: sensitivity to mass splitting is lost

Neutrino Oscillations in matter

Many neutrino oscillation experiments involve neutrinos propagating in matter (Earth for atmospheric neutrinos or accelerator experiments, Sun for solar neutrinos)





Wolfenstein

Index of refraction (coherent forward scattering) different for electron and μ/τ neutrinos

Neutrino propagation in matter

$$M_\nu^2 \longrightarrow \pm 2V_m E + M_\nu^2$$

+: neutrinos, -: antineutrinos

In the flavour basis: $$V_m = \begin{pmatrix} V_{NC} + \sqrt{2}G_F N_e & & \\ & V_{NC} & \\ & & V_{NC} \end{pmatrix}$$

Earth: $$V_m \simeq 10^{-13} eV \to 2V_m E \simeq 10^{-4} eV^2 \left[\frac{E}{1GeV}\right]$$

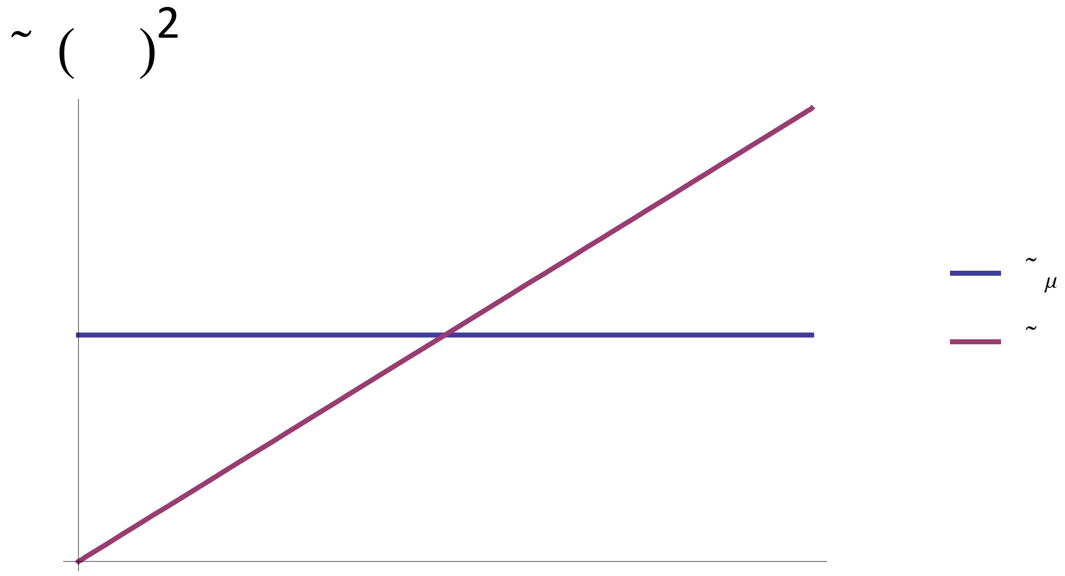
Sun: $$V_m \simeq 10^{-12} eV \to 2V_m E \simeq 10^{-6} eV^2 \left[\frac{E}{1MeV}\right]$$

Effective masses in matter

Solar neutrinos are produced in the center of the sun and as the exit they encounter a decaying matter density:

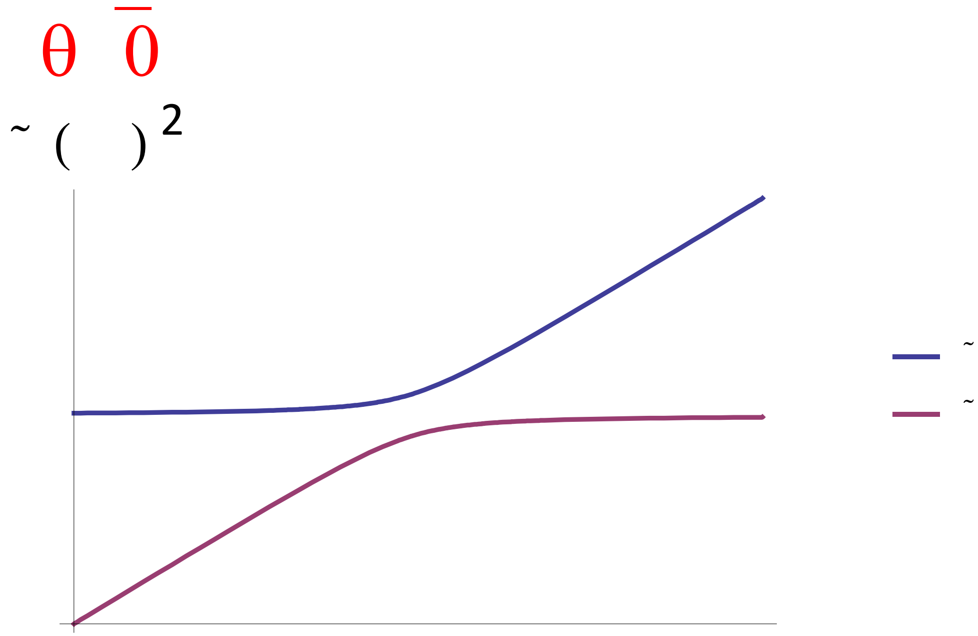
$$N_e(r) \propto N_e(0) e^{-r/R}$$

θ=0



MSW resonance

Mikheyev, Smirnov ‘85



$$\Delta m^2 \cos 2\theta \pm 2\sqrt{2}\, G_F E\, N_e = 0$$

MSW Resonance:

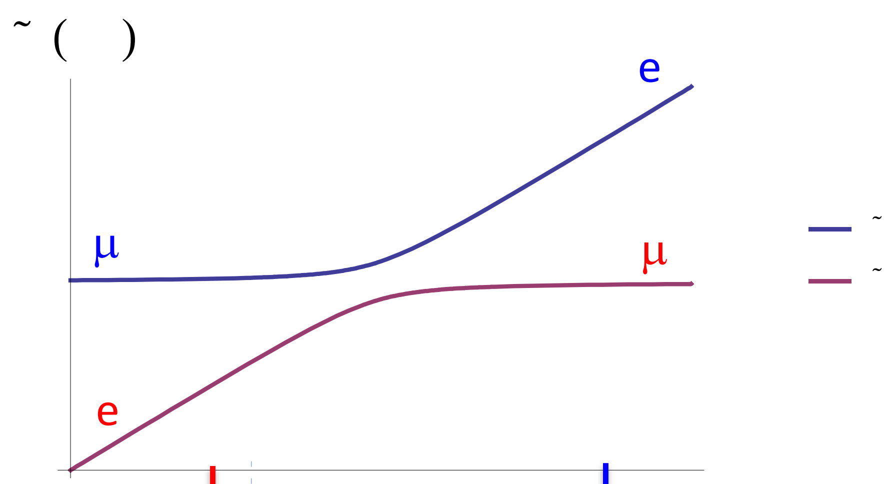
-Only for ν or $\bar{\nu}$, not both

-Only for one sign of $\Delta m^2 \cos 2\theta$

Neutrinos in variable matter

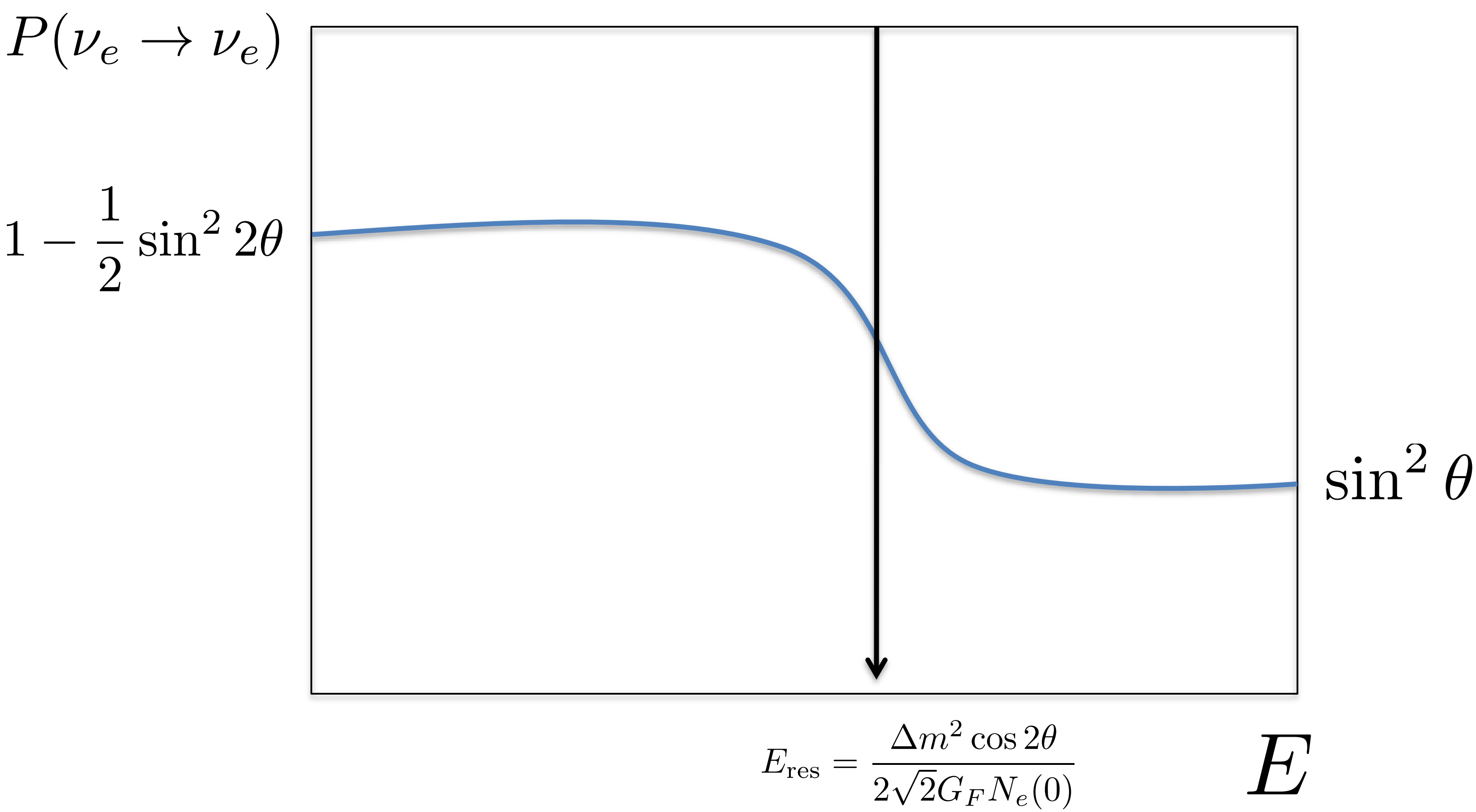
If the variation is slow enough: adiabatic approximation (if a state is at r=0 in an eigenstate $\tilde{m}_i^2(0)$ it remains in the i-th eigenstate until it exits the sun)

$$P(\nu_e \to \nu_e) = \sum_i |\langle \nu_e | \tilde{\nu}_i(\infty)\rangle|^2 |\langle \tilde{\nu}_i(0)|\nu_e\rangle|^2$$



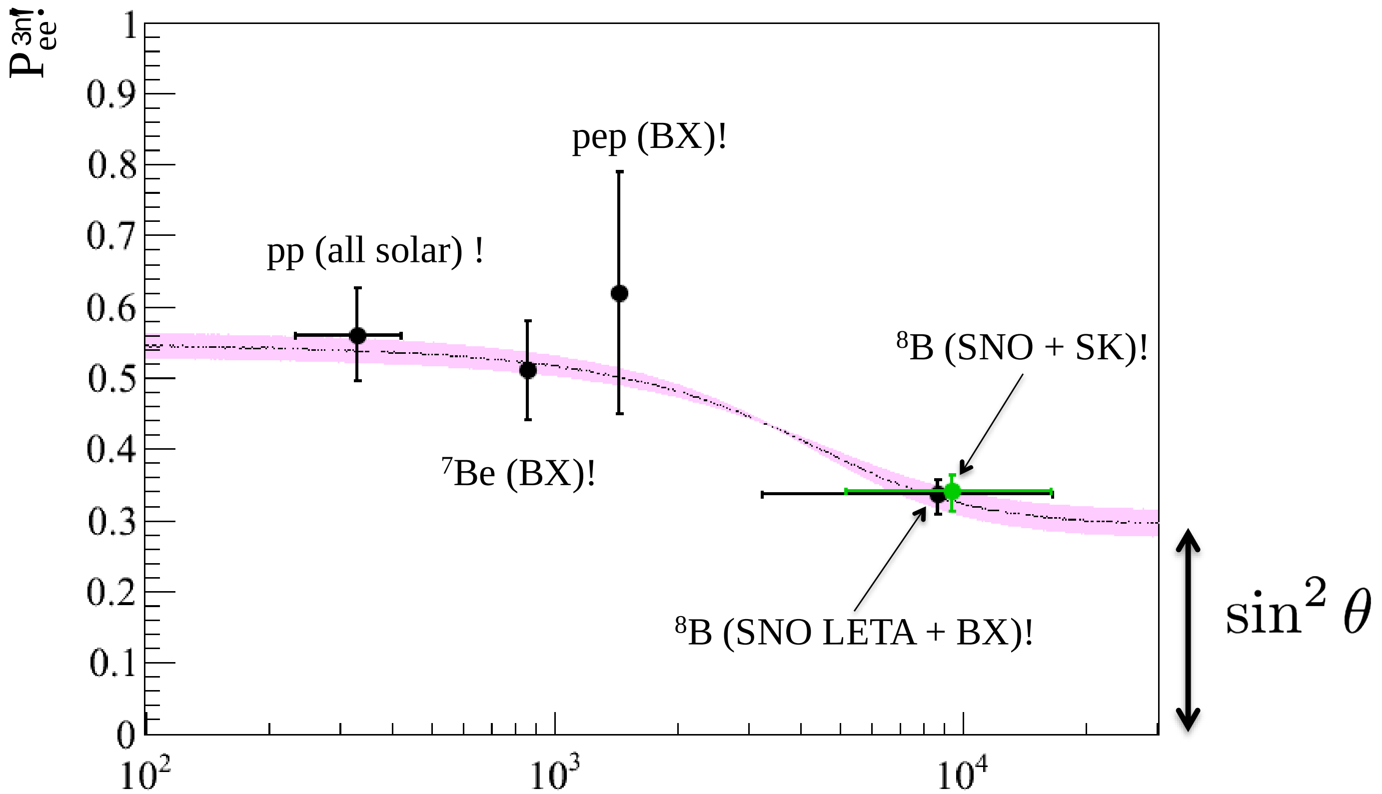
$P(\nu_e \to \nu_e) \simeq 1 - \frac{1}{2}\sin^2 2\theta$ $\quad$ $P(\nu_e \to \nu_e) \simeq \sin^2\theta$

Solar neutrinos



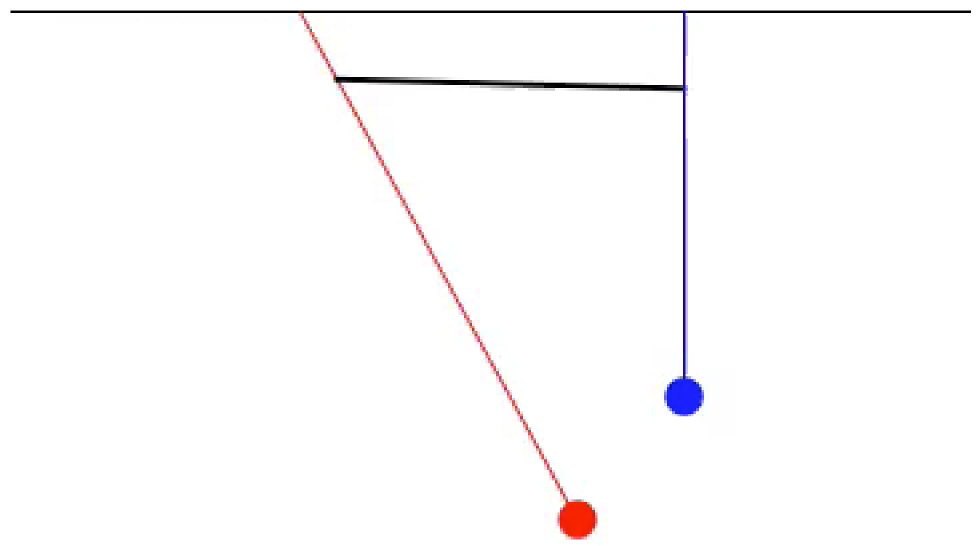
MSW resonance energy

Solar neutrinos and MSW

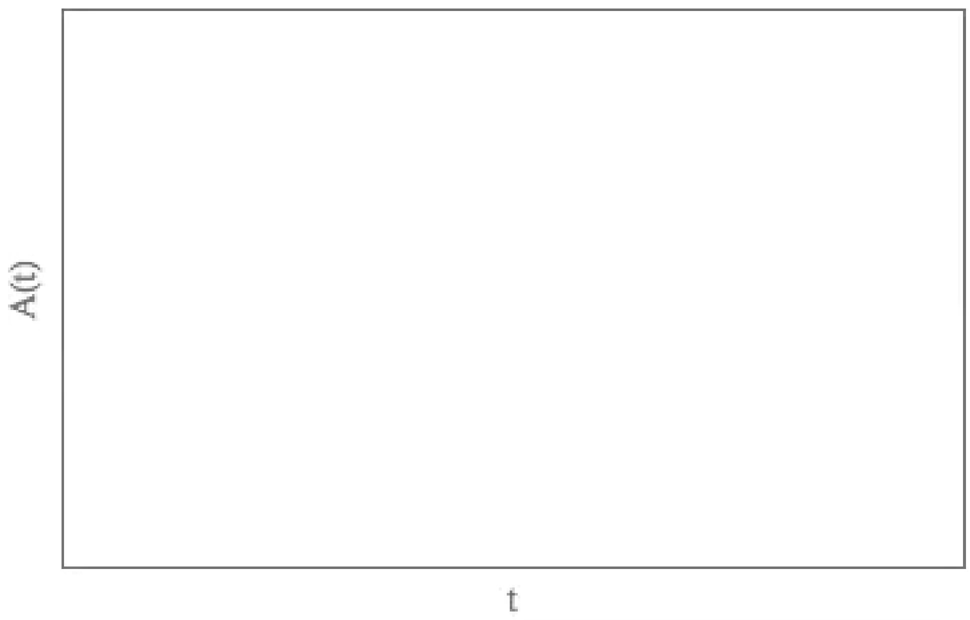


Borexino

Classical analogy IV:MSW resonance



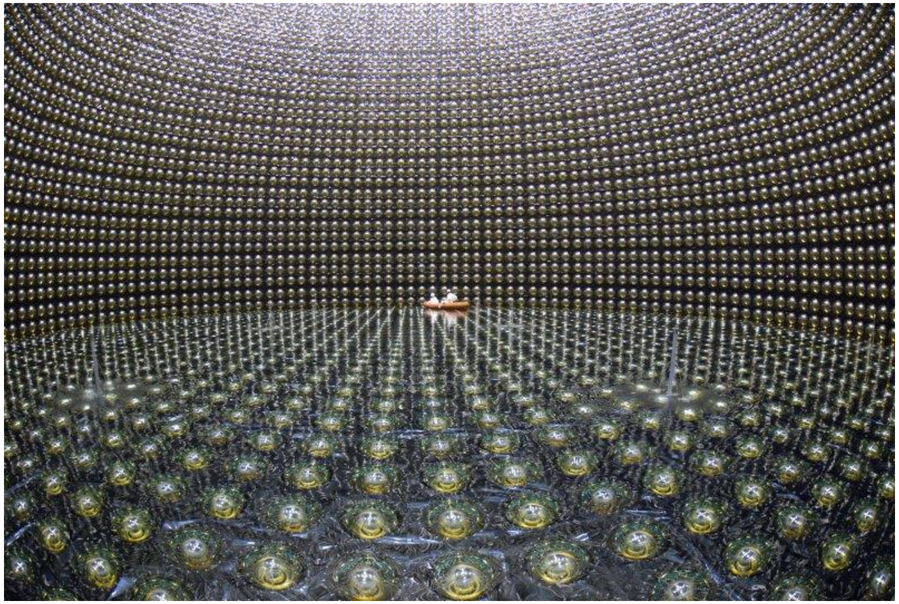
Classical analogy IV:MSW resonance



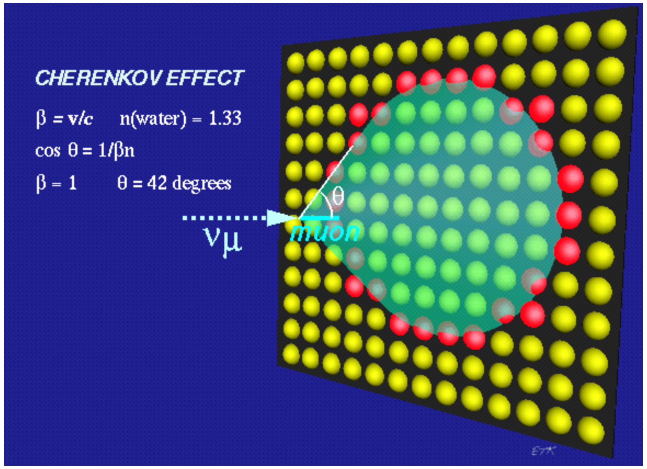
As we cross the resonance (when the two lengths are the same), there is a maximal flavour conversion: what was mostly ν_e is now mostly ν_μ

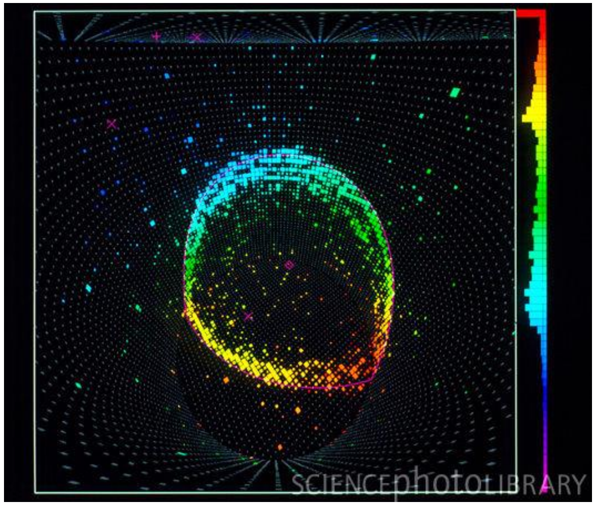
Underground cathedrals of light

Superkamiokande



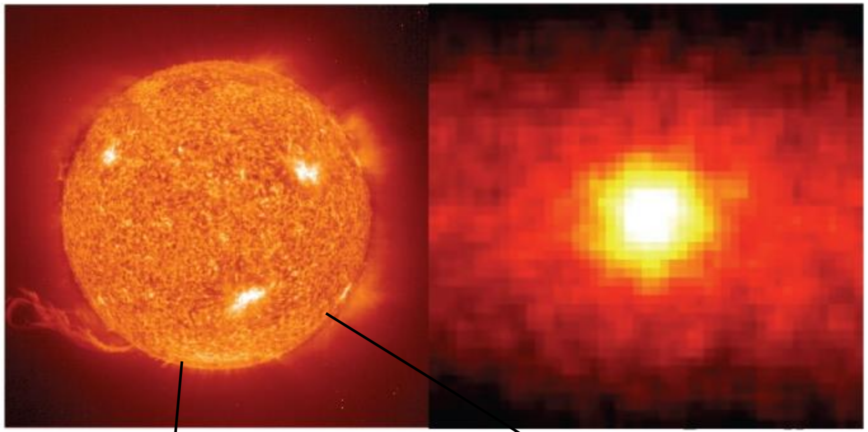
Koshiba (Nobel 2002)





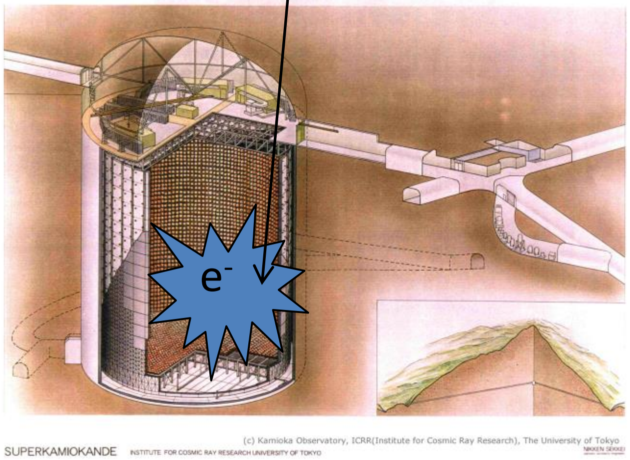
Allows to reconstruct velocity and direction, e/μ particle identification

Solar Neutrinos



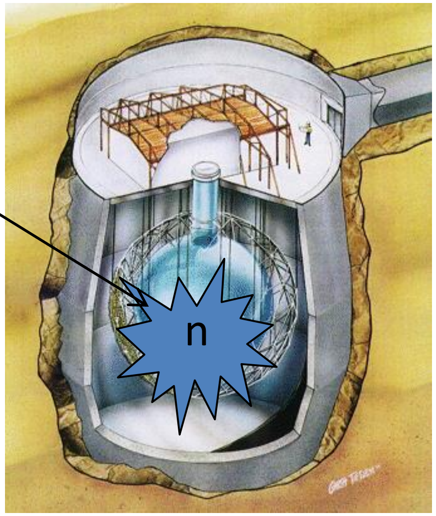
Neutrinography of the sun

SuperKamiokande (22.5 kton!)



$$\nu_e + e^- \rightarrow \nu_e + e^-$$

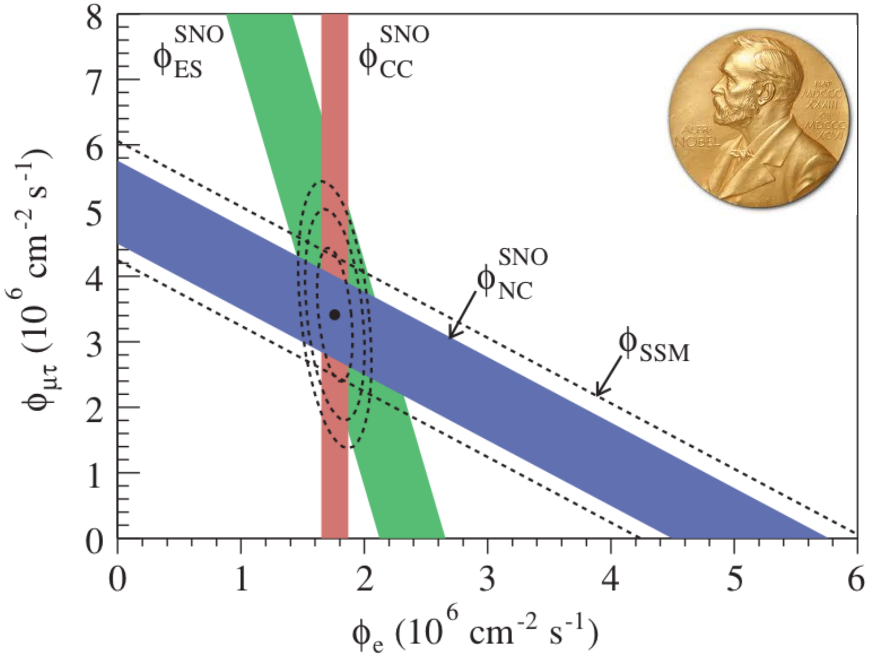
SNO

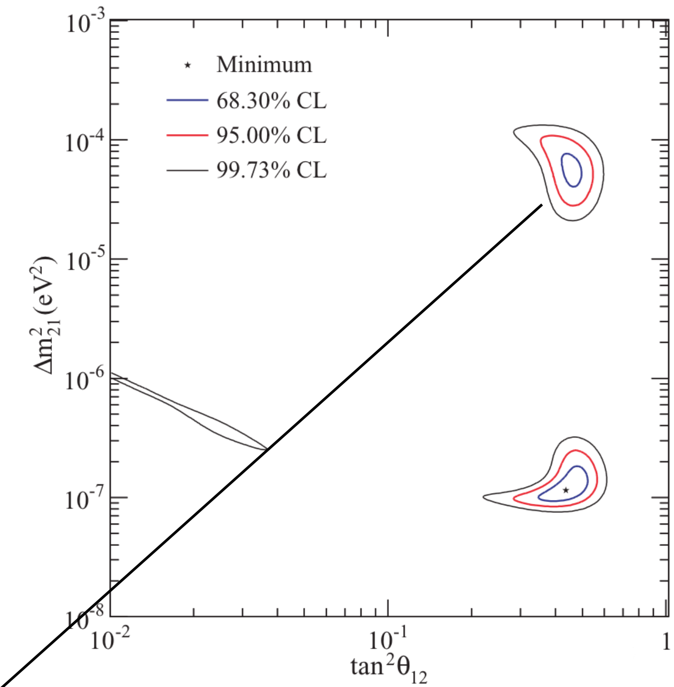


$$NC: \quad \nu_i + d \rightarrow p + n + \nu_i$$

$$CC: \quad \nu_e + d \rightarrow p + p + e^-$$

Flavour of solar neutrinos





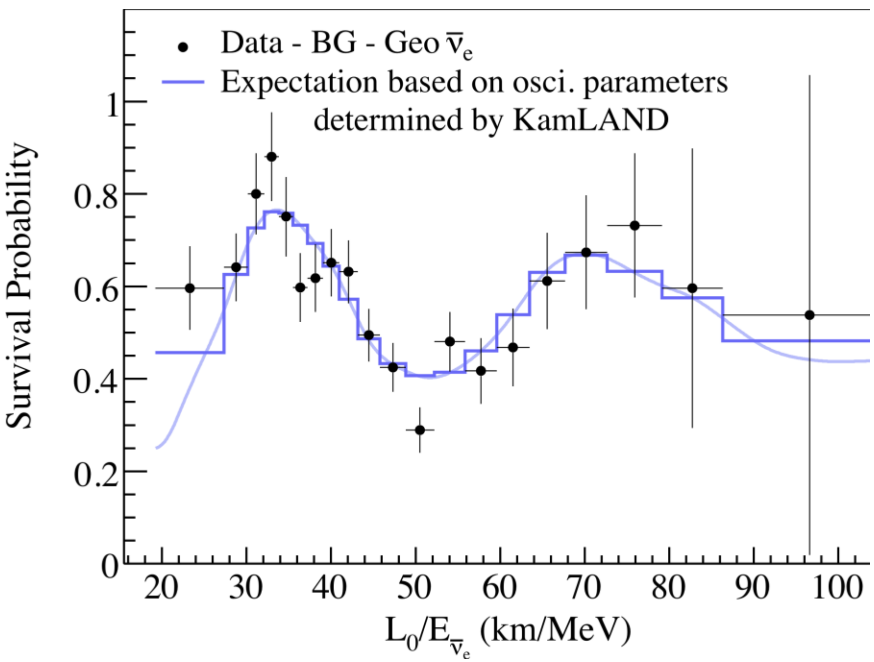
$$|\Delta m^2| \overset{-1}{\sim} \frac{O(100Km)}{O(MeV)}$$

Can be tested in the Earth with Reines&Cowen experiment !

KamLAND: solar oscillation

$$\overline{\nu}_e \rightarrow \overline{\nu}_e$$

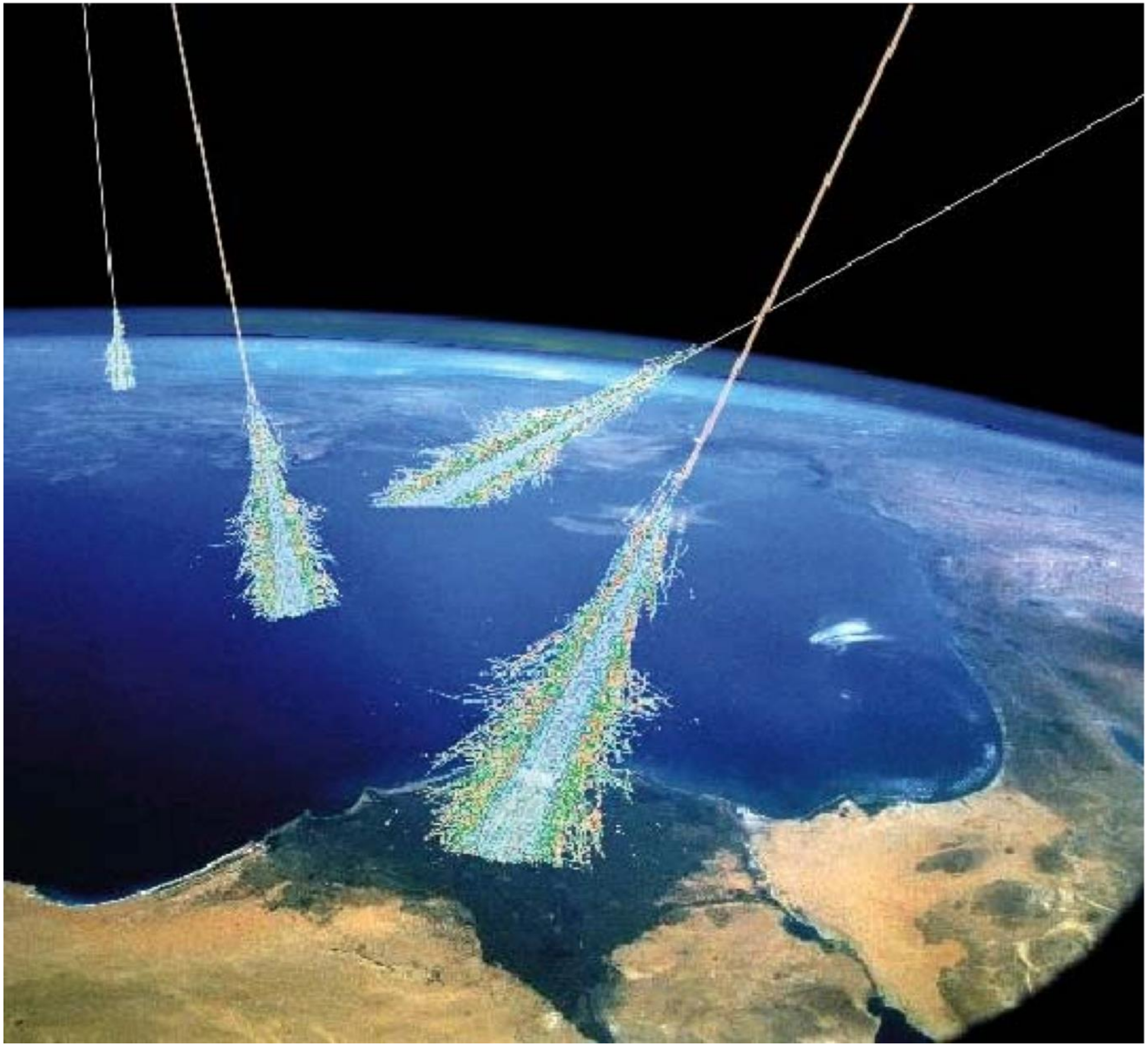
Reines&Cowan experiment ½ century later
at 170 km from Japenese reactors ...



$$\boxed{\Delta m^2_{\rm solar} \simeq 8 \times 10^{-5}\ eV^2}$$

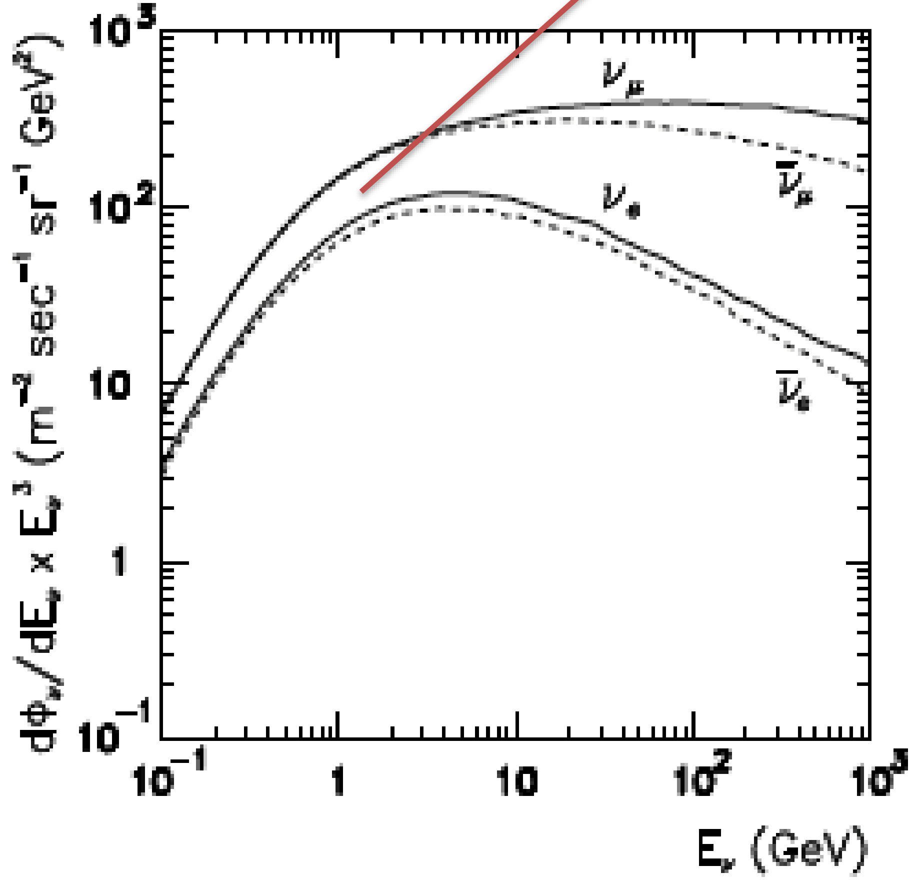
Large mixing

Atmospheric Neutrinos



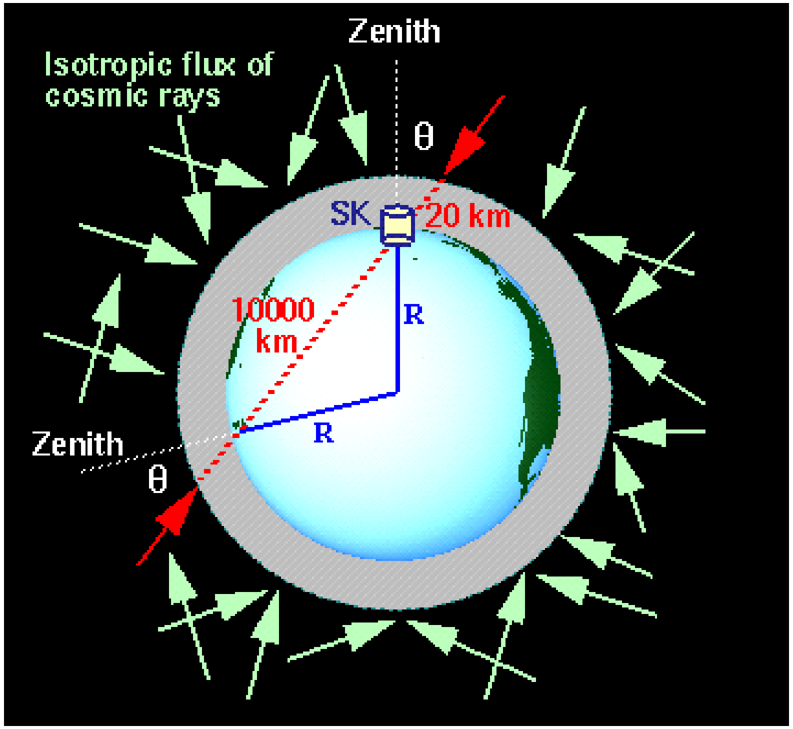
Atmospheric Neutrinos

$\nu_\mu/\nu_e \sim 2$

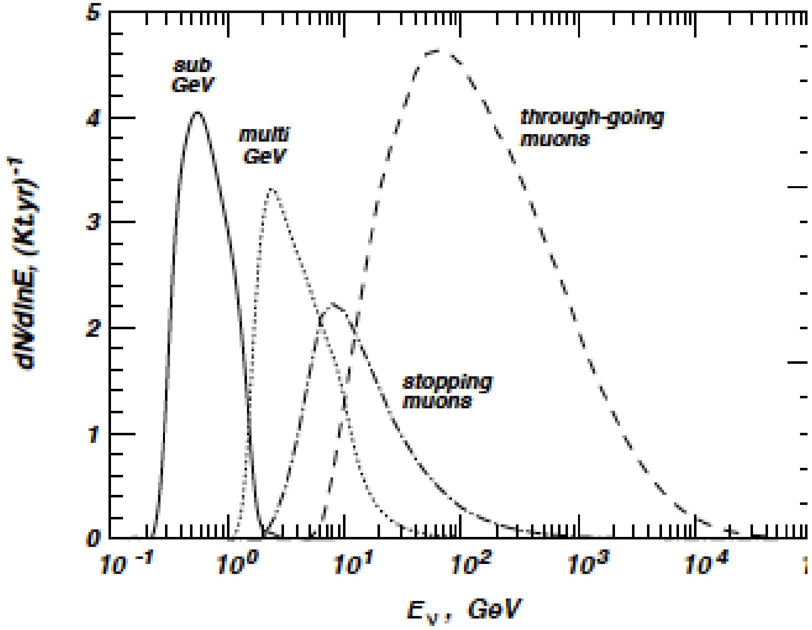


Produced in the atmosphere when primary cosmic rays collide with it, producing π, K

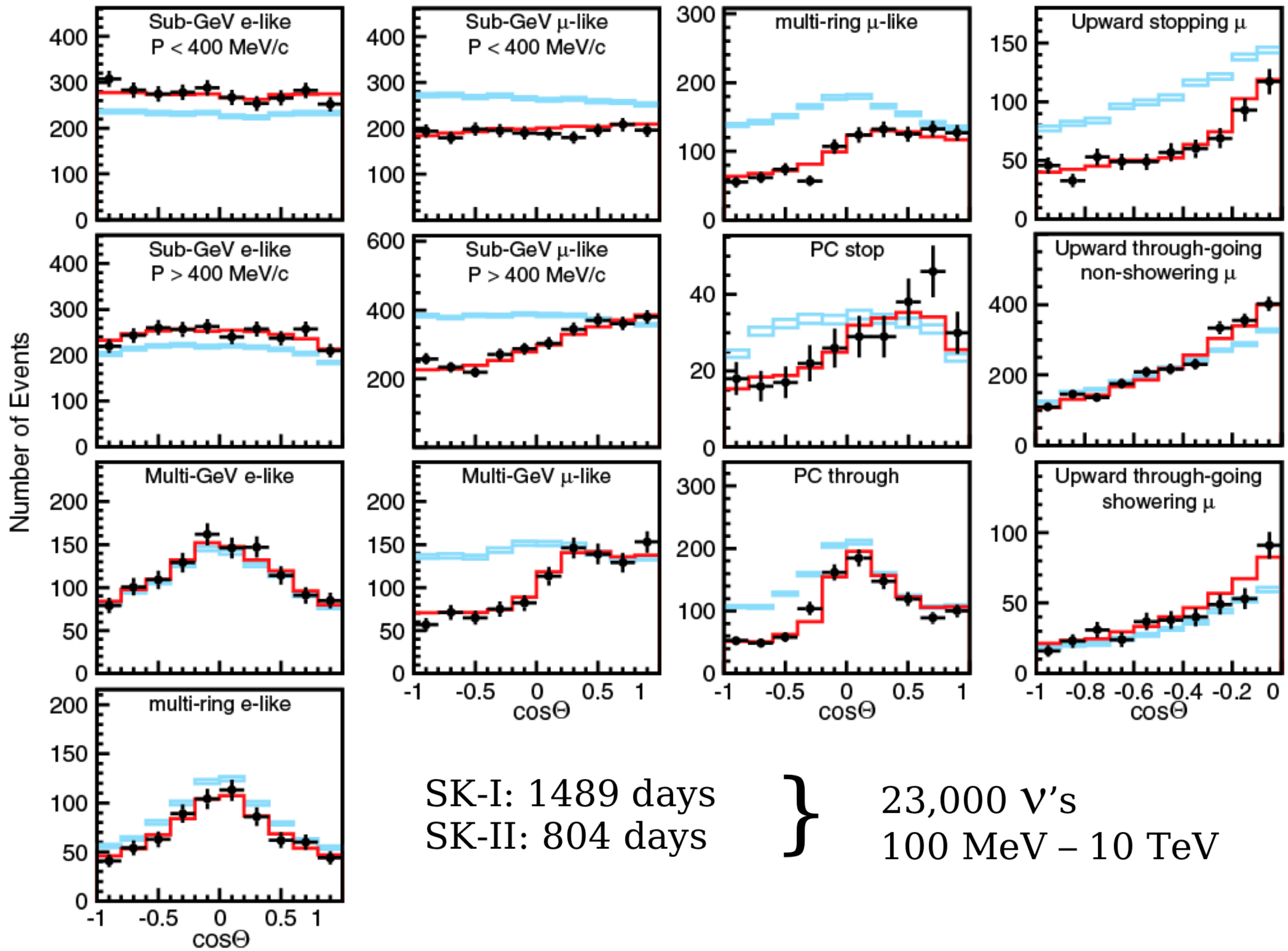
Atmospheric Neutrinos



$$L = 10 - 10^4 \text{ Km}$$



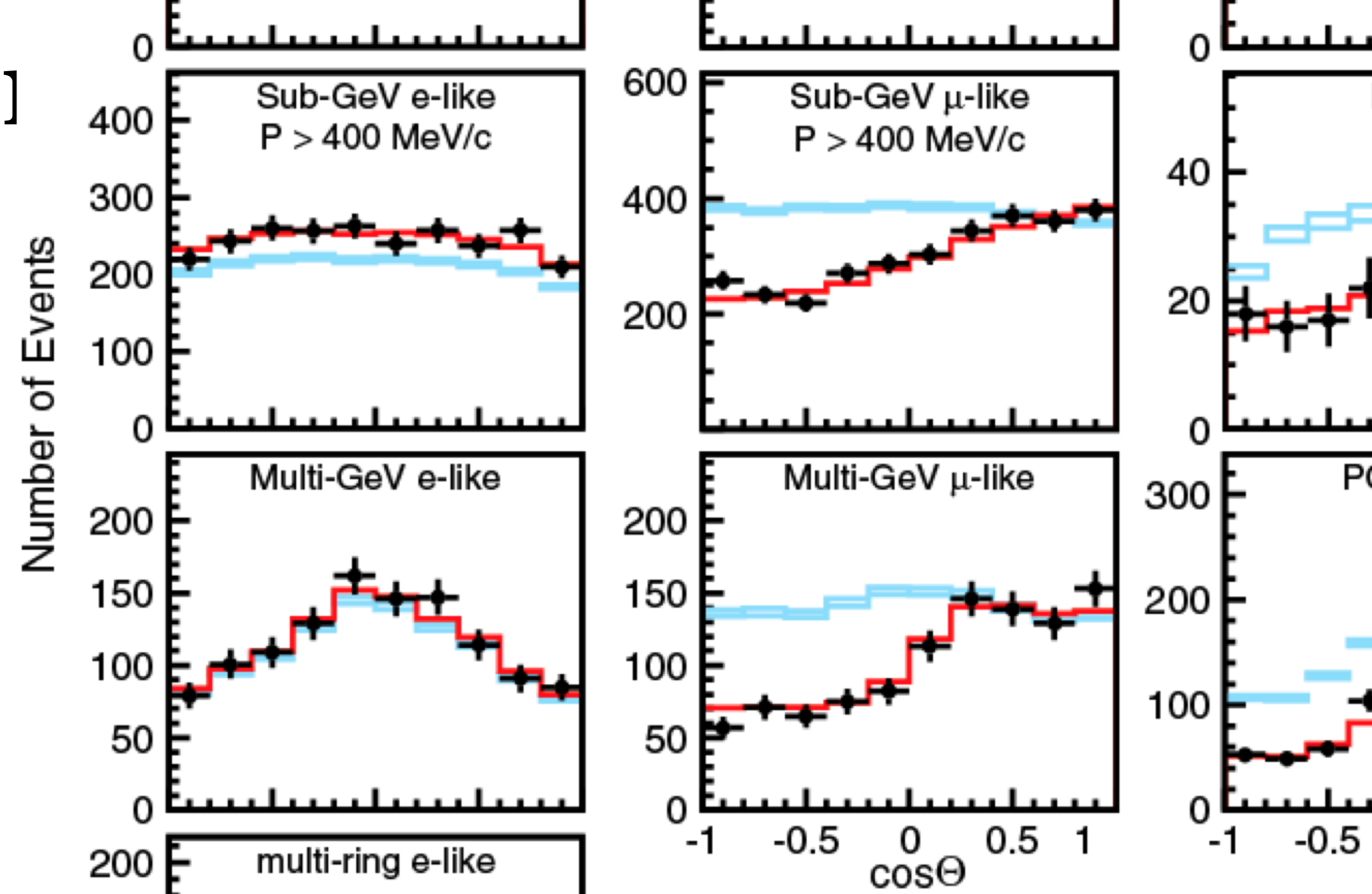
Atmospheric Neutrinos

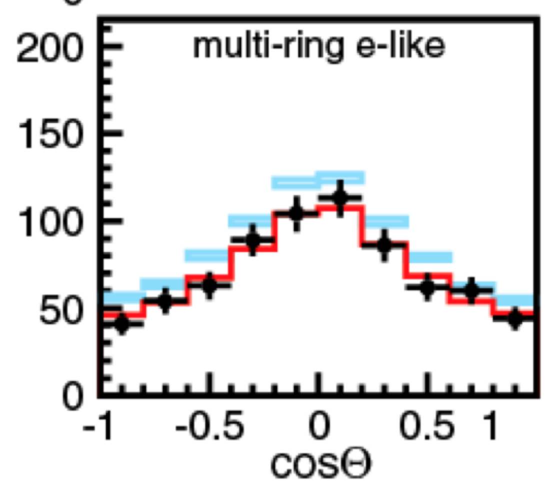


SK-I: 1489 days
SK-II: 804 days

} 23,000 ν's
100 MeV – 10 TeV

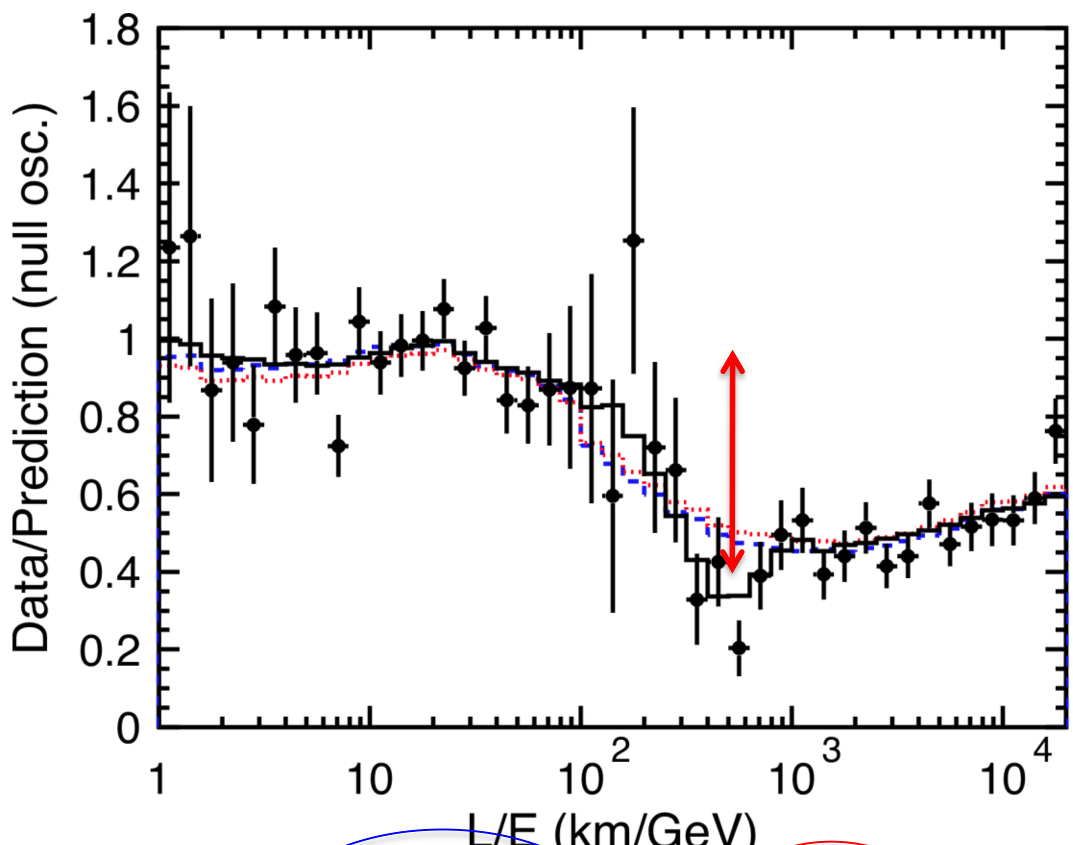
At





SK-I: 1489 days
SK-II: 804 days

Atmospheric Oscillation



$$\Delta m^2_{\rm atm} = 2.5 \times 10^{-3} eV^2$$

Large mixing

$$|\Delta m^2|^{-1} \sim \frac{O(1000Km)}{O(GeV)} \sim \frac{O(1km)}{O(MeV)}$$

Reines&Cowan experiment at 1km!

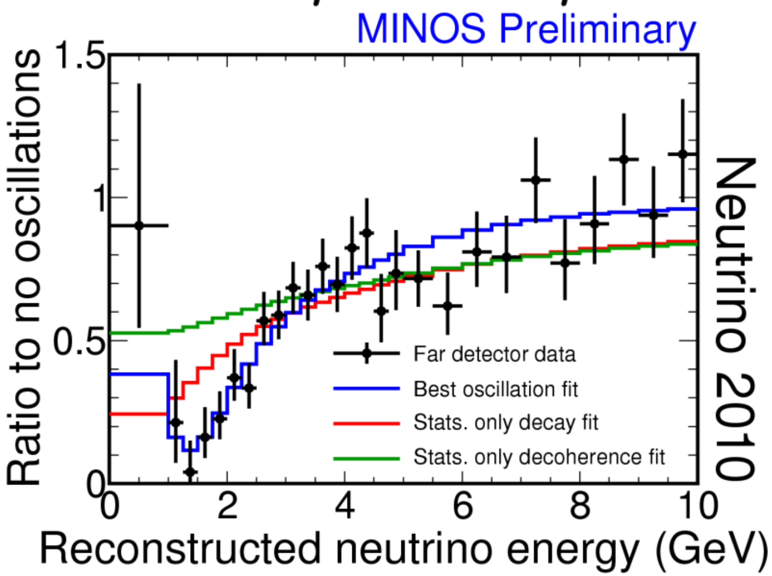
Lederman&co experiment at 1000km!

Lederman&co neutrinos oscillate with the atmospheric wave length

Pulsed neutrino beams to 700 km baselines

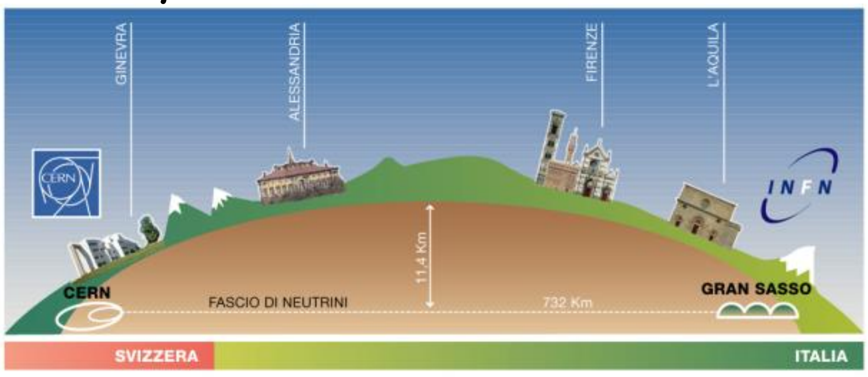
MINOS

$$\nu_\mu \to \nu_\mu$$



$$\nu_\mu \to \nu_\tau$$

OPERA

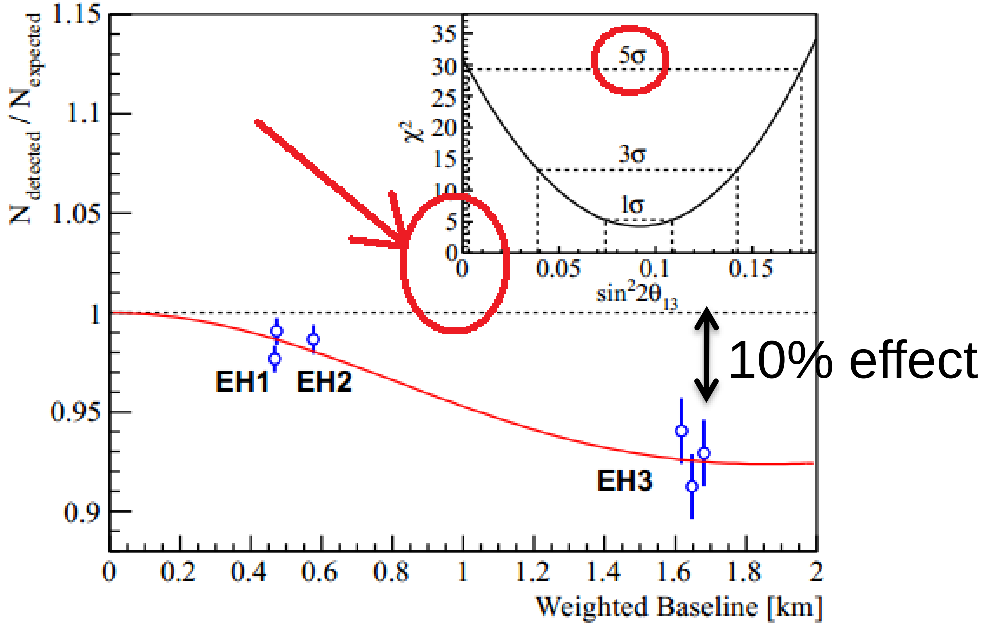


$$|\Delta m^2_{\rm atmos}| \simeq 2.5 \times 10^{-3}\ eV^2$$

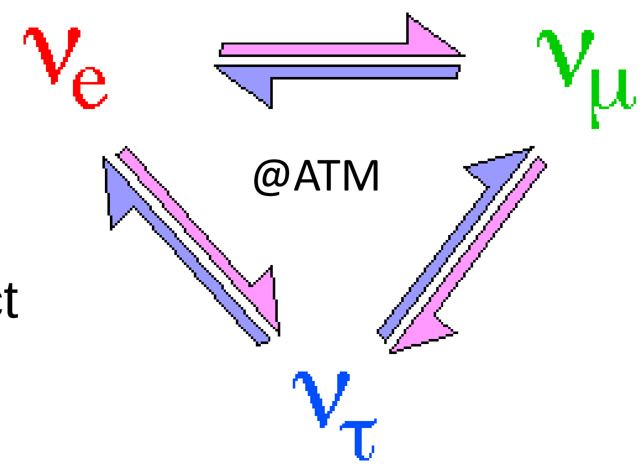
$$\sin^2 2\theta_{\rm atmos} \simeq 1$$

Reactor neutrinos oscillate with atmospheric wave length

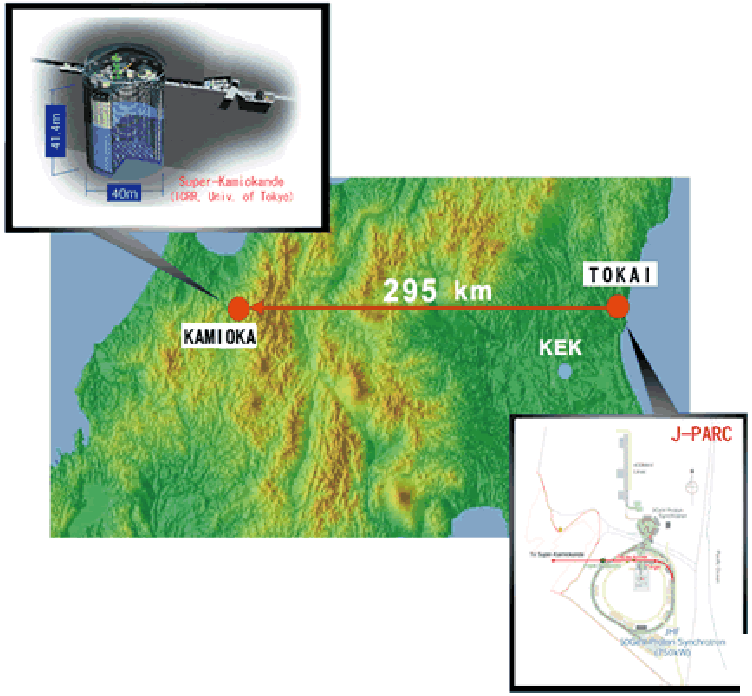
Double Chooz, Daya Bay, RENO

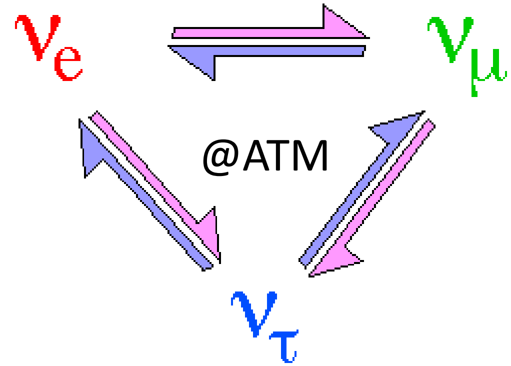


$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

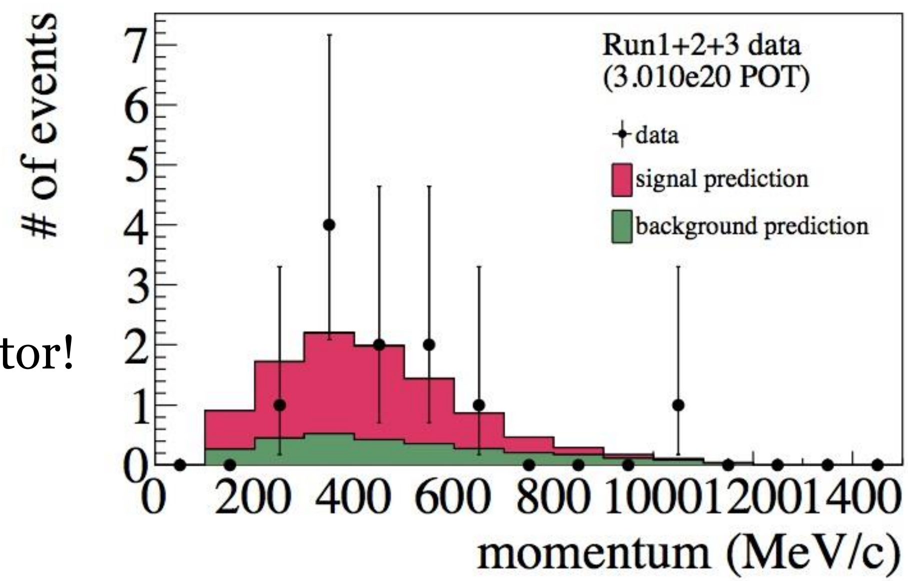


T2K





$$\nu_\mu \to \nu_e$$



Using the SuperKamiokande detector!

Standard 3ν scenario

$$\Delta m^2_{23} = m^2_3 - m^2_2 \equiv \Delta m^2_{atm}$$

$$\Delta m^2_{12} = m^2_2 - m^2_1 \equiv \Delta m^2_{sol}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{23}(\theta_{23})U_{13}(\theta_{13},\delta)U_{12}(\theta_{12}) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Solar and atmospheric osc. decouple as 2x2 mixing phenomena:

- hierarchy $\dfrac{|\Delta m^2_{atm}|}{|\Delta m^2_{sol}|} > 10$ $\qquad \theta_{23} \simeq \theta_{\mathrm{atm}}$
- small θ_{13} $\qquad \theta_{12} \simeq \theta_{\mathrm{sol}}$

SM+3 massive neutrinos: Global Fits

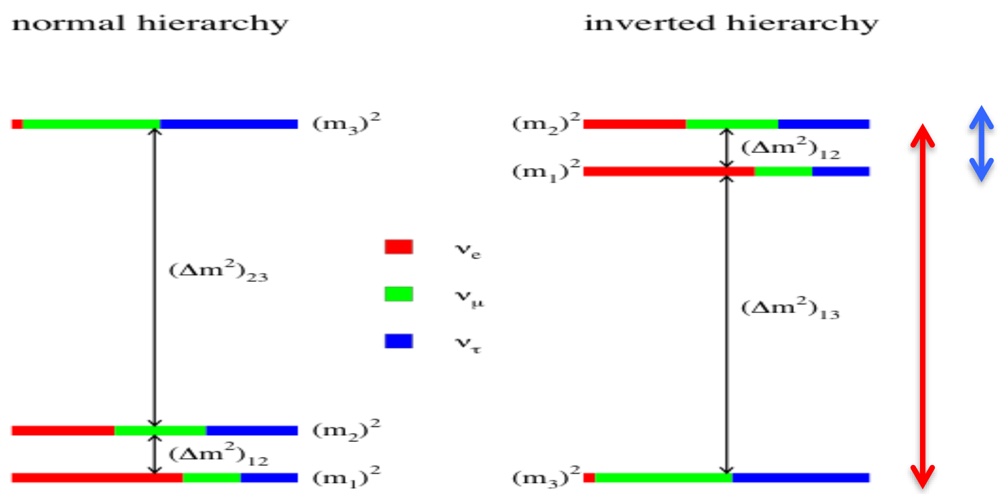
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \ldots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{12} \sim 34^\circ$

$\theta_{23} \sim 42^\circ$ o 48°

$\theta_{13} \sim 8.5^\circ$

$\delta \sim \ ?$



$7.5 \cdot 10^{-5} \mathrm{eV}^2$

$2.5 \cdot 10^{-3} \mathrm{eV}^2$