Cosmology I: Measuring and weighing the Universe

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Contents

Distances

Radius of the Earth: $\simeq$ 6000km

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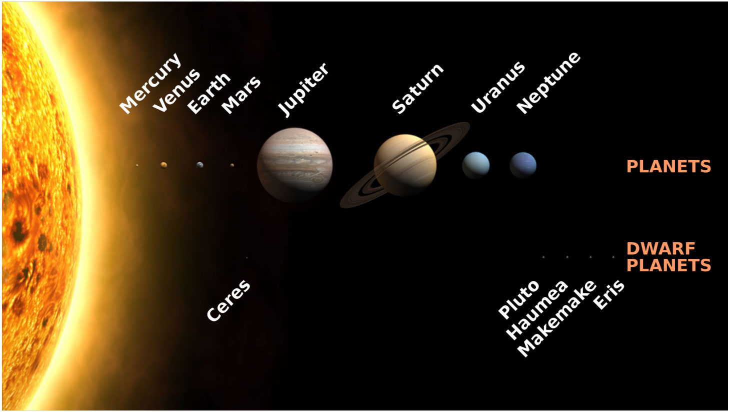
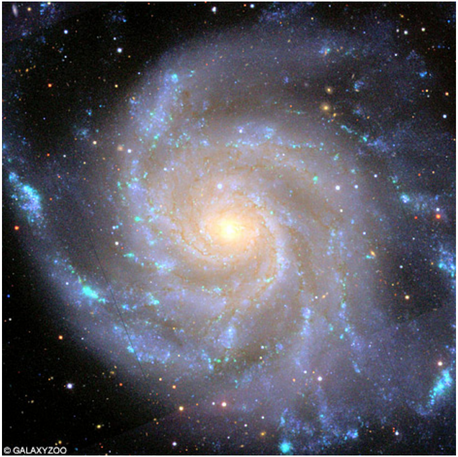
The solar system $\simeq 7 \times 10^9$km ($\simeq$ 50au =50 'astronomical units')
1 au $\simeq 1.5 \times 10^8$km is the average distance between the earth and the sun

(source: wikipedia)



Distances

The Milky Way (visible part)$\simeq 10^{18}$km $\simeq 10^5$ light years $\simeq 30'000$parsec



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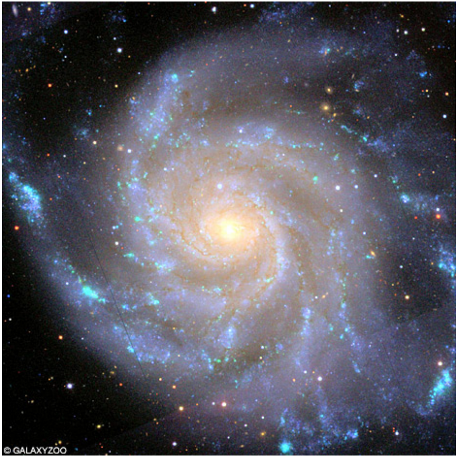
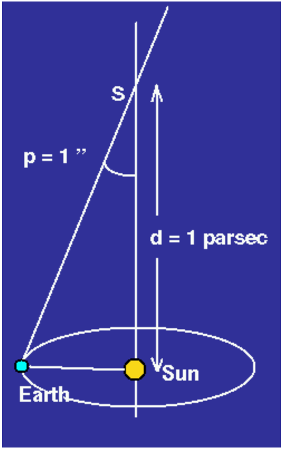


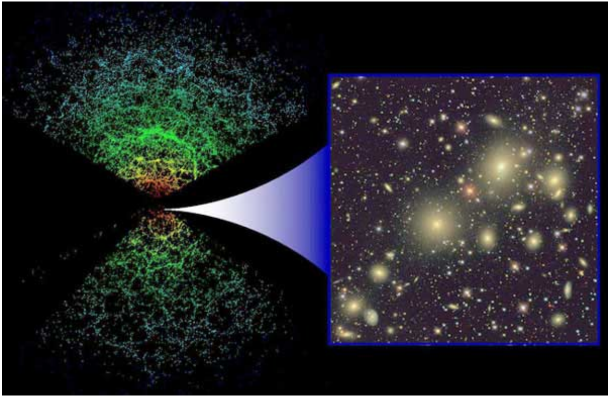


$1'' = 1^o/3600 = 1$arc second

1parsec $\simeq 3.26$ light years

Distances

The size of the 'visible Universe' (Hubble scale) $\simeq$ 28000Mpc $= 2.8 \times 10^{10}$parsec
(Contains about 0.5×10^{12} galaxies like the Milky Way with mass of about $10^{12} M_\odot$)



Each point represents a galaxy
(Sloan digital sky survey, SDSS)

The Universe is expanding

As you know, Newtonian gravity is an attractive force.
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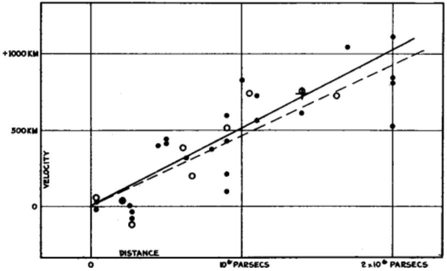
Despite this fact, observations show that the Universe is expanding.
Galaxies recede from each other with a speed proportional to their distance,

$$v = \dot{R} = H_0 \cdot R \qquad \text{(Hubble's law, } H_0 \simeq 70\text{km/s/Mpc)}$$



(Hubble 1932)

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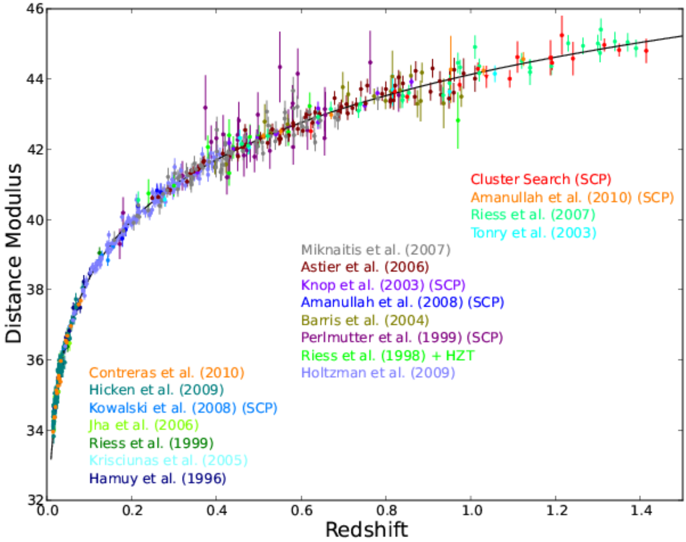

(The Supernova Cosmology Project)

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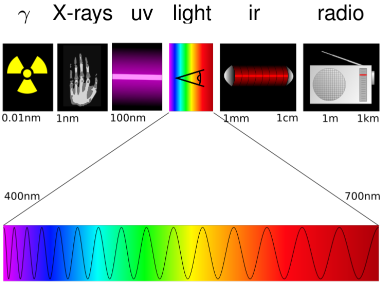
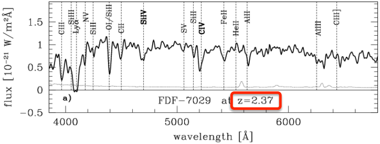
- To mesure the speed we mesure the redshift. This is the Doppler effect for light:

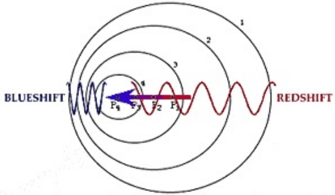
$$z = \frac{\lambda - \lambda_e}{\lambda_e} \simeq v/c\,, \qquad \text{if } \; z \ll 1 \qquad \left(z = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1 \right).$$

Astronomical observations can be made in different wavelengths bands of the electromagnetic spectrum. In the optical band specific spectral lines (atomic transitions) are at fixed wavelength





In a source moving away from us these spectral lines are shifted towards the red, 'redshifted'



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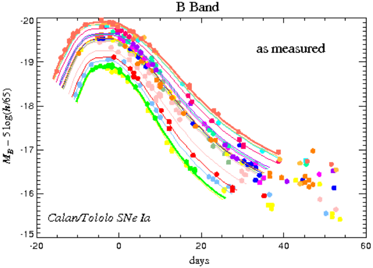
- A moment in the past can be characterized by its redshift z.
- The present expansion rate of the Universe is $H_0 \simeq 70$km/s/Mpc. In the past it has been different. We want to determine the expansion rate as function of the redshift, $H(z)$. For this we have to measure the redshift z and the distance d of far away galaxies.

The most powerful standard candles are supernovae of type Ia.

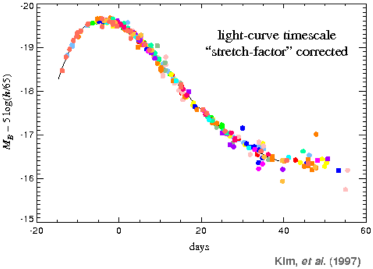
(SN1994D)

SNIa light curve

After application of a 'stretch factor' the maximum of the light curve, i.e. the maximum of the luminosity is nearly the same for all supernovae Ia.

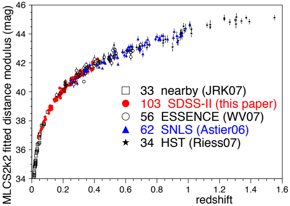


Without correction.

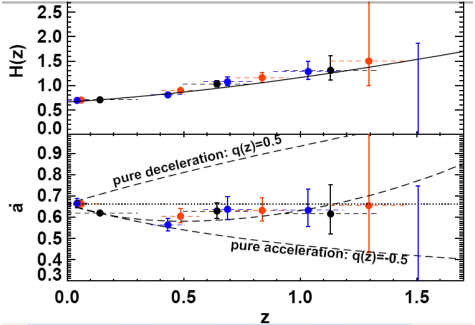


After correction.

The Universe is in *accelerated* expansion



(Kessler et al. 2009)



(Riess et al. 2007)

Luminosity distance from apparent flux, F, and known intensity, I:
$F = I/(4\pi d_L^2)$.

Distance modulus
$= -\log(\text{app. flux}) + \text{constant}$
$= \log(d_L^2) + \text{constant}$

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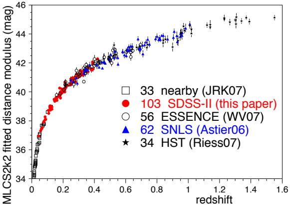

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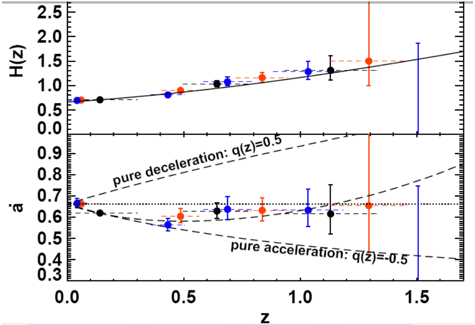

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$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$(K = 0)$

$$H = \frac{\dot R}{R} = (1+z)\dot a(z)$$

$$a = R/R_0$$

$\dot R(z) > 0\,, \quad \ddot R(z) > 0$ for $z < 0.5$.

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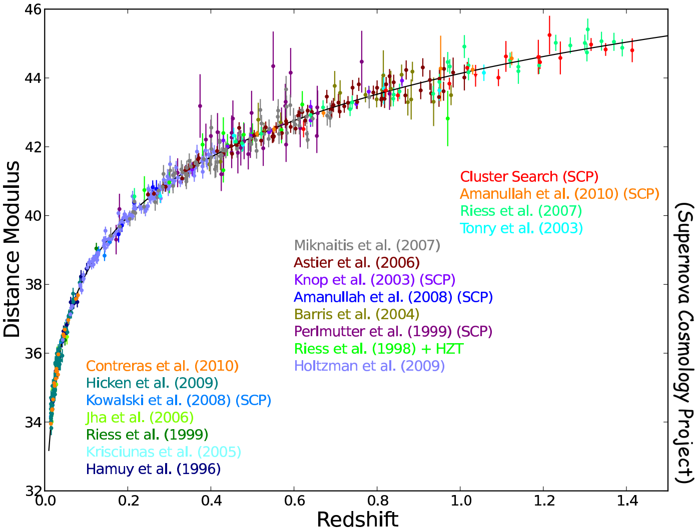
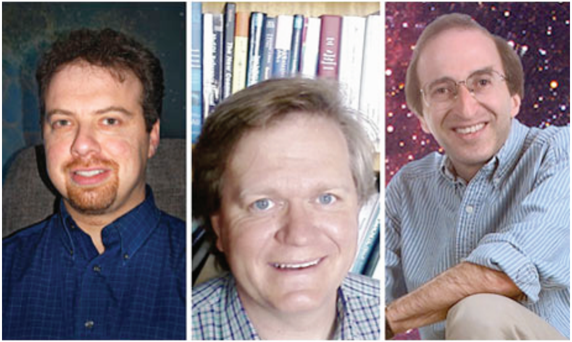


Present Hubble diagram

Nobel Prize in Physics 2011

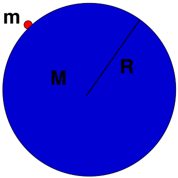


Adam G. Riess   Brian P. Schmidt   Saul Perlmutter

The Nobel Prize in Physics 2011 was awarded to Saul Perlmutter, Brian P. Schmidt and Adam G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

Understanding the expansion of the Universe within Newtonian gravity

We consider a test mass m at the border of a homogeneous sphere of density ρ, which is expanding with velocity $v = \dot{R}$.



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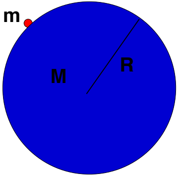

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Its energy is

$$E = \frac{m}{2}v^2 + U = \frac{m}{2}v^2 - \frac{GmM}{R} = \frac{m}{2}v^2 - \frac{4\pi G}{3}m\rho R^2$$

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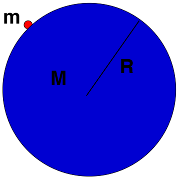

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As energy is conserved, $2E/m =: -K = \text{constant} = \dot{R}^2 - 8\pi G\rho R^2/3$. With $H^2 = \left(\frac{\dot{R}}{R}\right)^2$ we obtain

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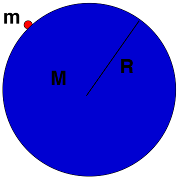

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This is the Friedmann equation (1922).

Due to the expansion, the density decreases,

$$\rho = \frac{M}{\frac{4\pi}{3}R^3}\,, \qquad \dot{\rho} = -3\rho\frac{\dot{R}}{R}$$

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$$\frac{d}{dt}\left[\left(\frac{\dot R}{R}\right)^2 + \frac{K}{R^2}\right] = 2\left[\frac{\ddot R}{R}\underbrace{-\left(\frac{\dot R}{R}\right)^2 - \frac{K}{R^2}}_{-8\pi G\rho/3}\right]\frac{\dot R}{R} = \frac{8\pi G}{3}\dot\rho = -8\pi G\rho\frac{\dot R}{R}$$

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This is the 2nd Friedmann equation (1922). It requires that the expansion decelerates!

Including general relativity these equations are modified:

$$H^2 + \frac{K}{R^2} = \left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G}{3c^2}\rho_E + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho_E + 3P) + \frac{\Lambda}{3}$$

P is the pressure and Λ is the cosmological constant,
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Introducing the 'density' parameters

$$\Omega_m = \frac{8\pi G\rho_E}{3c^2H^2}\,, \qquad \Omega_K = -\frac{K}{R^2H^2}\,, \qquad \Omega_\Lambda = \frac{\Lambda}{3H^2}\,,$$

the first Friedmann eqn. becomes

$$\Omega_m + \Omega_\Lambda + \Omega_K = 1\,.$$

To solve the Friedmann equations we need to know K, Λ and the equation of state, $P(\rho)$

$$P = \begin{cases} 0\,, & \text{matter} \\ 1/3\rho\,, & \text{radiation, massless particles} \\ -\rho\,, & \text{cosmological constant, vacuum energy} \\ -1/3\rho\,, & \text{curvature.} \end{cases}$$

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For $P = w\rho$ with constant w this is solved by $\rho = \rho_0 a^{-3(1+w)}$.
If this is the dominant term in the Friedmann eqn. ($K \sim \Lambda \sim 0$), the latter is solved by

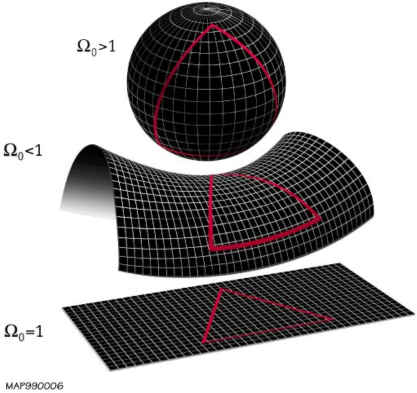
$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho\,, \qquad a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} \quad .$$

Curvature

$K > 0$ ($\Omega_K < 0$): spherical space,

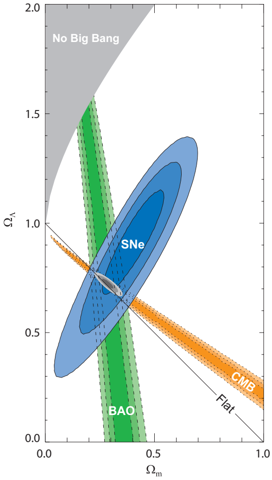
$K < 0$ ($\Omega_K > 0$): pseudo-spherical space (saddle),

$K = 0$ ($\Omega_K = 0$): flat space.

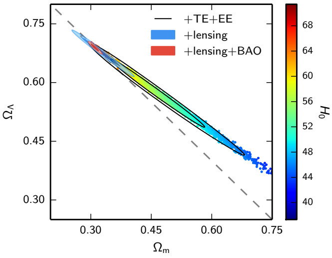


The Universe is in *accelerated* expansion

Matter, Ω_m, and cosmological constant, Ω_Λ (dark energy).



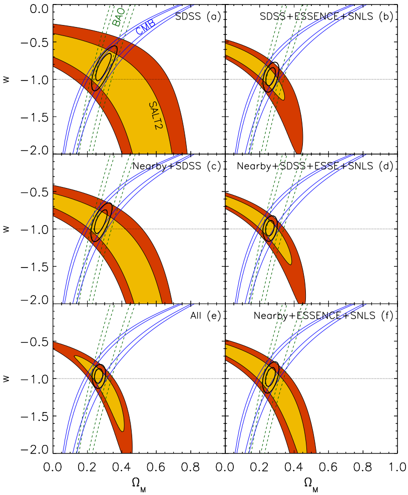
Supernova Cosmology Project, Suzuki et al. 2011



Planck 2015

The Universe is in *accelerated* expansion

If pressure is negative, $P = w\rho_E$ with $w < -1/3$ we can have accelerated expansion ($\ddot{R} > 0$) without a cosmological constant. Such a component is called dark energy. A cosmological constant corresponds to a dark energy component with $w = -1$.



The matter fraction and the parameter w of dark energy (Kessler et al. ’09).

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- The distance to a galaxy, or the time at which its light which is currently reaching us has been emitted is determined by its redshift.
- The Hubble diagram gives the distance of objects as function of their redshift.
- Recent observations have shown that the expansion of the Universe is accelerated. Understanding this within general relativity requires 'dark energy'.