Parton Saturation

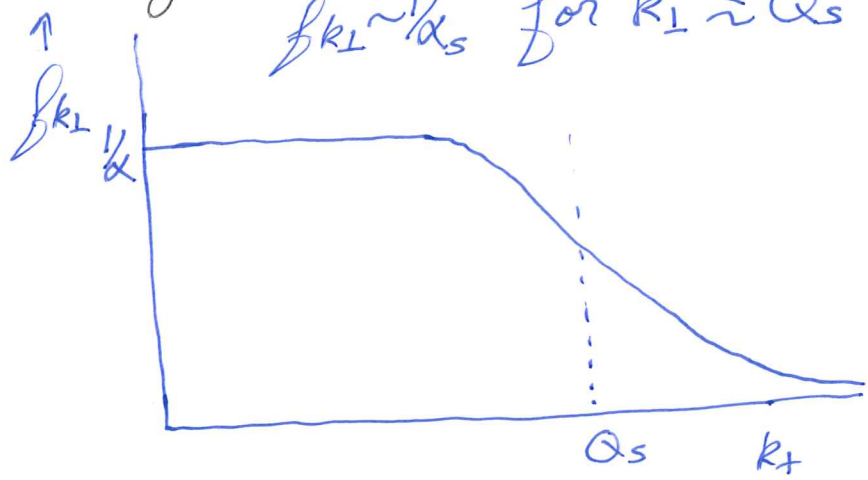
I. The ideas

Gribov, Levin, Ryskin ...

Dual description of saturation

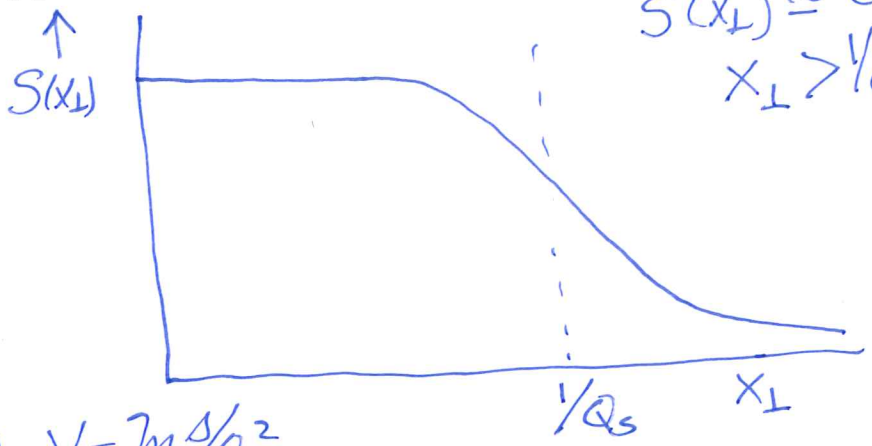
(i) A high occupancy state of gluons in a high momentum nucleus.

$f_{k_\perp} \sim 1/\alpha_s$ for $k_\perp \lesssim Q_s$



(ii) $S(x_\perp) \equiv 1 - T(x_\perp) \simeq 0$ for scattering a dipole of size $x_\perp$ on the high momentum nucleus.

$S(x_\perp) \simeq 0$ for $x_\perp > 1/Q_s$

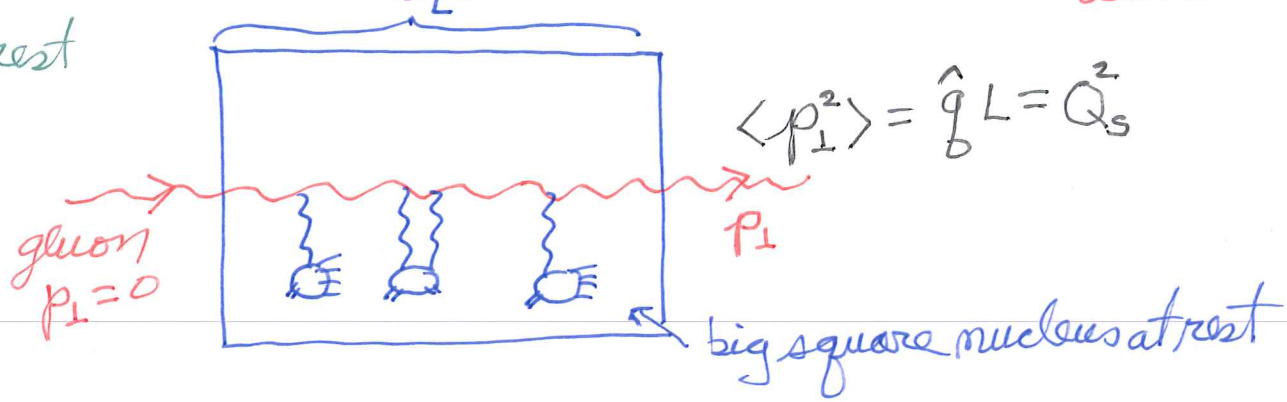


$Q_s = Q_s(Y)$; $Y = \ln s/Q_s^2$ or $Y = \ln 1/x_{BJ}$

2. Saturation and $\hat{q}$ of cold nuclear matter

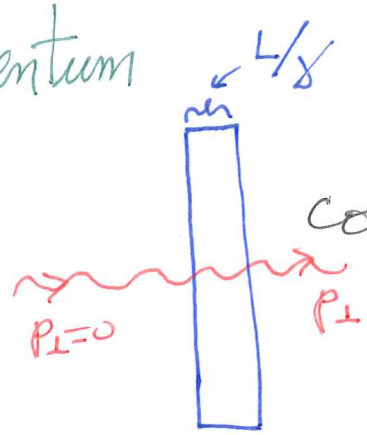
BDMPS

At rest



$$\langle p_\perp^2 \rangle = \hat{q} L = Q_s^2$$

At high momentum

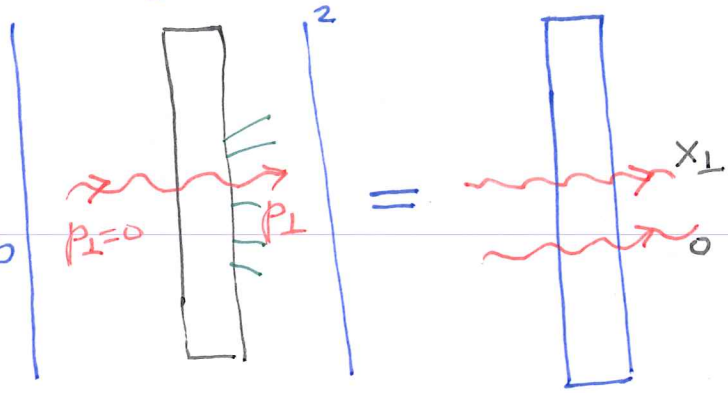


Here, in a partonic gauge, $p_\perp$ comes from absorbing a gluon from the saturated nuclear wavefunction. $f_{Q_s} \sim 1/\alpha$ allows that absorption to occur with high probability.

When little or no small-x evolution is present this is the MV picture

$$Q_s^2 = \frac{4\pi^2 \alpha_s N_c}{N_c^2 - 1} \underbrace{L \overset{\text{nuclear density}}{\rho} x G_p}_{\frac{dN_g}{d^2 b}} \Rightarrow \boxed{f_{Q_s} \simeq \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN_g}{Q_s^2 d^2 b} = \frac{\pi}{\alpha N_c}}$$

3. Equivalence of $p_\perp$-broadening and dipole scattering

$\sum_{\text{states}}$ 

$$= \int \frac{d^2 x_\perp}{4\pi^2} e^{-i p_\perp \cdot x_\perp} S(x_\perp)$$

S-matrix for dipole-nucleus scattering

$$\boxed{\frac{dN}{d^2 p_\perp} = \int \frac{d^2 x_\perp}{4\pi^2} e^{-i p_\perp \cdot x_\perp} S(x_\perp)}$$

Dipole scattering occurs in other processes

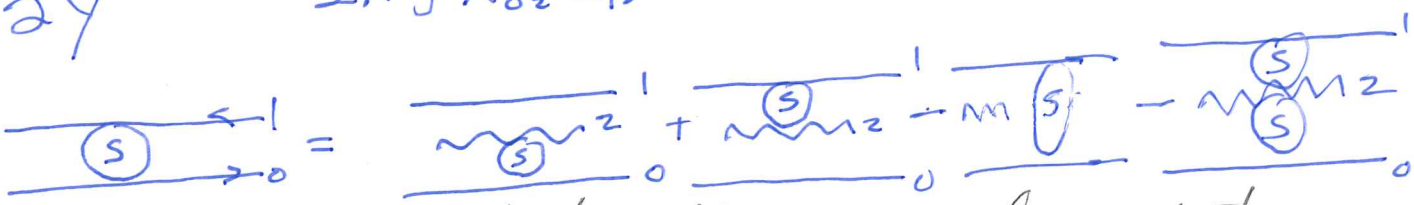
DIS (especially F_L)

Diffractive vector meson production — Munier, Stasto..

Diffractive dijet production

S obeys BK equation

$$\frac{\partial S(Y, x_{01})}{\partial Y} = \frac{\bar{\alpha}}{2\pi} \int \frac{d^2 x_2 \, x_{01}^2}{x_{02}^2 \, x_{12}^2} \left[S(Y, x_{02}) + S(Y, x_{12}) - S(Y, x_{01}) - S(Y, x_{02}) S(Y, x_{12}) \right]$$

$\frac{\partial}{\partial Y}$ 

For proton mean field equation. For nucleus exact.

Higher order kernel is known — Balitsky, Chirilli; Fadin, Lipatov

4. Saturation as initial condition for heavy ion collisions

4.1 General discussion, say for central unity/rapidity

BMSS

$$\frac{dN}{dy} = c(Q_s^2 R^2)\cdot\frac{1}{\alpha_s}$$ is initial density of gluons produced, mostly with $k_\perp \sim Q_s$

(liberation coefficient)

Small α_s but high occupancy allows high rate of collisions.

$$\tau_{coll} \sim \frac{1}{Q_s}(Q_s\tau)^{2/3}$$ at moderately early times

At early times either kinetic theory or classical field theory applies

Entropy: Initially $S \simeq (Q_s^2R^2)\ln 1/\alpha$

At equilibrium $S \simeq (Q_s^2R^2)\,1/\alpha^{7/5}$

$\frac{\Delta S}{Q_s^2R^2} \sim 1/\alpha$ comes from decoherence of saturated state

Factor of $1/\alpha^{2/5}$ comes from additional particle production

4.2 Numerical simulations

(a) Kurkela and Zhu use kinetic theory with

$$C_{coll} = C_{2\to 2} + C_{2\to 3}$$

Follow expanding system from $Q_s\tau \simeq 1$ to equilibration

Find rapid thermalization for moderate values of α.

Hydrodynamics works well before thermalization, maybe at $\tau \simeq 1$ fm

For very weak coupling the bottom-up picture seems good.

(b) Berges, Boguslavski, Schlichting, Venugopalan

Use classical field theory and very small coupling

Find early time scaling ($Q_s\tau \lesssim 1/\alpha^{3/2}$)

$$f(p_z, p_\perp, \tau) = (Q_s\tau)^{-2/3} f_s((Q_s\tau)^{1/3} p_z, p_\perp)$$

Also seen, roughly, in (a).

5. Measuring $\hat{q}$ (for hot matter) in heavy ion collisions

$\hat{q}$ is the fundamental parameter of a gauge theory medium. (Equivalent to "transport cross-section" in Landau and Lifshitz.) $\hat{q}$ describes small angle quark or gluon scattering with the constituents of the medium.

In cold matter $\hat{q} \simeq \frac{1}{25}$ GeV^2/fm from μ-pair broadening at fixed target energies. E772 E866 Kopeliovich et al

For hot matter the JET collaboration, from fits to energy loss finds

$\hat{q} \simeq 1$ GeV^2/fm at RHIC at $\tau_0 = 0.6$ fm

$\hat{q} \simeq 2$ GeV^2/fm at LHC at $\tau_0 = 0.6$ fm

Determining $\hat{q}$ from $p_\perp$-broadening of jets would be more direct. However, Sudakov effects dominate $\hat{q}$ effects in the 100 GeV jet regime. For jets of 30 GeV, say, $\hat{q}$ becomes important. SPHENIX? Xiao, Yuan ...

6 Nuclei vs Protons

Nuclei have many advantages

(i) Get enhancement due to higher parton densities

(ii) Proton-Proton interactions dominated by edges and likely fluctuations are important. Maybe good triggers can help

(iii) BK equation not so reliable for proton, only a mean field equation.