1. The ideas

(i) A high occupancy state of gluons in a high momentum nucleus.
\[ f_{k_1} \approx \frac{1}{x_s} \text{ for } k_1 \leq Q_s \]

(ii) \( S(x_\perp) = 1 - T(x_\perp) \approx 0 \) for scattering a dipole of size \( x_\perp \) on the high momentum nucleus.
\[ S(x_\perp) = 0 \text{ for } x_\perp > \frac{1}{Q_s} \]

\[ Q_s = Q_s(y) \; ; \; y = \frac{m^2}{Q_s^2} \; ; \; y \geq \frac{2m}{x_{Bj}} \]
2. Saturation and \( \hat{q} \) of cold nuclear matter

\[ \langle p_1^2 \rangle = \hat{q} L = Q_s^2 \]

At rest

\[ p_1 = 0 \]

big square nucleus at rest

At high momentum

Here, in a partonic gauge, \( p_1 \) comes from absorbing a gluon from the saturated nuclear wavefunction, \( f_{Q_s} \sim 1/x \) allows that absorption to occur with high probability.

When little or no small-\( x \) evolution is present this is the MV picture

\[
Q_s^2 = \frac{4\pi a_s N_c}{N_c^2 - 1} \left( \int \mathcal{P} \times G_0 \right) \Rightarrow f_{Q_s} \approx \frac{2\pi^3}{2(N_c^2 - 1)} \frac{dN_g}{Q_s^2 d^2 b} = \frac{2\pi}{\alpha N_c} \]

\( dN_g \)

\( d^2 b \)
3. Equivalence of $p^2$-broadening and dipole scattering

\[
\sum_{\text{states}} |p^2| = \int |x_1| = \int \frac{d^2x_1}{4\pi^2} \cdot S(x_1)
\]

S-matrix for dipole-nucleus scattering

\[
\frac{dN}{d^2p} = \int \frac{d^2x_1}{4\pi^2} \cdot S(x_1)
\]

Dipole scattering occurs in other processes:
- DIS (especially $F_2$)
- Diffractive vector meson production
- Diffractive dijet production

Sachs BK equation

\[
\frac{\partial S(y, x_{01})}{\partial y} = \frac{1}{2\pi} \int \frac{d^2x_2}{x_{02}} \left[ S(y, x_{02}) + S(y, x_{12}) - S(x_{02}) - S(x_{12}) \right]
\]

For proton mean field equation. For nucleus exact:

Balitsky, Chirilli

Higher order kernel is known: Fedin, Lipatov
4. Saturation as initial condition for heavy ion collisions

4.1 General discussion, say for central unit rapidity

\[ \frac{dN}{dy} = C(Q_s^2 R^2) \cdot \frac{1}{\alpha_s} \quad \text{is initial density of gluons} \]

produced, mostly with \( k_t \approx Q_s \)

Small \( \alpha_s \) but high occupancy allows high rate of collisions.

\[ T_{\text{coll}} \sim \frac{1}{Q_s} (Q_s R)^{2/3} \quad \text{at moderately early times} \]

At early times either kinetic theory or classical field theory applies.

\[ S \approx (Q_s^2 R^2) \ln \frac{1}{\alpha_s} \quad \text{Initially} \]

\[ S \approx (Q_s^2 R^2)^{1/7.5} \quad \text{At equilibrium} \]

\[ \frac{\Delta S}{Q_s^2 R^2} \sim \frac{1}{\alpha_s} \text{comes from decoherence of saturated state} \]

Factor \( 1/\alpha_s \) comes from additional particle production.
4.2 Numerical Simulations

(a) Kurkela and Hu, cneutronic theory with

\[ C_{\text{cut}} = C_{2\rightarrow 2} + C_{2\rightarrow 3} \]

Follow expanding system from \( \mathcal{E} \approx 1 \) to equilibration

Find rapid thermalization for moderate values of \( \mathcal{E} \)

Hydrodynamics works well before thermalization, maybe \( \mathcal{E} \approx 1 \mathrm{fm} \)

For very weak coupling, the bottom-up picture seems good.

(b) Berge, Boguslawski, Schlachting, Venugopalan

Use classical field theory and very small coupling

Find early time scaling (\( \mathcal{E} \approx 1/\mathcal{E}_3^{3/2} \))

\[ f(P_3, P_1, P_2) = (Q_{\mathcal{E}})^{-2/3} \frac{15}{15} f_s(Q_{\mathcal{E}}, P_3, P_2) \]

Also seen, roughly, in (a).
5. Measuring $\hat{g}$ (for hot matter) in heavy ion collisions

$\hat{g}$ is the fundamental parameter of a

gauge theory medium. (Equivalent to

"transport cross-section" in Landau and Lifshitz.)

$\hat{g}$ describes small angle quark or gluon

scattering with the constituents of the medium.

In cold matter $\hat{g} \approx \frac{1}{25} \text{GeV}^2/\text{fm}$ from

$\mu$-pair broadening at fixed target energies.

For hot matter the JET Collaboration, from jets

to energy loss finds

$\hat{g} \approx 1 \text{GeV}^2/\text{fm}$ at RHIC at $\tau_0 = 0.6 \text{fm}$

$\hat{g} \approx 2 \text{GeV}^2/\text{fm}$ at LHC at $\tau_0 = 0.6 \text{fm}$

Determining $\hat{g}$ from $p_t$-broadening of

jets would be more direct. However, Sudakov

effects dominate $\hat{g}$ effects in the 100 GeV jet

regime. For jets of 30 GeV, say, $\hat{g}$ becomes

important? SPHENIX?

Xiao Yuan...
Nuclei as Protons

Nuclei have many advantages

(i) Get enhancement due to higher proton densities

(ii) Proton-Proton interactions dominated by edges and likely fluctuations are important. Maybe good triggers can help

(iii) BK equation not so reliable for proton, only a mean field equation.