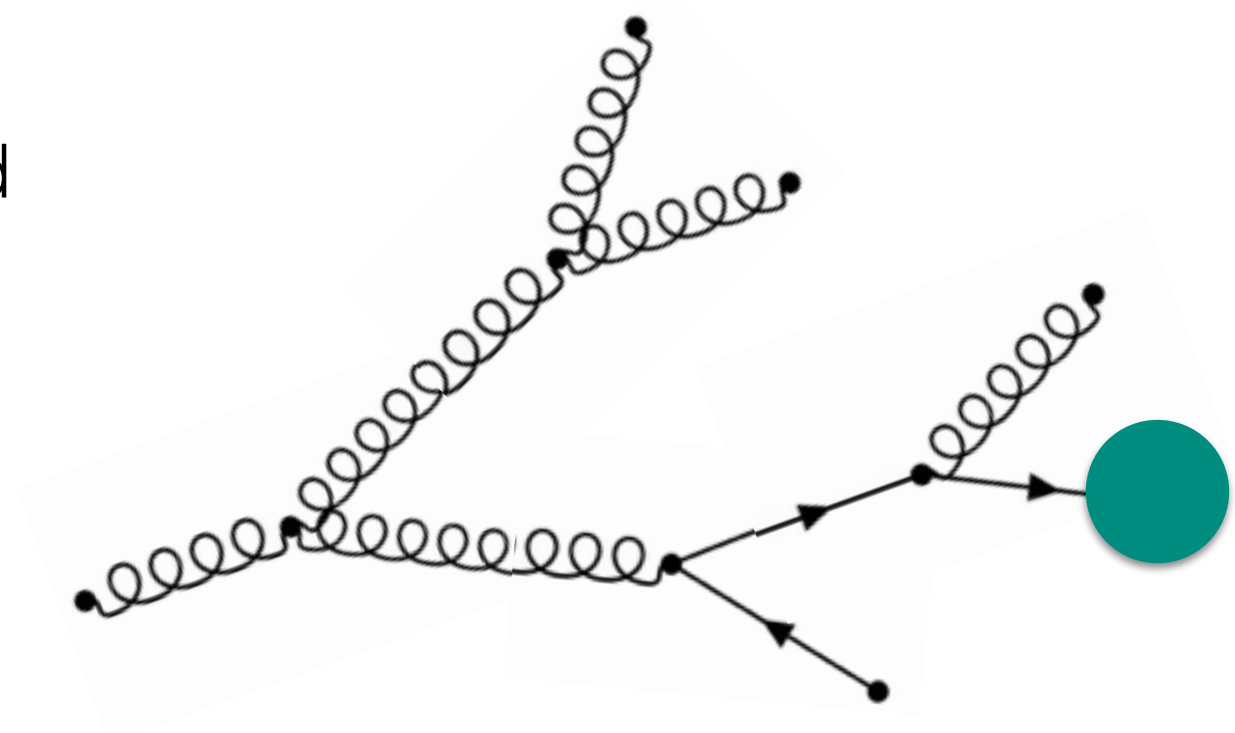
# Early time dynamics and hard probes in heavy-ion collisions (theory)

Sören Schlichting | Universität Bielefeld



Hard Probes 2018:
International Conference on
Hard and Electromagnetic Probes
of High-Energy Nuclear Collisions

Aix-les-Bains, Oct 2018

CRC-TR 211
Strong-interaction matter
under extreme conditions

Universität Bielefeld

# Outline

Early time dynamics & equilibration process

— microscopic dynamics & connections to jet physics

Description of early-time dynamics by macroscopic d.o.f.

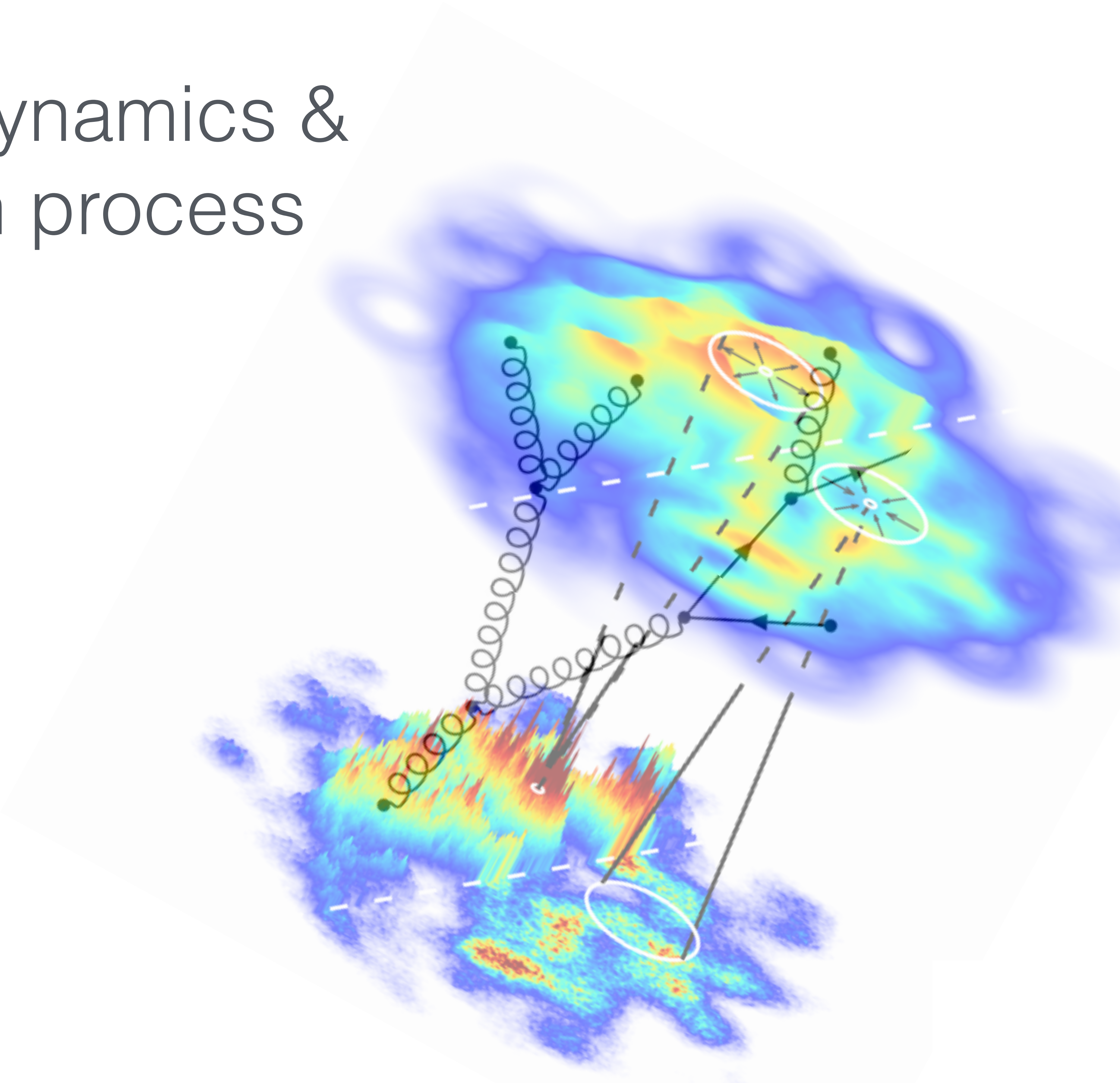
— energy momentum tensor & non-eq. response function

Effects of the pre-equilibrium phase in small & large systems

— entropy production; soft & hard probes in small systems

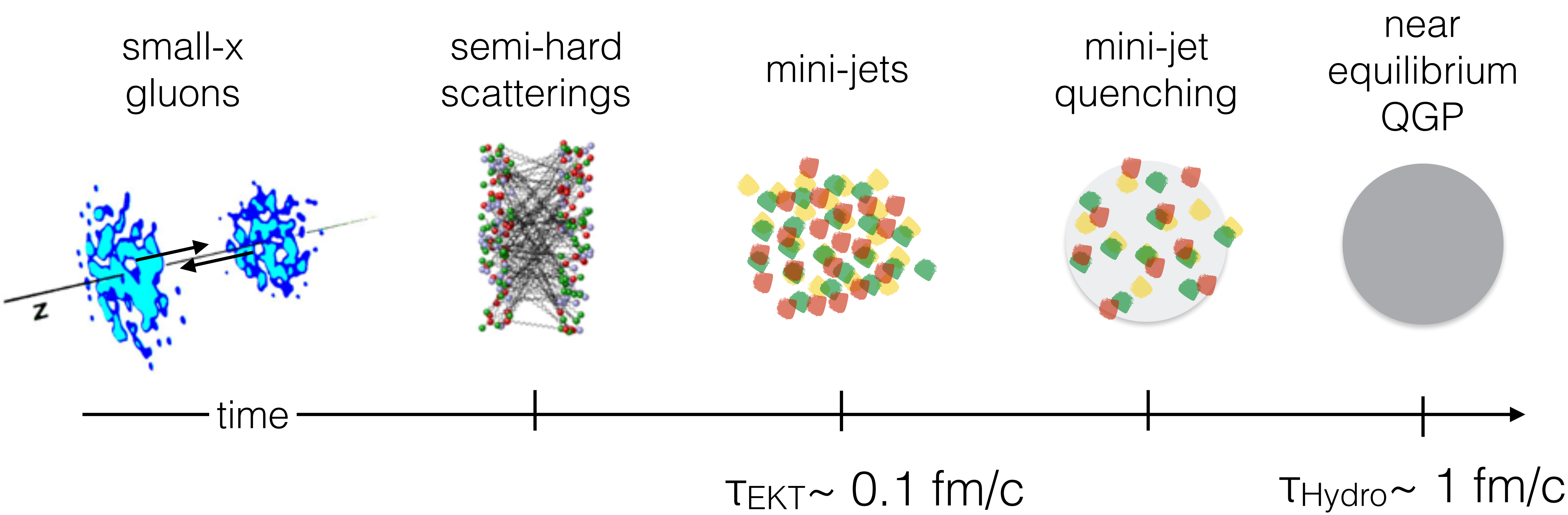
Conclusions & Outlook

# Early time dynamics & equilibration process



# Early time dynamics & equilibration process

Canonical picture at weak coupling:




Starting with the collision of heavy-ions a sequence of processes eventually leads to the formation of an equilibrated QGP

# Effective kinetic description

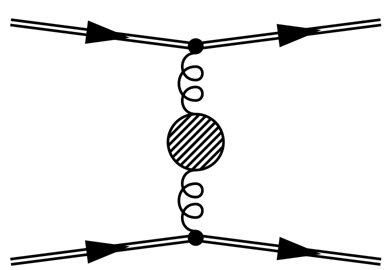
Beyond (very) early times, kinetic interactions dominate and subsequent equilibration process can be efficiently described in kinetic theory

Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) no.7, 074011

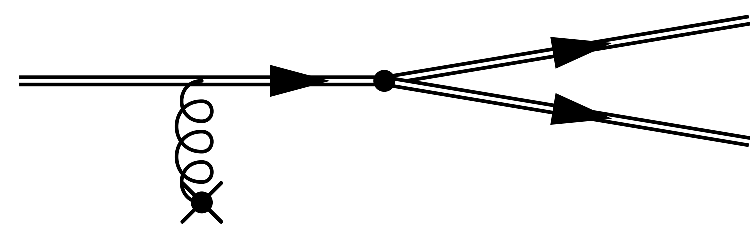
Effective kinetic theory of QCD at leading order weak coupling

Arnold,Moore,Yaffe JHEP 0301 (2003) 030

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$



elast. 2<->2 scattering
screened by Debye mass



collinear 1<->2 Bremsstrahlung
incl. LPM efffect
via eff. vertex re-summation

in-medium matrix elements for 2<->2 and 1<->2 processes, self-consistently determined assuming isotropic screening

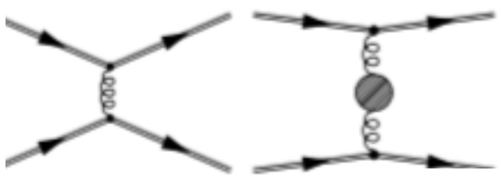
( c.f. talk by S. Hauksson Tue 15:20 for possible improvements),

see Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171 for details on numerics

# Effective kinetic description

Basic theoretical framework for equilibration studies is the same as in weakly coupled jet-energy loss calculations (implemented e.g. in MARTINI, JEWEL, …)

screened elast. scattering



collinear Bremsstrahlung

Differences between pre-equilibrium & parton/jet energy loss calculations

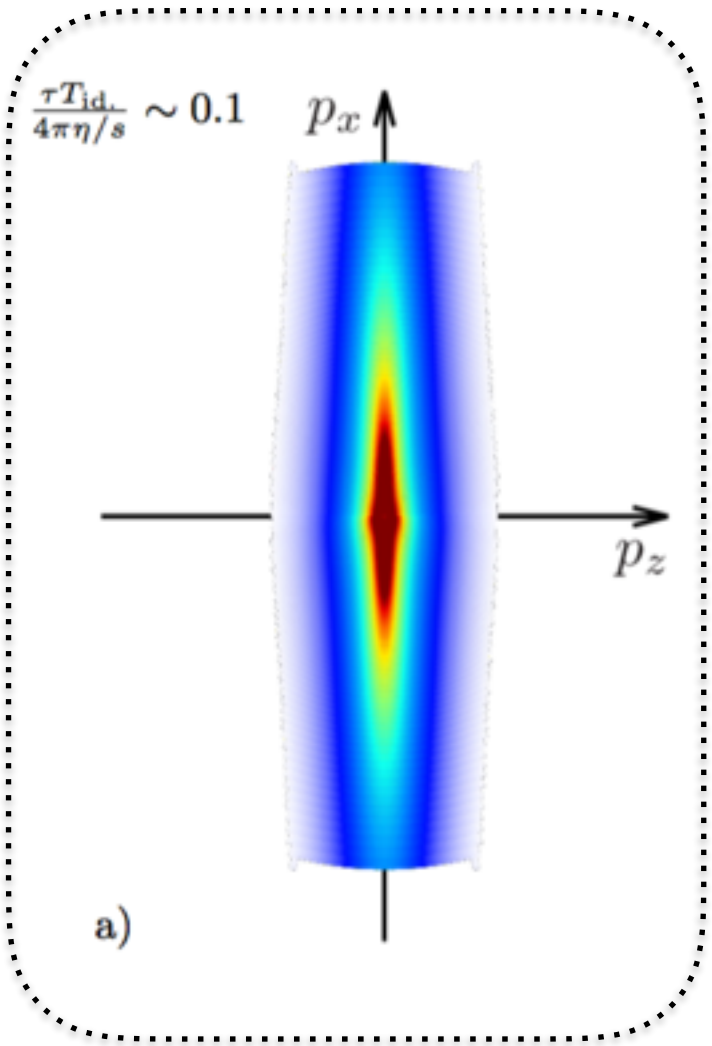
- No hierarchy of scales as in jet physics $E_{Jet} >> T$

+ soft & (semi-)hard degrees of freedom all treated within same kinetic framework

+ no "background" medium -> non-linear treatment of interactions between mini-jets

- Description of pre-equilibrium dynamics requires kinetic description down to $p_T \sim T$

# Equilibration process at weak coupling



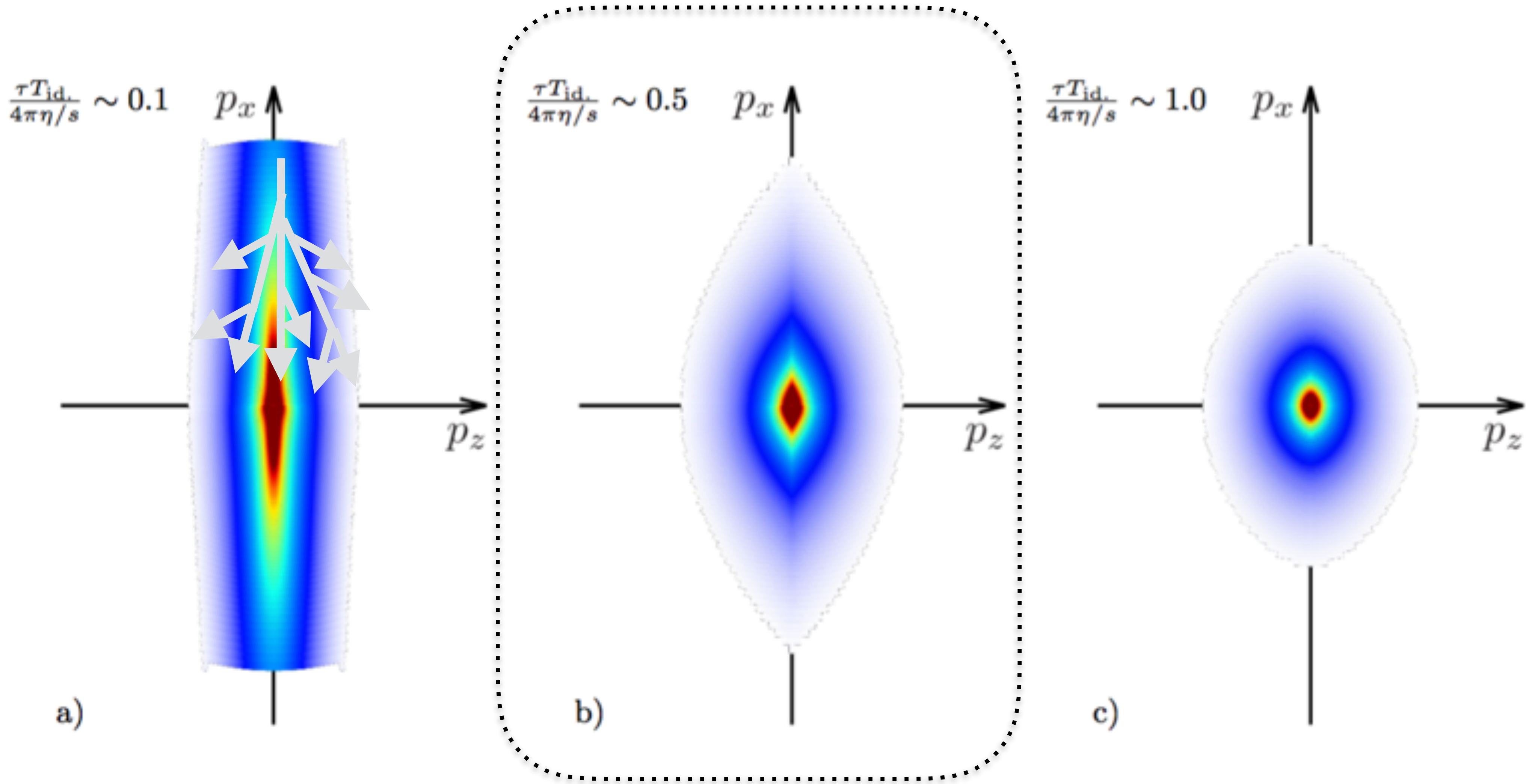

Semi-hard gluons produced around mid-rapidity have $p_T >> p_z$ in l.r.f.
-> initial phase-space distribution is highly anisotropic

Early on the non-equilibrium plasma subject to rapid long. expansion
-> continuous increase of anisotropy & depletion of phase space density

Basic microscopic picture of equilibration process developed in "bottom-up" scenario along with first estimates of the relevant time scales Baier, Mueller, Schiff, Son PLB502 (2001) 51-58

# Equilibration process at weak coupling

Numerical solutions of QCD kinetic theory confirm “bottom-up”picture

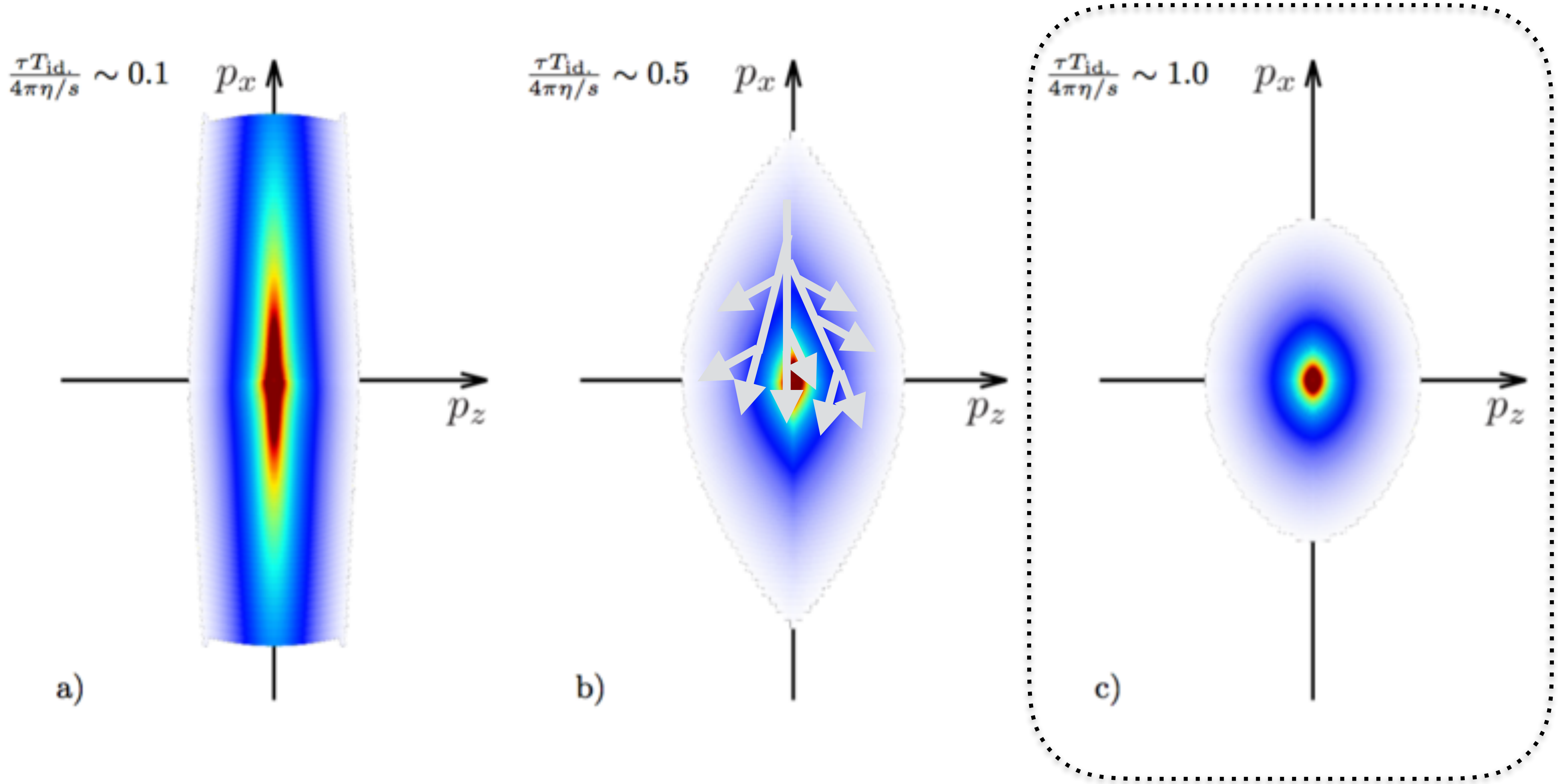



Mini-jets undergo a radiative break-up cascade
eventually leading to formation of soft thermal bath

c.f. Kurkela, Zhu PRL 115 (2015) 182301; Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171;
Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# Equilibration process at weak coupling

Quenching of left-over mini-jets in soft thermal bath transfers energy to soft sector leading to isotropization of plasma

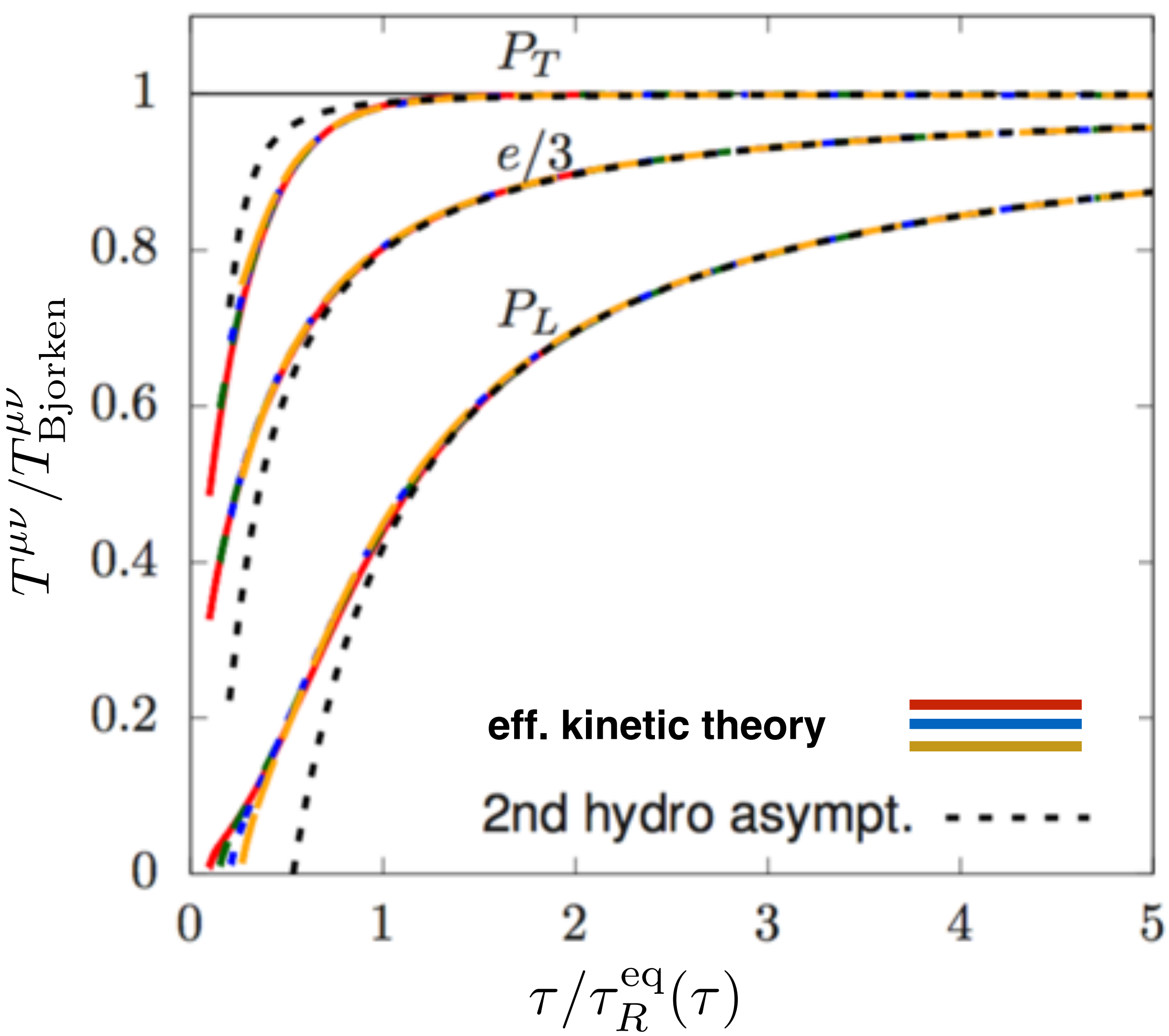



Equilibration process completed once mini-jets (~$Q_S$) are fully quenched

c.f. Kurkela, Zhu PRL 115 (2015) 182301; Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171; Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# Hydrodynamic behavior

Since the system is highly anisotropic initially $P_L << P_T$, one key question is to understand evolution of $T^{\mu\nu}$ towards local equilibrium ($P_L = P_T$)




Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney
arXiv:1805.01604; arXiv:1805.00961

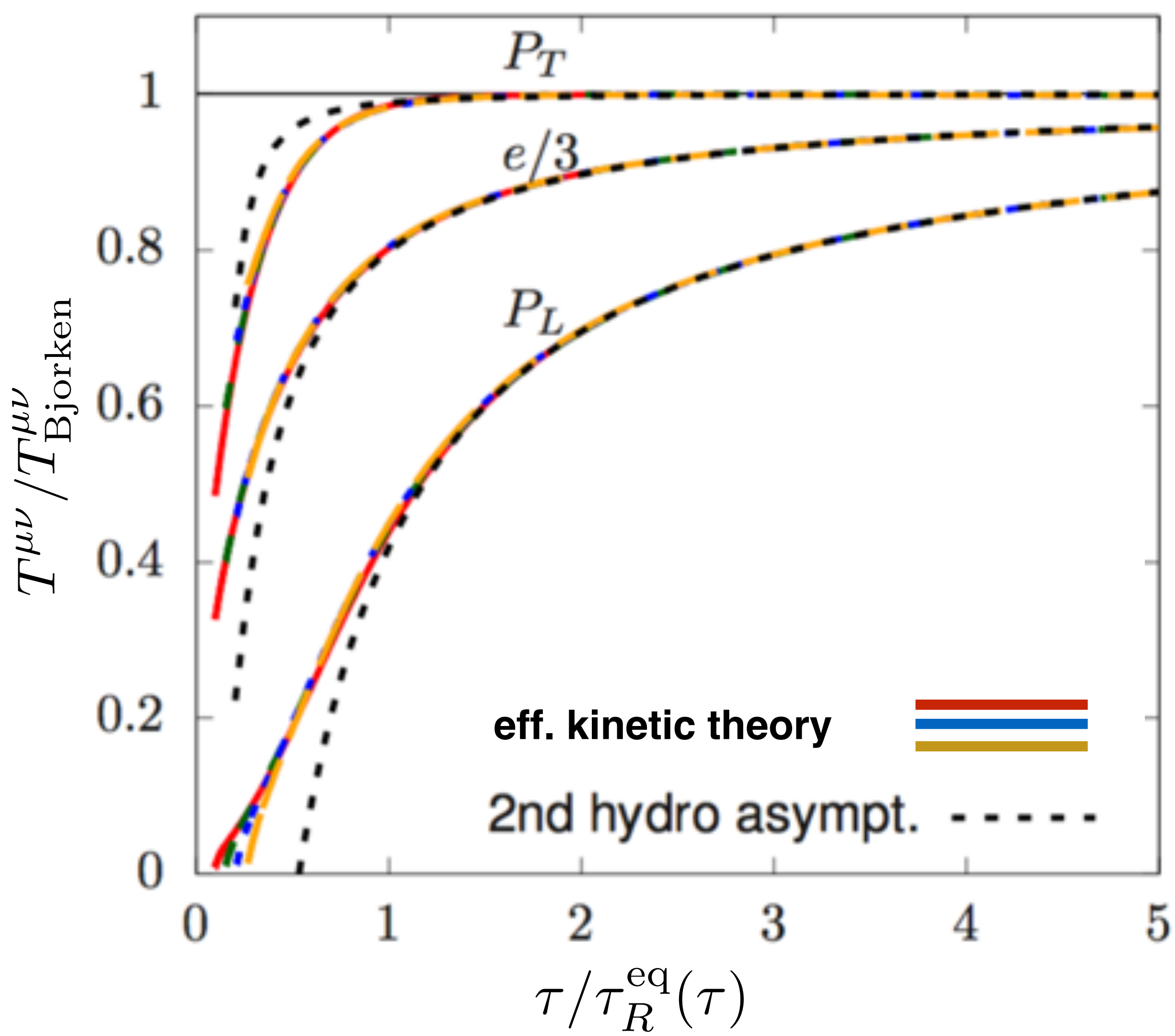
Evolution of the energy momentum tensor determined by universal scaling curve, interpolating from approx. free-streaming at early times to visc. hydrodynamics at late times.

Hydrodynamization time controlled by equilibrium relaxation rate

$$\tau_R^{\mathrm{eq}}(\tau) = \frac{4\pi\eta/s}{T_{\mathrm{eff}}(\tau)}$$

# Hydrodynamic behavior

Extrapolations of weak-coupling dynamics to realistic coupling strength yields realistic estimates in line with phenomenology




Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney
arXiv:1805.01604; arXiv:1805.00961

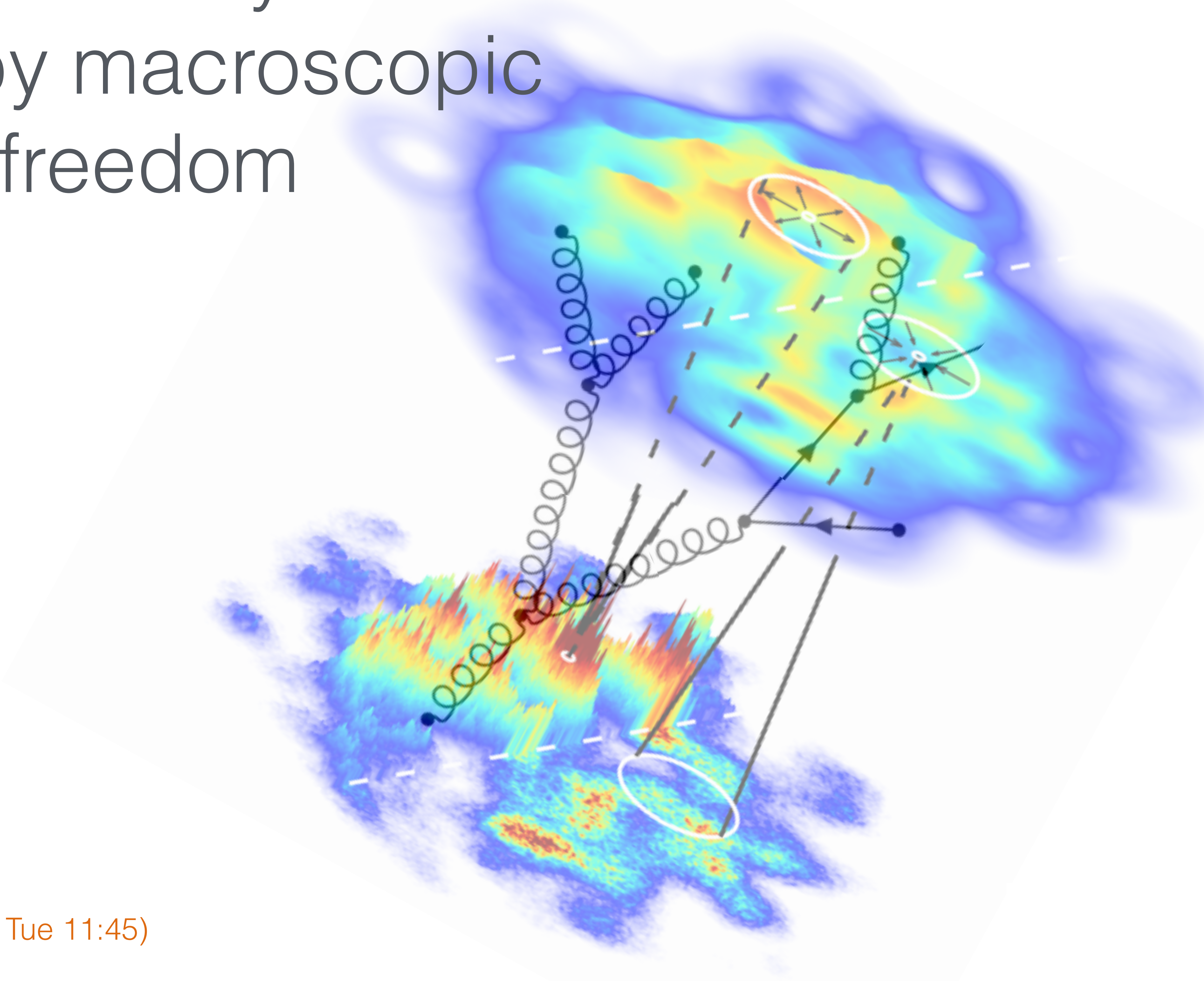
$$\tau_{\rm hydro} \approx 1.1\,{\rm fm}\left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}}\left(\frac{\langle \tau s\rangle}{4.1\,{\rm GeV}^2}\right)^{-1/2}$$

Viscous hydrodynamics becomes applicable when pressure anisotropies are still O(1) and microscopic physics is still somewhat jet-like

Similar findings also at strong coupling and in the context of non-equilibrium attractors

Heller,Janik,Witaszczyk PRL 108 (2012) 201602
Romatschke PRL 120 (2018) no.1, 012301
…

# Description of early-time dynamics by macroscopic degrees of freedom



(c.f. talk by A.Mazeliauskas, Tue 11:45)

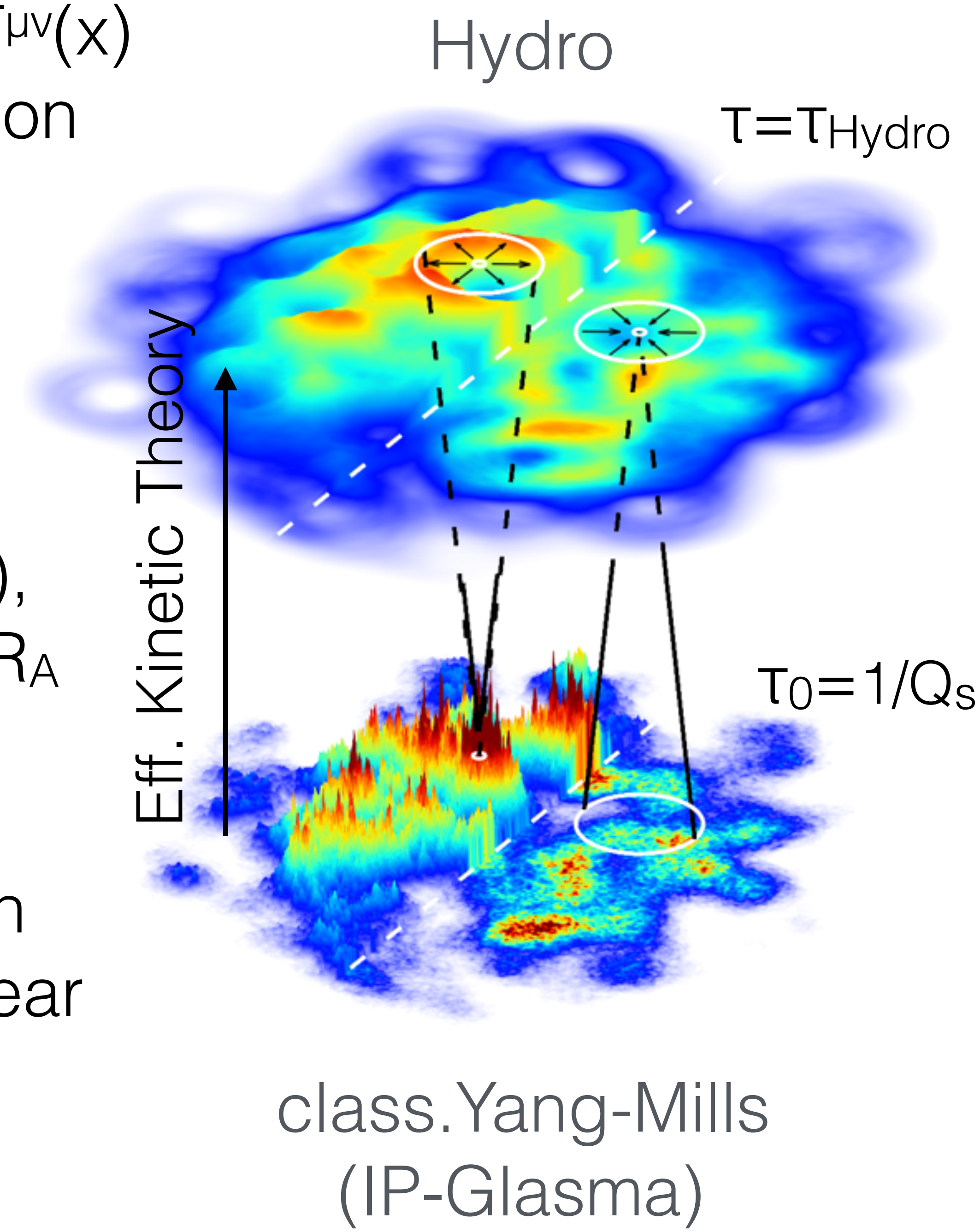
# Macroscopic pre-equilibrium evolution

Starting with $T^{\mu\nu}(x)$ from initial state model (e.g. IP-Glasma) at initial time $\tau_0 \sim 1/Q_s$, propagate $T^{\mu\nu}(x)$ up to $\tau_{Hydro}$ using eff. kinetic theory description

Separation of scales $\tau_{Hydro}-\tau_0 << R_A$:

Causality restricts contributions to $T^{\mu\nu}(x)$ to be localized in causal disc $|x-x_0|< c(\tau_{Hydro}-\tau_0)$, whereas relevant gradients of $T^{\mu\nu}(x)$ are $\sim 1/R_A$

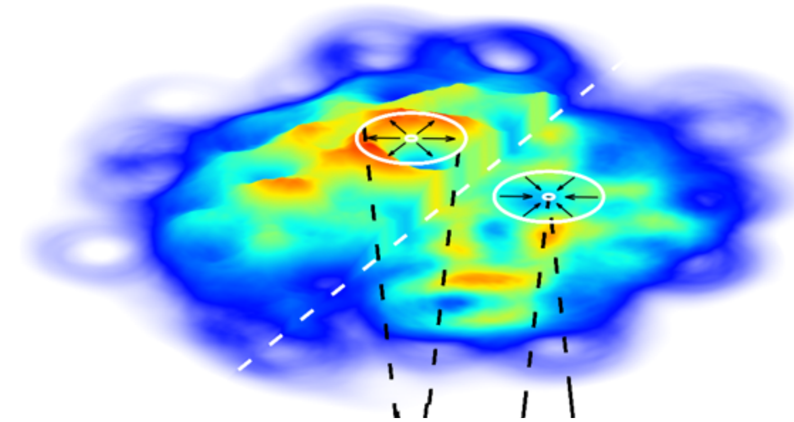
Decomposing $T^{\mu\nu}(x)$ into a local average $T^{\mu\nu}_{BG}(x)$ and fluctuations $\delta T^{\mu\nu}(x)$, are small on scales $c(\tau_{Hydro}-\tau_0)$, they can be treated in linear response theory




Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# Non-equilibrium linear response

Energy-momentum tensor at $\tau_{Hydro}$ can be reconstructed directly from initial conditions

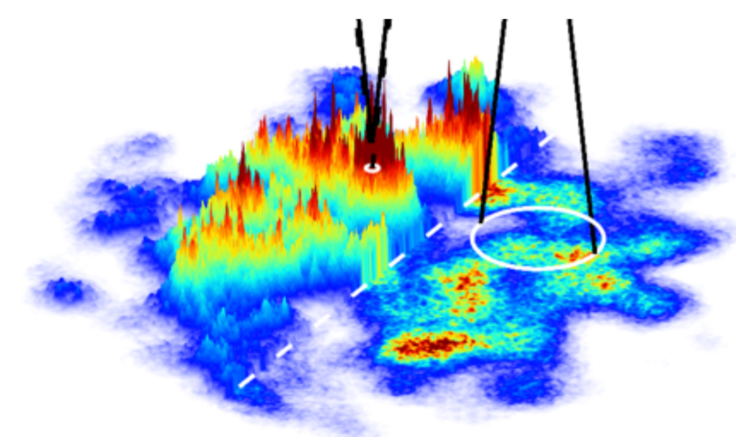


$$T^{\mu\nu}(\tau,x) = T^{\mu\nu}_{BG}(\tau) + \int_{\odot} G^{\mu\nu}_{\alpha\beta}(\tau,\tau_0,x,x_0)\delta T^{\alpha\beta}(\tau_0,x_0)$$

non-equilibrium evolution of (local) average background

non-equilibrium Greens function of energy-momentum tensor

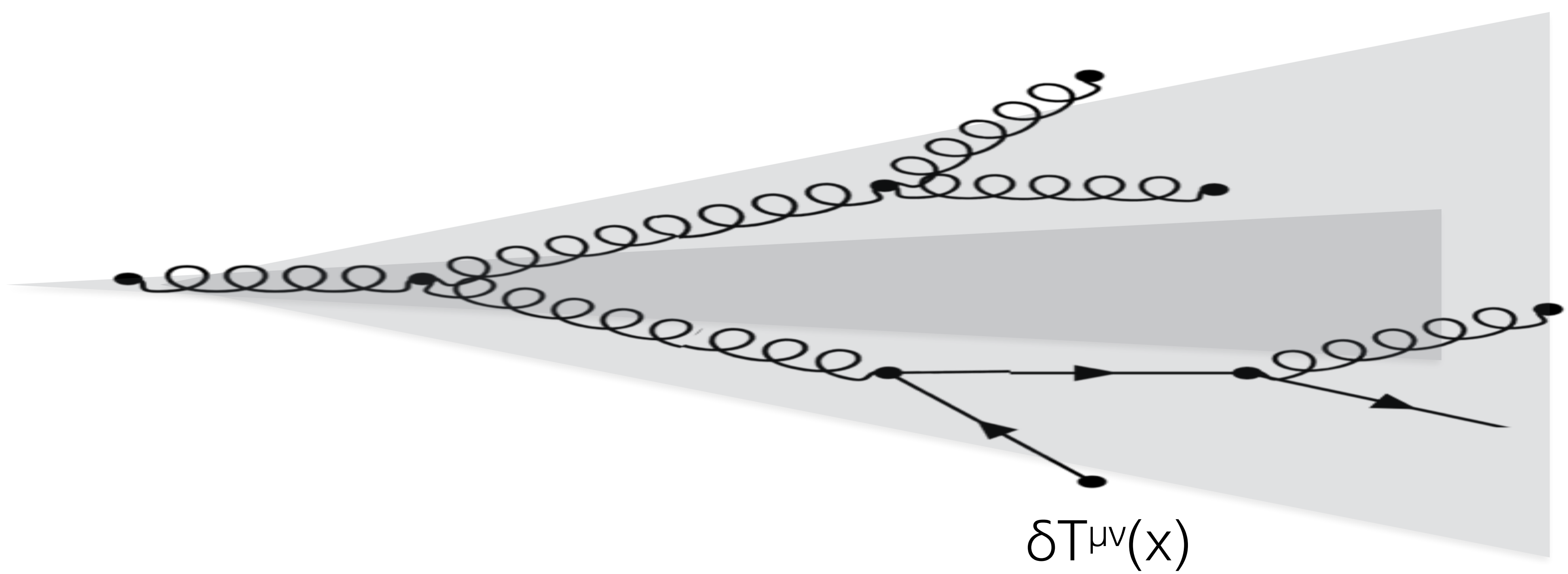
Instead of event-by-event Monte-Carlo, effective kinetic theory simulations performed only once to compute evolution of background $T^{\mu\nu}{}_{BG}$ and Greens functions $G^{\mu\nu}{}_{\alpha\beta}$



Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# Non-equilibrium linear response

Linear response formalism could be useful in coupled jet +hydro simulations (CoLBT,MARTINI,…) to describe energy-momentum deposition of jet's



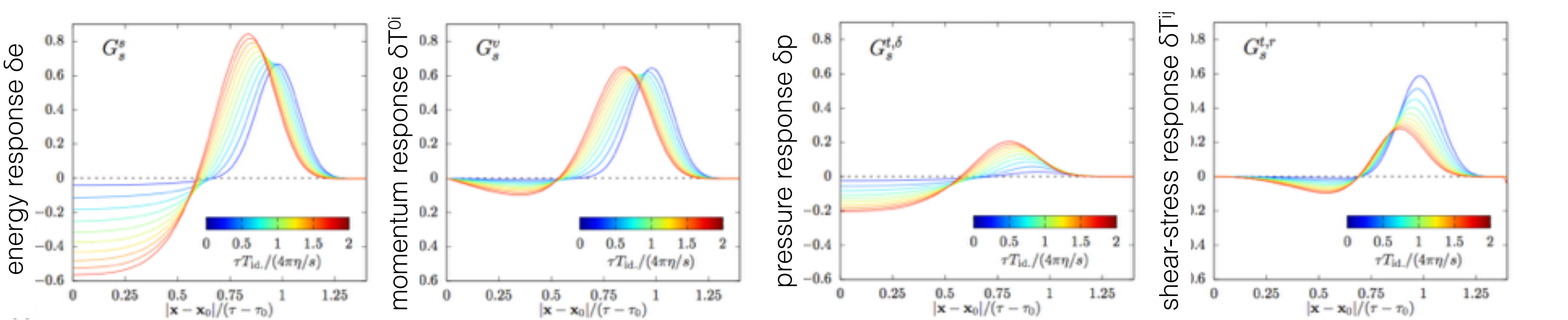

(c.f. talks by Y. He Thu 09:40; D. Pablos Thu 12:05; C. Park Thu 12:05; …)

# Non-equilibrium linear response

Non-equilibrium Green's functions calculated from linearized Boltzmann equation, using representative form of $\delta f(x,p)$ to represent $\delta T^{\mu\nu}(x)$

Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

Energy-momentum response to initial energy perturbation

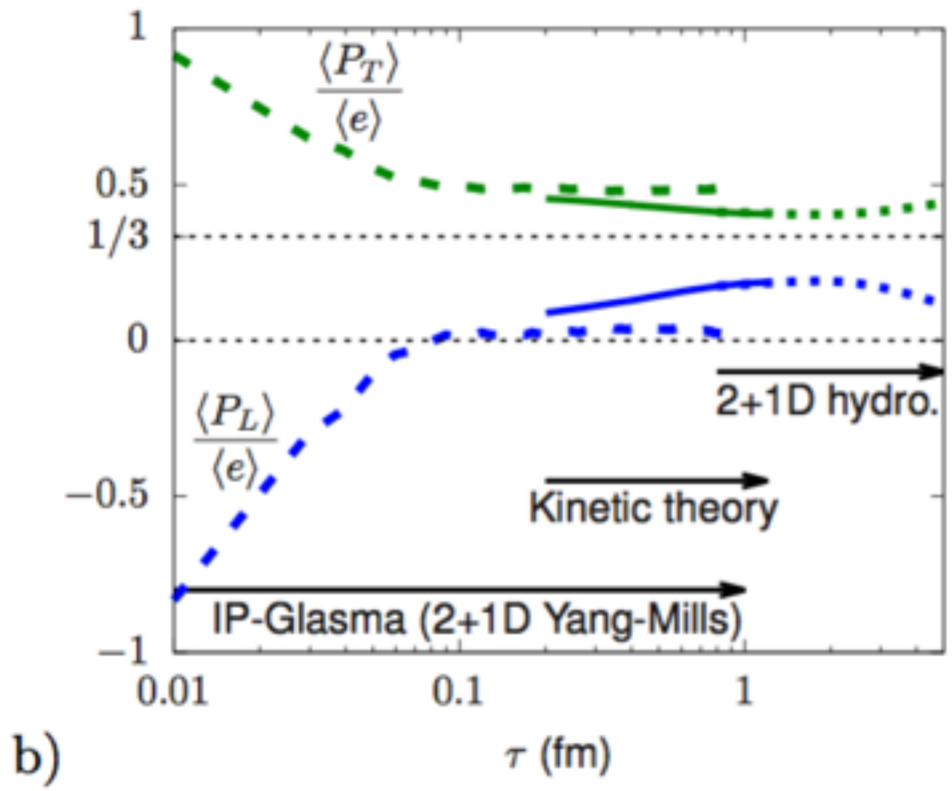


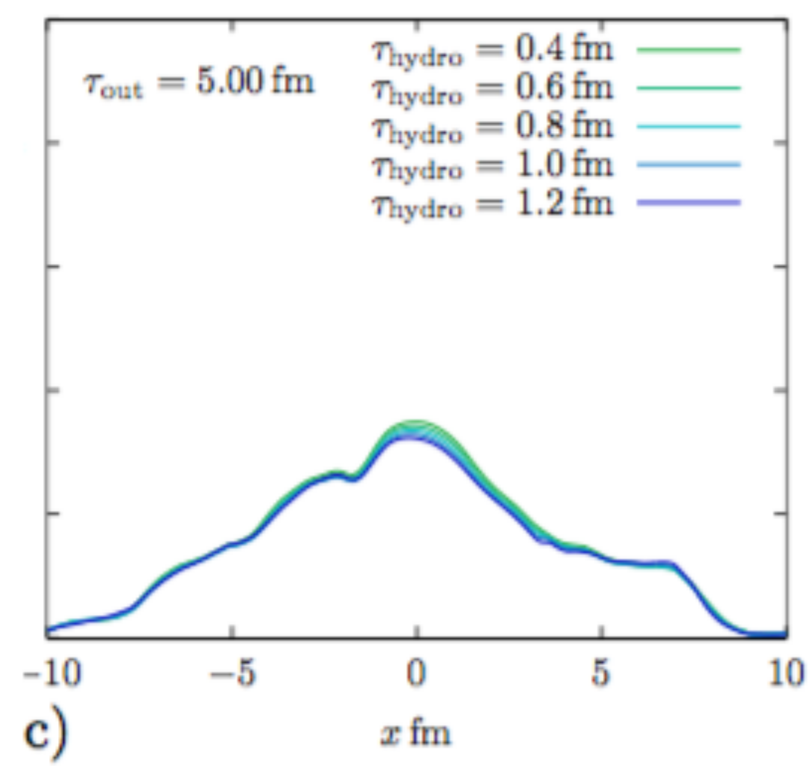

KoMPoST package* provides implementation for event-by-event description of pre-equilibrium dynamics

*Code publicly available at github.com/KMPST/KoMPoST

# Event-by-event pre-equilibrium evolution

Evolution of energy density in central Pb+Pb event



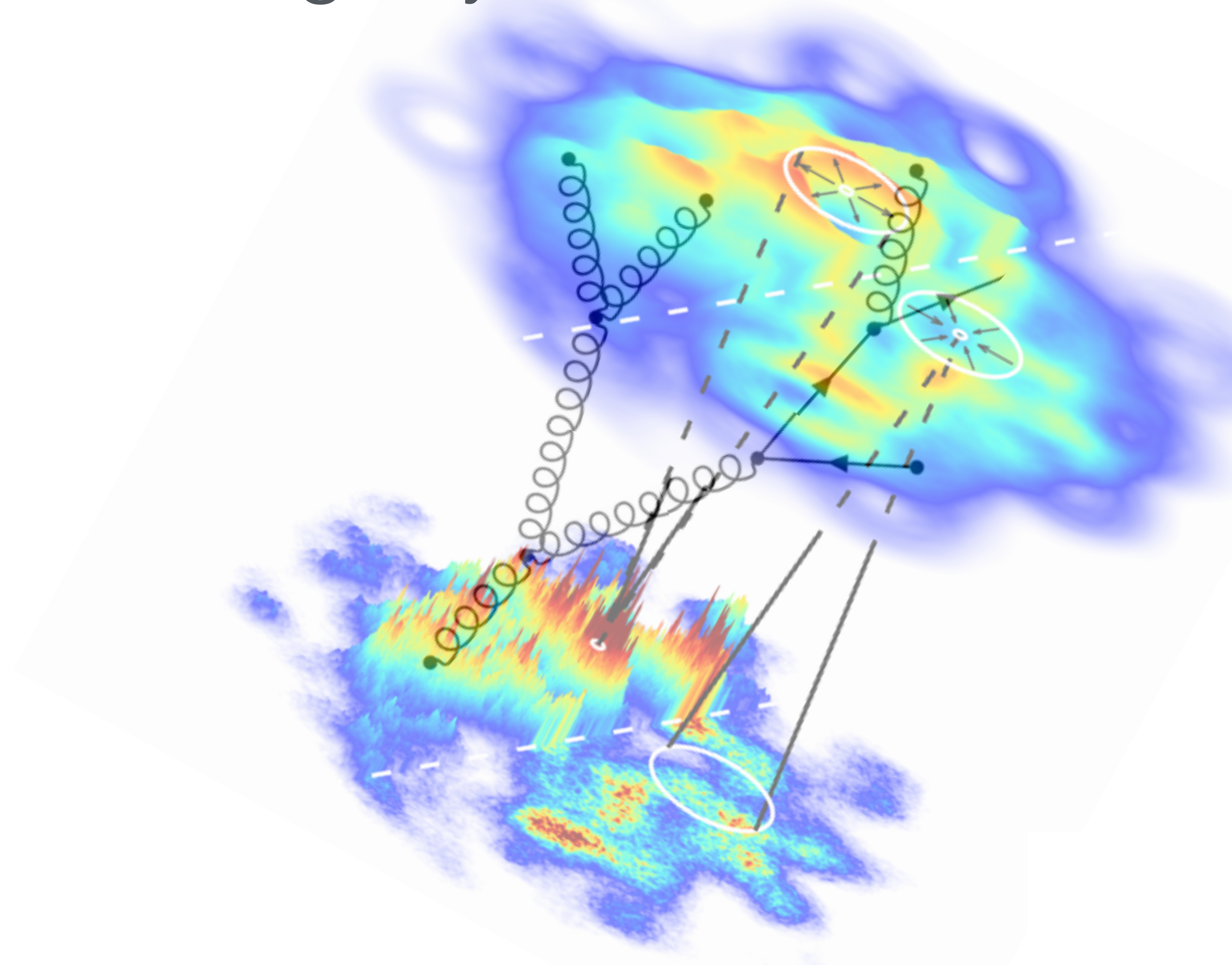

By combination of weak-coupling methods a complete description of early-time dynamics until the onset of hydrodynamics can now be achieved

class. Yang-Mills (IP-Glasma) → eff. kinetic theory (KoMPoST) → visc. hydrodynamics (Music)

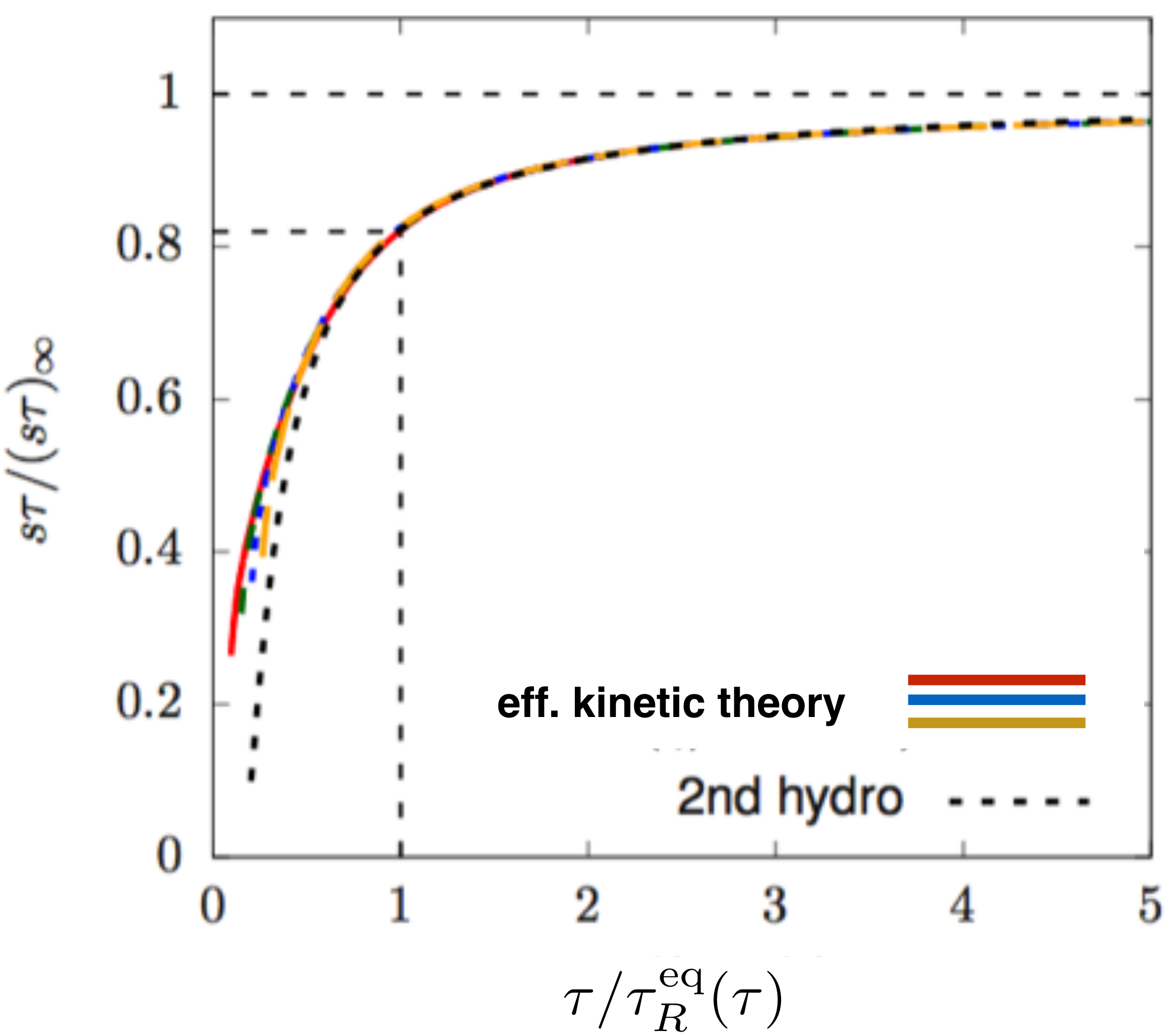
No sensitivity to matching time due to overlapping range of validity

Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# Effects of pre-equilibrium phase in small & large systems



# Entropy production




Significant amount of entropy production (x2-3) during the pre-equilibrium phase

Since there is an almost one to one correspondence between entropy and multiplicity

$$\langle s\tau \rangle_\infty \simeq \frac{S}{N_{\mathrm{ch}}} \frac{1}{\pi R^2} \frac{dN_{\mathrm{ch}}}{d\eta}$$

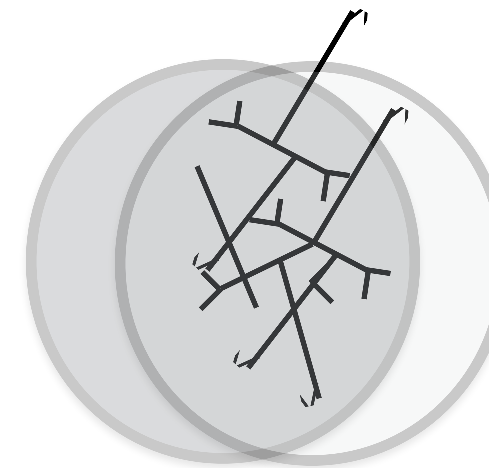
important to take into account when relating properties of initial state (e.g. $Q_s$) to experimental data

Calulcuations of dN/dy vs. $N_{part}$ should be revisited

Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

# QGP in small systems?

Importance of final state effects determined by
equilibration time ($\tau_{\mathrm{Hydro}}$), system size (R)




If $\tau_{\mathrm{Hydro}}$ >> R almost no final state interactions

If $\tau_{\mathrm{Hydro}}$ << R long lived hydrodynamic phase

Based on new estimates of the equilibration time, one obtains

$$\frac{\tau_{\mathrm{Hydro}}}{R} \simeq \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{dN_{\mathrm{ch}}/d\eta}{63}\right)^{-\frac{1}{2}} \left(\frac{S/N_{\mathrm{ch}}}{7}\right)$$

where $\left.\frac{dN_{ch}}{d\eta}\right|_{min.\ bias}^{p+p\ 7TeV} \sim 6$ , $\left.\frac{dN_{ch}}{d\eta}\right|_{min.\ bias}^{p+Pb\ 5.02TeV} \sim 16$ , $\left.\frac{dN_{ch}}{d\eta}\right|_{0-5\%}^{Pb+Pb\ 2.76TeV} \sim 1600$

Experimental results in small systems mostly fall into
the regime $\tau_{\mathrm{Hydro}}$/R~1 dominated by the pre-equilibrium phase

# Dynamics of small systems

Non-equilibrium description will be needed to describe dynamics of small systems across wide range of multiplicities

New popularity of transport models
& semi-analytic transport calculations

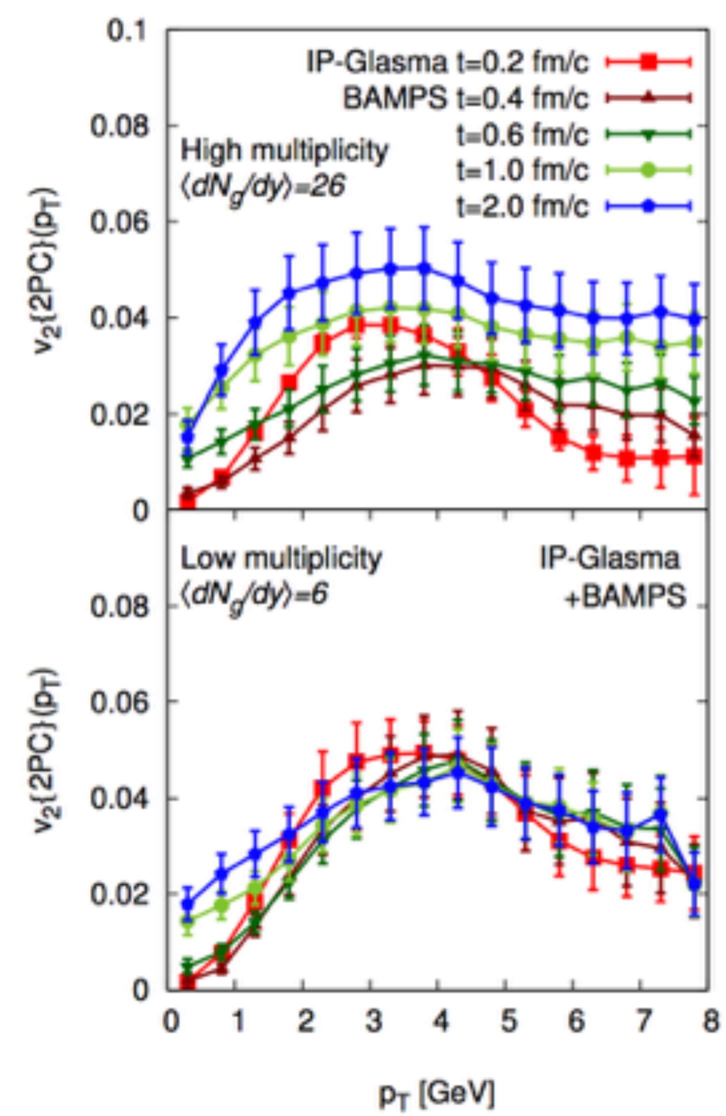
- many of the models are not QCD

Orjuela Koop, Adare, McGlinchey, Nagle PRC 92 (2015) no.5, 054903
Borghini, Gombeaud, EPJC 71 (2011) 1612
Kurkela,Wiedemann,Wu, PLB 783 (2018) 274-279
Römatschke EPJC78 (2018) no.8, 636
..

Event-by-event class. Yang-Mills (IP-Glasma)
+ pQCD transport (BAMPS)

Greif, Greiner, Schenke, SS, Xu, PRD96 (2017) no. 9, 091504

(c.f. talk by F. Senzel Tue 17:25 on possible improvements)

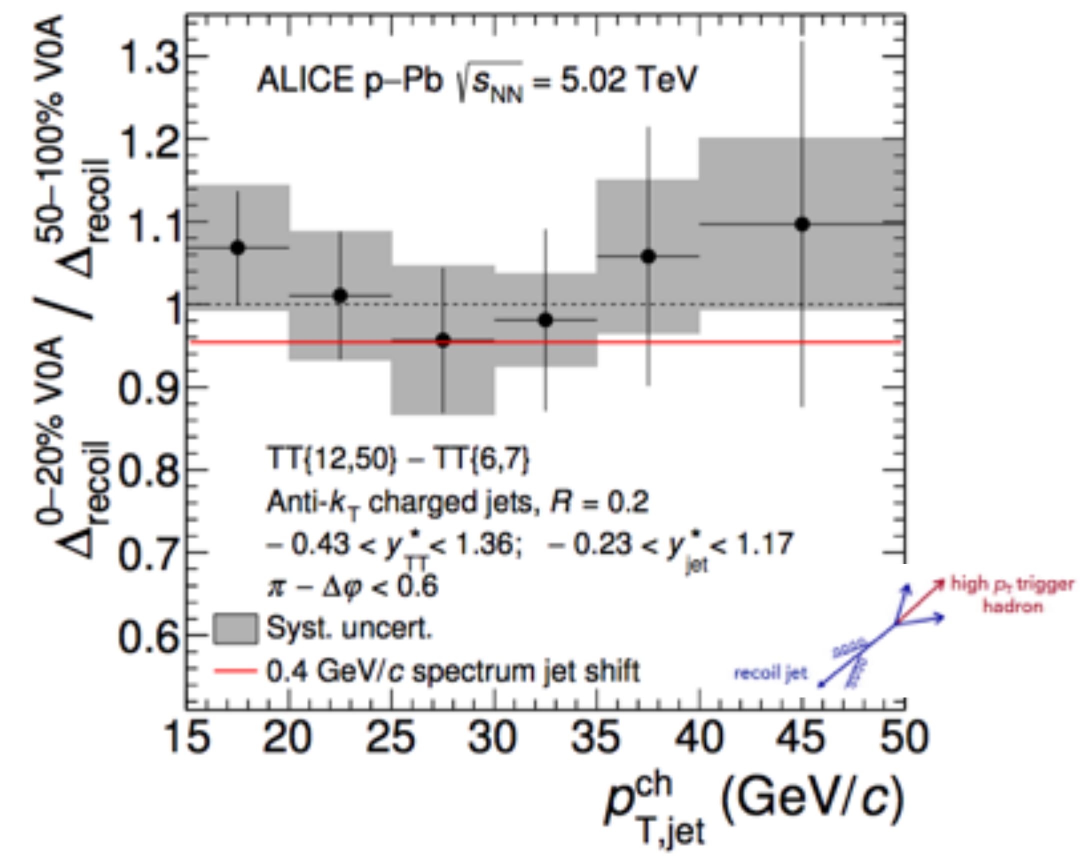


# Soft & Hard probes in small systems

Based on microscopic picture, one expects onset of thermalization and jet-quenching go hand in hand

Experiments starting to provide stringent constraints on hard probes in small systems

ALICE PLB 783 (2018) 95-113




<u>Key question:</u> How and to what extent can we constrain the bulk dynamics of small systems by hard probes?

INT Program: “Origins of Correlations in High Energy Collisions“, April 29 - May 24, 2019
http://www.int.washington.edu/PROGRAMS/19-1b/

# Conclusions & Outlook

Significant progress in understanding early time dynamics of heavy-ion collisions from weak-coupling perspective

-> similarities between equilibration and parton energy loss

Development of macroscopic description of pre-equilibrium dynamics which enables event-by-event description of heavy-ion collisions from beginning to end

-> could be interesting for jet-energy disposition into medium

So far focus of equilibration studies has been on typical d.o.f. semi-hard gluons; next up

Quark production & chemical equilibration (c.f. talk by S. Schlichting Thu 11:05)

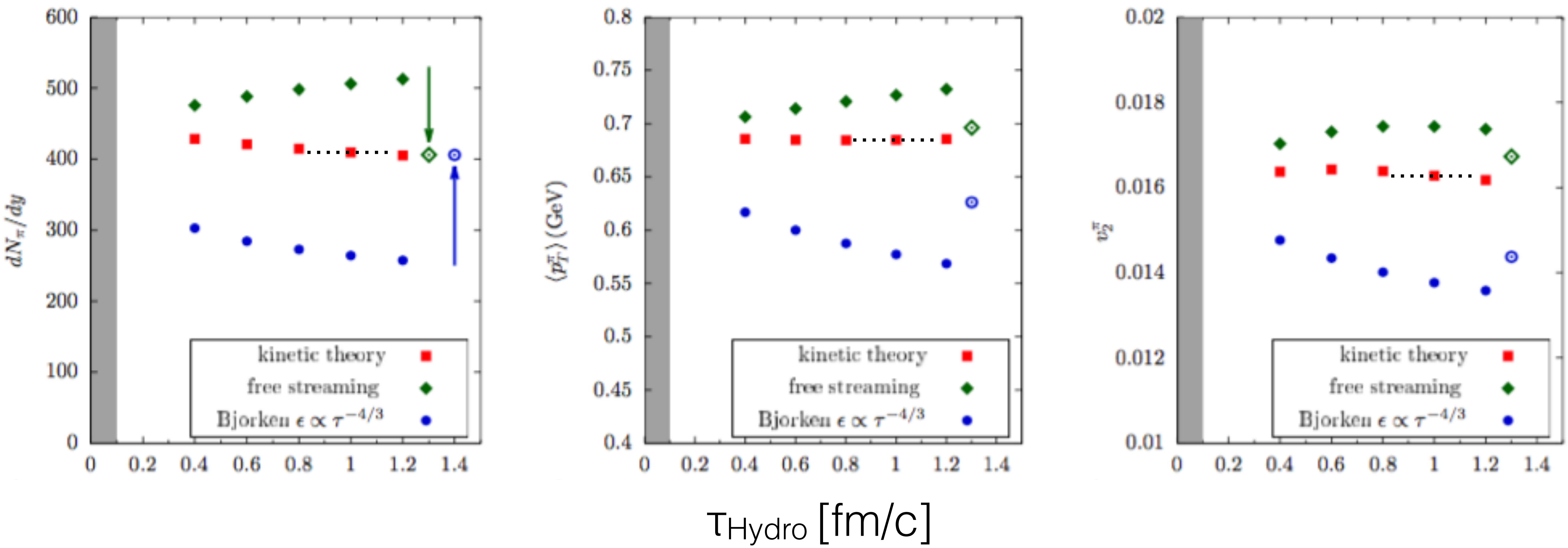
Electro-magnetic and hard probes (c.f. talk by S. Mrowczynski Tue 12:05)

Non-equilibrium phase is key in understanding dynamics of small systems; need for unified description of bulk dynamics & hard probes

# Backup

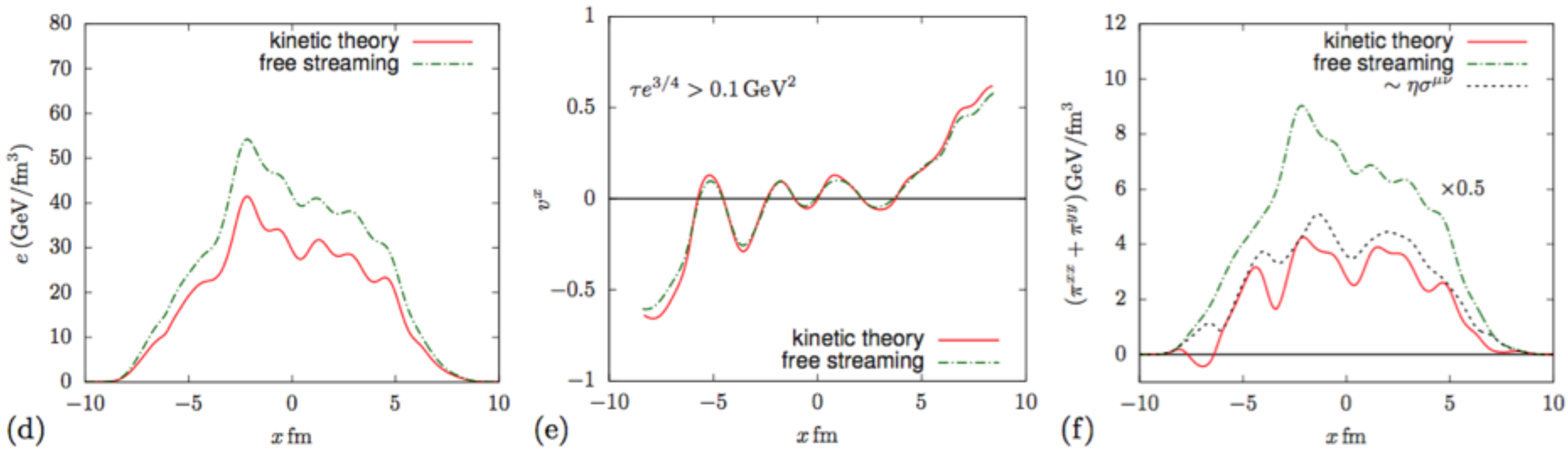
# Event-by-event pre-equilibrium evolution

Hadronic observables in single (MC-Glauber) Pb+Pb event:




Very little to no sensitivity to switching time $\tau_{Hydro}$ from pre-equilibrium to hydro for dN/dy, <$p_T$>, <$v_2$>, …

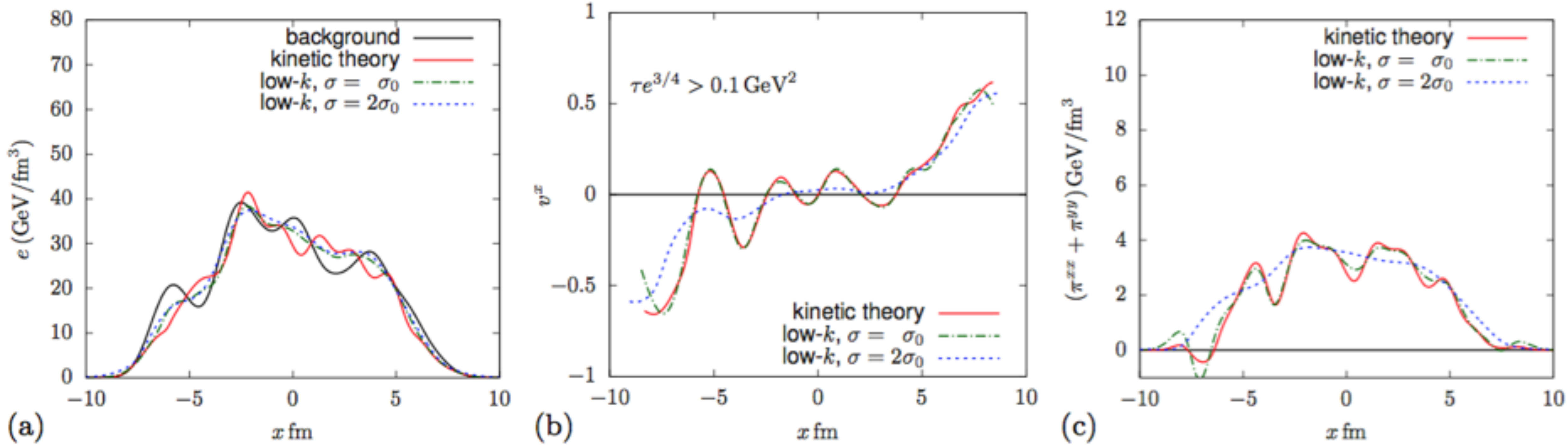
# Comparison with free-streaming




Velocities well reproduced due to universality of long wavelength response

Energy density decreases too fast;
Shear-stress never equilibrates to Navier-Stokes

# Comparison with long wavelength limit




Energy/momentum response in long wavelength limit:

$$\frac{\delta T^{\tau\tau}(\tau,\mathbf{x})}{\bar{T}^{\tau\tau}_{\mathbf{x}}(\tau)} \approx \frac{\tilde{G}^0_s(\tau,\tau_0)}{\bar{T}^{\tau\tau}_{\mathbf{x}}(\tau_0)} \left[\frac{1}{2}\tilde{s}^{(2)}_s(\tau-\tau_0)^2\partial_k\partial^k\right] \bar{T}^{\tau\tau}(\tau_0,\mathbf{x}) \tag{67a}$$

$$\frac{\delta T^{\tau i}(\tau,\mathbf{x})}{\bar{T}^{\tau\tau}_{\mathbf{x}}(\tau)} \approx \frac{\tilde{G}^0_s(\tau,\tau_0)}{\bar{T}^{\tau\tau}_{\mathbf{x}}(\tau_0)} \left[-\tilde{s}^{(1)}_v(\tau-\tau_0)\partial^i\right] \bar{T}^{\tau\tau}(\tau_0,\mathbf{x}) \tag{67b}$$

Note: Scale dependence of coefficients

# Macroscopic pre-equilibrium evolution

Effective kinetic description needs phase-space distribution $f(\tau, p, x)$

Memory loss: Details of initial phase-space distribution become irrelevant as system approaches local equilibrium

Can describe evolution of $T^{\mu\nu}$ in kinetic theory in terms of a representative phase-space distribution

$$f(\tau, p, x) = f_{BG}(Q_s(x)\tau, p/Q_s(x)) + \delta f(\tau, p, x)$$

where $f_{BG}$ characterizes typical momentum space distribution, and $\delta f$ can be chosen to represent local fluctuations of initial energy momentum tensor, e.g. energy density $\delta T^{\tau\tau}$ and momentum flow $\delta T^{\tau i}$

Energy perturbations:

$$\delta f_s(\tau_0, p, x) \propto \frac{\delta T^{\tau\tau}(x)}{T^{\tau\tau}_{BG}(x)} \times \frac{\partial}{\partial Q_s(x)} f_{BG}\Big(\tau_0, p/Q_s(x)\Big)$$

local amplitude

representative form of phase-space distribution

# Scaling variables

Background evolution and Greens functions still depend on variety of variables e.g. $Q_s(x)$ (local energy scale), $\alpha_s$, (coupling constant) …

-> Identify appropriate scaling variables to reduce complexity

Since ultimately evolution will match onto visc. hydrodynamics, check wether hydrodynamics admits scaling solution

1st order hydro: $$T^{\tau\tau}(\tau) = T^{\tau\tau}_{Ideal}(\tau)\Big(1 - \frac{8}{3}\frac{\eta/s}{T_{eff}\tau} + ...\Big)$$

where $T^{\tau\tau}_{Ideal}(\tau)$ is the Bjorken energy density and $T_{eff} = \tau^{-1/3} \lim_{\tau\to\infty} T(\tau)\tau^{1/3}$

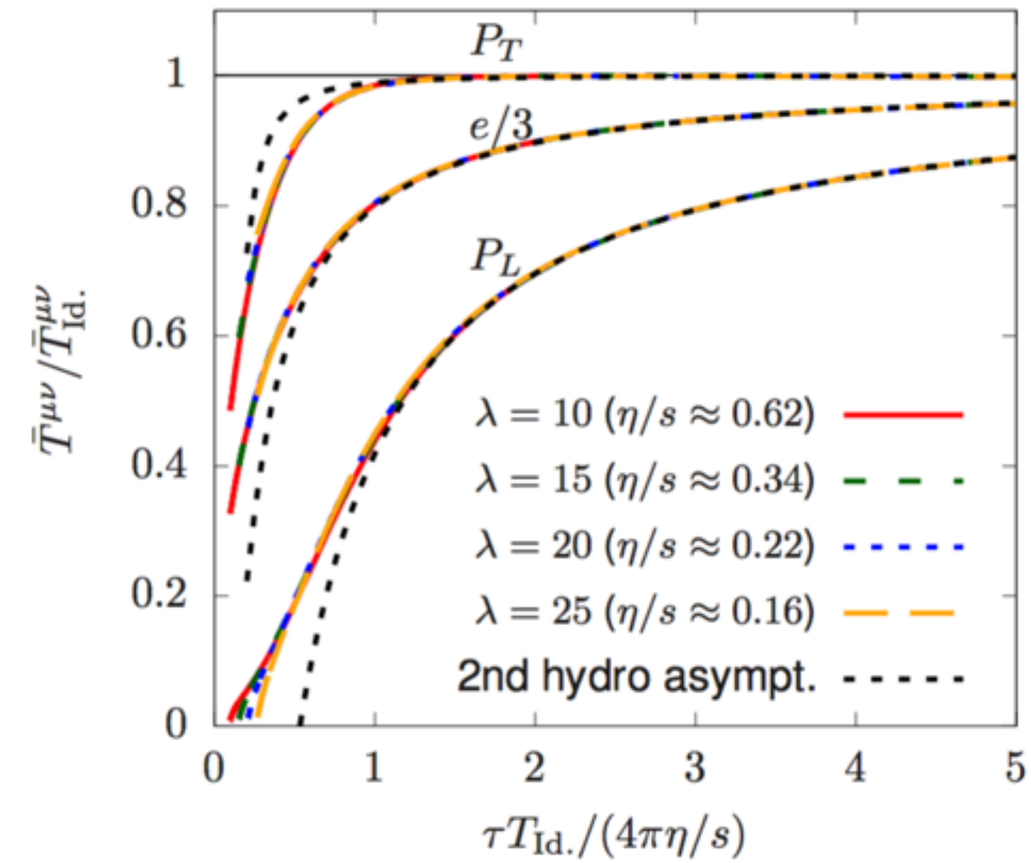
Natural candidate for scaling variable is $x_s = T_{eff}\tau/(\eta/s)$

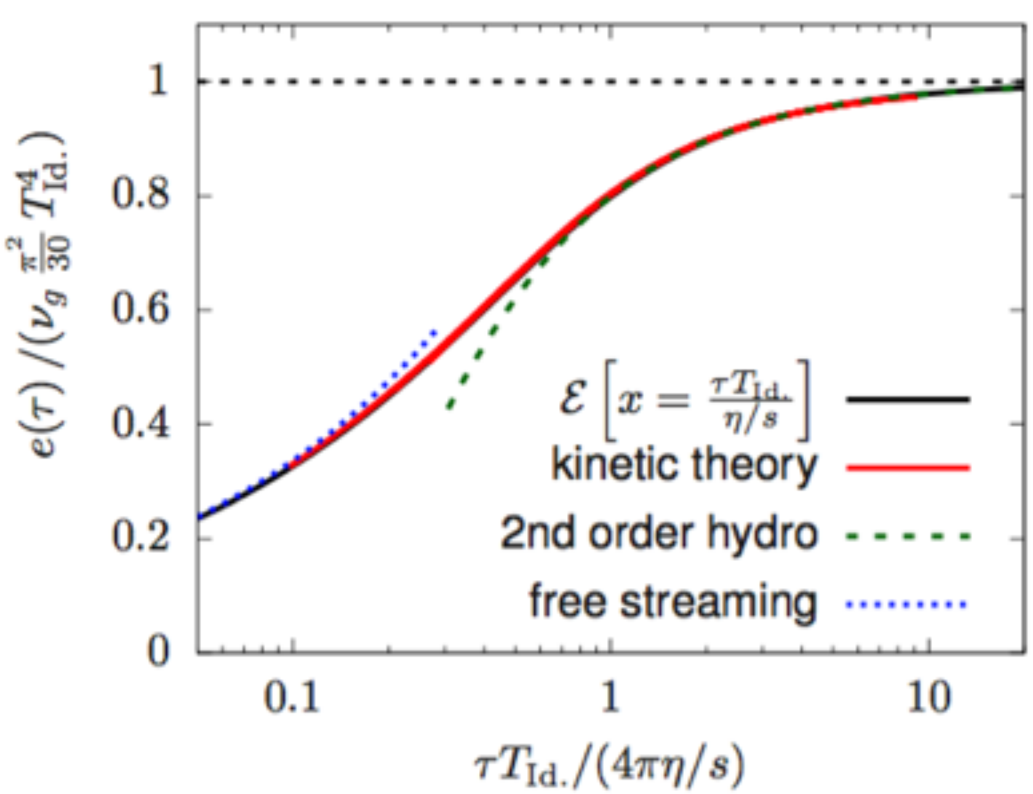
(evolution time / equilibrium relaxation time)

# Background — Scaling & Equilibration time

Scaling property extends well beyond hydrodynamic regime

Non-equilibrium evolution of background $T^{\mu\nu}$ is a unique function of $x_s = T_{eff}\tau/(\eta/s)$




Near equilibrium physics (η/s) determines time scale for mini-jet quenching for relevant values of coupling strength (η/s)

Kurkela, Zhu PRL 115 (2015) 182301
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

# Greens functions

Greens functions describe evolution of energy/momentum perturbations on top of a (locally) homogenous boost-invariant background

-> Description of perturbations in Fourier space

Decomposition in a complete basis of tensors leaves a total of 10 independent functions, e.g. for energy perturbations

energy response

$$\tilde{G}^{\tau\tau}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{s}_{s}(\tau,\tau_0,|\mathbf{k}|) \ ,$$

momentum response

$$\tilde{G}^{\tau i}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \frac{\mathbf{k}^i}{|\mathbf{k}|}\tilde{G}^{v}_{s}(\tau,\tau_0,|\mathbf{k}|) \ ,$$

shear stress/pressure response

$\tau\tau$

$$\tilde{G}^{ij}_{s}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{t,\delta}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \delta^{ij} + \tilde{G}^{t,k}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \frac{\mathbf{k}^i\mathbf{k}^j}{|\mathbf{k}|^2}$$

Numerically computed in eff. kinetic theory by solving linearized Boltzmann equation on top of non-equilibrium background

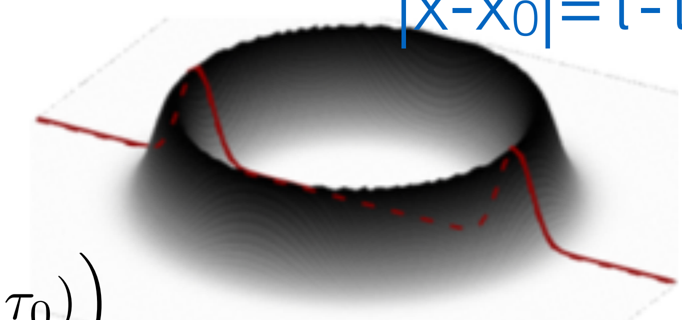
$$\left(\partial_\tau + \frac{i\mathbf{p}_\perp\mathbf{k}_\perp}{p} - \frac{p_z}{\tau}\right)\delta\widetilde{f}(\tau,|\mathbf{p}_\perp|,p_z;\ \mathbf{k}_\perp) = \delta\mathcal{C}[f,\widetilde{\delta f}]$$

# Greens functions

Free-streaming:

Energy-momentum perturbations propagate as a concentric wave traveling at the speed of light

coordinate space




energy/momentum response:

$$G_s^{s/v}(\tau,\tau_0,\mathbf{x}-\mathbf{x}_0) = \frac{1}{2\pi(\tau-\tau_0)}\delta\Big(|\mathbf{x}-\mathbf{x}_0|-(\tau-\tau_0)\Big)$$

Hydrodynamic response functions in the limit of large times $x_s >> 1$ and small wave-number $k\ (\tau-\tau_0) << x_s^{1/2}$

(c.f. Vredevoogd,Pratt PRC79 (2009) 044915, Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171)

energy response: $$\tilde{G}_s^s(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(1-\frac{1}{2}k^2(\tau-\tau_0)^2\ \tilde{s}_s^{(2)}+...\right),$$

momentum response: $$\tilde{G}_s^v(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(\quad k(\tau-\tau_0)\ \tilde{s}_v^{(1)}+...\right),$$

shear response: determined by hydrodynamic constitutive relations

$$\tilde{G}_s^s(\tau,\tau_0,k=0) = \left(\frac{T^{\tau\tau}(\tau_0)}{T^{\tau\tau}(\tau)}\right)\left(\frac{3T^{\tau\tau}(\tau)-T^{\eta}_{\eta}(\tau)}{3T^{\tau\tau}(\tau_0)-T^{\eta}_{\eta}(\tau_0)}\right)$$

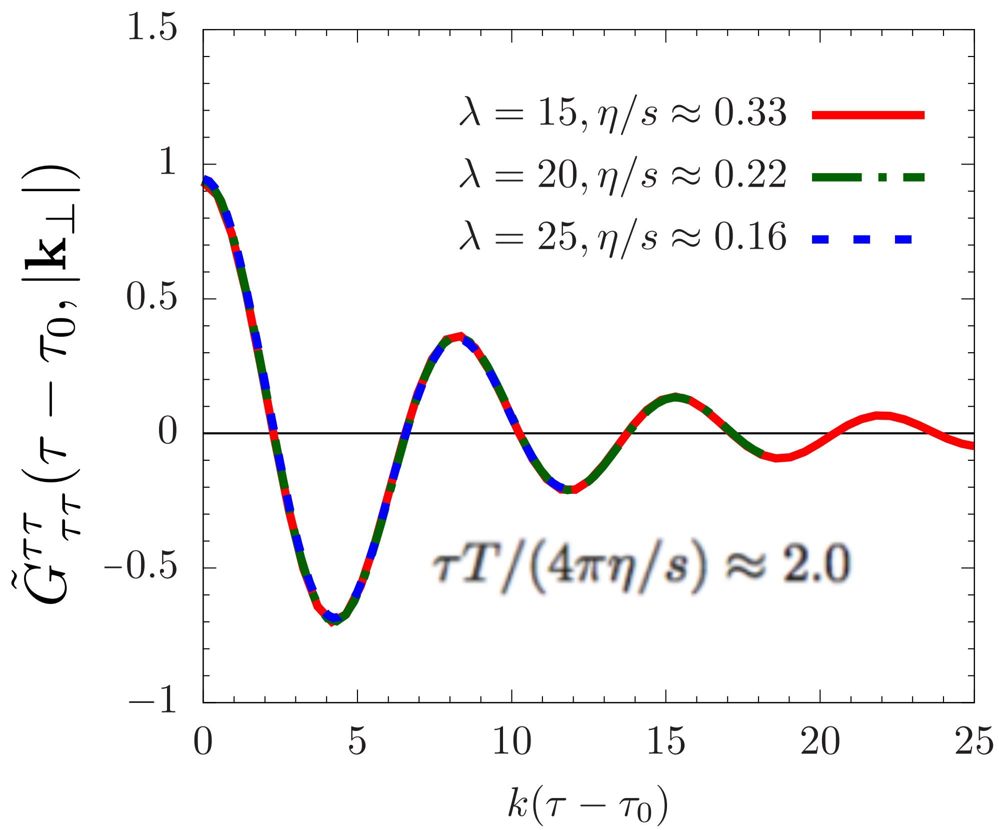
background evolution

$$\tilde{s}_s^{(2)} = \frac{1}{2}+\frac{1}{2}\frac{\eta/s}{\tau T_{\text{id.}}},\qquad \tilde{s}_v^{(1)} = \frac{1}{2},$$

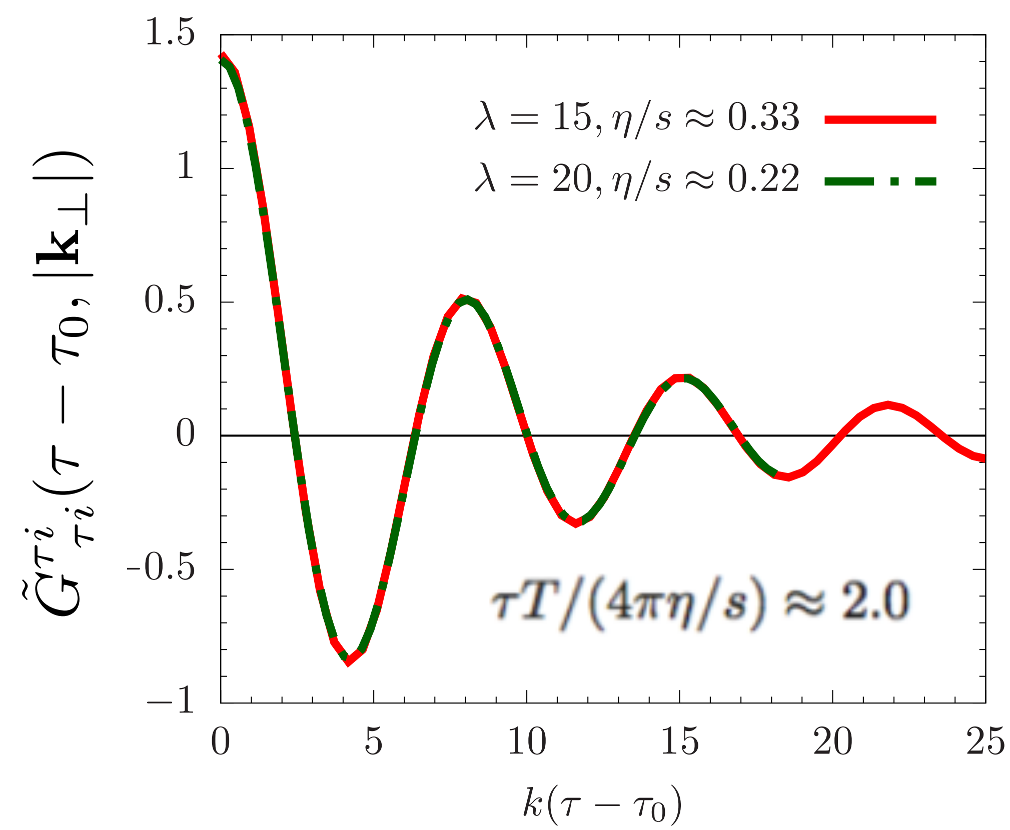
"long wave-length constants"

# Greens functions — Scaling variables

Energy response
to energy perturbation



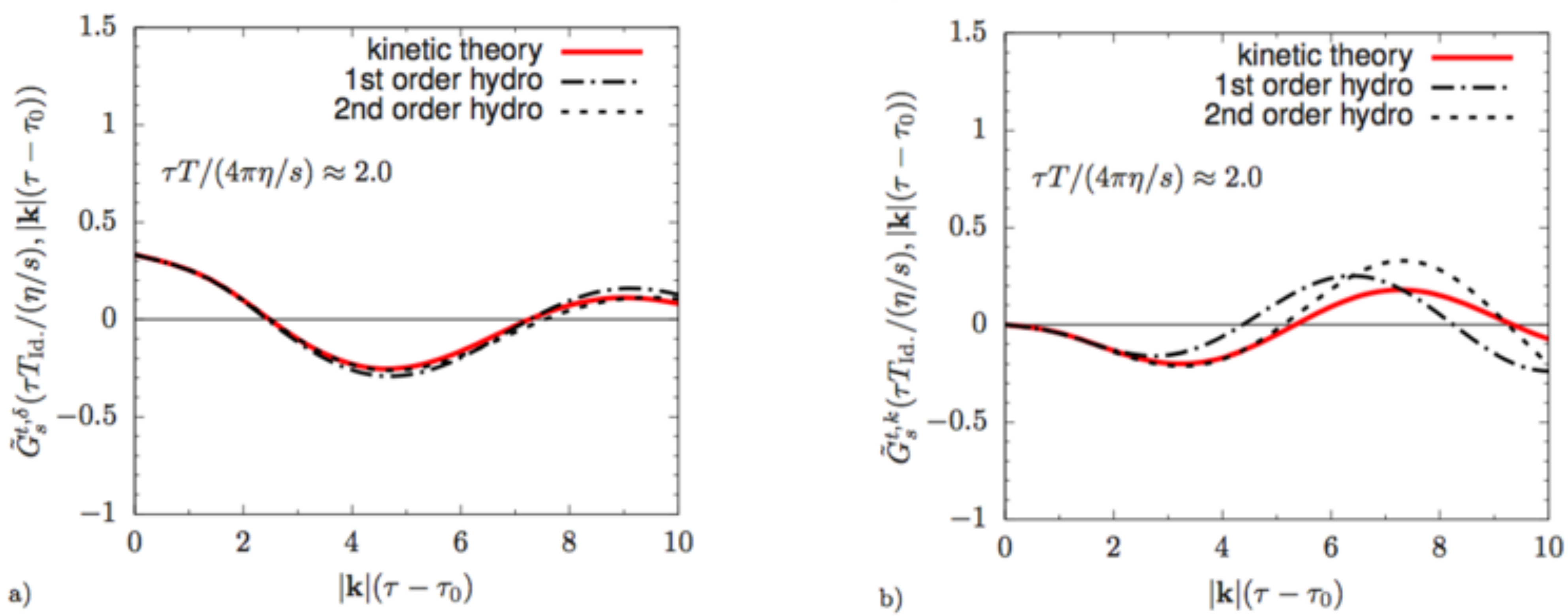

Momentum response
to momentum perturbations




Non-equilibrium Greens functions show universal scaling in $x_s = T_{eff}\tau/(\eta/s)$ and $k(\tau - \tau_0)$ beyond hydro limit

# Greens functions — Scaling variables

Shear stress & pressure response to energy perturbation




Satisfy hydrodynamic constitutive relations for sufficiently large times $x_s >> 1$ and long wave-length $k\ (\tau-\tau_0) << x_s^{1/2}$

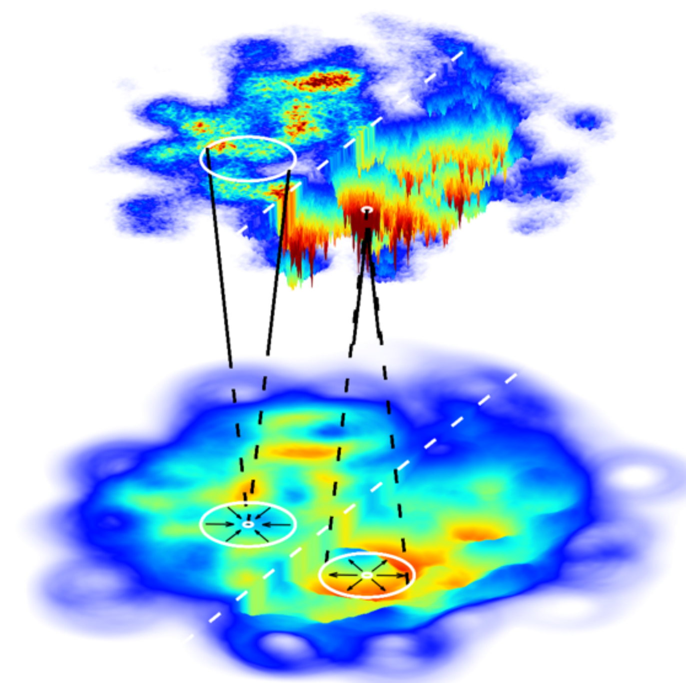
# KoMPoST

General framework for event-by-event pre-equilibrium dynamics (KoMPoST):

Input: Out-of-equilibrium energy-momentum tensor & value of η/s

Non-equilibrium evolution in linear response formalism

$$T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{hydro}})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{EKT}})} \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}').$$



Output: Energy-momentum tensor at $\tau_{\text{Hydro}}$ when visc. hydro becomes applicable

Easy to use & publicly available at github.com/KMPST/KoMPoST