



Parton Energy Loss in Generalized High-twist Approach

Yuan-Yuan Zhang

Central China Normal University (CCNU)

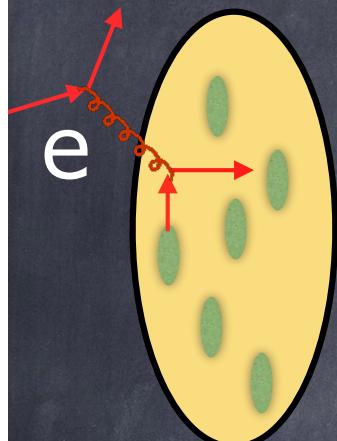
Collaborators: Guang-You Qin, Xin-Nian Wang



*Hard Probes 2018
Aix-Les-Bains, France*

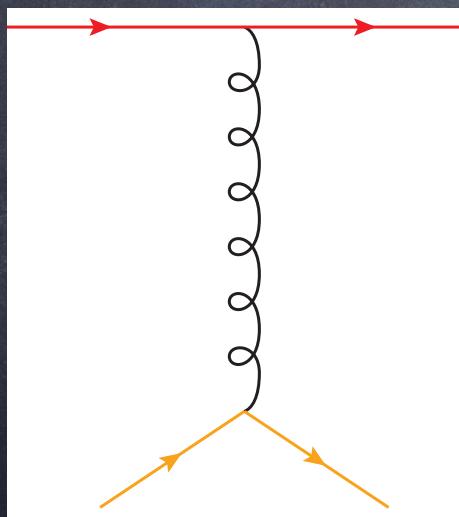


- ⦿ Introduction
- ⦿ Generalized High Twist approach
- ⦿ Results and approximations
- ⦿ Summary

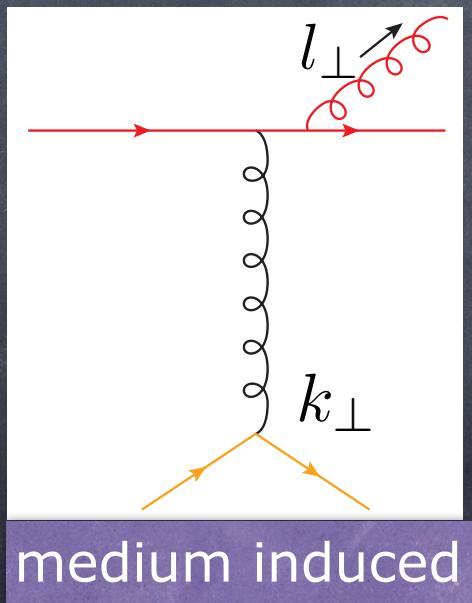


- High energy partons travel through hot QGP or cold nuclei (eA DIS process)
- Energy loss mechanisms reveal medium properties

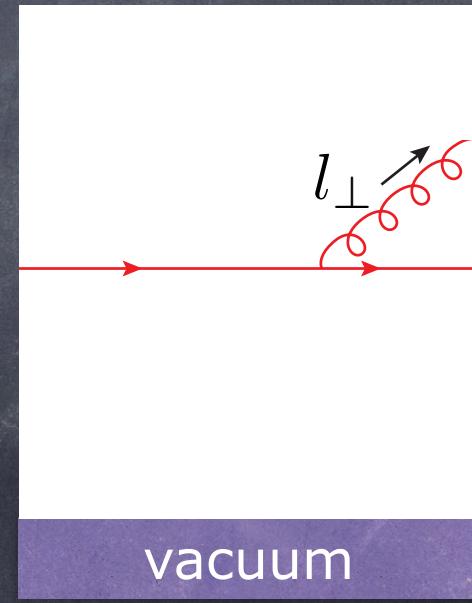
Parton energy loss mechanisms



collisional energy loss



radiative energy loss



Radiative energy loss: assumptions



Approaches to radiative energy loss : BDMPS-Z, GLV,
AMY, SCET, High Twist

• Scattering Center : Static or Dynamic?

Static: no energy transfer
(BDMPS-Z, GLV)

Extension of GLV to dynamic S.C.
Djordjevic, Heinz PRL 101, 022302

Dynamic: both momentum and energy transfer



• Radiated Gluon : Soft or hard ?

$z \rightarrow 0$ (BDMPS-Z, GLV, SCET)

Discussion on soft appr. of GLV
Blagojevic *et al.* arXiv:1804.07593

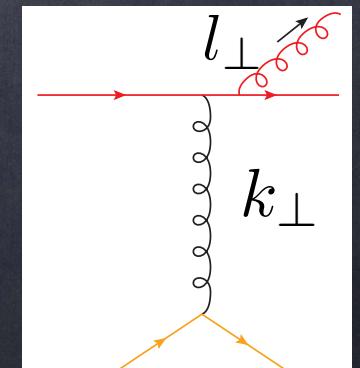
z finite



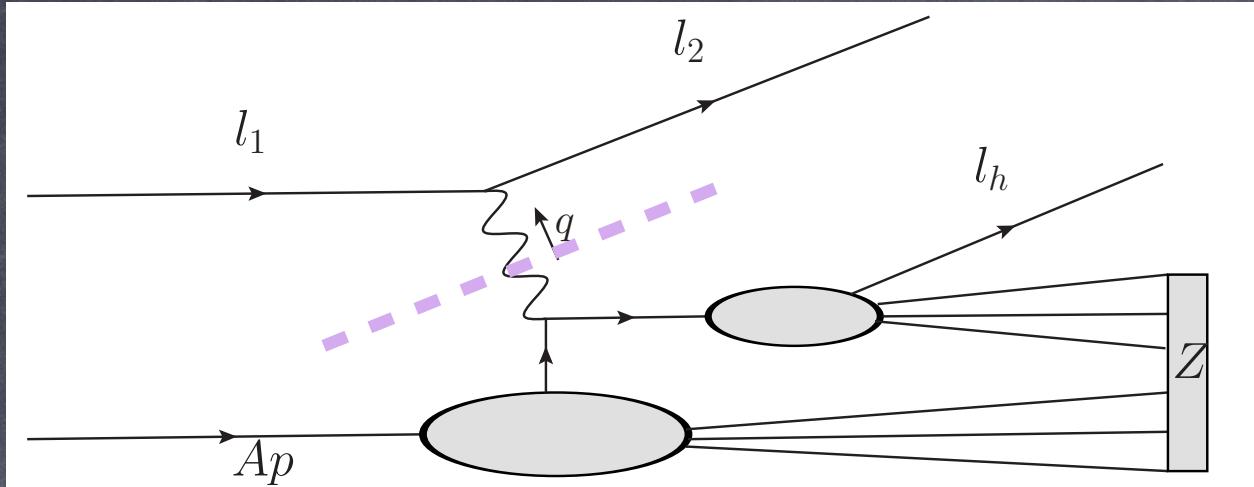
• Transverse momentum transfer : smaller or same order l_\perp ?

$k_\perp \ll l_\perp$ (High Twist)

$k_\perp \sim l_\perp$



Semi-inclusive Deeply Inelastic Scattering



- lepton-nucleus scattering

$$e(l_1) + A(Ap) \rightarrow e(l_2) + h(l_h) + Z$$

identify one hadron in final state

- cross section $d\sigma$ and hadronic tensor $W^{\mu\nu}$

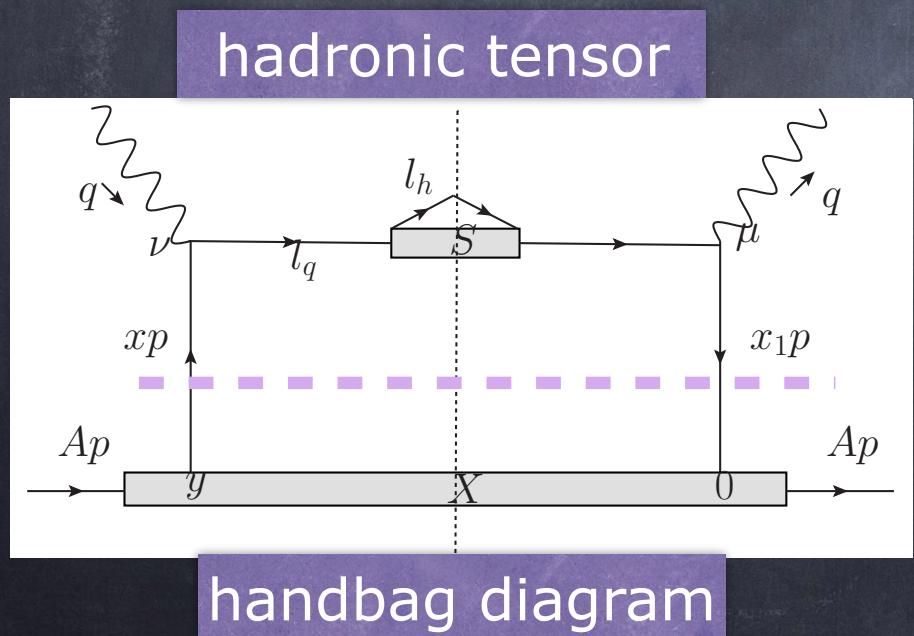
$$d\sigma = \frac{e^4}{2s} \frac{\sum_q e_q^2}{q^4} \int \frac{d^4 l_2}{(2\pi)^4} 2\pi \delta(l_2^2) \frac{1}{2} L_{\mu\nu} W^{\mu\nu}$$

Factorization

Separate non-perturbative part (pdf, fragmentation function) from perturbative part (hard scattering)

SIDIS process

Collinear factorization when final hadron $l_{h\perp}$ integrated



$$\frac{dW_{S(0)}^{\mu\nu}}{dz_h} = \int dx f_q^A(x) H_{(0)}^{\mu\nu} D_{q \rightarrow h}(z_h)$$

$f_q^A(x)$ quark distribution function

$D_{q \rightarrow h}(z_h)$ quark fragmentation function

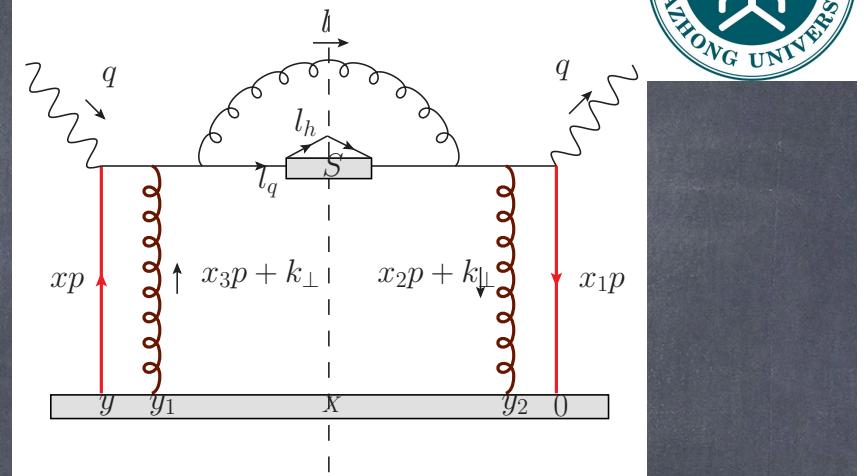
Factorization of Medium Induced Radiation



High Twist approach

Collinear factorization

XF Guo, XN Wang (2000) PRL 85(17), 3591
 XN Wang and XF Guo (2001) Nucl. Phys. A 696, 788



$$\frac{dW_{D(1)q}^{\mu\nu}}{dz_h} = \int_{z_h}^1 \frac{dz}{z} D_{q \rightarrow h}(z_h/z) \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{d^2 y_{1\perp}}{(2\pi)^2} d^2 y_{2\perp} d^2 k_\perp e^{-i \vec{k}_\perp \cdot (\vec{y}_{1\perp} - \vec{y}_{2\perp})}$$

$$\frac{1}{2} < A | \bar{\psi}_q(0) \gamma^+ A^+(y_2^-, \vec{y}_{2\perp}) A^+(y_1^-, \vec{y}_{1\perp}) \psi_q(y^-) | A > H_D^{\mu\nu}(k_\perp, y^-, y_1^-, y_2^-, p, q, z)$$

Collinear expansion of hard part

$$H_D^{\mu\nu}(k_\perp, y^-, y_1^-, y_2^-, p, q, z) = H_D^{\mu\nu}(k_\perp = 0) + \left. \frac{\partial H_D^{\mu\nu}}{\partial k_\perp^\alpha} \right|_{k_\perp=0} k_\perp^\alpha$$

$k_\perp \ll l_\perp$ approximation

$$+ \left. \frac{1}{2} \frac{\partial^2 H_D^{\mu\nu}}{\partial k_\perp^\alpha \partial k_\perp^\beta} \right|_{k_\perp=0} k_\perp^\alpha k_\perp^\beta + \dots$$

$k_\perp = 0$ contribute to gauge link of initial quark PDF
 k_\perp^α contribute zero for unpolarized beam

Factorization of Medium Induced Radiation



- Generalized High Twist approach

relax $k_\perp \ll l_\perp$ without collinear expansion

factorize quark PDF and gluon PDF directly

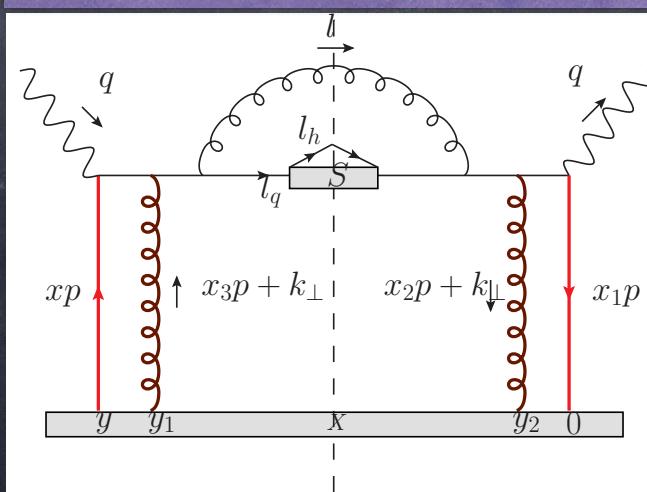
$$\frac{dW_{D(1)q}^{\mu\nu}}{dz_h} \sim \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \frac{1+z^2}{1-z} H_{(0)}^{\mu\nu}(x) \otimes \rho(y_1^-, \vec{y}_{1\perp}) \otimes \frac{D_{q \rightarrow h}(z_h/z)}{z}$$

splitting function nucleon density

$$\otimes f_q^A(x) \frac{\pi}{[\vec{l}_\perp - (1-z)\vec{k}_\perp]^2} \frac{\phi(x_L + x_D, \vec{k}_\perp)}{k_\perp^2} \text{ TMD gluon pdf}$$

quark pdf

One example diagram



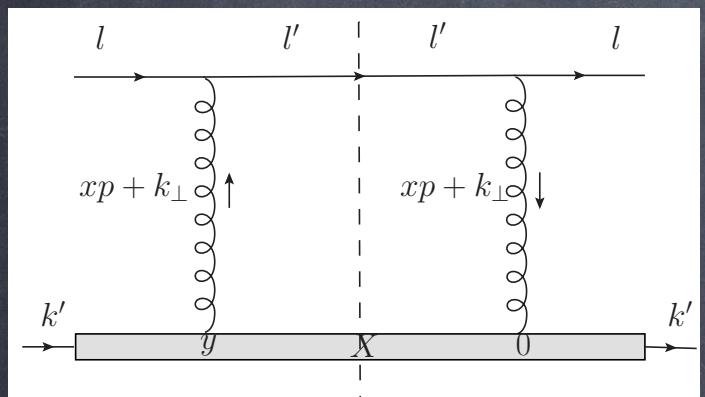
TMD gluon distribution function



- TMD gluon distribution function emerge from TMD transport parameter $\hat{q}(\vec{k}_\perp)$

$$\hat{q} = \rho \int dk_\perp^2 \frac{\langle d\sigma \rangle}{dk_\perp^2} k_\perp^2$$

color source density*average kT broadening per scattering



$$\begin{aligned}\hat{q} &\equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \int dx \delta(x - \frac{k_\perp^2}{2p^+ l^-}) \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \phi(x, \vec{k}_\perp) \\ &\equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \hat{q}(\vec{k}_\perp)\end{aligned}$$

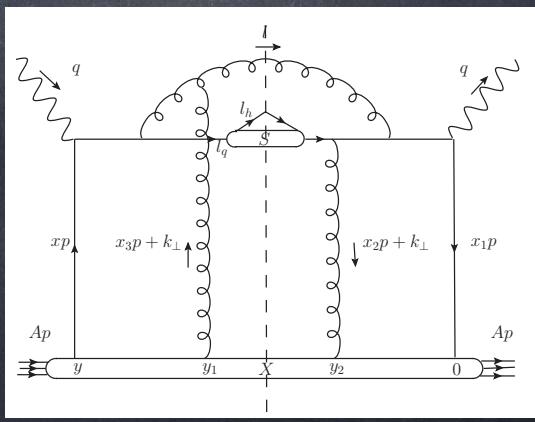
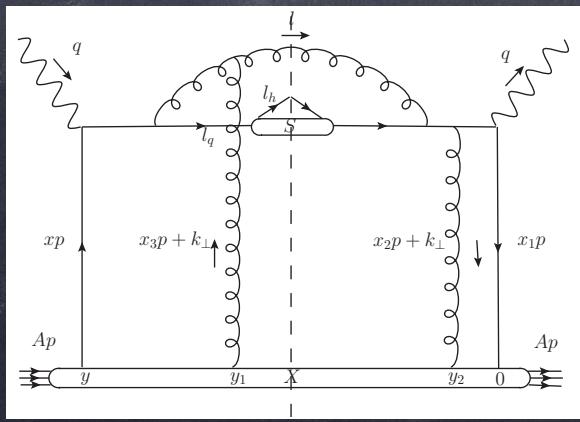
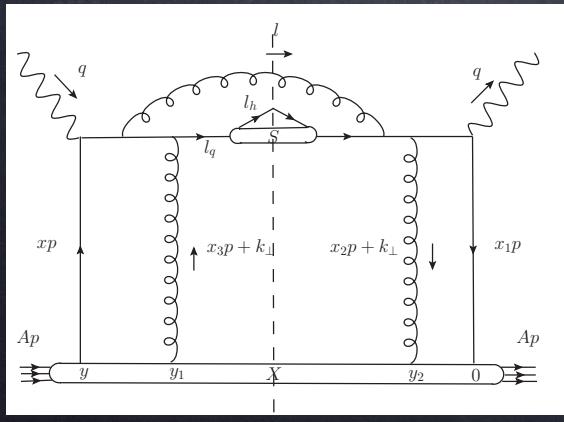
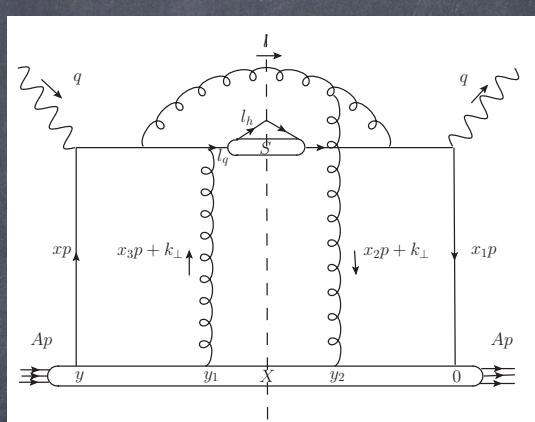
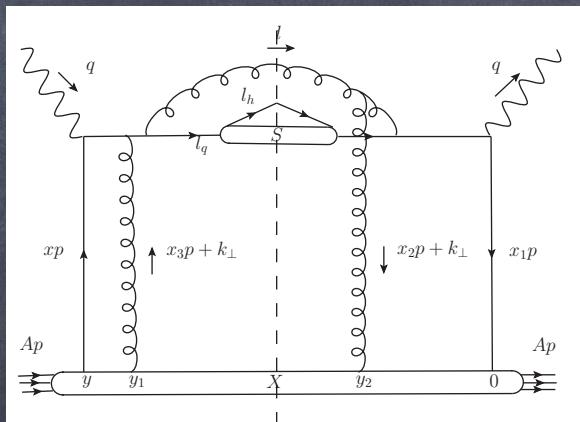
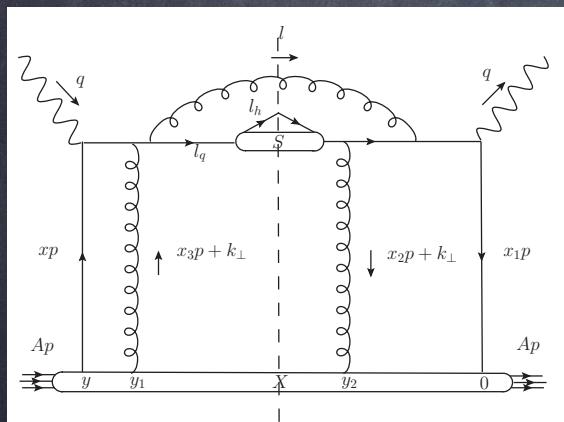
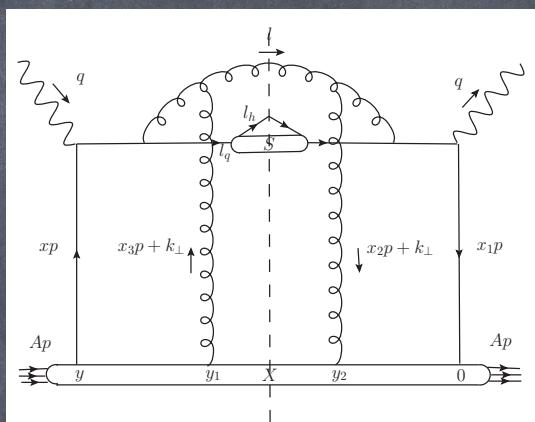
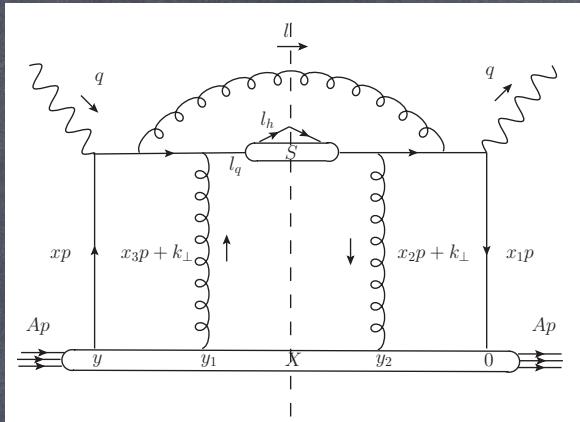
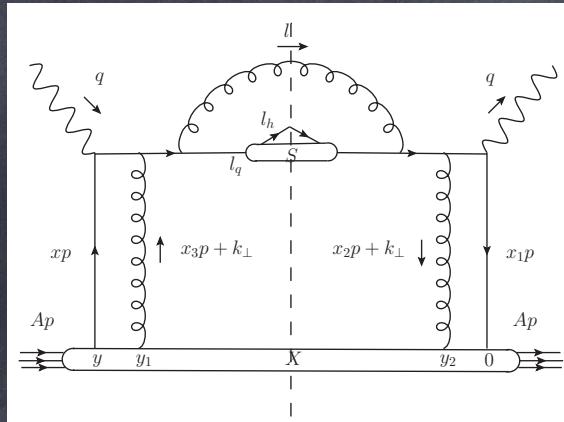
$\phi(x, \vec{k}_\perp)$ emerge naturally

$$\phi(x, \vec{k}_\perp) \equiv \int \frac{dy^-}{2\pi p^+} \int d^2 y_\perp e^{ixp^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle p | F_\alpha + (0) F^{+\alpha}(y^-, y_\perp) | p \rangle$$

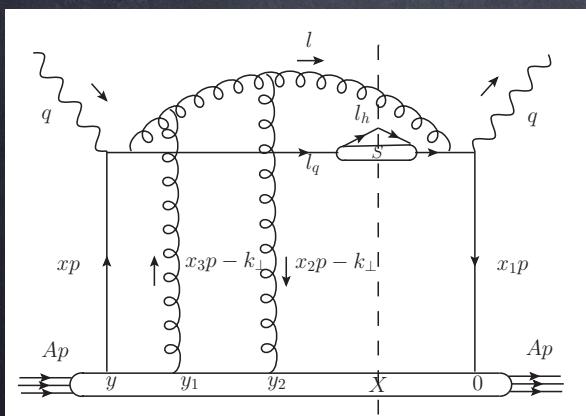
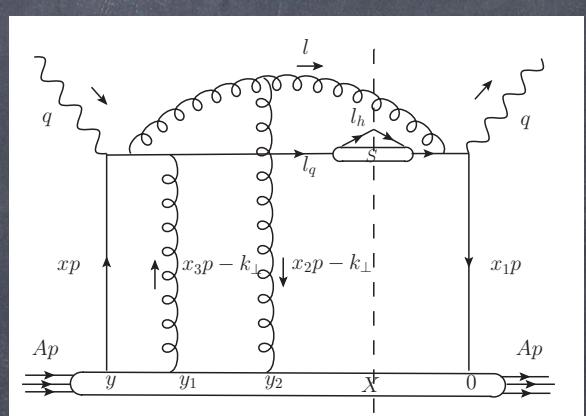
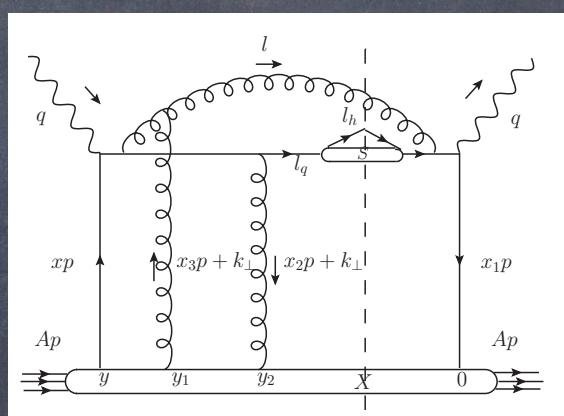
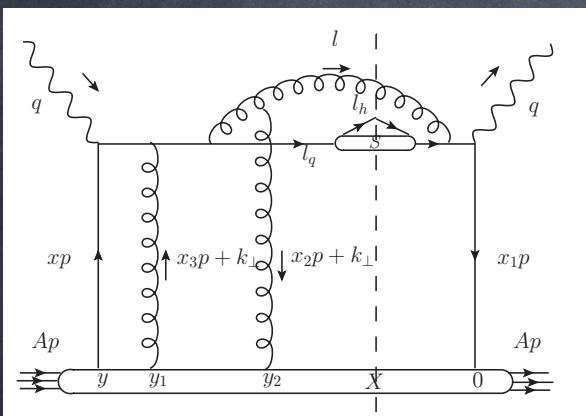
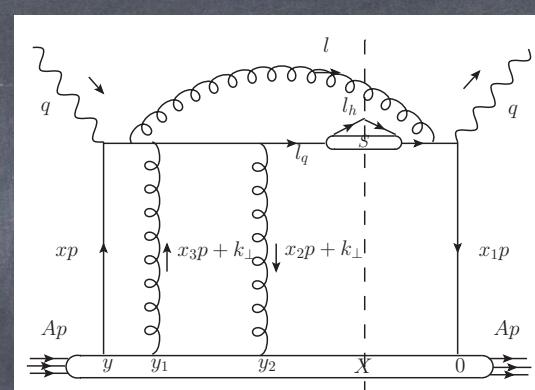
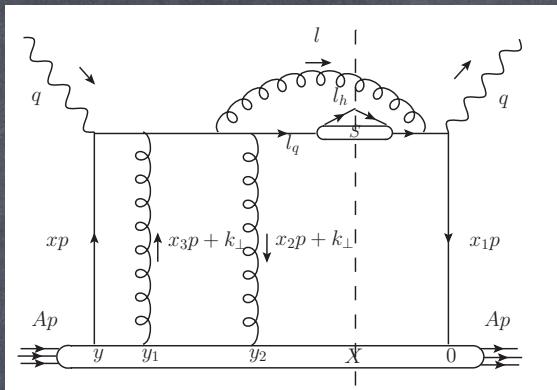
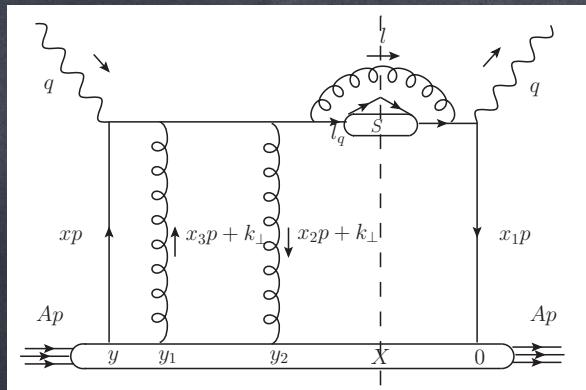
$\hat{q}(\vec{k}_\perp)$ $\phi(x, \vec{k}_\perp)$ depend on the parton energy l^- and medium color source energy p^+ via x energy transfer via x

Gluon Spectrum Result

summing up all the diagrams



Gluon Spectrum Result



- vacuum + medium induced radiation interference

- left cut diagrams symmetric to right cut diagrams

Gluon Spectrum Result: Full Result



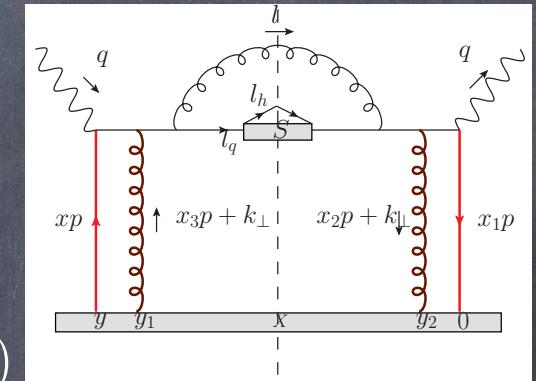
We get gluon spectrum from the hadronic tensor

$$\frac{dW_{D(1)g}^{\mu\nu}}{dz_h} = \int dx H_{(0)}^{\mu\nu}(x) \int \frac{dz}{z} D_{g\rightarrow h}(z_h/z) \int dl_\perp^2 T_{qg}(x, z, l_\perp^2)$$

Quark-gluon correlation function $T_{qg}(x, z, l_\perp^2)$

nuclear enhancement

$$T_{qg} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \\ [H_C^D \theta(y^- - y_1^-) \theta(-y_2^-) + H_L^D \theta(y_1^- - y_2^-) \theta(y^- - y_1^-) + H_R^D \theta(y_2^- - y_1^-) \theta(-y_2^-)]$$



Gluon spectrum : probability of quark to radiate one gluon with longitudinal momentum zq^- transverse momentum \vec{l}_\perp

$$\frac{dN}{dl_\perp^2 dz} \sim \frac{T_{qg}}{f_q^A(x)}$$

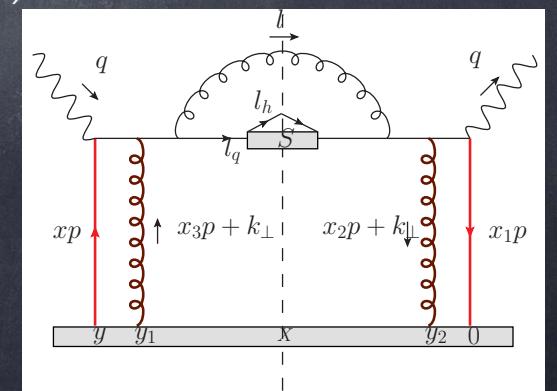
Gluon Spectrum Result: Full Result

$$\begin{aligned}
 H_C^D = & \left\{ \left[\frac{C_A}{(l_\perp - k_\perp)^2} \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L + \frac{x_T}{z}) - \frac{C_A}{l_\perp^2} \frac{\phi(\frac{1-z}{z}x_T, -\vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L) \right] \right. \\
 & + \left[\left(\frac{C_F}{l_\perp^2} + C_A \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_F}{[\vec{l}_\perp - z\vec{k}_\perp]^2} + \frac{1}{N} \frac{\vec{l}_\perp \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{l_\perp^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right. \right. \\
 & \left. \left. - C_A \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x) \right] \\
 & \left[\left(-\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \right. \\
 & \left. \frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} e^{-i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x + x_L + \frac{x_T}{z}) \right] \\
 & \left[\left(-\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \right. \\
 & \left. \left. \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} e^{i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x) \right] \right\}
 \end{aligned}$$

x : energy transfer

$$H_L^D = \dots$$

$$H_R^D = \dots$$



Gluon Spectrum Result: Contact Terms



There are contact terms, which are negligible

$$\frac{dN_{contact}}{dl_\perp^2 dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) H_{contact} [\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_1^- - y_2^-)\theta(y^- - y_1^-) - \theta(y_2^- - y_1^-)\theta(-y_2^-)]$$

The θ functions constrain the integration region

$$\begin{aligned} & \int dy_1^- dy_2^- [\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_1^- - y_2^-)\theta(y^- - y_1^-) - \theta(y_2^- - y_1^-)\theta(-y_2^-)] f(y_1^-, y_2^-) \\ &= \int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^- f(y_1^-, y_2^-) \end{aligned}$$

gives $0 < y_2^- < y_1^- < y^-$, quark and gluon comes from same nucleon, no nuclear enhancement.



Gluon Spectrum Approximations

1. Static scattering center approximation:

no energy transfer for medium scattering

$$\phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \dots \approx \phi(0, \vec{k}_\perp) \quad \text{no } x \text{ dependence}$$

$$f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B)$$

$$Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)} \quad \text{or} \quad x_B \gg x_L, \frac{x_T}{z}$$

gluon spectrum reduces to

$$\frac{d\bar{N}}{dl_\perp^2 dz} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \left[\bar{H}_C^D + \frac{1}{2} \bar{H}_L^D + \frac{1}{2} \bar{H}_R^D \right] \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$$\bar{H}_C^D = \left\{ \left[\frac{C_A}{(l_\perp - k_\perp)^2} - \frac{C_A}{l_\perp^2} \right] + \dots \right. \quad \bar{H}_R^D = \dots$$

$$\bar{H}_L^D = \dots$$



Gluon Spectrum Approximations

2. Soft radiated gluon approximation:

the radiated gluon momentum fraction $z \rightarrow 0$
gluon spectrum reduces to

$$\frac{dN_{soft}}{dl_\perp^2 dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) [(H_C^D)_{soft} \theta(y^- - y_1^-) \theta(-y_2^-) \\ + (H_L^D)_{soft} \theta(y_1^- - y_2^-) \theta(y^- - y_1^-) + (H_R^D)_{soft} \theta(y_2^- - y_1^-) \theta(-y_2^-)]$$

no dependence on z except splitting function, f_q^A and ϕ

$$(H_C^D)_{soft} = \left\{ \left[\frac{C_A}{(l_\perp - k_\perp)^2} \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L + \frac{x_T}{z}) - \frac{C_A}{l_\perp^2} \frac{\phi(\frac{1-z}{z}x_T, -\vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L) \right] + \dots \right.$$

$$(H_R^D)_{soft} = \dots \quad (H_L^D)_{soft} = \dots$$



Gluon Spectrum Approximations

3. Static scattering + Soft radiated gluon approximation:

$$\left\{ \begin{array}{l} \phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \dots \approx \phi(0, \vec{k}_\perp) \\ f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B) \end{array} \right. \quad \text{and } z \rightarrow 0 \text{ give}$$

notice here $Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$, or $x_B \gg x_L, \frac{x_T}{z}$ as $z \rightarrow 0$

the gluon spectrum is

$$\frac{d\tilde{N}}{dl_\perp^2 dz} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) C_A \frac{2\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left(1 - \cos[(x_L + \frac{x_T}{z}) p^+ y_1^-] \right) \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$\phi(x, \vec{k}_\perp)$

non-perturbative

Gluon Spectrum Approximations



3. Static scattering + Soft radiated gluon approximation:

TMD gluon pdf relation to transport parameter \hat{q}

$$\hat{q} = \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \int \frac{d^2 k_\perp}{(2\pi)^2} \phi(0, \vec{k}_\perp) \quad \star$$

One method : Static potential model to calculate \hat{q}

$$\langle d\sigma \rangle = \frac{C_2(R)C_2(T)}{d_A} \frac{4\pi\alpha_s^2}{t^2} dt$$

Casimir $C_2(R)$ $C_2(T)$

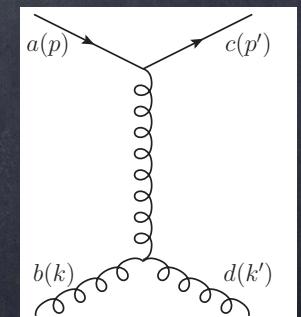
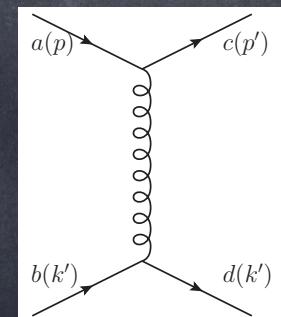
Mandelstam variable t

$$\hat{q} = \rho \int dk_\perp^2 \frac{C_2(R)C_2(T)}{d_A} \frac{4\pi\alpha_s^2}{k_\perp^2 + \mu_0^2} \quad \star$$

$$t = k_\perp^2 + \mu_0^2$$

compare two \hat{q}

$$\phi(0, \vec{k}_\perp) = C_2(T) \frac{4\alpha_s}{k_\perp^2 + \mu_0^2}$$



Gluon Spectrum Approximations



3. Static scattering + Soft radiated gluon approximation:

substitute $\phi(0, \vec{k}_\perp)$ into gluon spectrum, one get

$$\frac{d\tilde{N}}{dl_\perp^2 dz} = 8\pi\alpha_s^3 \frac{C_2(T)C_A}{N_c} \frac{1 + (1-z)^2}{z} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \rho(y_1^-) \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left(1 - \cos\left[(x_L + \frac{x_T}{z})p^+ y_1^-\right]\right) \frac{1}{(k_\perp^2 + \mu_0^2)^2}$$

agrees with of GLV formalism at first order in opacity

arguments in Cosine function $\left\{ \begin{array}{l} \omega_1 \approx \sqrt{2}(x_L + \frac{1}{z}x_T)p^+ \\ y_{10} = y_1 - y_0 \approx \frac{y_1^-}{\sqrt{2}} \end{array} \right.$

- Radiative energy loss in semi-inclusive DIS process
- Generalized High Twist Approach
 - no soft radiated gluon approximation
 - dynamic scattering center (energy transfer)
 - ★ - relax $k_\perp \ll l_\perp$ approximation, $k_\perp \sim l_\perp$
 - ★ - transverse momentum dependent (TMD) gluon pdf TMD transport parameter
- Relation of Generalized High Twist approach result with GLV result

Further work

- gauge link of TMD gluon distribution
- implementation into CoLBT-Hydro Model

Thanks!