

# Multi-particle correlations in small systems from the initial state

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October, 2018



# Outline

1. Introduction and motivation
2. Simple power counting argument for  $v_n$  multiplicity dependence at LHC

MM, V. Skokov, P. Tribedy, R. Venugopalan PLB (in press) [arXiv:1807.00825]

4. Demonstration of hierarchy of  $v_2$  and  $v_3$  across small systems in CGC EFT at RHIC

MM, V. Skokov, P. Tribedy, R. Venugopalan PRL 121 (2018) [arXiv:1805.09342], and in preparation

# Initial State Flow

At high energy  $\rightarrow$  high density gluon matter described by the **Color Glass Condensate** Effective Field Theory

*McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204*

High gluon density in QCD generates dynamical saturation scale,  $Q_s$

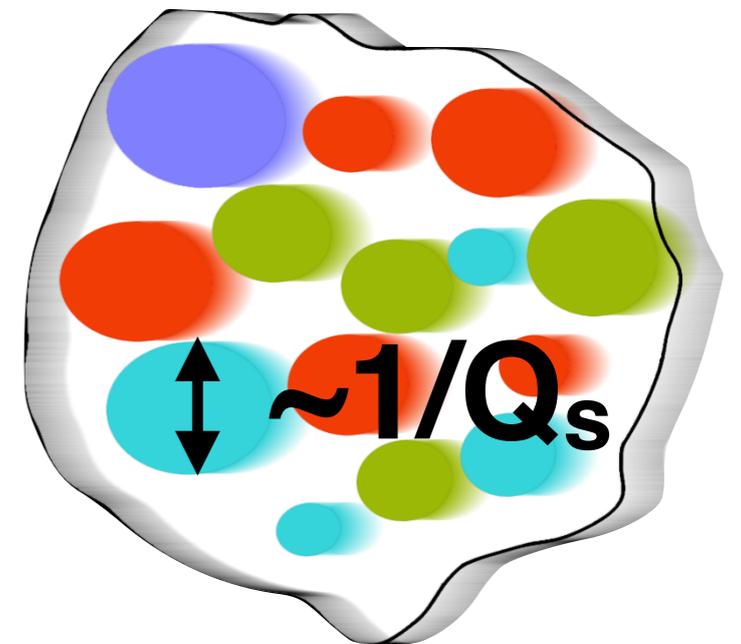
**c.f. plenary talk on Monday by A. Mueller**

Intuitive picture of CGC:  
Nucleus becomes saturated with high occupancy gluons for  $k_T < Q_s$   
For  $k_T \gg Q_s$  smooth matching of framework to pQCD

Note: *Very strongly correlated system*. Dependence on coupling drops out, effective classical description

**This talk: CGC has “flow” in line with observations**

$$Q_s^2 \sim A^{1/3} s^\lambda$$



# Dilute-dense for gluons

CGC EFT: solve QCD CYM with static color sources

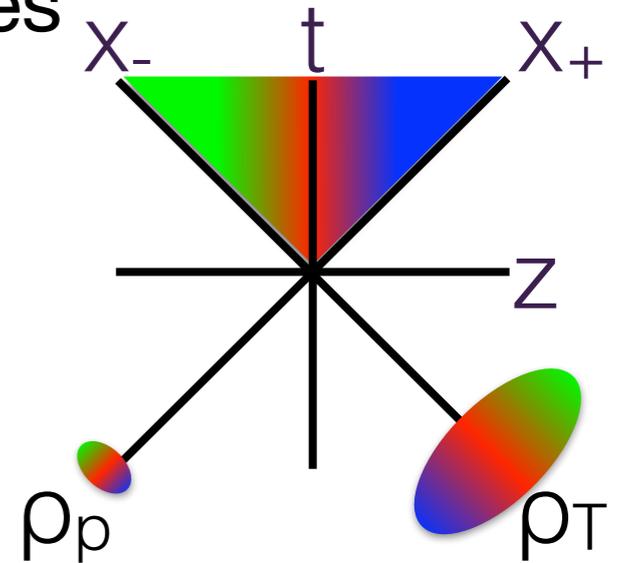
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

Dilute-dense regime:  $\rho_T/k_T^2 \gg \rho_p/k_T^2$

*Kovchegov, Mueller NPB 529 (1998), Kovner, Wiedemann PRD 64 (2001), Dumitru, McLerran NPA 700 (2002), Blaizot, Gelis, Venugopalan NPA 743 (2004),...*

$$\frac{dN}{d^2k} \sim g^2 \rho_p^2 f_{(1)}(\rho_T) + g^4 \rho_p^4 f_{(2)}(\rho_T) + \dots$$



# Dilute-dense for gluons

CGC EFT: solve QCD CYM with static color sources

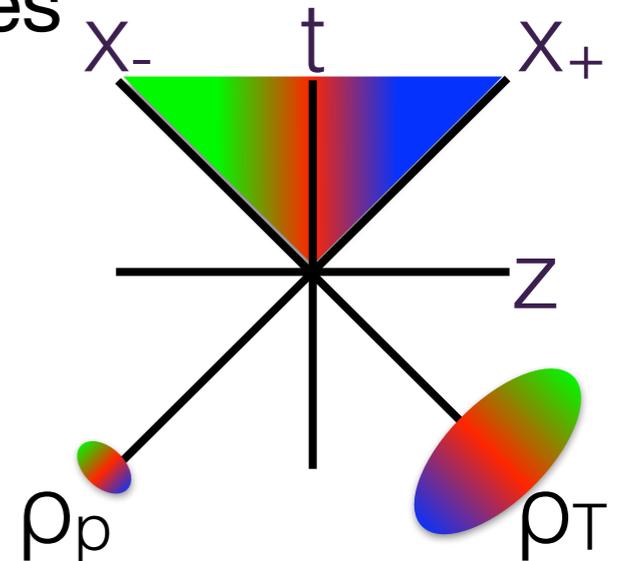
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$$\frac{dN}{d^2k} \sim g^2 \rho_p^2 f_{(1)}(\rho_T) + g^4 \rho_p^4 f_{(2)}(\rho_T) + \dots$$

Framework has been applied to study numerous final states at RHIC and LHC (quarkonia, photons, multiplicity dist...)

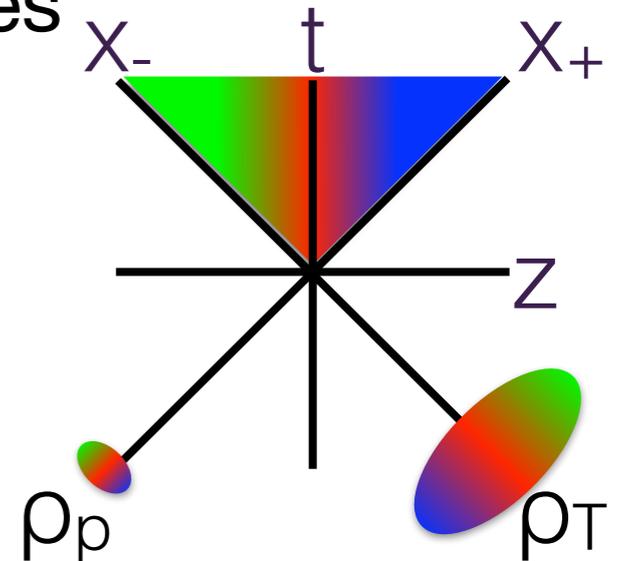
**e.g. see talk Tuesday by R. Venugopalan and references within**

Dense-dense (all  $O(\rho_p^\#)$ ) leads to IP-Glasma model

*Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)*

Includes genuine quantum correlations (BE, HBT)

**See previous talk by N. Armesto and references within**



# Dilute-dense CGC scaling

Even harmonics appear at LO dilute-dense

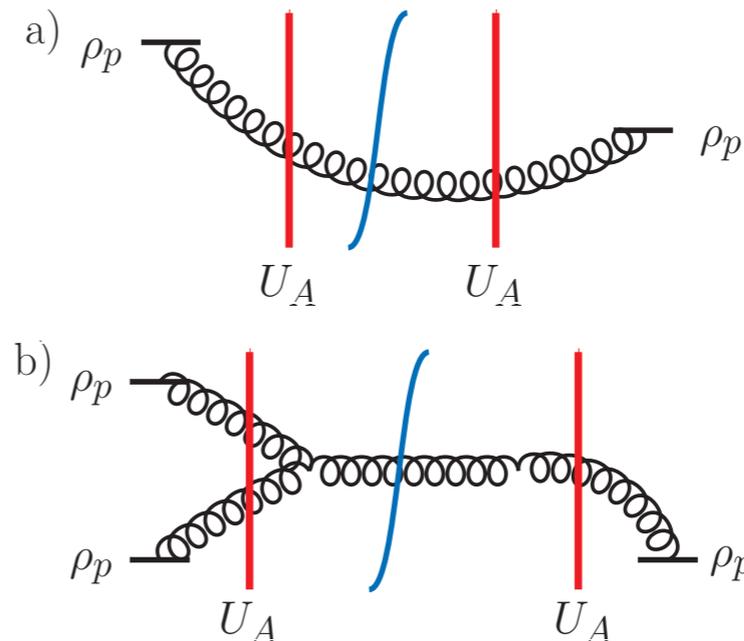
Odd harmonics only non-zero at next to leading order in  $\rho_p$   
 — **first saturation correction**

*McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)*

$$\frac{dN^{\text{even}}(\mathbf{k}_{\perp})}{d^2k dy} \sim \rho_p^2$$

$$\frac{dN^{\text{odd}}(\mathbf{k}_{\perp})}{d^2k dy} \sim \rho_p^3$$

Full expressions in [arXiv:1807.00825](https://arxiv.org/abs/1807.00825)

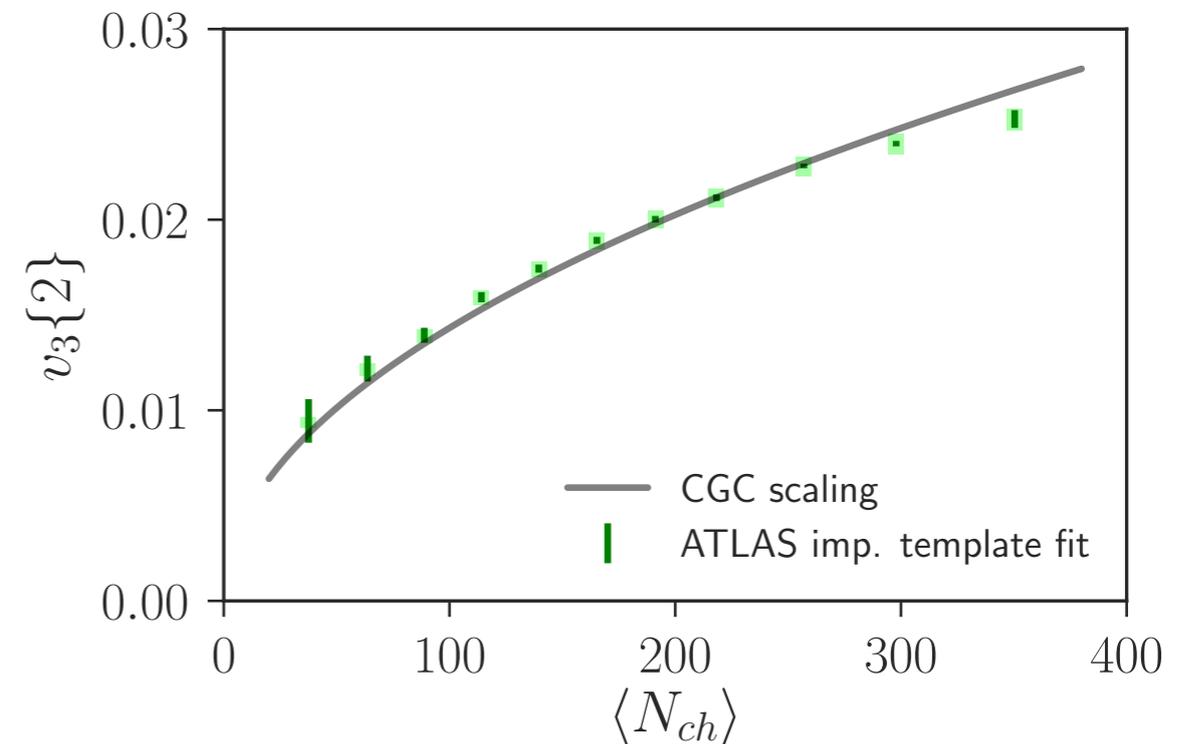
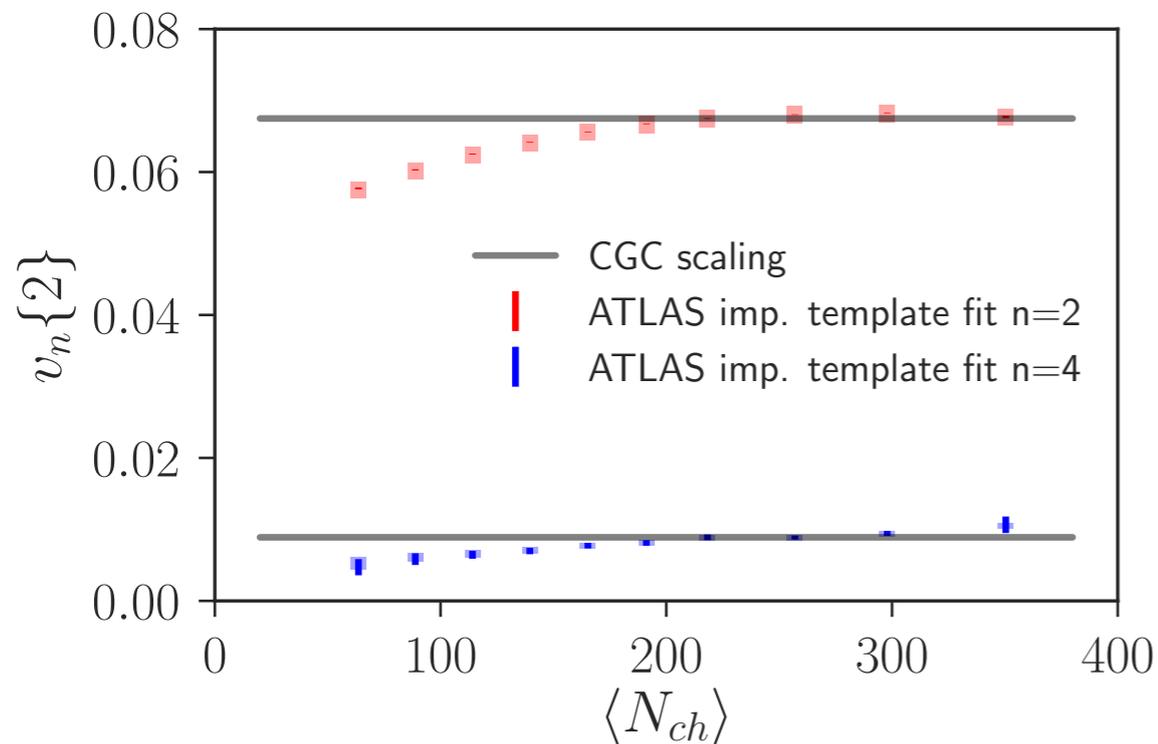


Multiplicity driven by  $\rho_p$ , so dilute-dense CGC expectation:

$$v_{2n}\{2\} \sim N_{ch}^0, \quad v_{2n+1}\{2\} \sim N_{ch}^{1/2}$$

# Dilute-dense CGC scaling

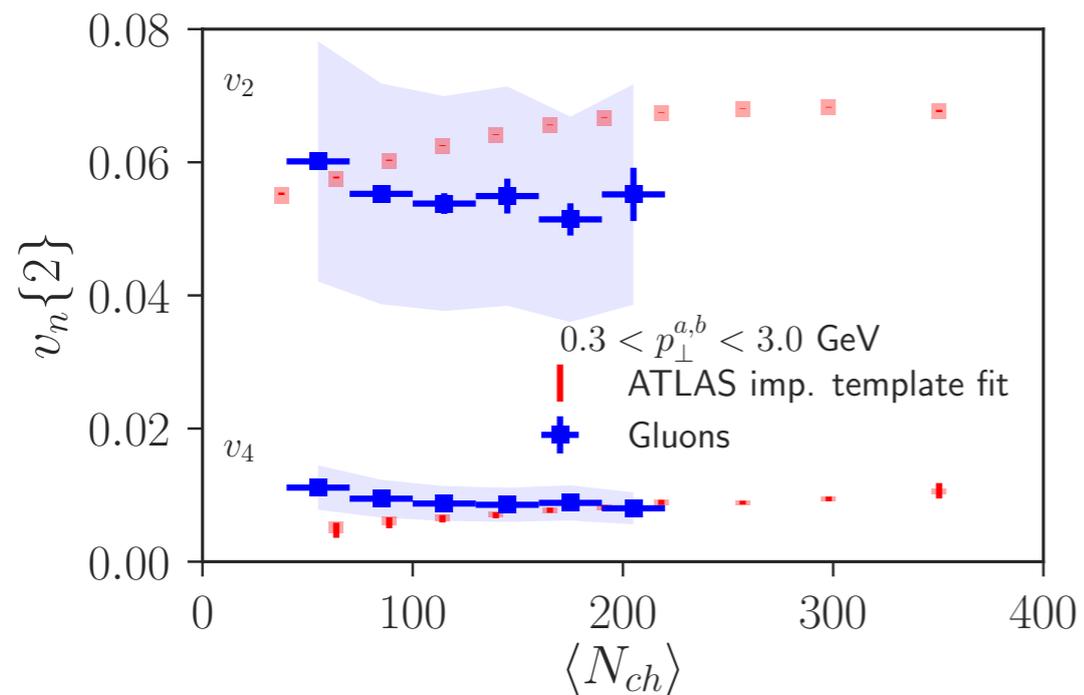
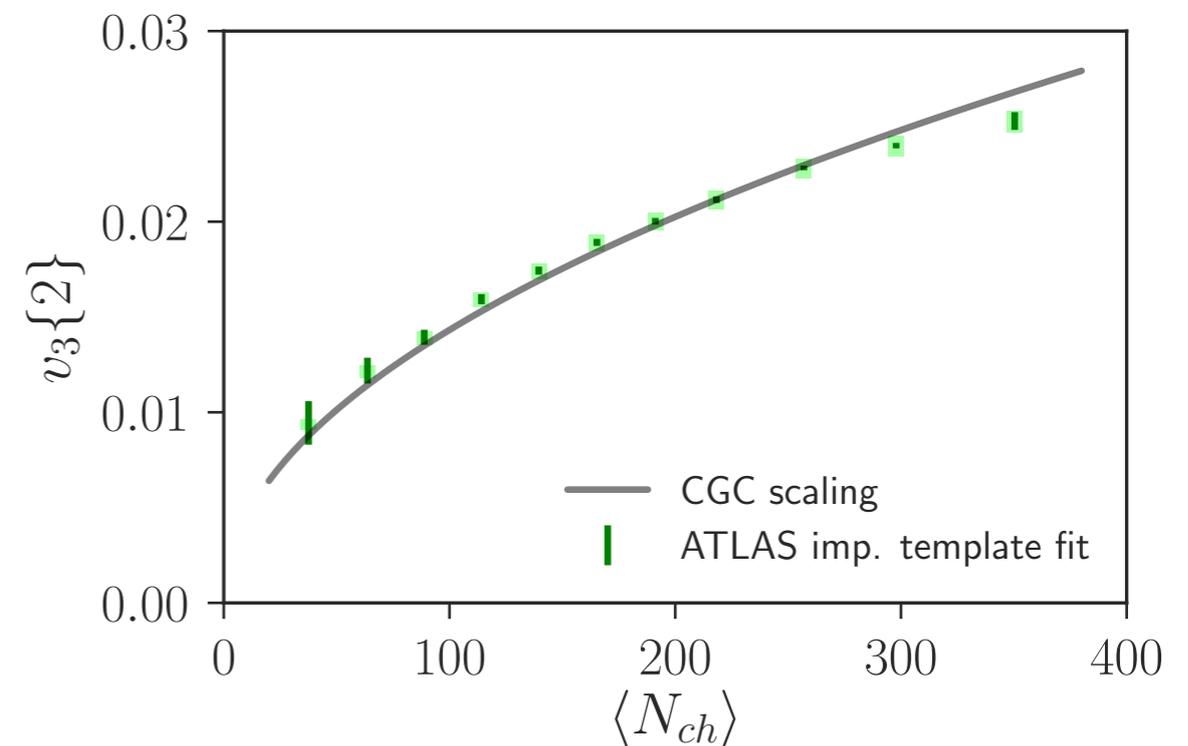
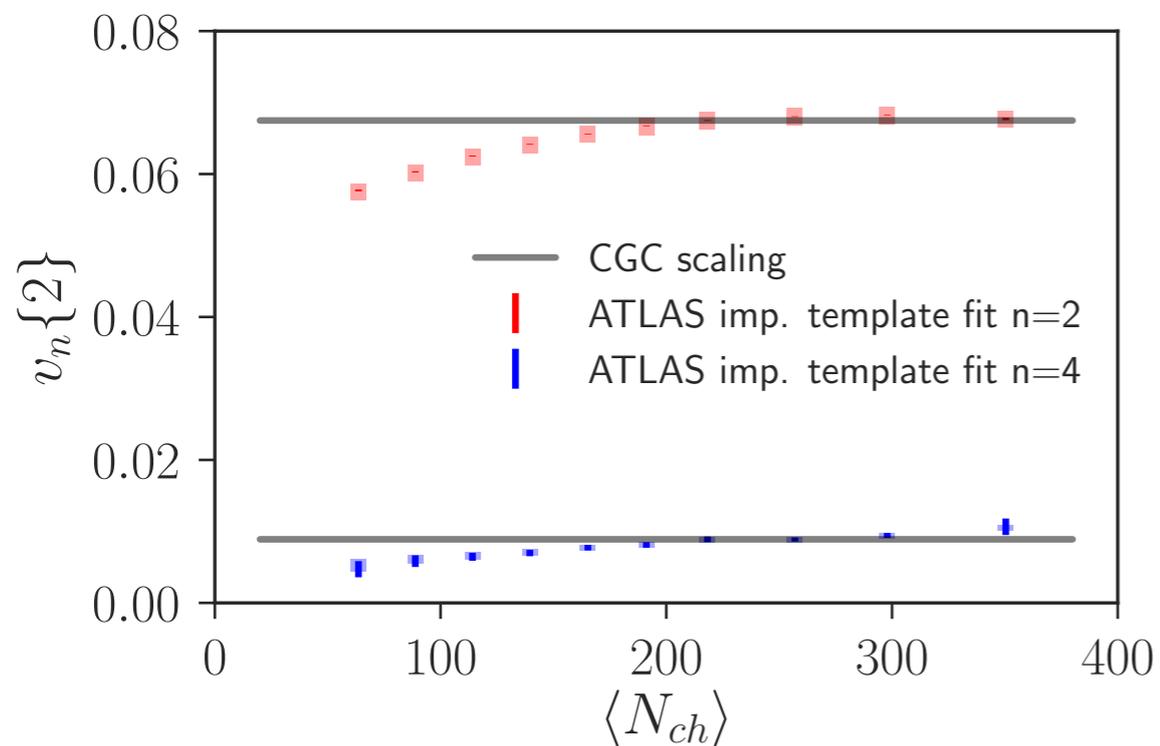
Fixing proportionality coefficient at a single multiplicity for each  $v_n$



High projectile density effects may explain large  $N_{ch}$  deviation

# Dilute-dense CGC scaling

Fixing proportionality coefficient at a single multiplicity for each  $v_n$



Realized with numerics!

# Initial configurations

For initial nuclear configurations, use data-guided approach similar to IP-Glasma model

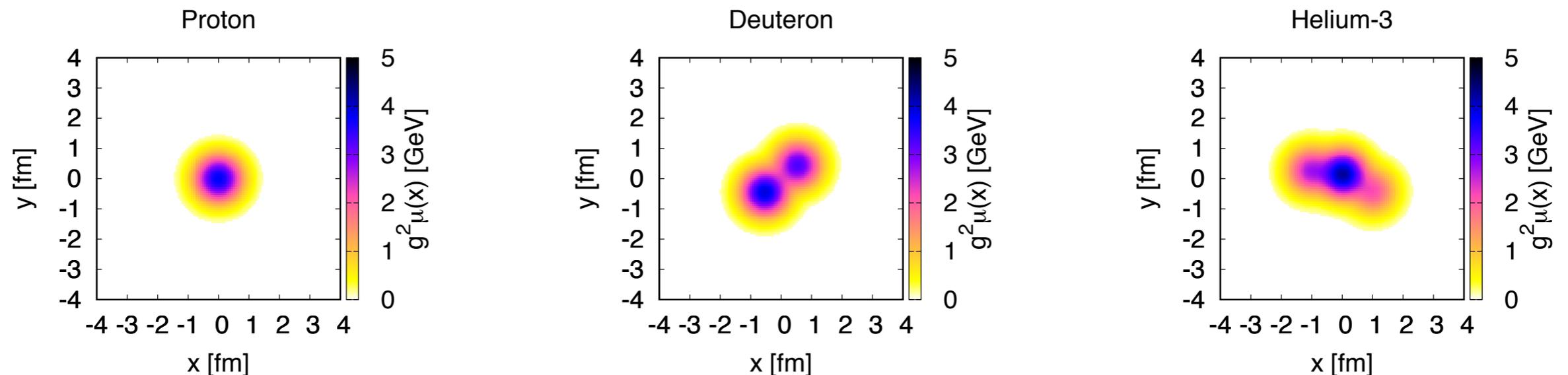
*Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)*

Sample nucleon positions as is done in Monte-Carlo Glauber

IP-Sat model (+fluctuations) provides  $Q_s^2(x, \mathbf{b})$  for each nucleon

*Kowalski, Teaney, Phys.Rev. D68 (2003) 114005, McLerran, Tribedy NPA 945 (2016)*

Example of three high multiplicity (0-5%) configurations



Color charge fluctuations sampled event-by-event with MV

model:  $\langle \rho_{p/T}^a(\mathbf{x}_\perp) \rho_{p/T}^b(\mathbf{y}_\perp) \rangle = g^2 \mu^2(x, \mathbf{b} = (\mathbf{x}_\perp + \mathbf{y}_\perp)/2) \delta^{ab} \delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)$

# Dilute-dense CGC EFT framework

From initial  $\rho$ 's, calculate particle production — includes quantum effects (BE, HBT)

Essential to account for color charge fluctuations; in particular for p+p

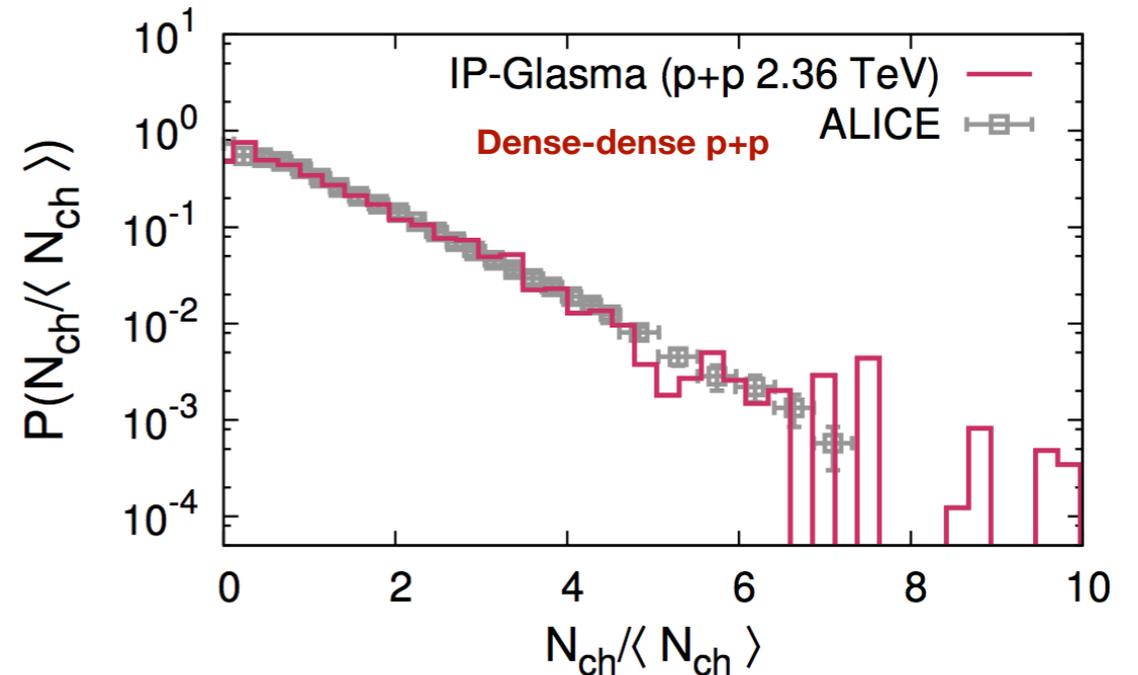
Generates negative binomial distributions from first principles, not an input!

*Gelis, Lappi, McLerran NPA 828 (2009)*

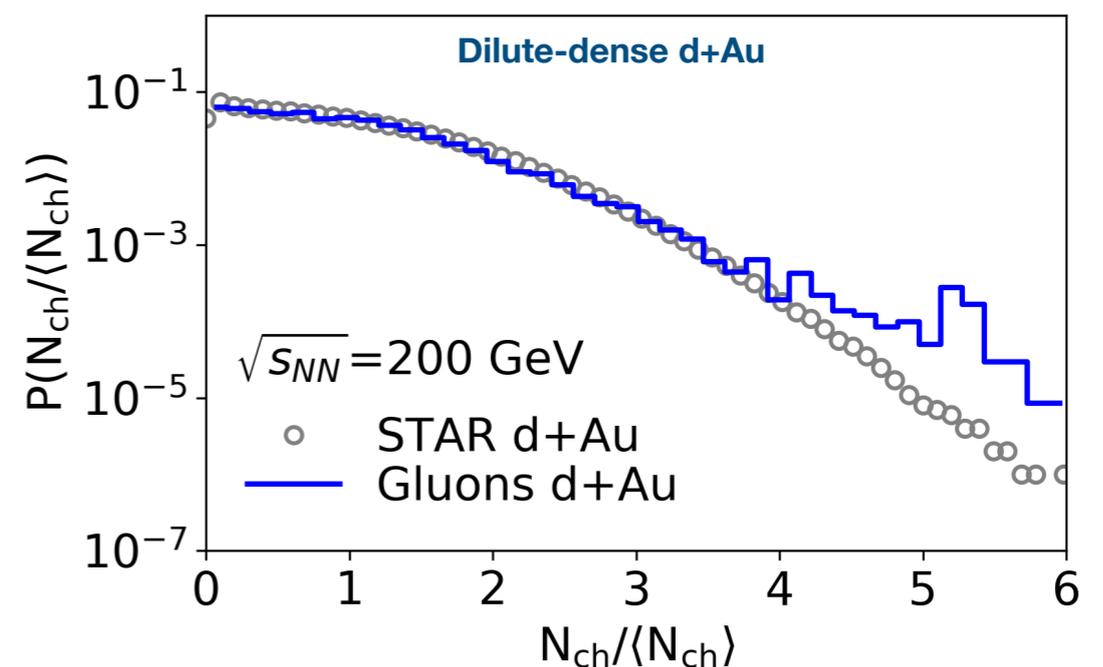
*Schenke, Tribedy, Venugopalan PRC 86 (2012)*

*McLerran, Tribedy NPA 945 (2016)*

Reasonable agreement found with STAR d+Au multiplicity distribution

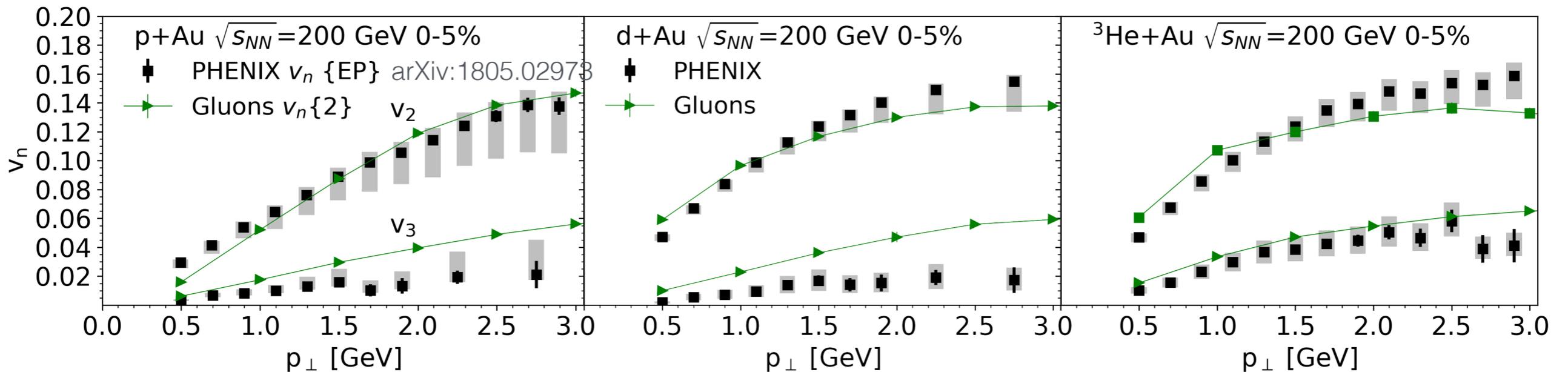


*McLerran, Tribedy NPA 945 (2016)*



*MM, Skokov, Tribedy, Venugopalan, arXiv:1805.09342  
STAR PRC 79 (2009)*

# Gluon correlations vs RHIC data for small systems



*MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018) arXiv:1805.09342*

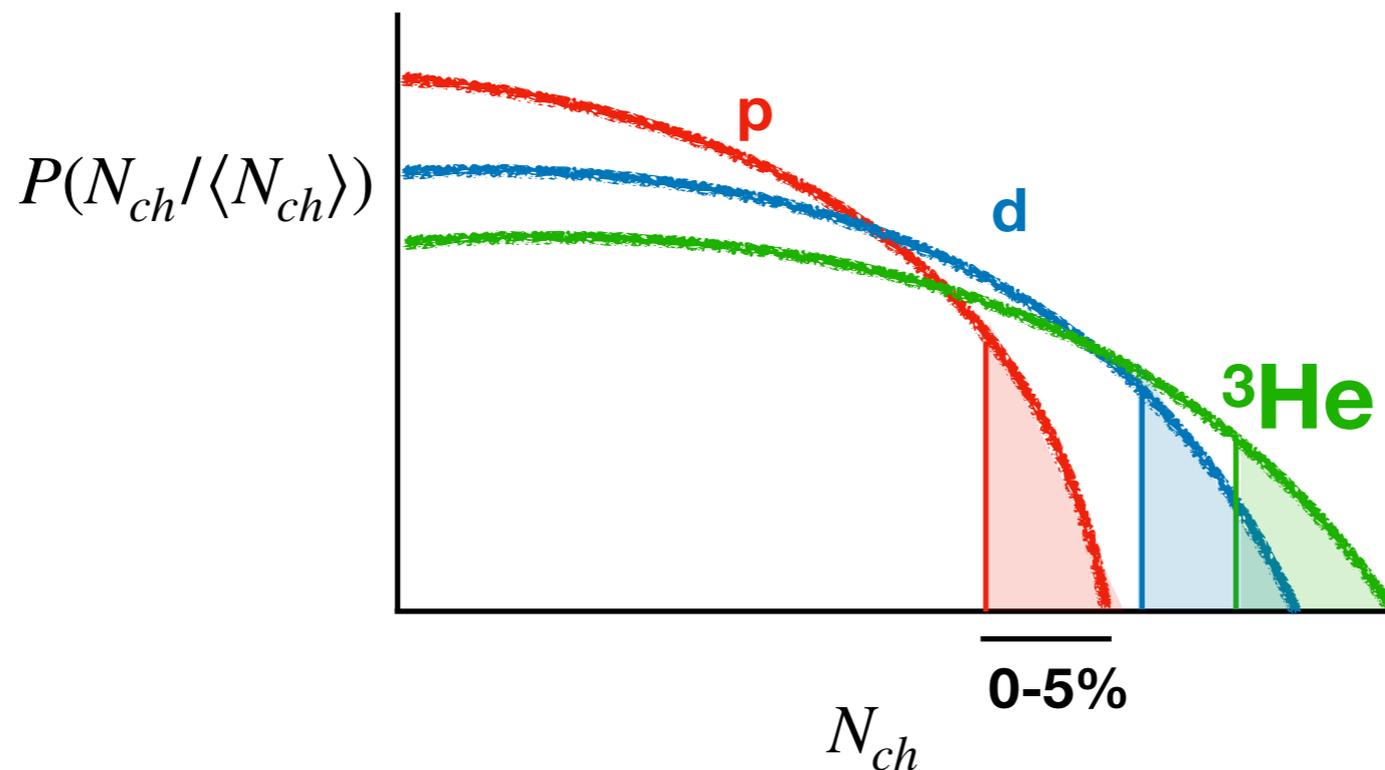
Key features of system dependence captured by initial state gluon correlations

$v_3$  known to be fluctuation dominated — mismatch on high multiplicity tail needs to be better understood

*Alver, Roland PRC 81 (2010)*

# Qu'est-ce que c'est?

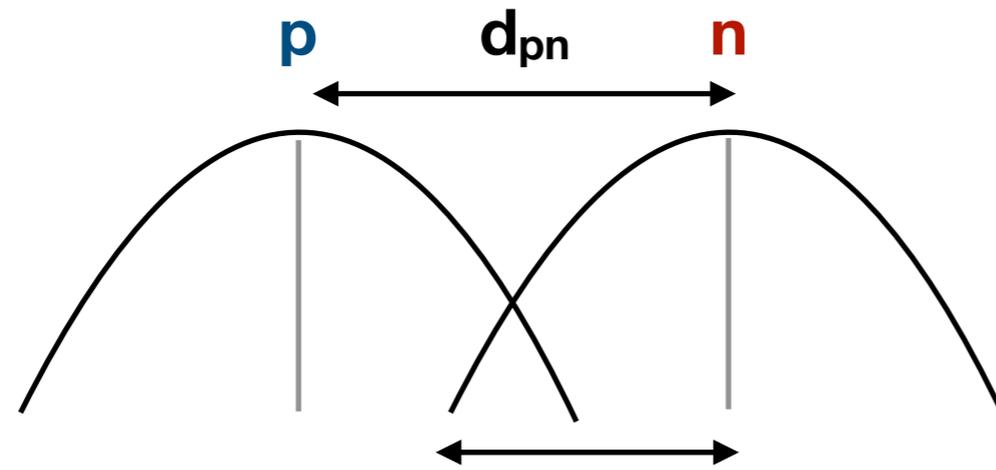
A naïve interpretation of results: Fixed multiplicity bin  $\mapsto$   
larger average  $N_{ch}$  for larger systems  $\mapsto$  larger average  $Q_s$   
 $\mapsto$  more correlations



Natural consequence: same multiplicity  $\sim$  similar correlations

However, we are considering non-linear QCD with quantum effects, classical intuition may be misleading

# Qu'est-ce que c'est?

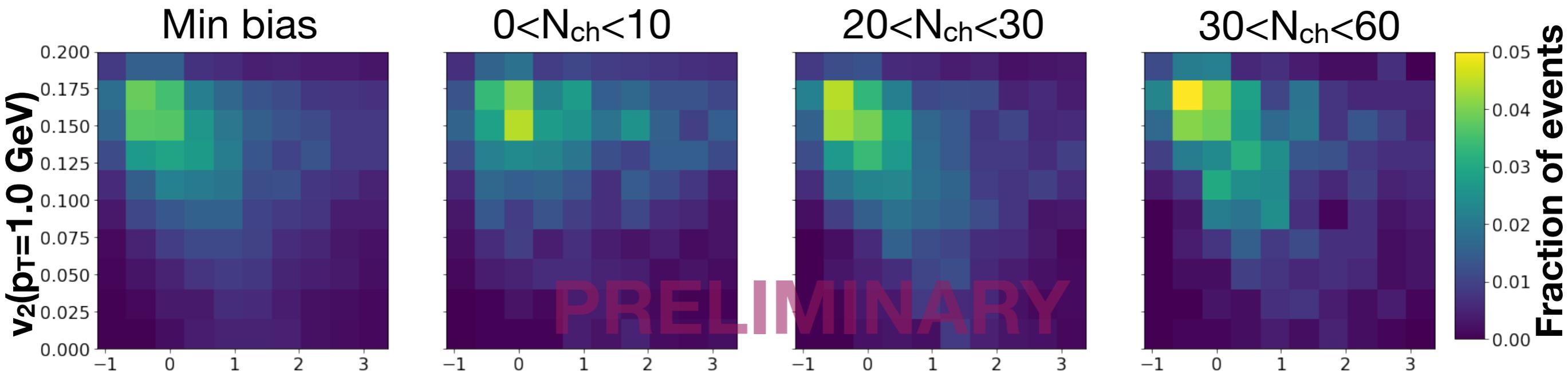


$$d_{\text{sep}} = |\mathbf{d}_{\text{pn}}| - 2d_{\text{gluon rms}}$$

$d_{\text{sep}} \approx -1 \text{ fm}$  — full overlap

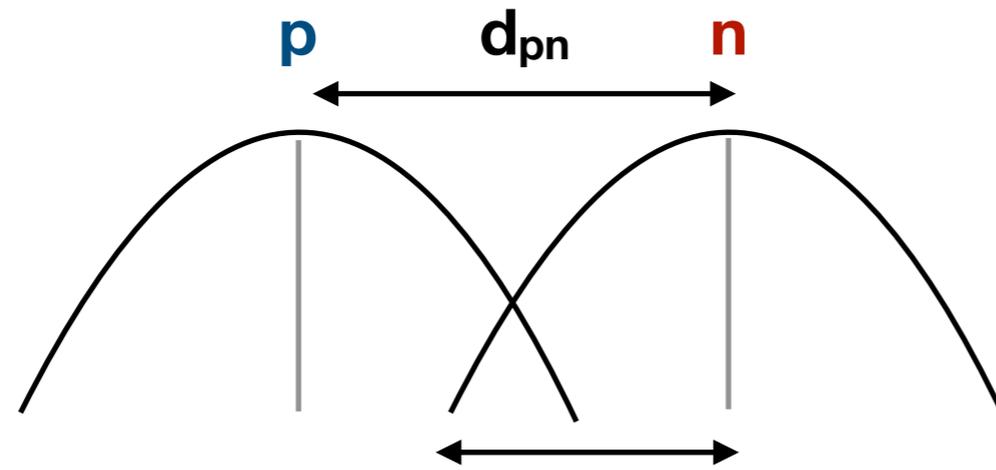
$d_{\text{sep}} \approx 0 \text{ fm}$  — overlapping tails

$$d_{\text{gluon rms}} = (2 B_G)^{1/2} \approx 0.56 \text{ fm}$$



Overlap of nucleon profiles:  $d_{\text{sep}} = |\mathbf{d}_{\text{pn}}| - 2d_{\text{gluon rms}}$  [fm]

# Qu'est-ce que c'est?

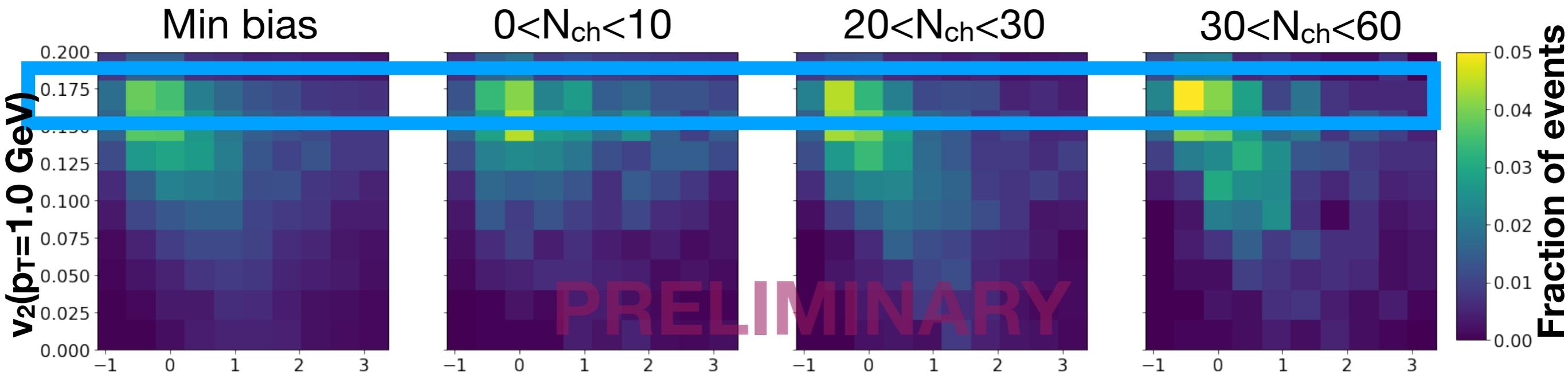


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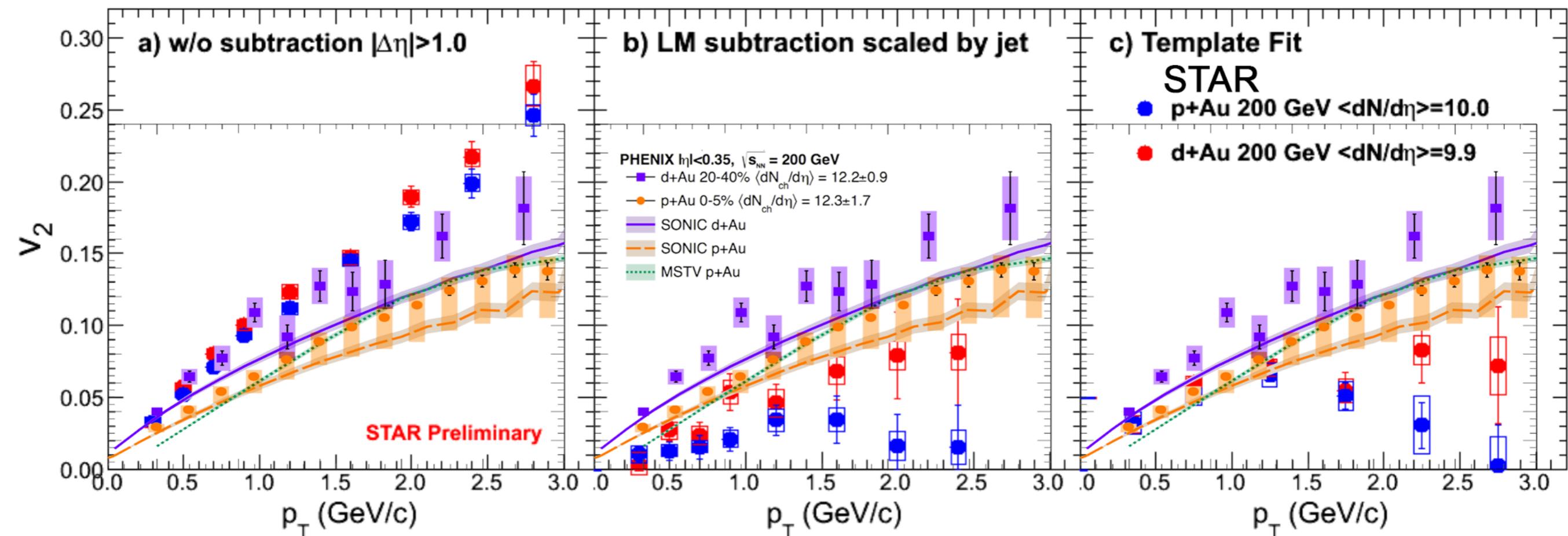
High multiplicity events bias towards overlapping nucleons in deuteron

Clearly distinct from spatial ellipticity+hydro response picture

# Same multiplicity p/d+Au

Dilute-dense CGC conjecture: same multiplicity  $\sim$  similar correlations

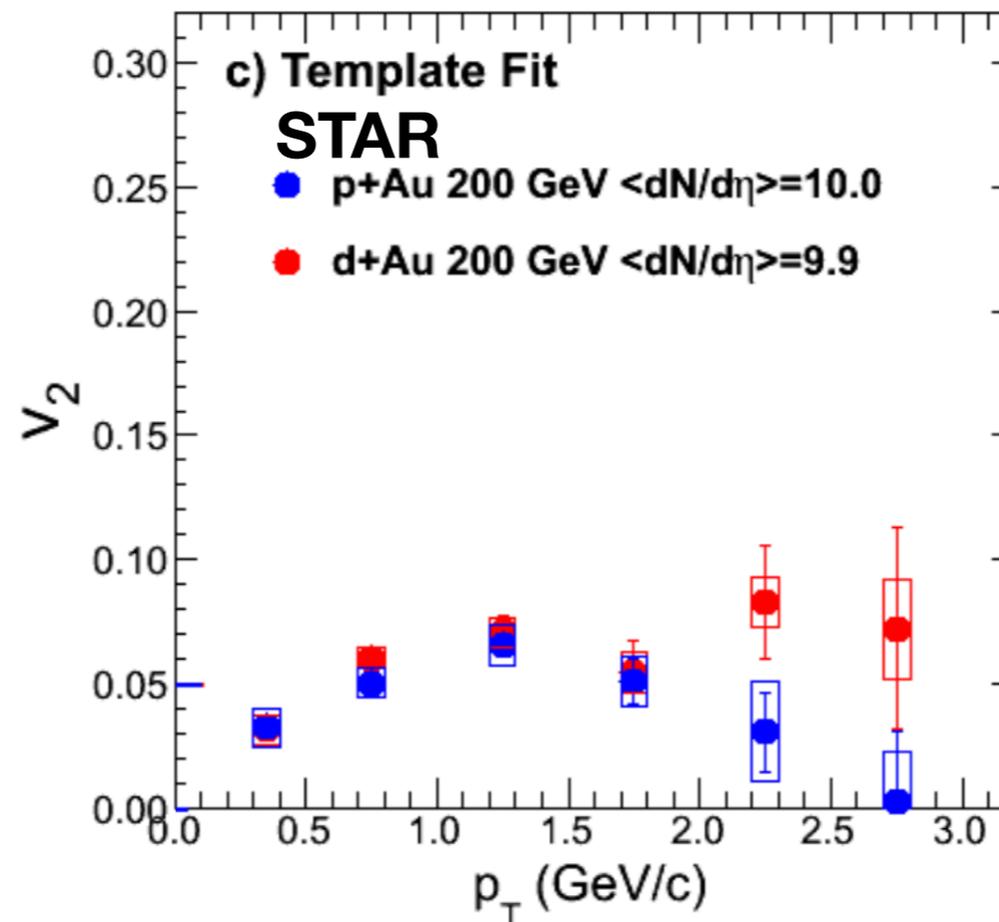
*MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018) arXiv:1805.09342*



*STAR from QM18 presentation by S Huang: <https://tinyurl.com/y95aeupu>, PHENIX arXiv:1805.02973*

Systematic uncertainties between experiments, methods

# Same multiplicity p/d+Au



STAR data from QM18 presentation by S Huang: <https://tinyurl.com/y95aeupu>

Intriguing result that p/d+Au at same multiplicity  
are *highly* compatible

# Conclusions

Dilute-dense CGC can explain multiplicity dependence of  $v_n$  at LHC

*MM, Skokov, Tribedy, Venugopalan, PLB (in press) arXiv:1807.00825*

Full dilute-dense CGC framework able to describe system size hierarchy of  $v_2$  and  $v_3$  at RHIC

*MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018) arXiv:1805.09342*

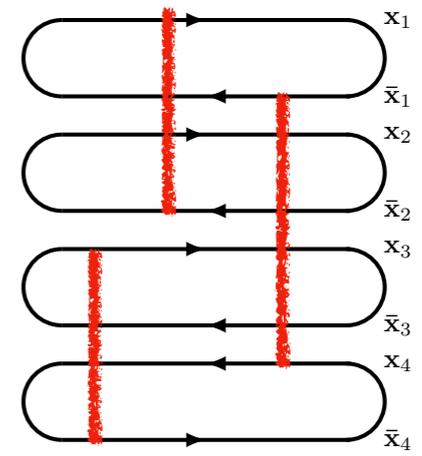
Color charge fluctuations and quantum correlations crucial features of framework, cannot be reproduced by classical intuitions

*MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018) arXiv:1805.09342, and in progress*

To quantify the roles of initial state and hydrodynamics, important to have  $p/{}^3\text{He}+\text{Au}$  multiplicity distributions and anisotropies in different event classes, different observables

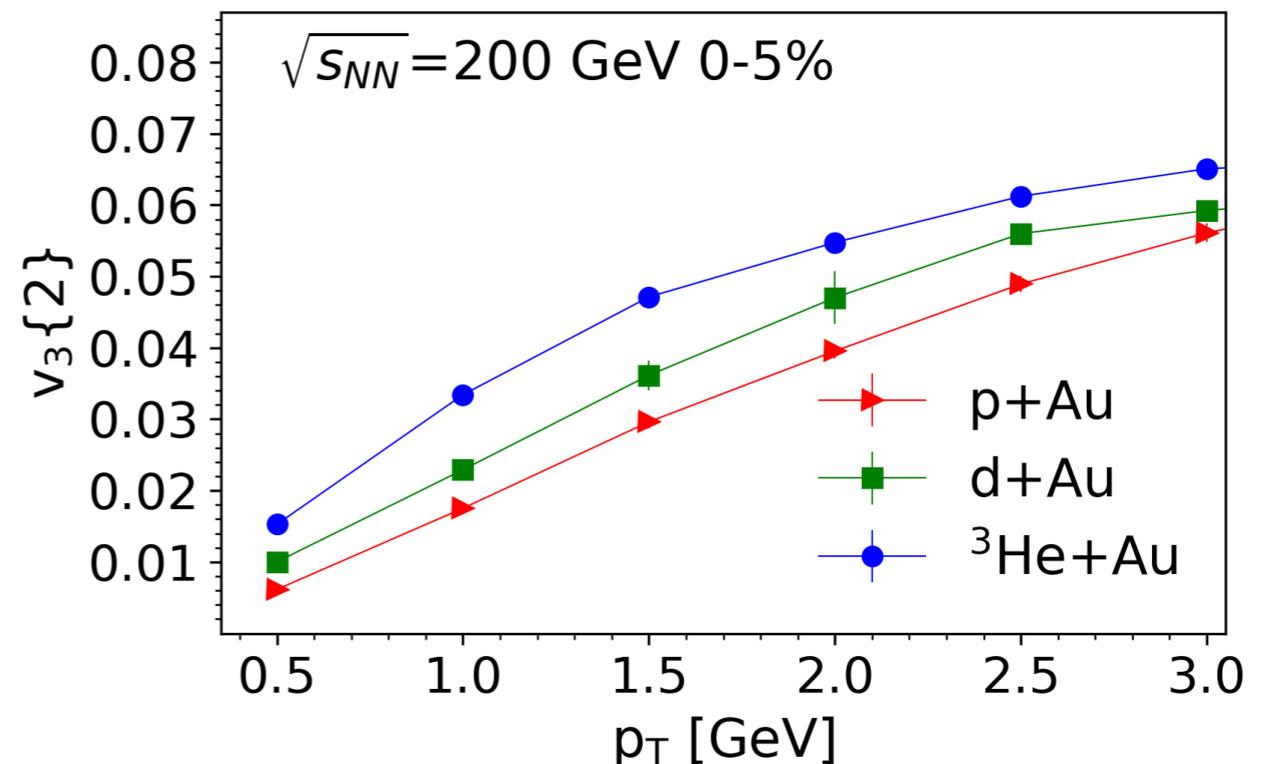
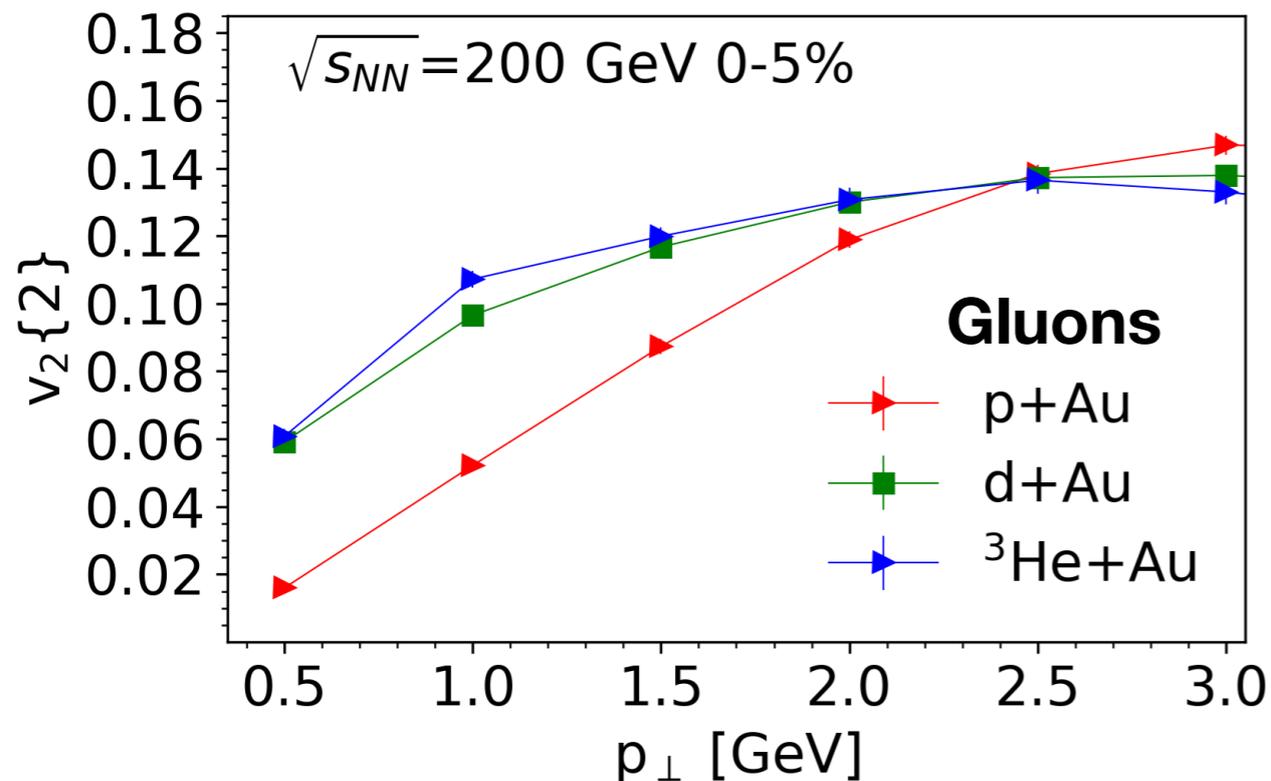


# BACKUP



# Hierarchy of anisotropies across systems

System size dependence at RHIC captured by CGC initial state  
gluon correlations



*MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018) arXiv:1805.09342*

# Quantifying systematic uncertainties

All parameters are fixed, even for p and  $^3\text{He}$ , by fit to STAR d+Au multiplicity distribution. **Would be useful to have p/ $^3\text{He}$ +Au multiplicity distributions**

Nuclear wave function: strong short-range correlations (measured at JLab). Exciting prospect; quantify influence on high multiplicity events

*c.f. Hen, Miller, Piasezky, Weinstein Rev.Mod.Phys. 89 (2017);  
Cruz-Torres, Schmidt, Miller, Weinstein, Barnea, Weiss, Piasezky, Hen arXiv:1710.07966  
Hen, MM, Schmidt, Venugopalan, in progress.*

Fragmentation — uncertainty which enters in multiplicity spectrum and  $v_n$ : CGC+Lund string model can be applied here

*e.g. Schenke, Schlichting, Tribedy, Venugopalan, PRL 117 (2016) no.16, 162301*

# Fluctuating initial shape

Constrain proton shape fluctuations from comparison to exclusive  $J/\Psi$  production (HERA)

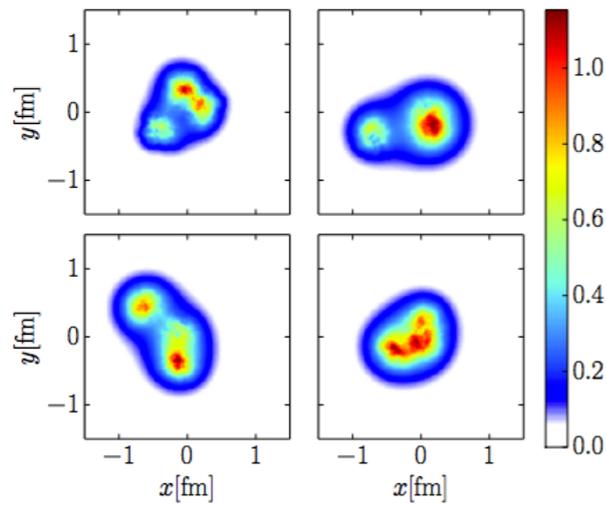


Fig. 3. Example of the proton density profiles at  $x \approx 10^{-3}$ . The quantity shown is  $1 - \text{Re Tr}V(\mathbf{x})/N_c$ .

Incoherent cross section sensitive to fluctuations

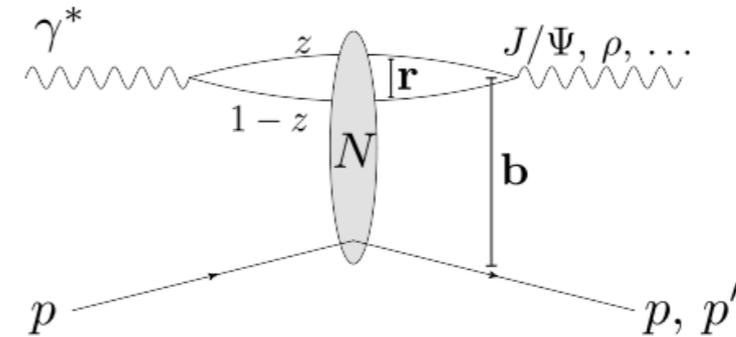
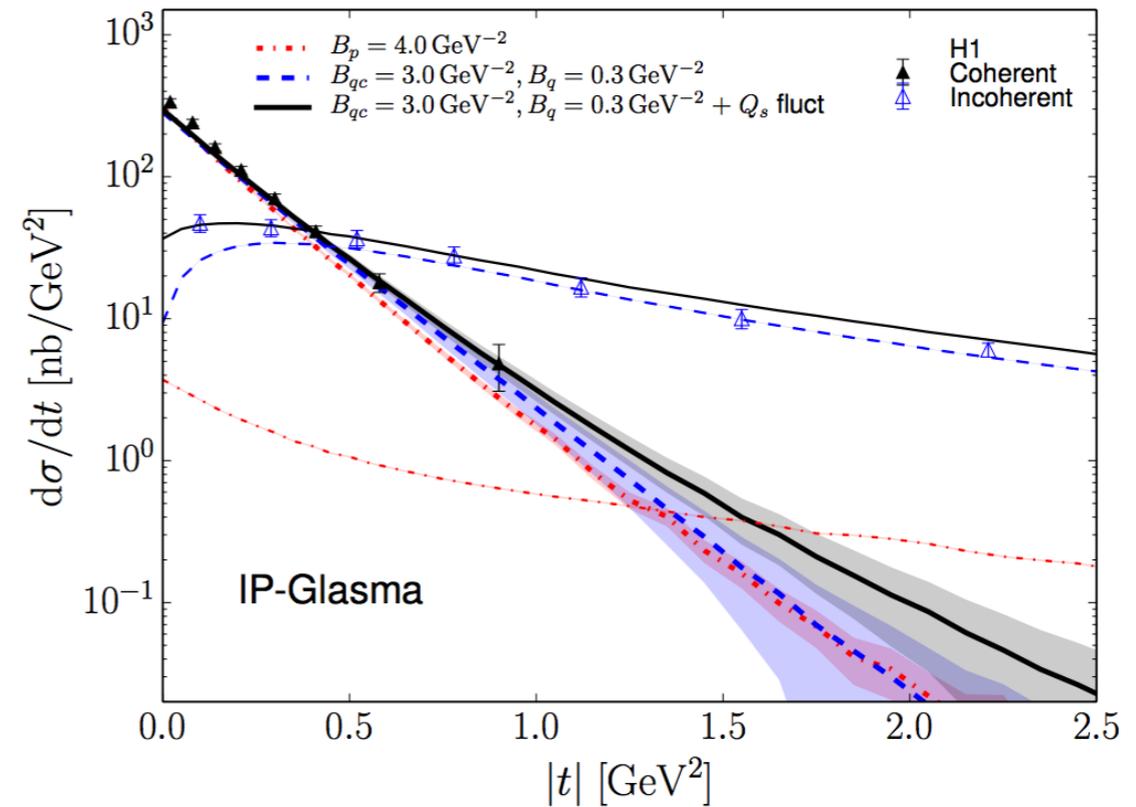


Fig. 1. Diffractive vector meson production in dipole picture.

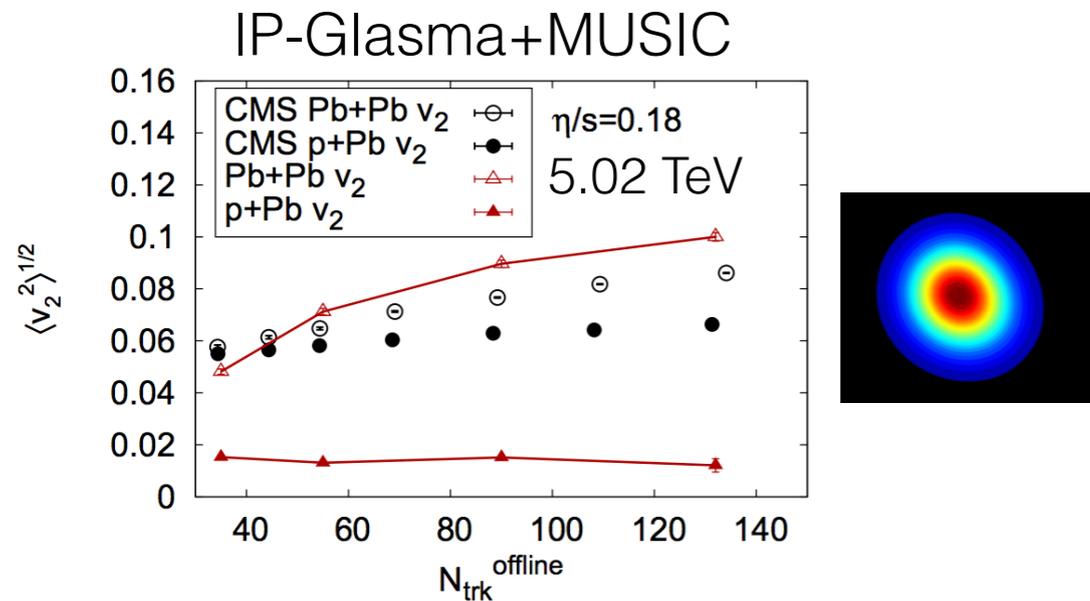


Mäntysaari, Schenke, PRL 117 (2016) 052301; PRD 94 (2016) 034042

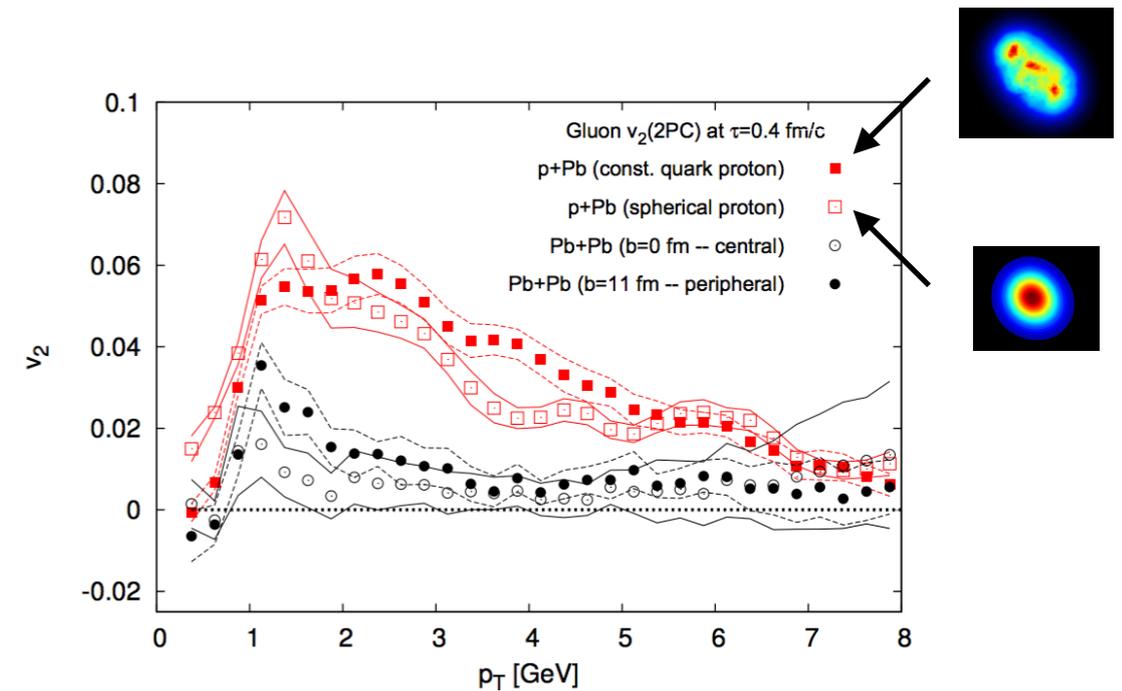
# Fluctuating projectile

Important for spatial eccentricity driven models (hydro)

CGC has only momentum-space correlations

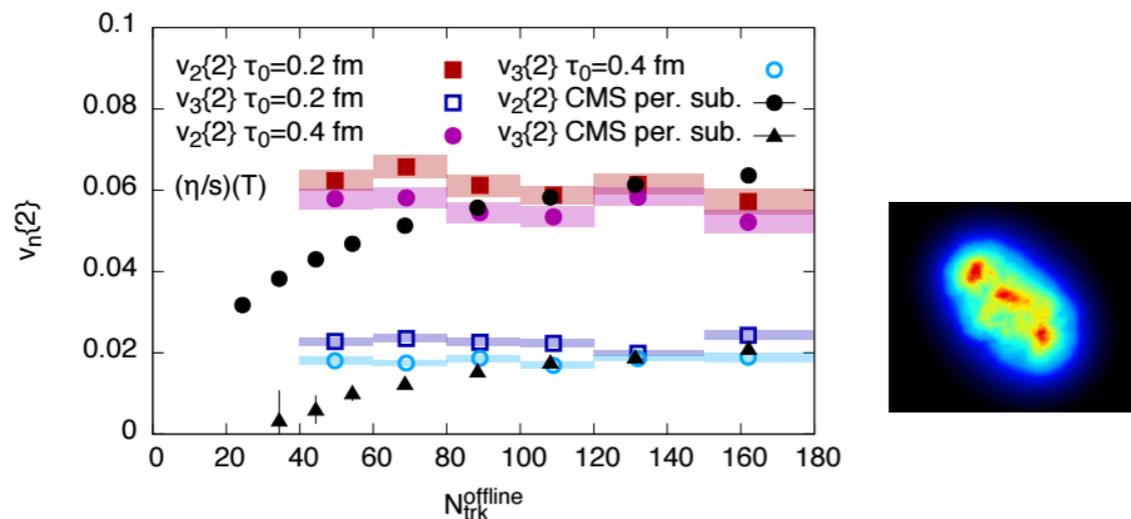


Schenke, Venugopalan PRL 113 (2014) 102301



Schlichting, Schenke, Venugopalan PLB 742 (2015)

IP-Glasma+**Fluct. proton**+MUSIC+UrQMD



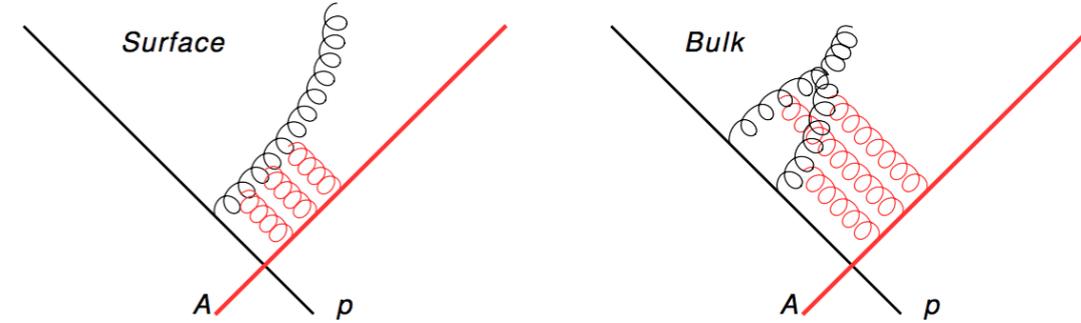
Mäntysaari, Schenke, Shen, Tribedy arXiv:1705.03177

No qualitative difference observed

# Dilute dense for gluons

Nonzero  $v_3$  at next order in  $\rho_p$   
 Symmetry broken in  $\frac{d^2 N}{d^3 k_1 d^3 k_2}$  by first  
 saturation correction  $O(\rho_p^6)$

*McLerran, Skokov NPA 959 (2017)*



*McLerran, Skokov NPA 959*

Event-by-event particle production given by

$$\frac{dN^{\text{even}}(\mathbf{k})}{d^2 k dy} [\rho_p, \rho_t] = \frac{2}{(2\pi)^3} \frac{\delta_{ij} \delta_{lm} + \epsilon_{ij} \epsilon_{lm}}{k^2} \Omega_{ij}^a(\mathbf{k}) [\Omega_{lm}^a(\mathbf{k})]^*$$

$$\frac{dN^{\text{odd}}(\mathbf{k})}{d^2 k dy} [\rho_p, \rho_T] = \frac{2}{(2\pi)^3} \text{Im} \left\{ \frac{g}{\mathbf{k}^2} \int \frac{d^2 l}{(2\pi)^2} \frac{\text{Sign}(\mathbf{k} \times \mathbf{l})}{l^2 |\mathbf{k} - \mathbf{l}|^2} f^{abc} \Omega_{ij}^a(\mathbf{l}) \Omega_{mn}^b(\mathbf{k} - \mathbf{l}) [\Omega_{rp}^c(\mathbf{k})]^* \times \right. \\ \left. [(\mathbf{k}^2 \epsilon^{ij} \epsilon^{mn} - \mathbf{l} \cdot (\mathbf{k} - \mathbf{l}) (\epsilon^{ij} \epsilon^{mn} + \delta^{ij} \delta^{mn})) \epsilon^{rp} + 2\mathbf{k} \cdot (\mathbf{k} - \mathbf{l}) \epsilon^{ij} \delta^{mn} \delta^{rp}] \right\}$$

Projectile Target

In terms of:  $\Omega_{ij}^a(\mathbf{x}) = g \left[ \frac{\partial_i}{\partial^2} \rho_p^b(\mathbf{x}) \right] \partial_j U^{ab}(\mathbf{x})$  Projectile gluon fields rotated by target

$$\frac{d^n N}{d^2 k_1 dy_1 \cdots d^2 k_n dy_n} = \left\langle \left\langle \frac{dN}{d^2 k_1 dy_1} \Big|_{\rho_p, \rho_T} \cdots \frac{dN}{d^2 k_1 dy_1} \Big|_{\rho_p, \rho_T} \right\rangle_p \right\rangle_T$$

# The $v_3$ problem

Leading order dilute-dense limit highly amenable to numerics

*Lappi EPJC 55 (2008)*

For double inclusive,  $\frac{d^2 N}{d^3 k_1 d^3 k_2}$ , leading order is also known

*Kovner, Lublinsky, IJMPE 22 (2013), Kovchegov, Wertepny, NPA 906, (2013)*

$$\frac{d^2 N}{d^2 k_1 dy_1 d^2 k_2 dy_2} = \frac{d^2 N}{k_1 dk_1 dy_1 k_2 dk_2 dy_2} (1 + 2v_2^2 \{2\} \cos 2(\phi_1 - \phi_2) + 2v_3^2 \{2\} \cos 3(\phi_1 - \phi_2) + \dots)$$

For a non-zero  $v_3$

*McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)*

$$\begin{aligned} \int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2 N}{d^2 k_1 d^2 k_2} (\delta\phi) &= \int_0^\pi d\Delta\phi \cos 3\Delta\phi \frac{d^2 N}{d^2 k_1 d^2 k_2} (\delta\phi) - \int_0^\pi d\Delta\phi \cos 3\Delta\phi \frac{d^2 N}{d^2 k_1 d^2 k_2} (\delta\phi + \pi) \\ &= \int_0^\pi d\Delta\phi \cos 3\Delta\phi \left[ \frac{d^2 N}{d^2 k_1 d^2 k_2} (\mathbf{k}_1, \mathbf{k}_2) - \frac{d^2 N}{d^2 k_1 d^2 k_2} (\mathbf{k}_1, -\mathbf{k}_2) \right] \end{aligned}$$

**Must be non-vanishing**

But, LO ( $O(\rho_p^4)$ ) exactly zero, but not in dense-dense (all  $O(\rho_p^\#)$ )

*Kovner, Lublinsky, PRD 83 (2011), Kovchegov, Wertepny, NPA 906 (2013),  
Lappi, Srednyak, Venugopalan JHEP 1001 (2010), Schenke, Schlichting, Venu*

# Dilute-dense CGC scaling

$$\rho_p \rightarrow c\rho_p \quad \longrightarrow \quad \Omega \rightarrow c\Omega$$

Multiplicity is then rescaled as  $\frac{dN}{dy} [\rho_p, \rho_t] \rightarrow c^2 \frac{dN}{dy} [\rho_p, \rho_t] + \mathcal{O}(c^3)$

Rescaling Fourier moments

$$V_{2n} = \frac{\int_{k,\phi,p} e^{i2n\phi} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2kdy} [\rho_p, \rho_t]}{\int_{k,\phi,p} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2kdy} [\rho_p, \rho_t]} \rightarrow c^0 V_{2n}, \quad V_{2n+1} = \frac{\int_{k,\phi,p} e^{i(2n+1)\phi} \frac{dN^{\text{odd}}(\mathbf{k}_\perp)}{d^2kdy} [\rho_p, \rho_t]}{\int_{k,\phi,p} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2kdy} [\rho_p, \rho_t]} \rightarrow c V_{2n+1}$$

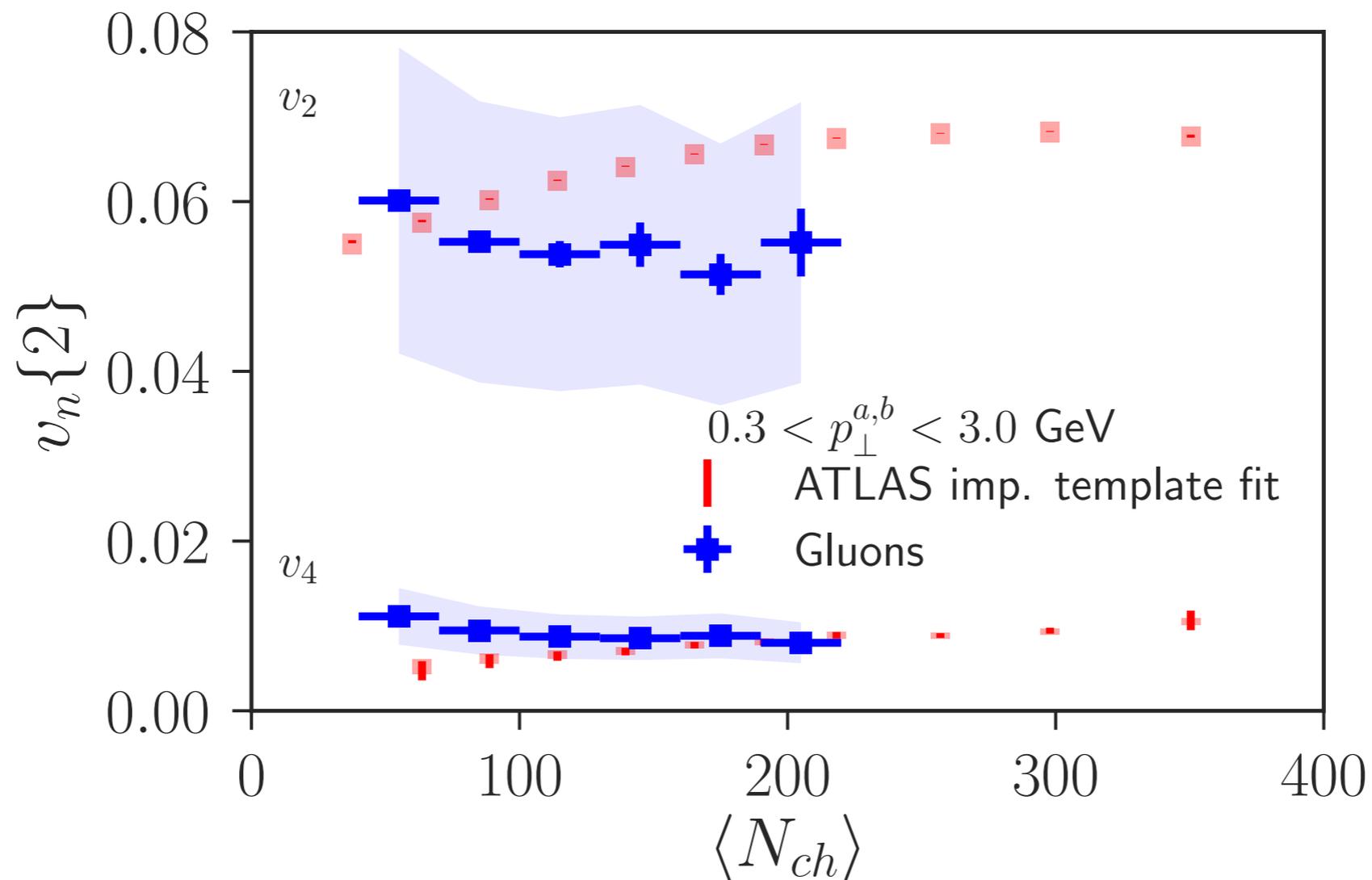
In terms of multiplicity

$$V_{2n}(p_1, p_2) \sim \left( \frac{dN}{dy} [\rho_p, \rho_t] \right), \quad V_{2n+1}(p_1, p_2) \sim \left( \frac{dN}{dy} [\rho_p, \rho_t] \right)^0$$

Dilute-dense CGC scaling is then

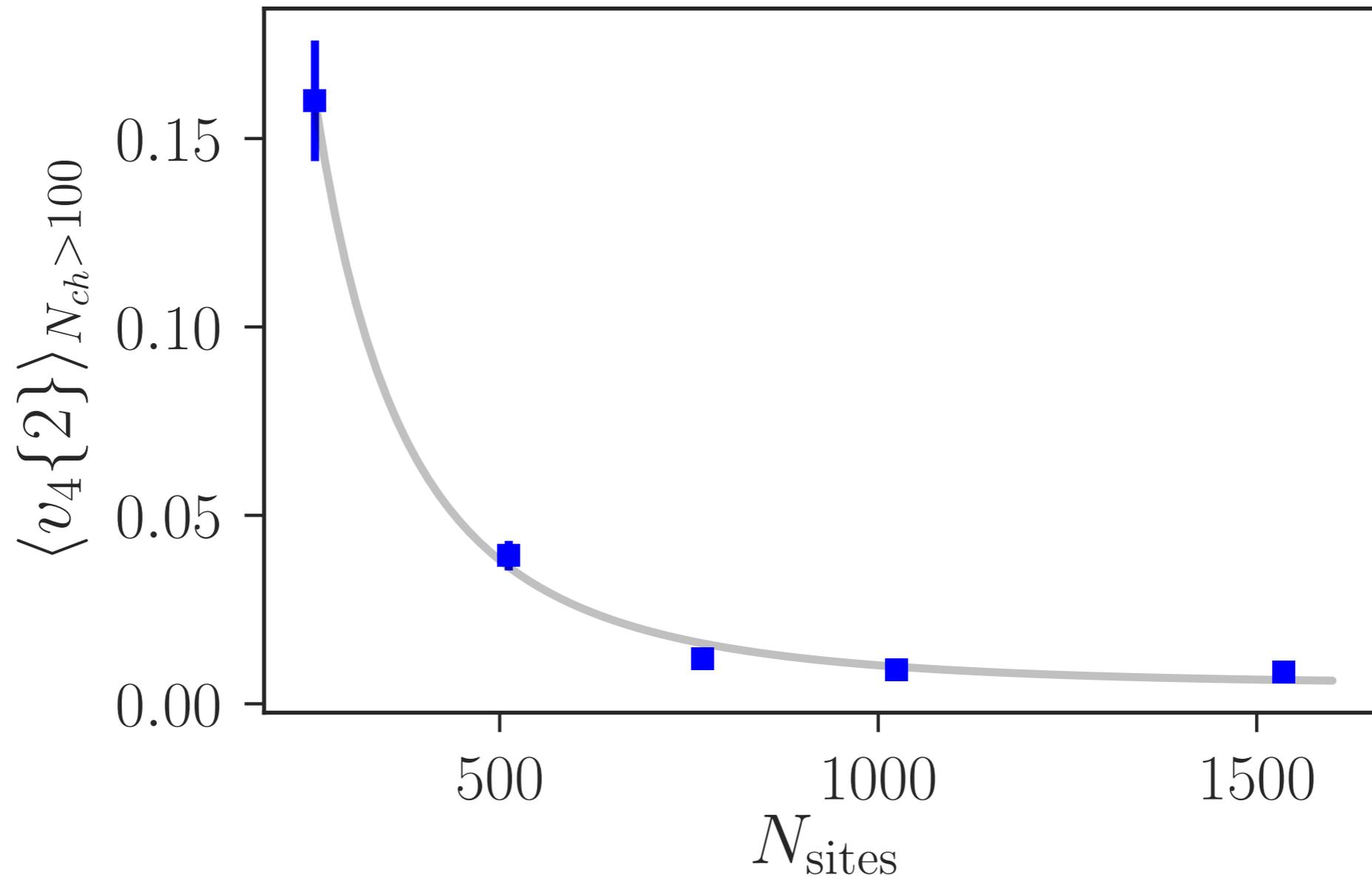
$$v_{2n}\{2\} \sim N_{ch}^0, \quad v_{2n+1}\{2\} \sim N_{ch}^{1/2}$$

# Numerics of scaling



$v_3$  extremely computationally intensive

# Continuum limit $v_4$



# A parton model

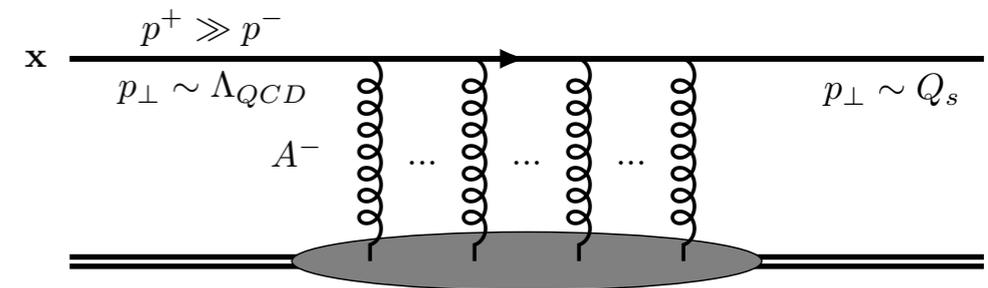
Working in dilute-dense limit:  $Q_s(\text{target}) \gg Q_s(\text{projectile})$ ,  
 consider eikonal quark scattering off dense nuclear target  
 with color domains of size  $\sim 1/Q_{s,T}$

*Lappi, PLB 744, 315 (2015); Lappi, Schenke, Schlichting, Venugopalan, JHEP 1601 (2016) 061; Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018)*

Quark coherent multiple scattering off target represented by  
 Wilson line phase

*Bjorken, Kogut, Soper, PRD (1971), Dumitru, Jalilian-Marian, PRL 89 (2002)*

$$U(\mathbf{x}) = \mathcal{P} \exp \left( -ig \int dz^+ A^{a-}(\mathbf{x}, z^+) t^a \right)$$



Single quark inclusive distribution

$$\left\langle \frac{dN_q}{d^2\mathbf{p}} \right\rangle \simeq \int_{\mathbf{b}, \mathbf{r}, \mathbf{k}} e^{-|\mathbf{b}|^2/B_p} e^{-|\mathbf{k}|^2 B_p} e^{i(\mathbf{p}-\mathbf{k}) \cdot \mathbf{r}} \left\langle \frac{1}{N_c} \text{Tr} \left( U\left(\mathbf{b} + \frac{\mathbf{r}}{2}\right) U^\dagger\left(\mathbf{b} - \frac{\mathbf{r}}{2}\right) \right) \right\rangle$$

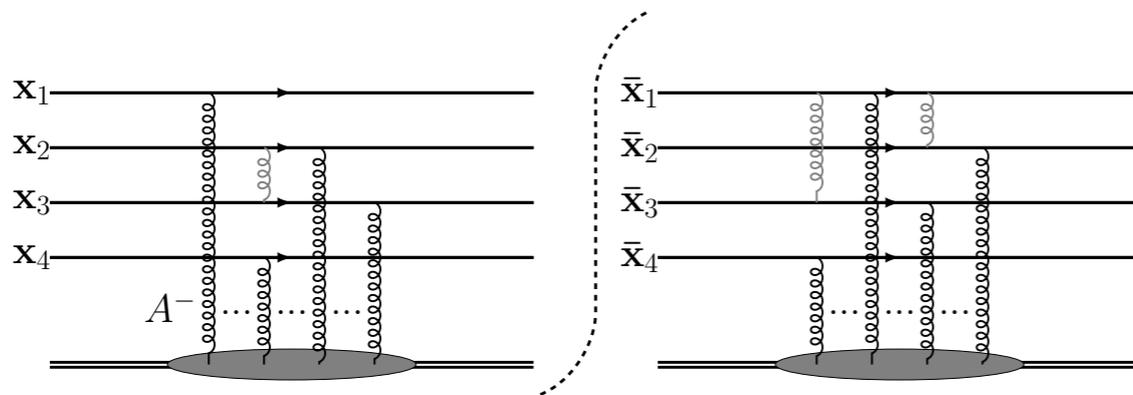
Projectile: Wigner function

Target scattering:  
 Dipole operator  $D(\mathbf{x}, \mathbf{y})$

\*Single scale to defines projectile  $B_p = 4 \text{ GeV}^{-2}$  from HERA DIS fits

# A parton model

Generalizing for multiple particle correlations for *simple* model of multi particle correlations  $\left\langle \frac{d^m N}{d^2 \mathbf{p}_1 \dots d^2 \mathbf{p}_m} \right\rangle = \left\langle \frac{dN}{d^2 \mathbf{p}_1} \dots \frac{dN}{d^2 \mathbf{p}_m} \right\rangle \sim \int \langle D \dots D \rangle$



**Novel method to compute arbitrary Wilson line correlators in MV - arXiv:1706.06260**

$dN/d^2 \mathbf{p}$  itself is not well defined. Average over classical configurations and over all events using MV model

McLerran, Venugopalan, PRD 49, 3352, 2233 (1994)

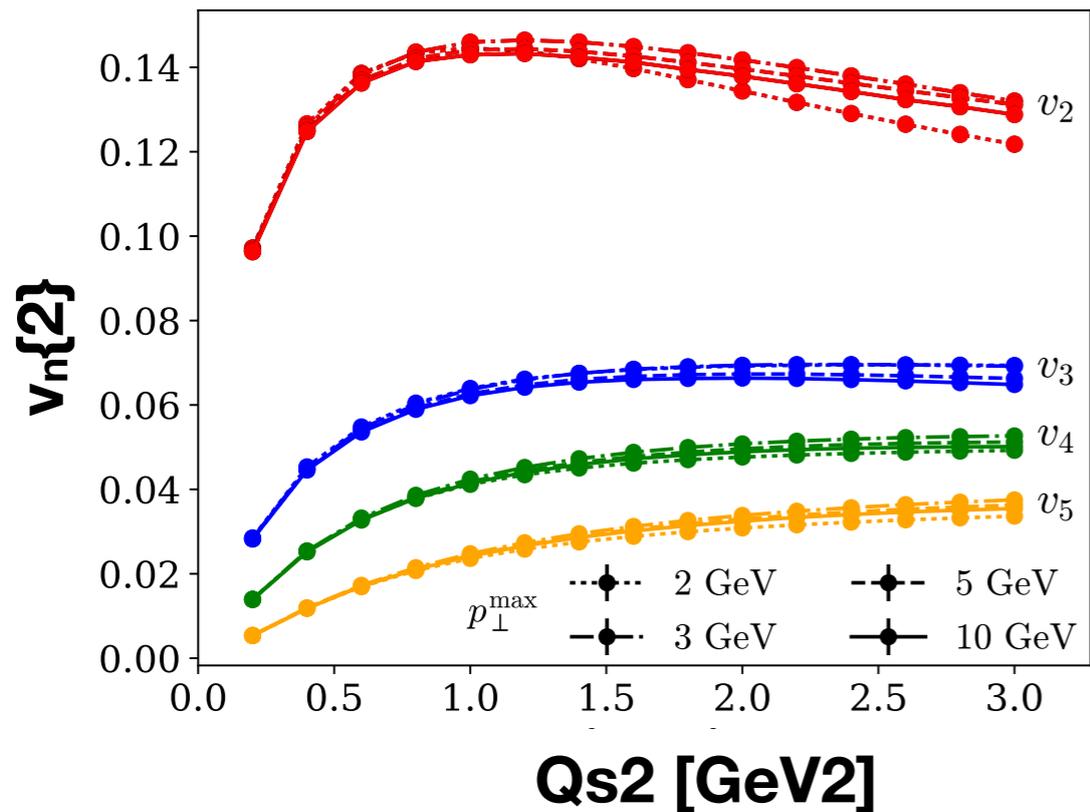
Generate cumulants, integrate to scale  $p_{\perp}^{max}$

$$\kappa_n \{m\} = \int_{\mathbf{p}_1 \dots \mathbf{p}_m} \cos(n(\phi_1^p + \dots - \phi_m^p)) \left\langle \frac{d^m N}{d^2 \mathbf{p}_1 \dots d^2 \mathbf{p}_m} \right\rangle$$

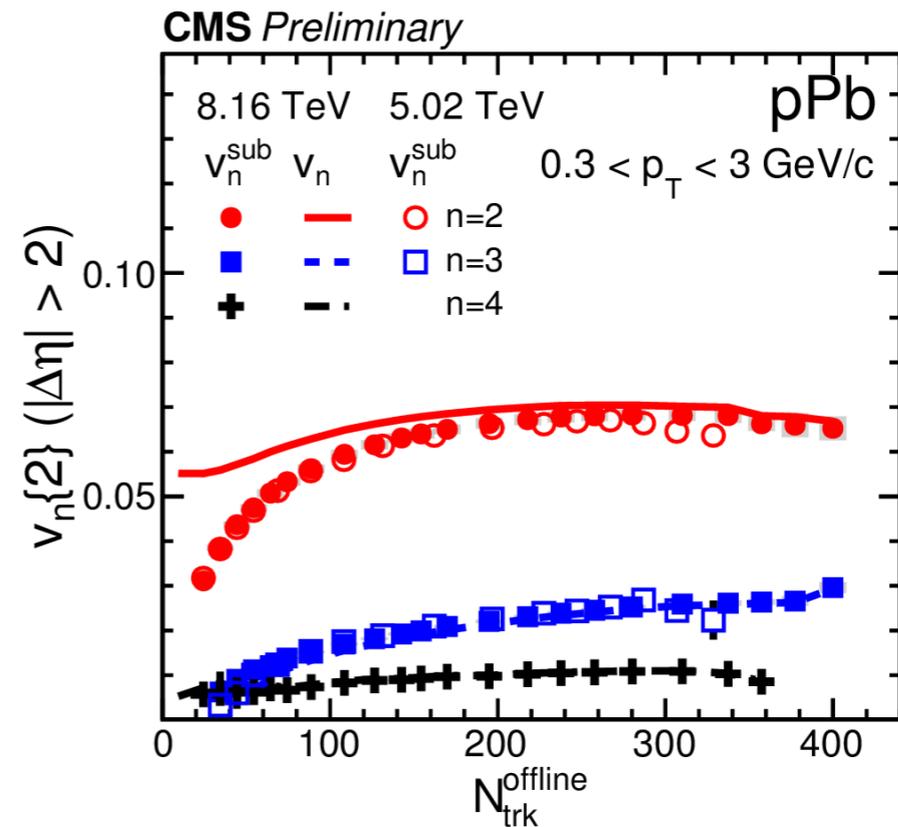
$$c_2 \{2\} = \frac{\kappa_2 \{2\}}{\kappa_0 \{2\}}, \quad c_2 \{4\} = \frac{\kappa_2 \{4\}}{\kappa_0 \{4\}} - 2 \left( \frac{\kappa_2 \{2\}}{\kappa_0 \{2\}} \right)^2, \dots$$

# Multi-particle quark correlations

Ordering in two particle Fourier harmonics similar to data



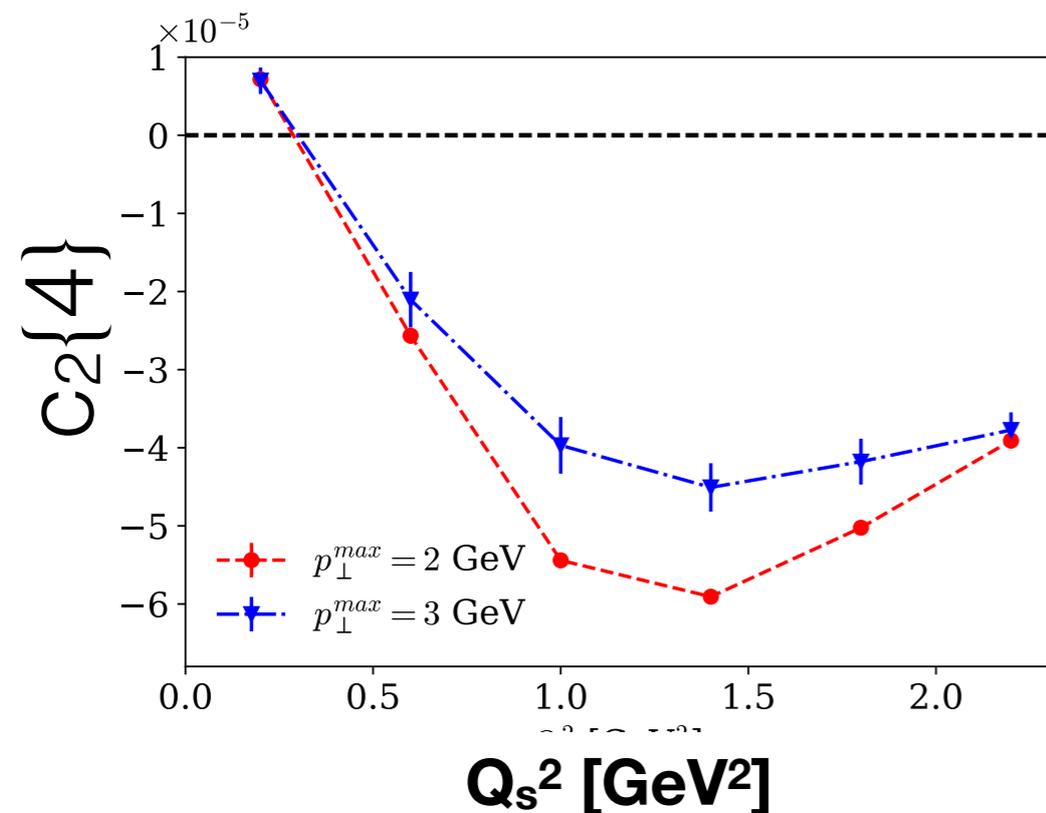
Dusling, MM, Venugopalan PRL 120 (2018)



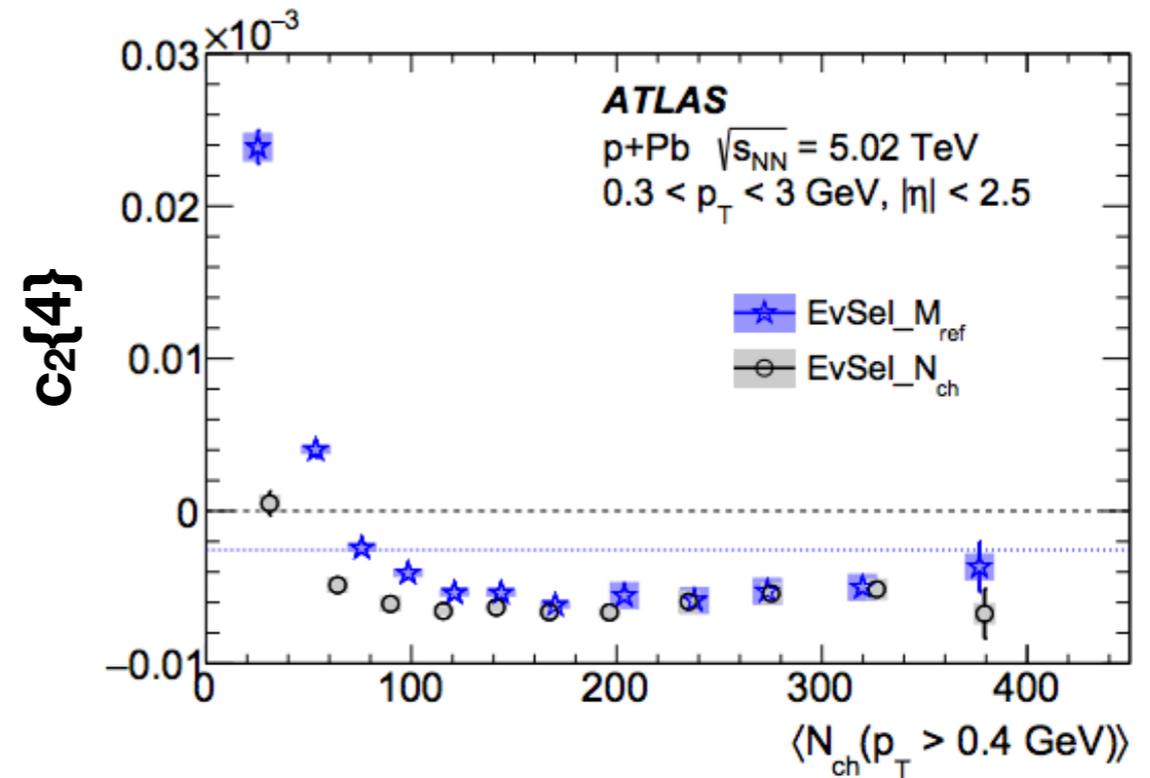
CMS-PAS-HIN-16-022

# Multi-particle quark correlations

$c_2\{4\}$  becomes negative for increasing  $Q_s$



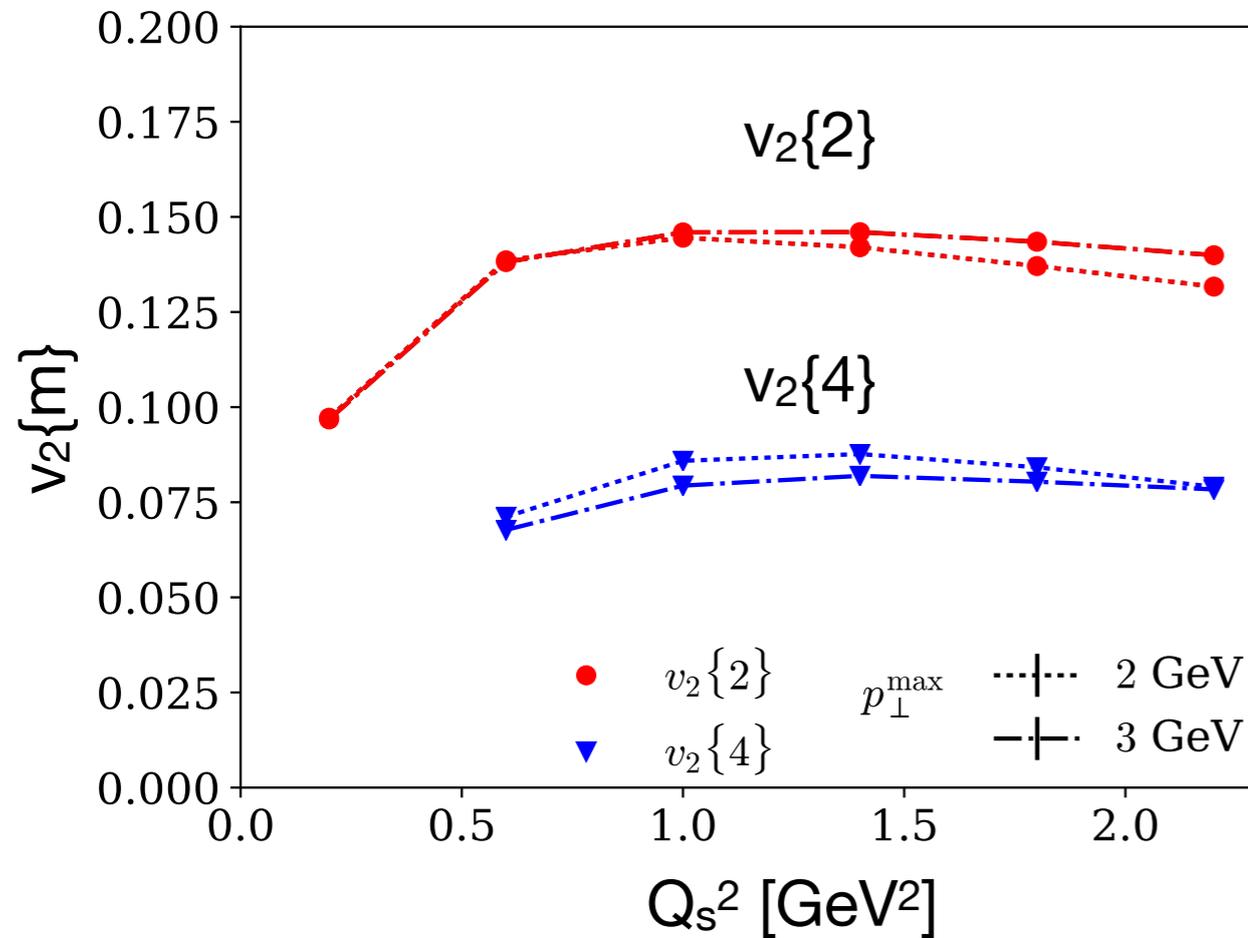
Dusling, MM, Venugopalan PRD 97 (2018)



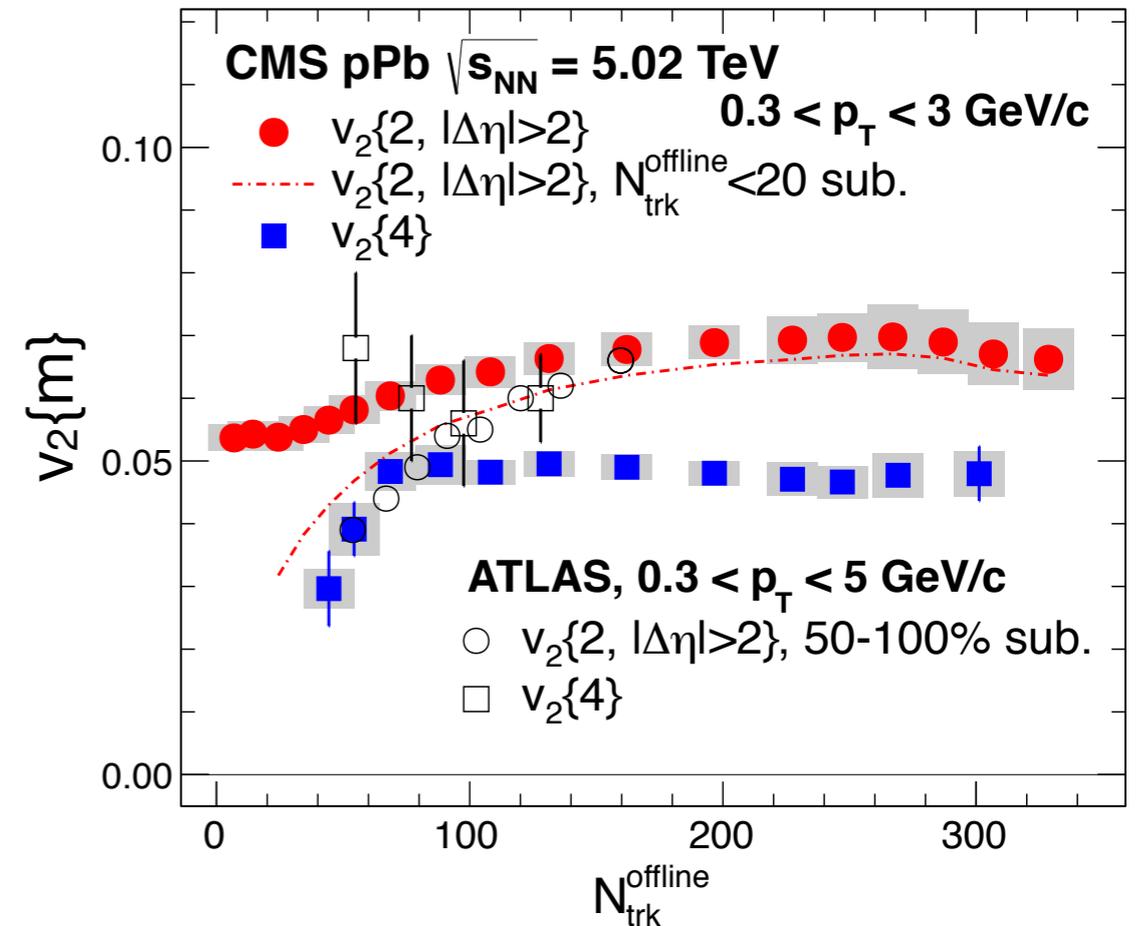
ATLAS EPJC 77 (2017)

Mild dependence on maximum integrated  $p_{\perp}$

# Multi-particle quark correlations



Dusling, MM, Venugopalan PRL 120 (2018)

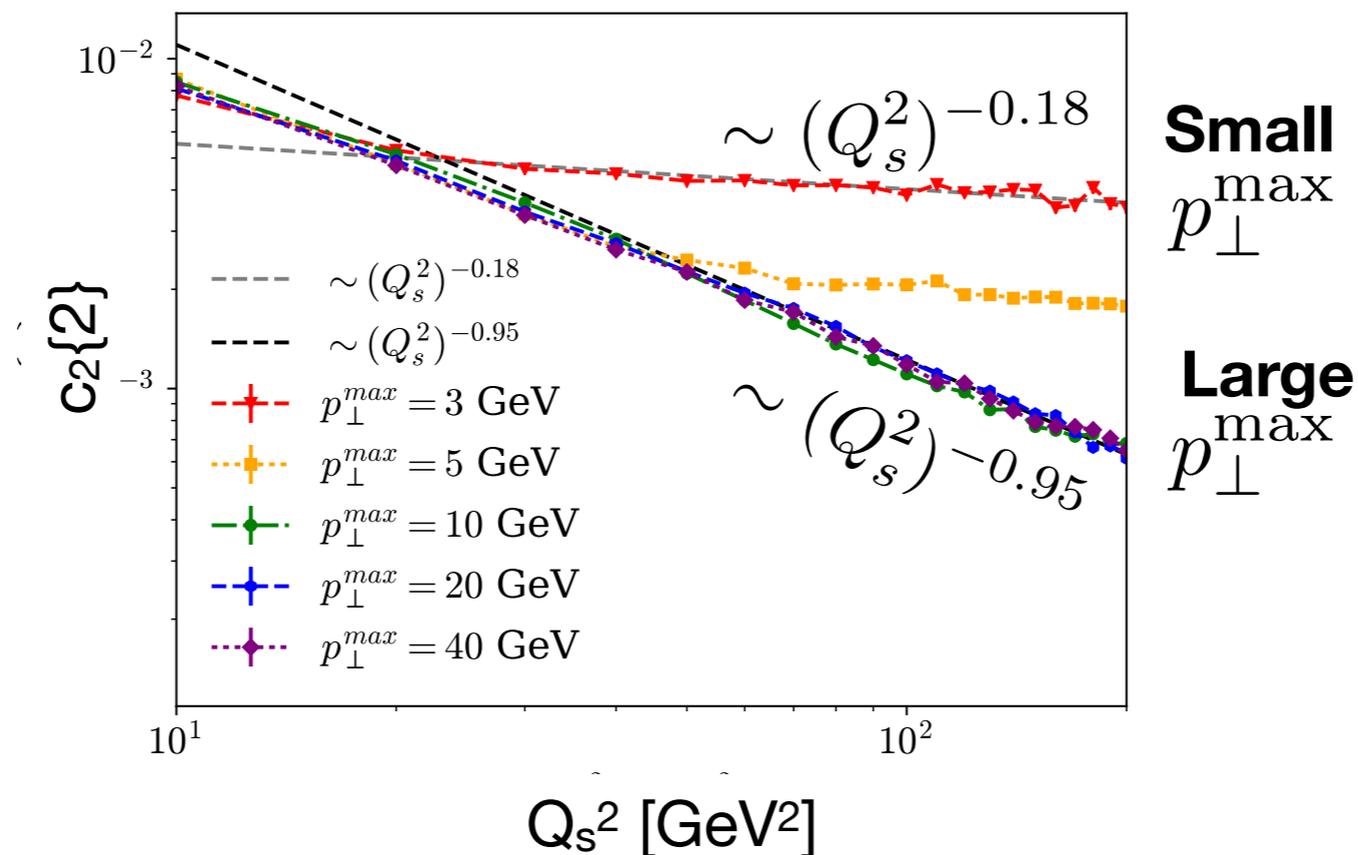


CMS PLB 724 (2013) 213

No inverse scaling by number of domains in CGC and data

# Scale dependence

Two dimensionless scales:  $Q_s^2 B_p$ , the number of domains, and the ratio of resolution scales,  $Q_s^2 / (p_{\perp}^{\max})^2$ .



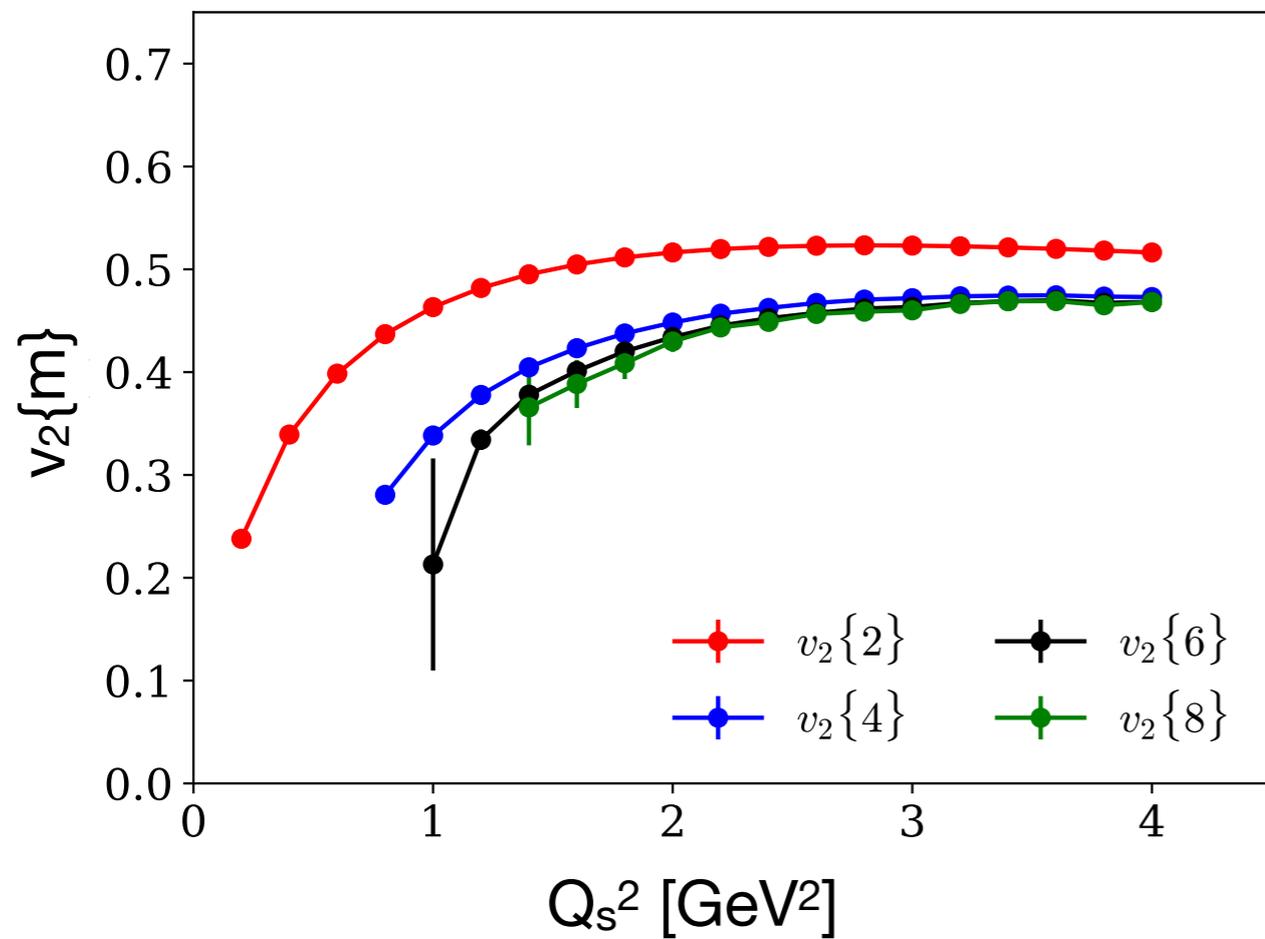
$(p_{\perp}^{\max})^2 \lesssim Q_s^2$  : probe coarse graining over multiple domains

$(p_{\perp}^{\max})^2 \gtrsim Q_s^2$  : probe resolves area less than domain size

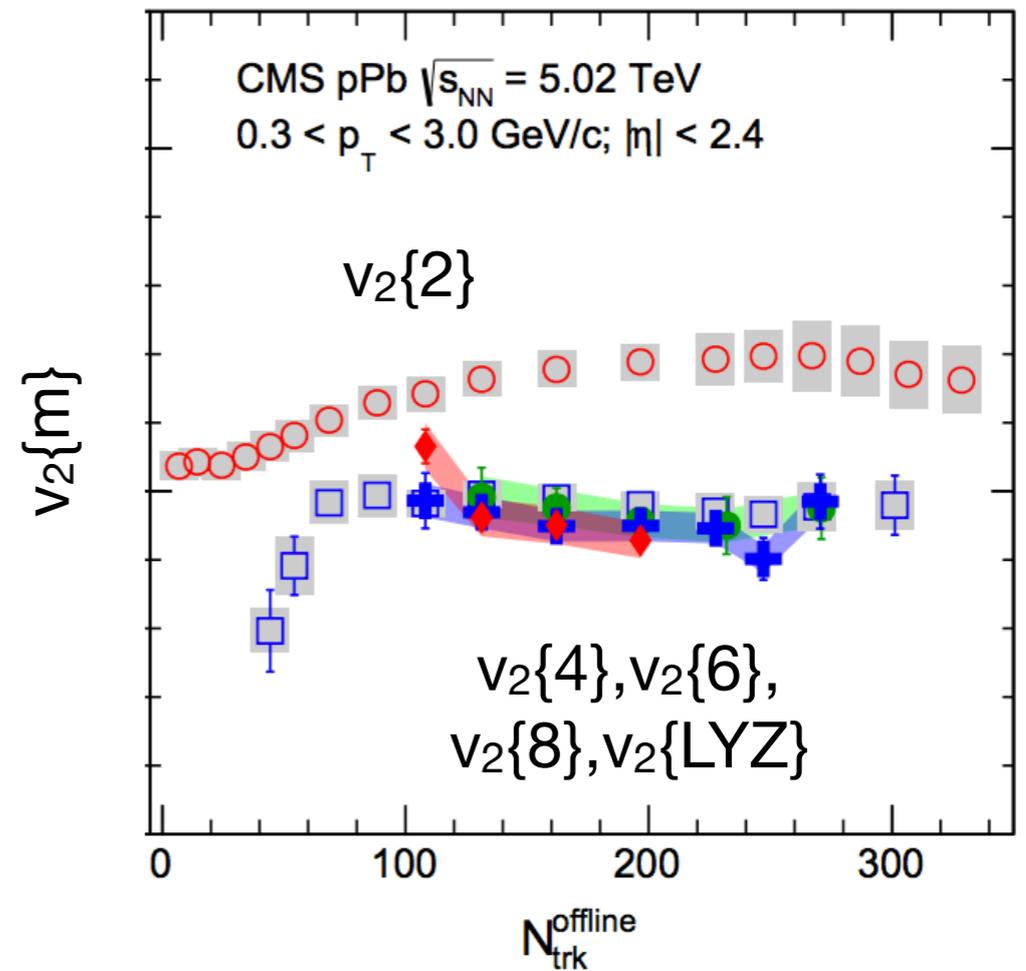
Scaling with inverse number of domains seen only for large  $p_{\perp}^{\max}$

# Collectivity from parton model

For computational reduction, consider Abelian version



*Dusling, MM, Venugopalan PRL 120 (2018)*



*CMS PRL 115 (2015) 012301*

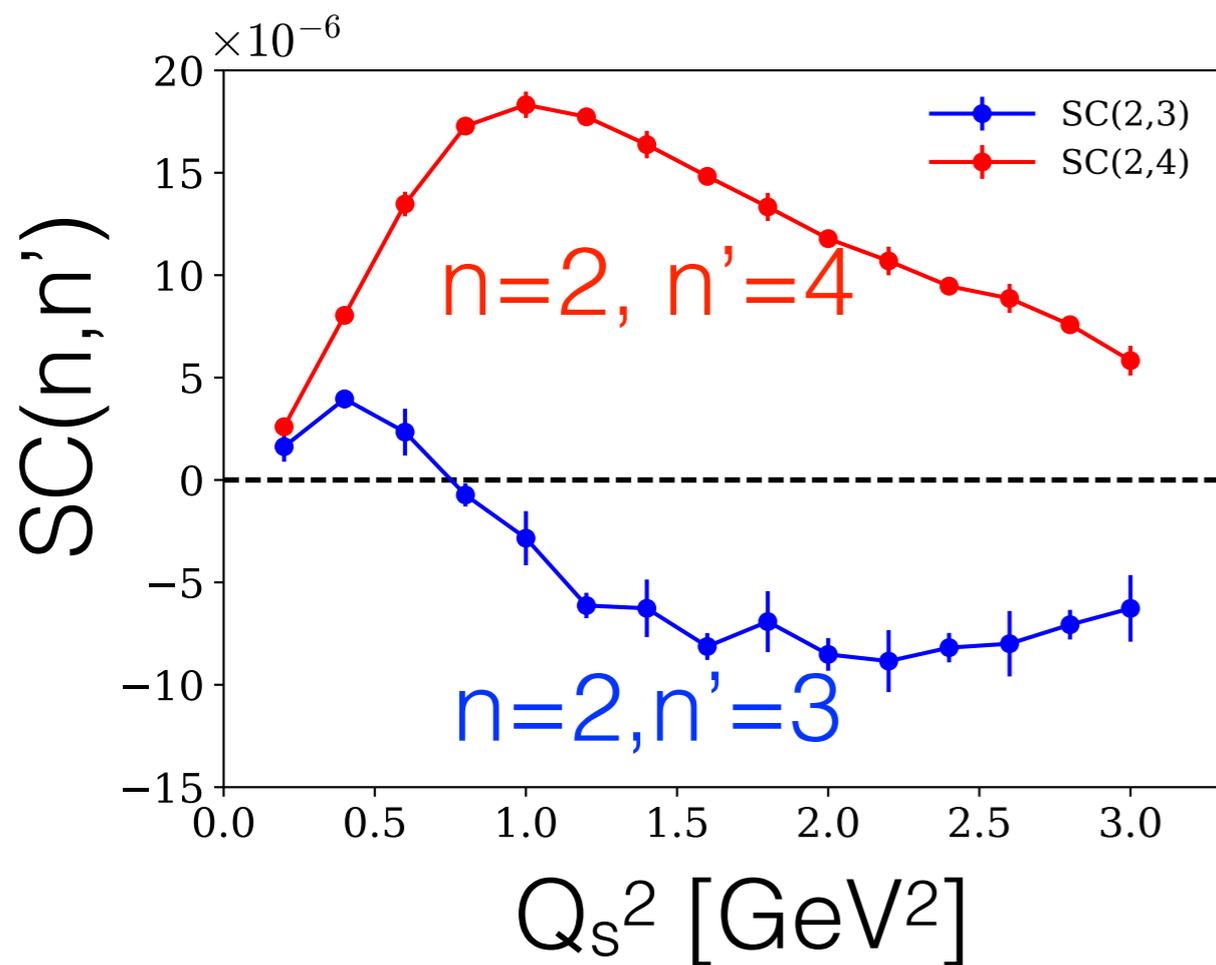
Clear demonstration that  $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$   
collectivity not unique to hydrodynamics

# Symmetric Quark Cumulants

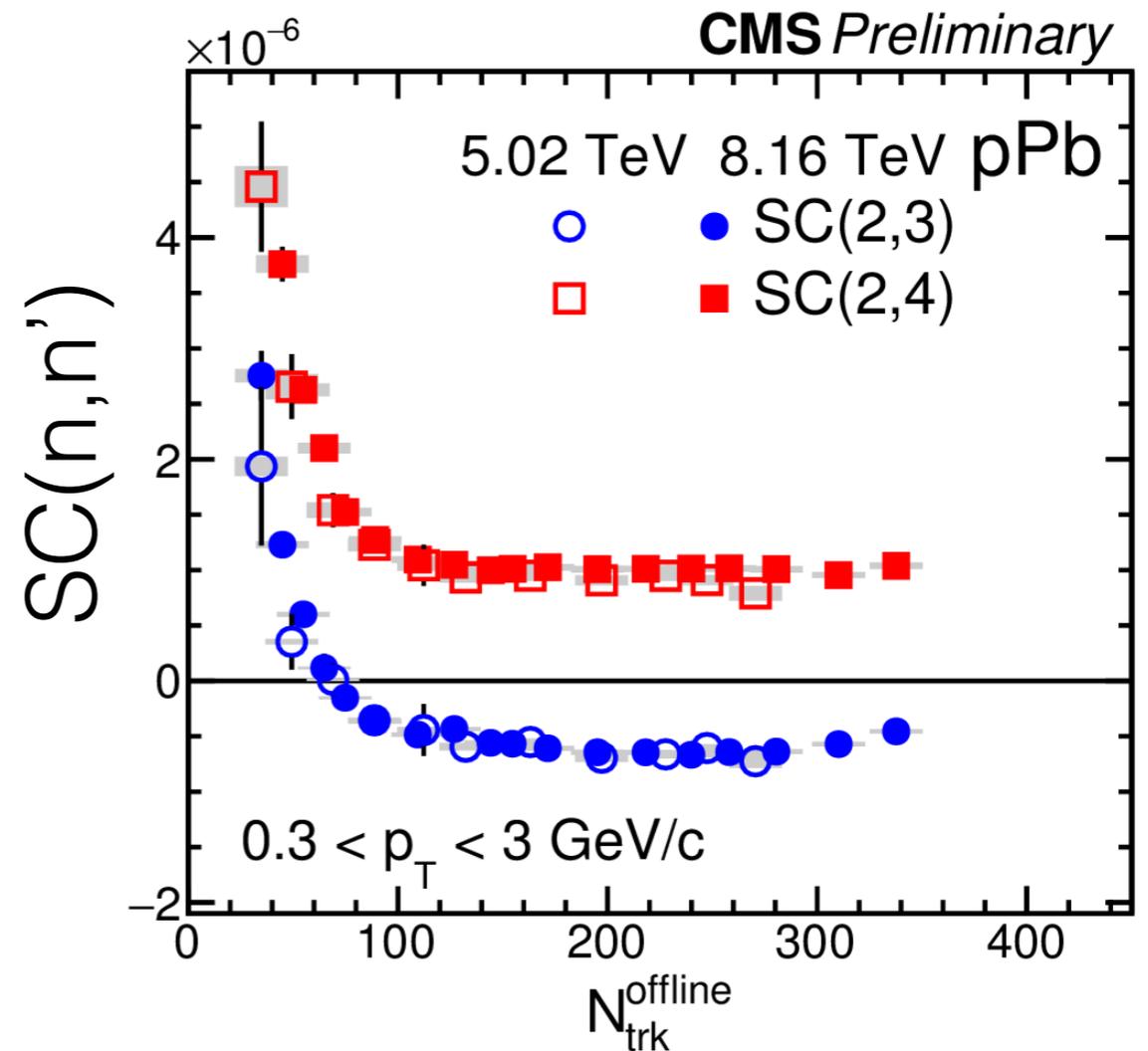
Symmetric cumulants: mixed harmonic cumulants

$$SC(n, n') = \langle e^{i(n(\phi_1 - \phi_3) - n'(\phi_2 - \phi_4))} \rangle - \langle e^{in(\phi_1 - \phi_3)} \rangle \langle e^{in'(\phi_2 - \phi_4)} \rangle$$

*Bilandzic et al, PRC 89, no. 6, 064904 (2014)*



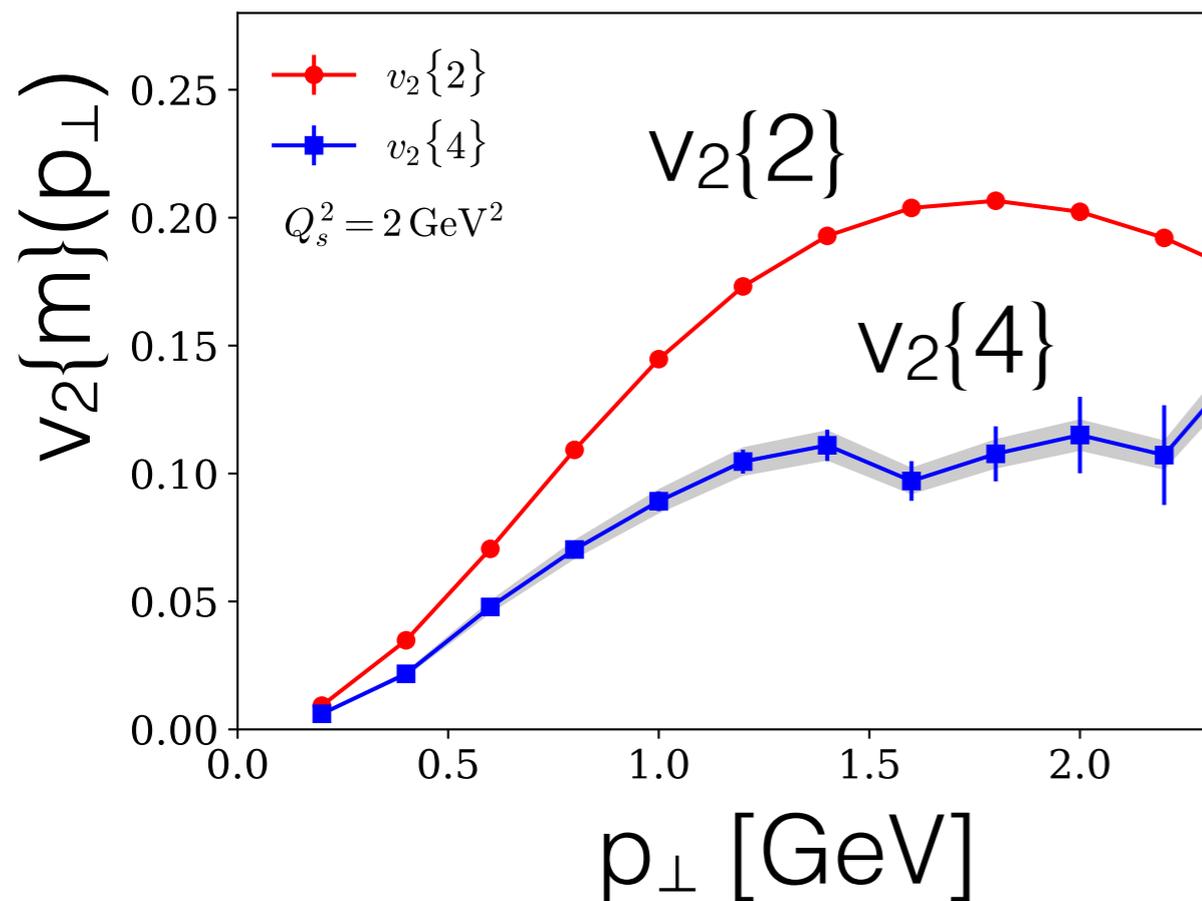
*Dusling, MM, Venugopalan PRD 97 (2018)*



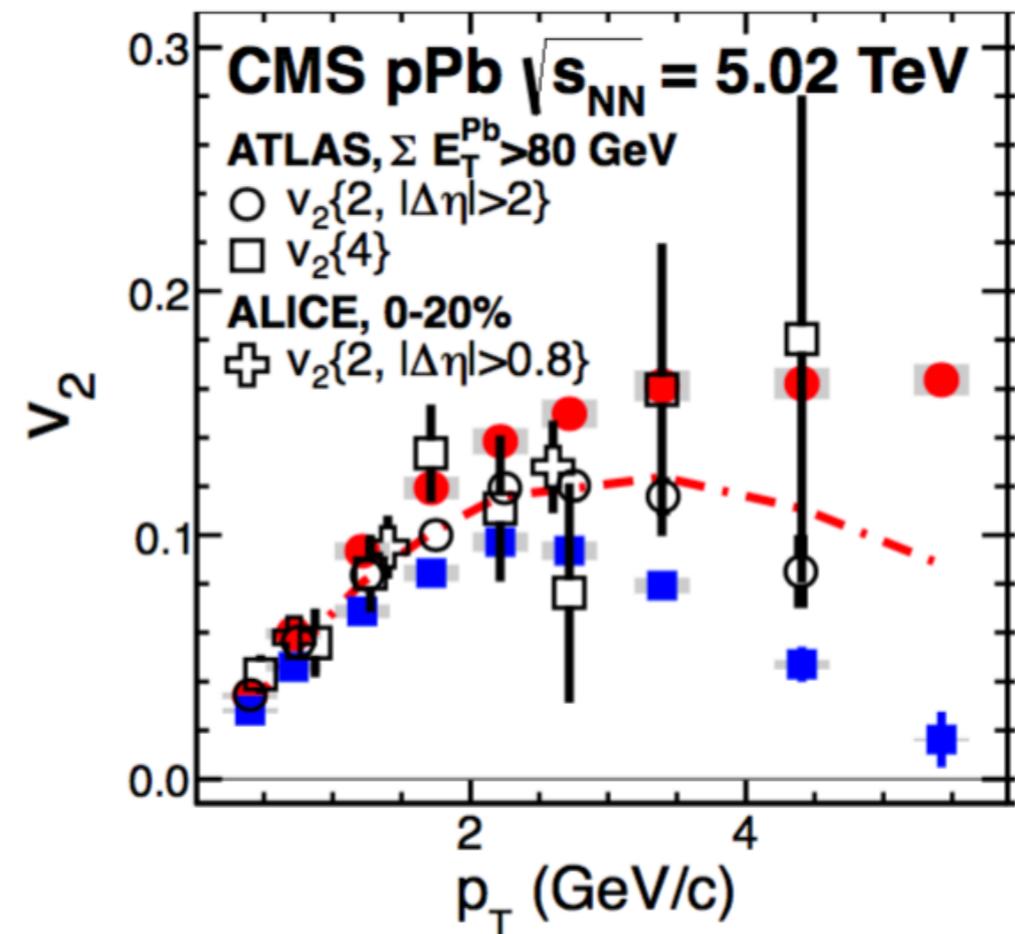
*CMS-PAS-HIN-16-022*

# Multiparticle correlations

Integrating momentum of  $m-1$  particles



Dusling, MM, Venugopalan PRD 97 (2018)



CMS PLB 724 (2013) 213

Similar characteristic shape

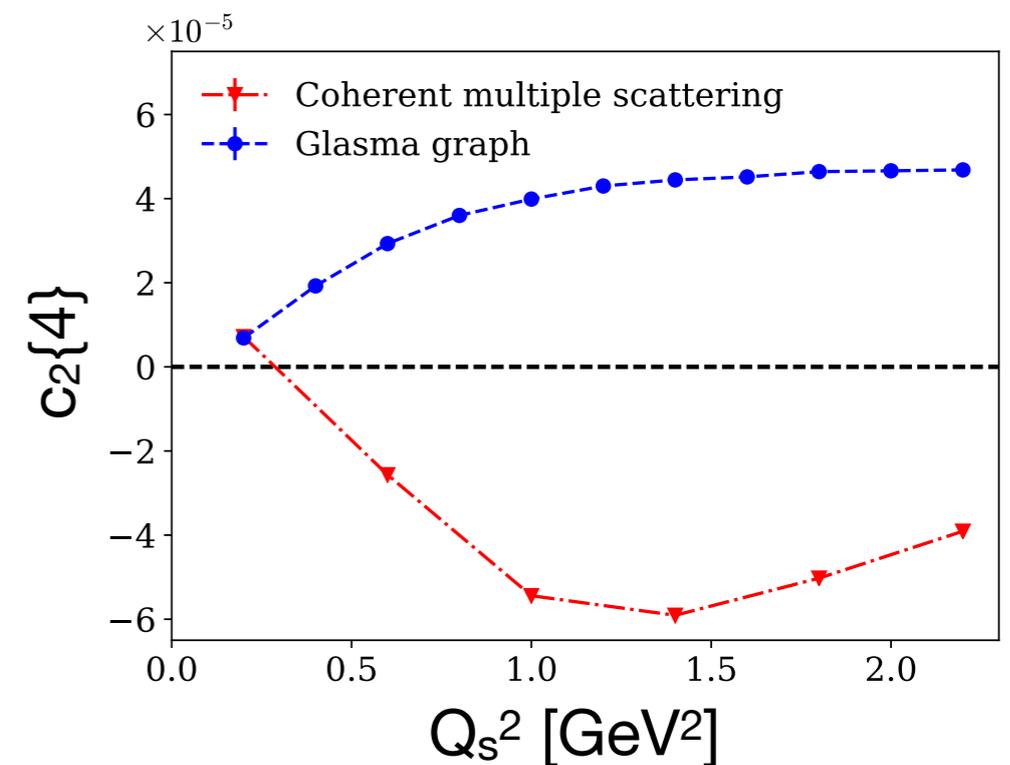
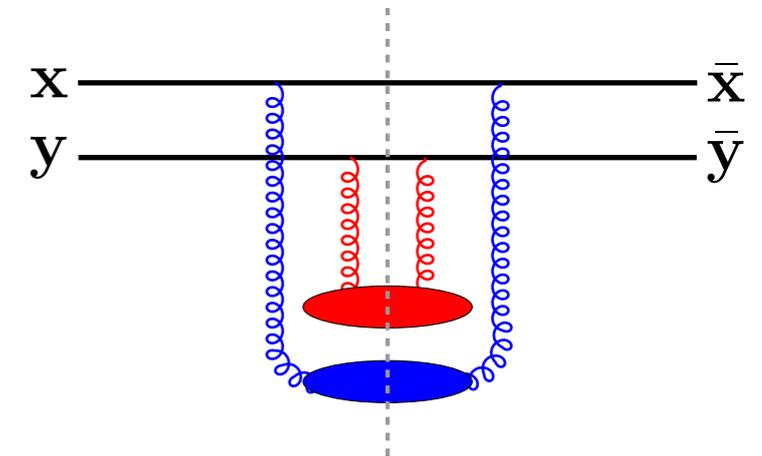
# Comparison to glasma graphs

Glasma graph approximation, valid only for  $p_{\perp} > Q_s$ , only considers single gluon exchange

*Dumitru, Gelis, McLerran, Venugopalan, NPA 810 (2008),  
Dusling, Venugopalan PRL 108 (2012), PRD 87 (2013)*

Glasma graphs have very strong correlations, close to a Bose distribution (as in a laser)

*Gelis, Lappi, McLerran NPA 828 (2009)*



Multiple scattering suppresses higher cumulants  $\rightarrow c_2\{2\} < 0$

*Dusling, MM, Venugopalan PRD 97 (2018)*