Quarkonium Production in Heavy Ion Collision: Coupled Boltzmann Transport Equations

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Oct. 3 2018, Hard Probes 2018, Aix-Les-Bains

Introduction

- Debye (static) screening on heavy quark bound state, not enough explain quarkonium production suppression
- Production complicated by many factors:
  - Cold nuclear matter effect (CNM) initial production
  - Static screening (real part potential suppressed) v.s. dynamical screening (imaginary part potential, related to dissociation, thermal width)
  - In-medium evolution: dissociation v.s. recombination (sensitive to open HQ dynamics)
  - Feed-down, etc.
- **Include all factors consistently:**
  - **Open quantum system (non-unitary, time irreversible dynamics from QCD)**
  - **Transport equations**

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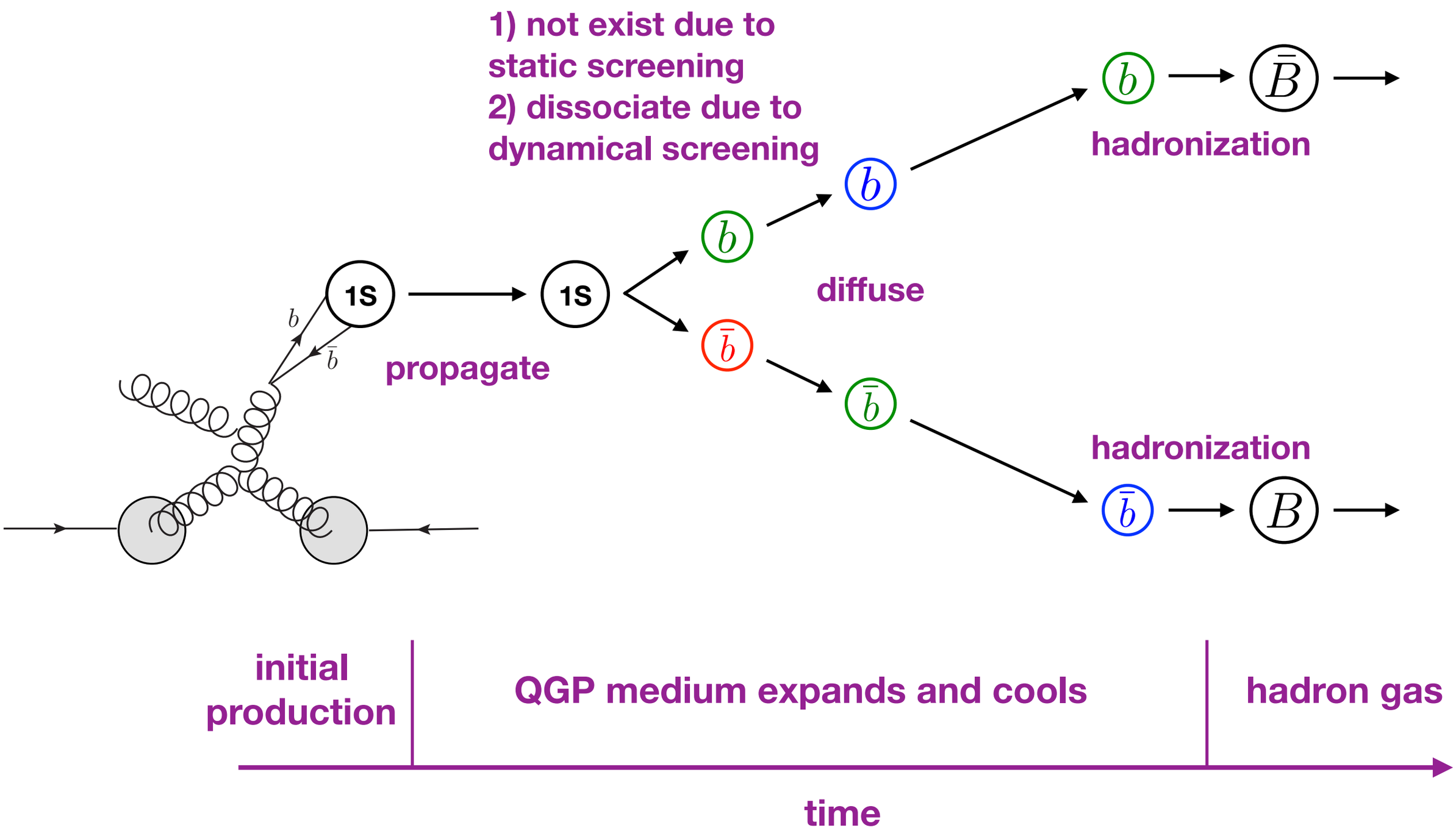
Dynamical Evolution: Dissociation



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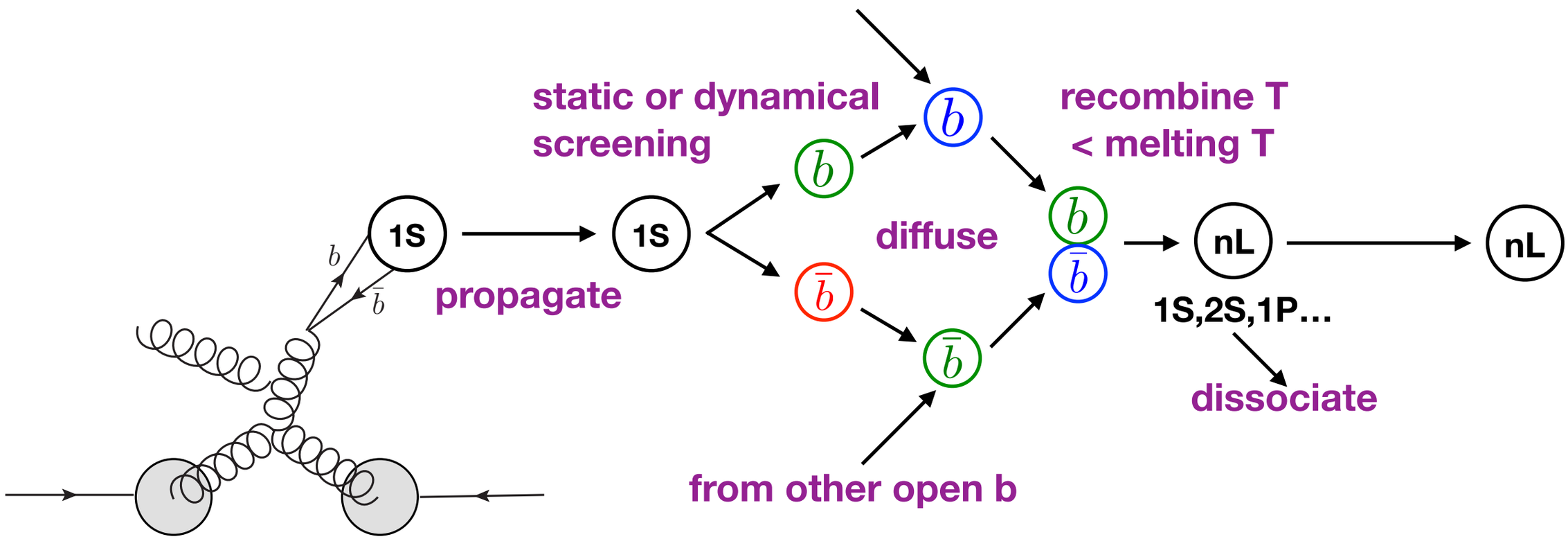
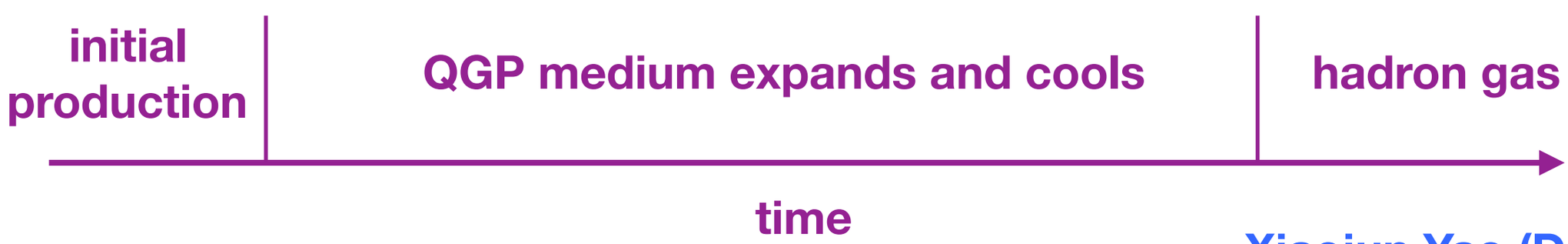
Dynamical Evolution: Recombination

melting temperature: above which a specific bound state
1) ill defined (thermal width too large)
2) not exists (potential not supports bound state)

in-medium formation

RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)





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Coupled Boltzmann Equations

heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_Q^+ + \mathcal{C}_Q^- + \mathcal{C}_Q$$

anti-heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^- + \mathcal{C}_{\bar{Q}}$$

each quarkonium state
nl = 1S, 2S,1P etc. $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = +\mathcal{C}_{nl}^+ - \mathcal{C}_{nl}^-$$

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Coupled Boltzmann Equations

heavy Q energy loss
see talk by Weiyao Ke
Tu 14:40 pm

heavy quark

anti-heavy quark

each quarkonium state
nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_Q^+ + \mathcal{C}_Q^- + \mathcal{C}_Q$$

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^- + \mathcal{C}_{\bar{Q}}$$

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = +\mathcal{C}_{nl}^+ - \mathcal{C}_{nl}^-$$

phase space evolution
of distribution function

recombination **dissociation**
quarkonium gain **quarkonium loss**
heavy quark loss **heavy quark gain**

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Dissociation, Recombination, pNRQCD

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\mathbf{r}\cdot g\mathbf{E}, \mathrm{O}\} + \cdots\Big)$$

- Separation of scales (bound state exists) $M \gg Mv \gg Mv^2, T, m_D$
- Systematic expansion in $\frac{1}{M},\ r \sim \frac{1}{Mv}$

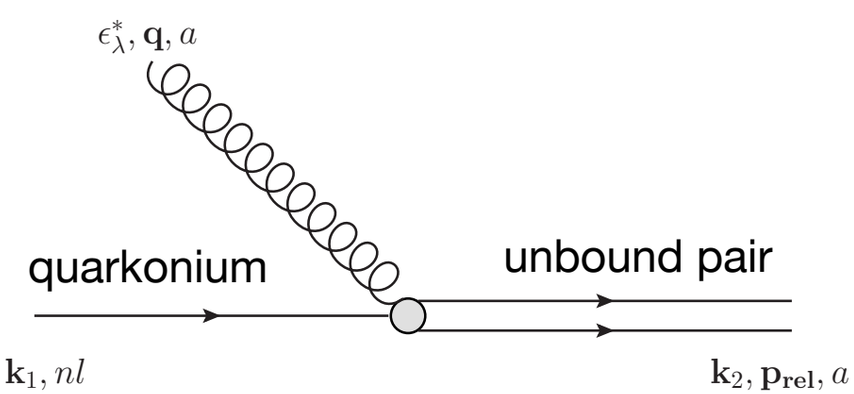
Brambilla, Ghiglieri, Vairo, Petreczky, Phys. Rev. D 78, 014017 (2008)
Brambilla, Escobedo, Ghiglieri, Vairo, JHEP1112,116(2011)JHEP1305,130(2013)

$$H_{s,o} = \frac{P^2_{\rm c.m.}}{4M} + \boxed{\frac{p^2_{\rm rel}}{M} + V^{(0)}_{s,o}} + \frac{V^{(1)}_{s,o}}{M} + \frac{V^{(2)}_{s,o}}{M^2} + \cdots$$

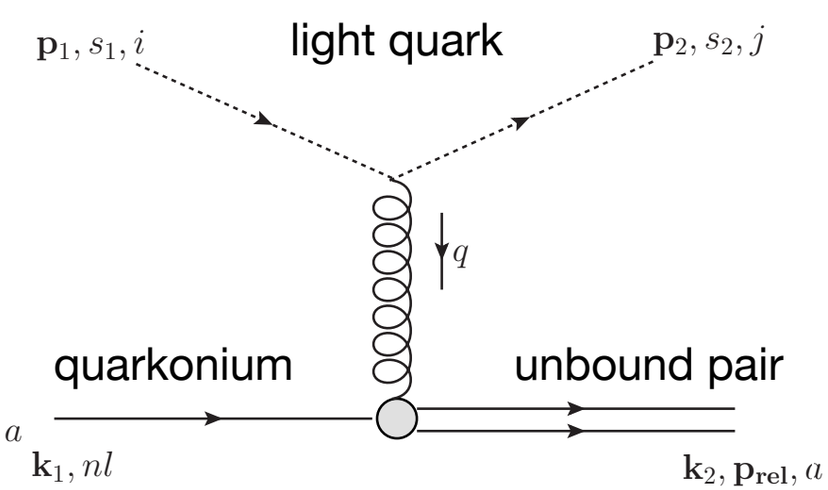
virial theorem

$$V^{(0)}_s = -C_F\frac{\tilde{\alpha}_s}{r} \qquad V^{(0)}_o = \frac{1}{2N_C}\frac{\tilde{\alpha}_s}{r}$$

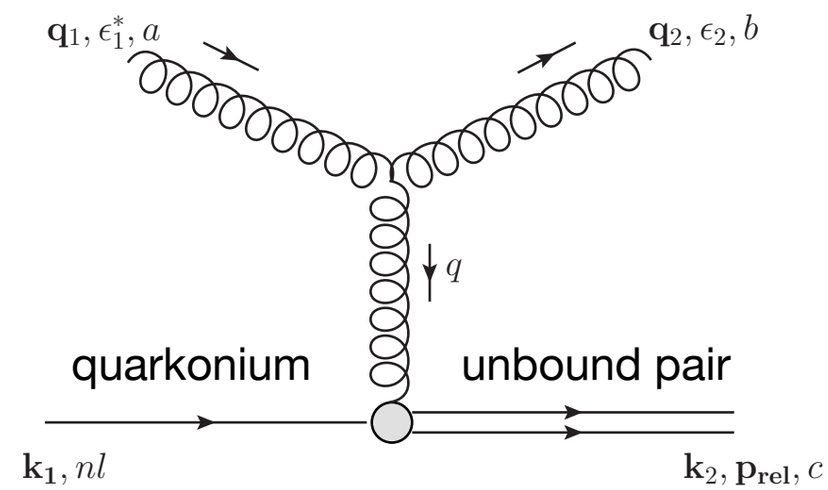
no imaginary potential
can be improve: lattice motivated potential



gluon absorption/emission



inelastic scattering w/ light quark



inelastic scattering w/ gluon

Approach Equilibrium

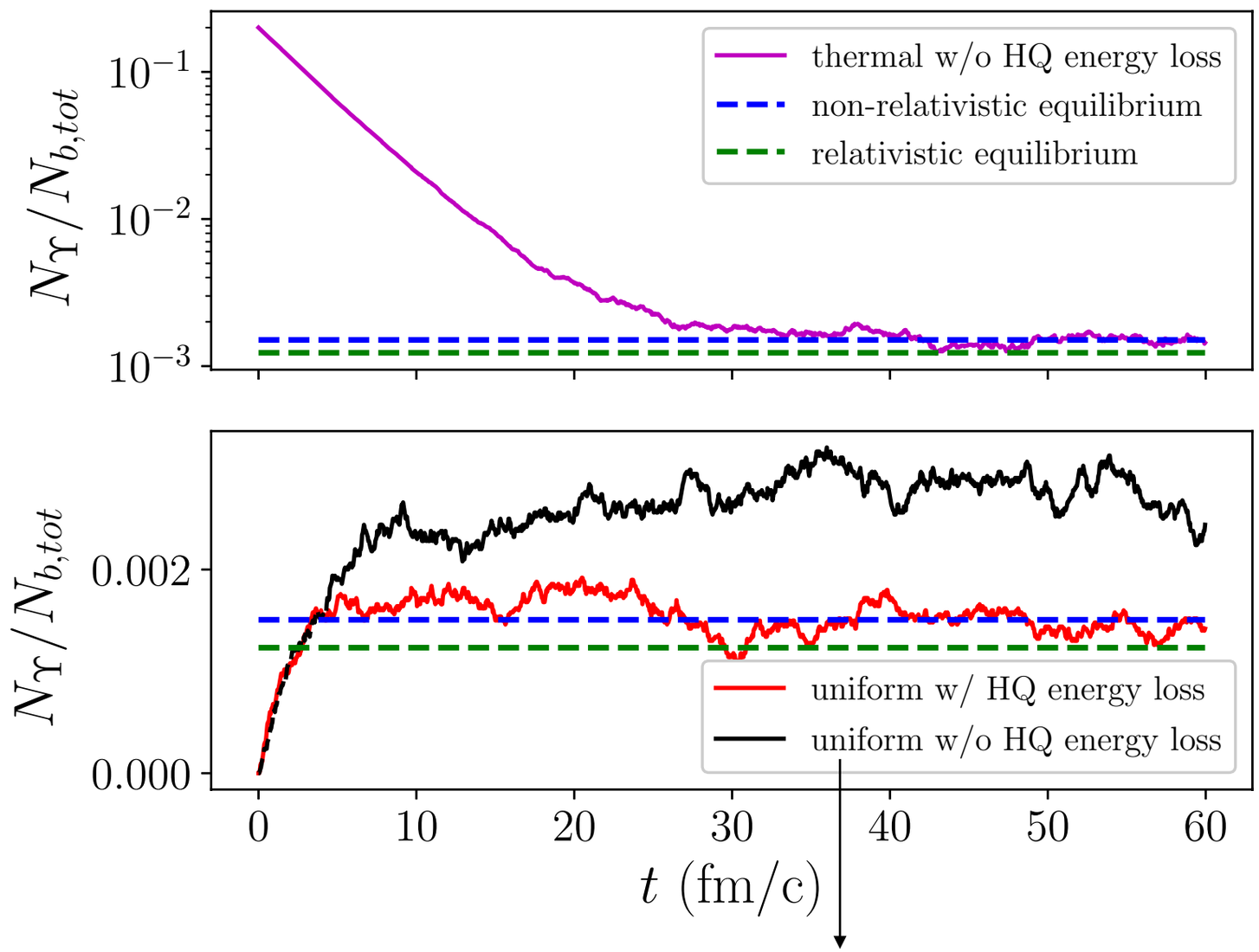
Setup:
QGP box w/ const T, 1S state and b quarks: total b flavor = 50 (fixed)
Initial momenta sampled from thermal or uniform distributions

XY, B.Mueller, Phys. Rev. C 97, no. 1, 014908 (2018)

Recombination from QCD effective field theory and real dynamics of HQ

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy loss necessary to drive kinetic equilibrium of quarkonium



sampled from uniform distribution between 0-1 GeV

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Collision Event Simulation

- Initial production:

PYTHIA 8.2: NRQCD factorization — Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159; Bodwin, Braaten, Lepage Phys. Rev. D 51, 1125 (1995)

Nuclear PDF: EPS09 (cold nuclear matter effect) — Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

Trento, sample position, hydro. initial condition — Moreland, Bernhard, Bass, Phys. Rev. C 92, no. 1, 011901 (2015)

- Medium background: 2+1D viscous hydrodynamics (calibrated)

Song, Heinz, Phys.Rev.C77,064901(2008)
Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)
Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94,no.2,024907(2016)

- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

- Effect of neglecting other states: feed-down v.s. in-medium recombination

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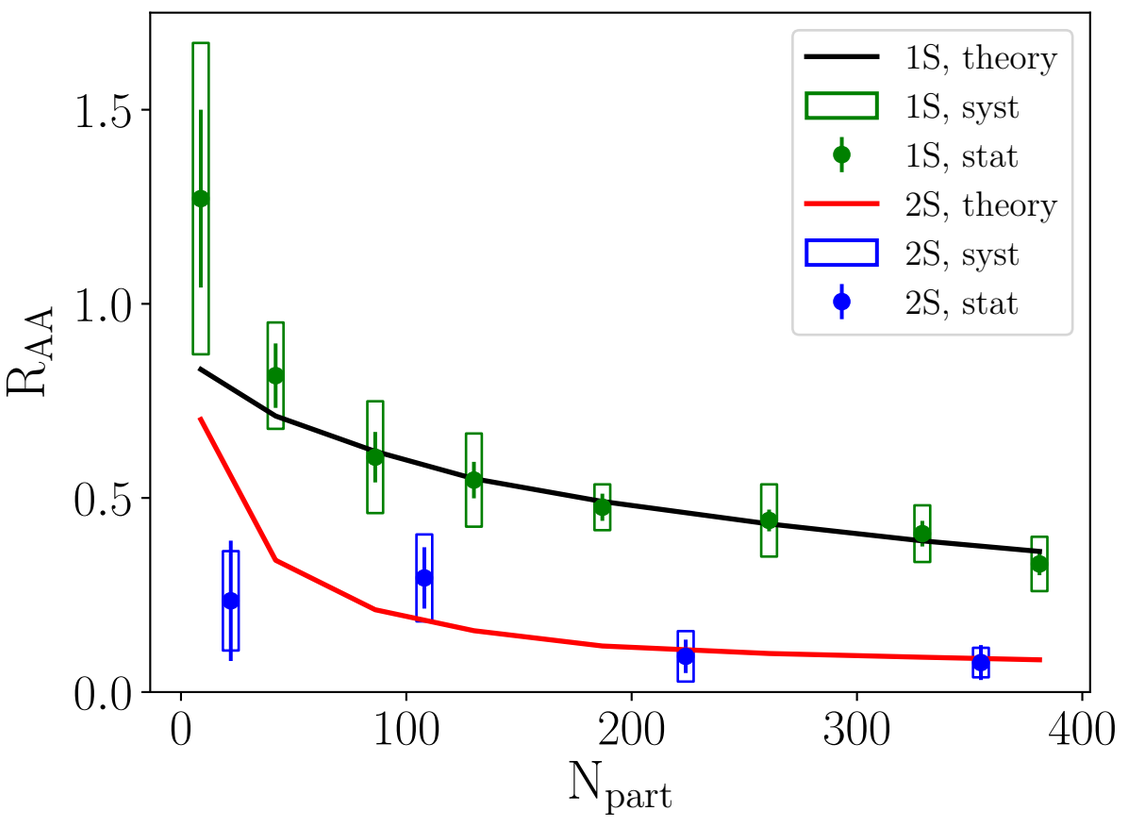
Upsilon in 2760 GeV PbPb Collision

Fix $\alpha_s = 0.3$

Tune $T_{\rm melt}(2S) = 210$ MeV

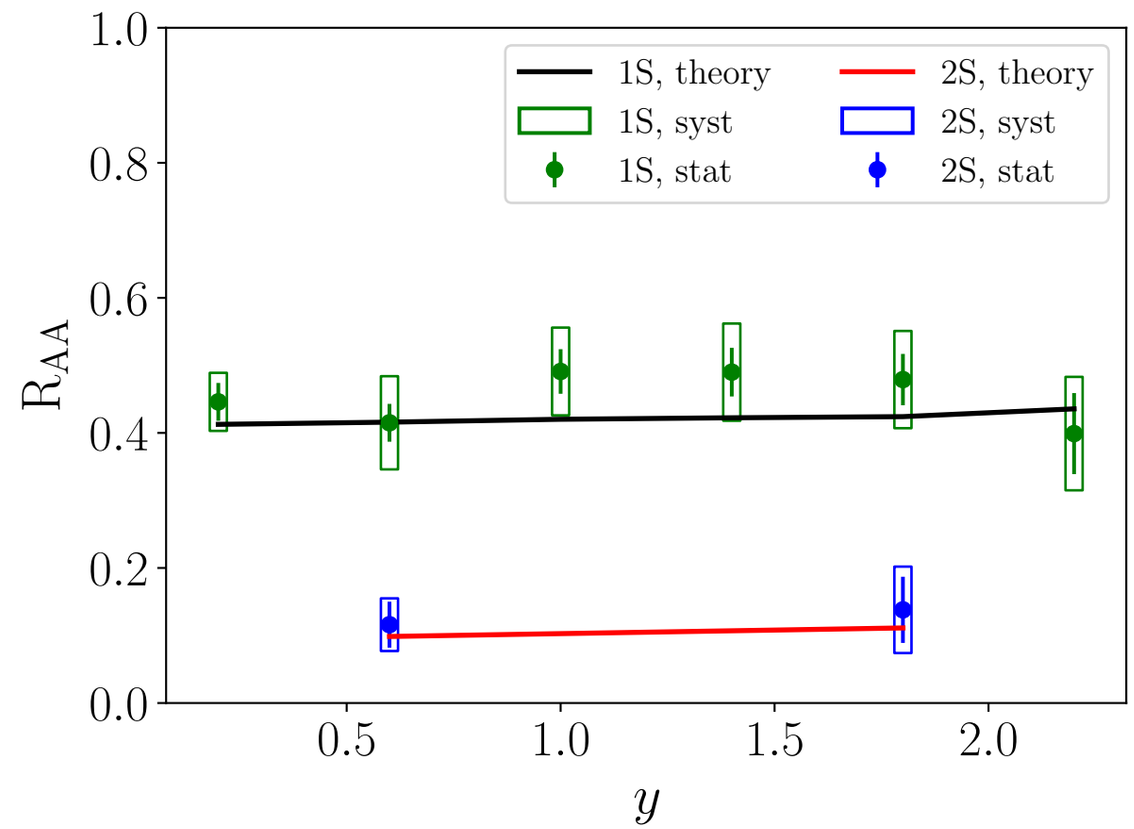
Tune $V_s = -C_F \dfrac{0.42}{r}$

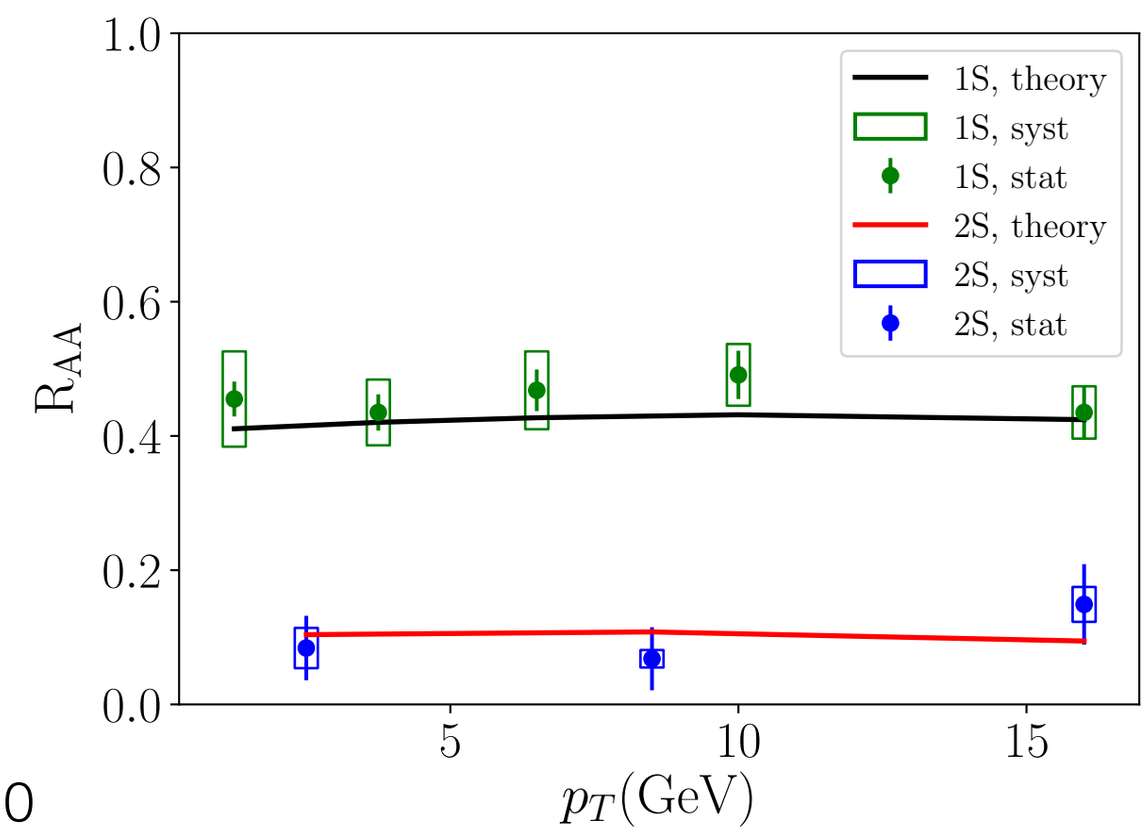
Cold nuclear matter effect ~ 0.87 (PYTHIA + nPDF)



CMS Phys.Lett. B
770 (2017) 357-379

X.Yao, W.Ke, Y.Xu, S.Bass
and B.Müller arXiv:1807.06199

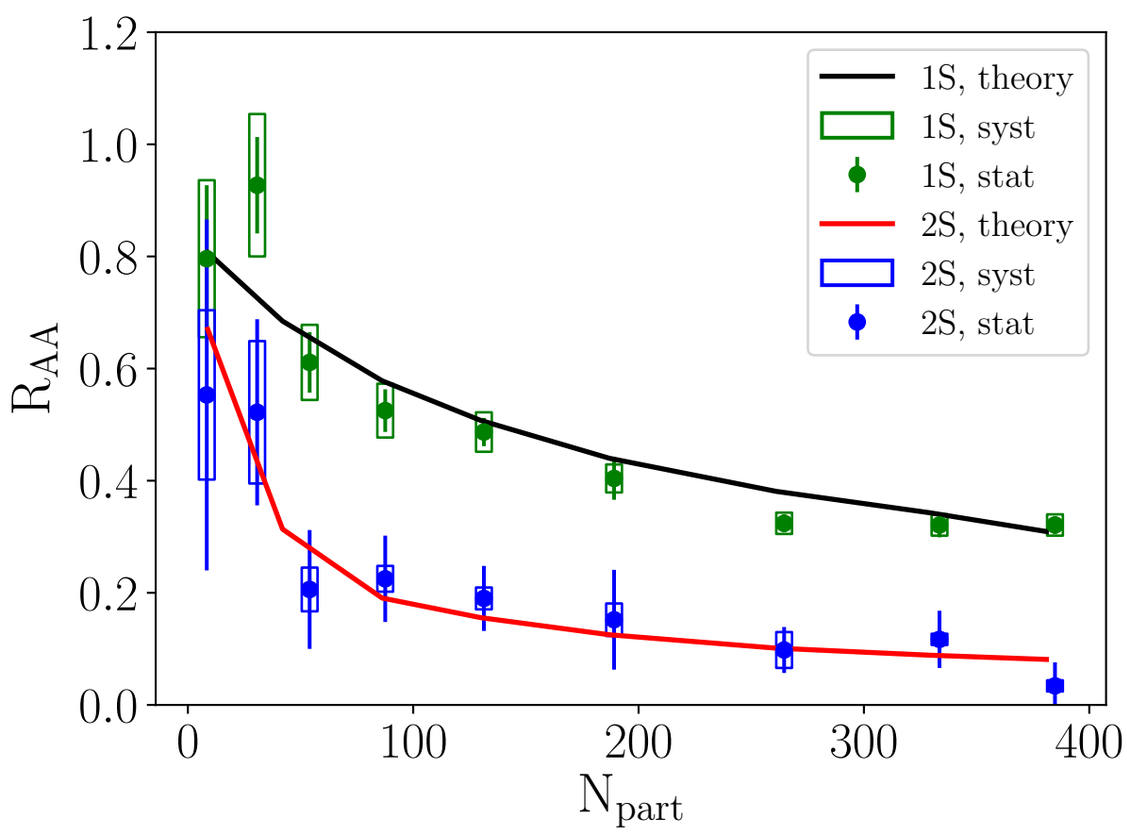




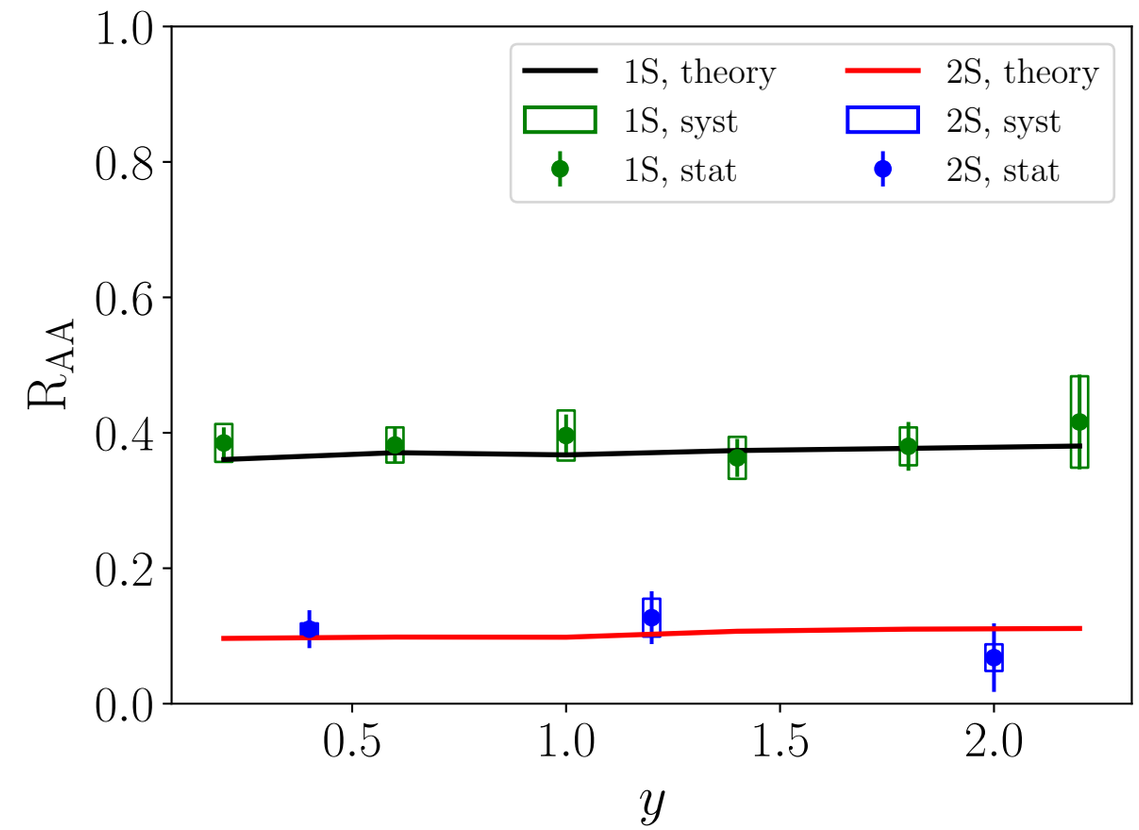
Upsilon in 5020 GeV PbPb Collision

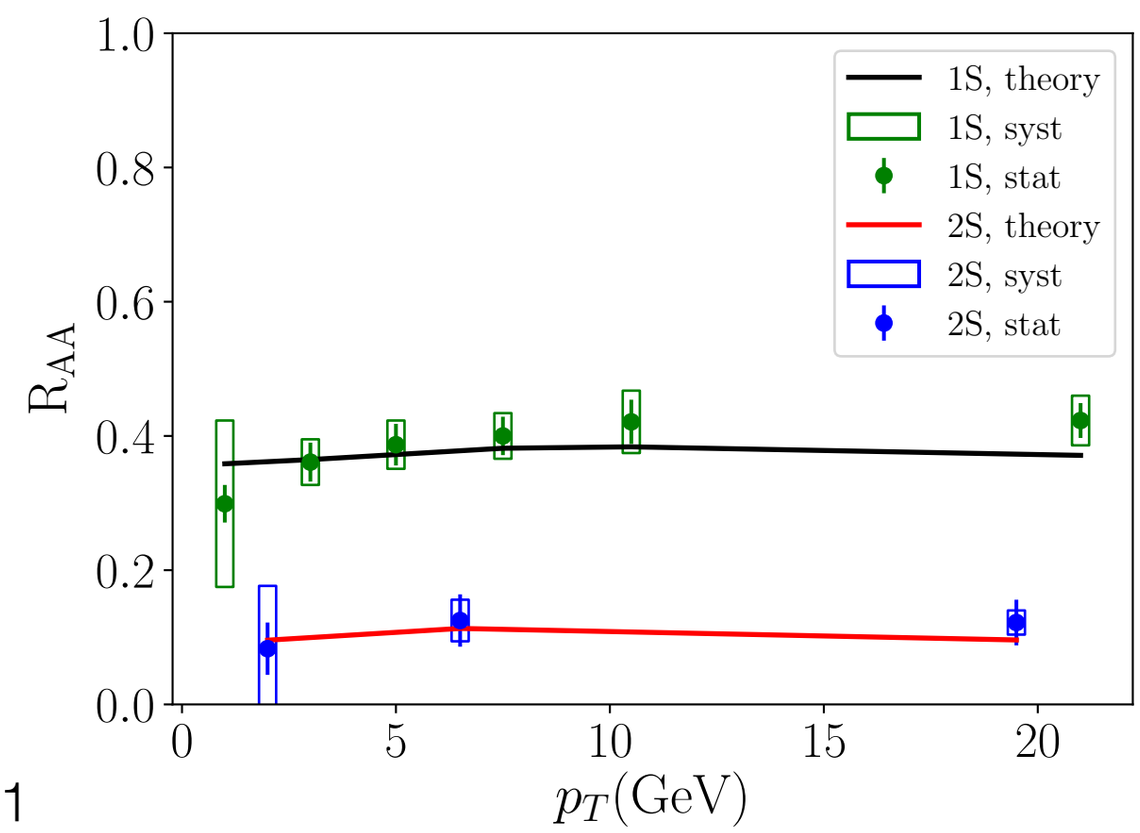
Use same set of parameters

Cold nuclear matter effect ~ 0.85 (PYTHIA + nPDF)



CMS arXiv:1805.09215

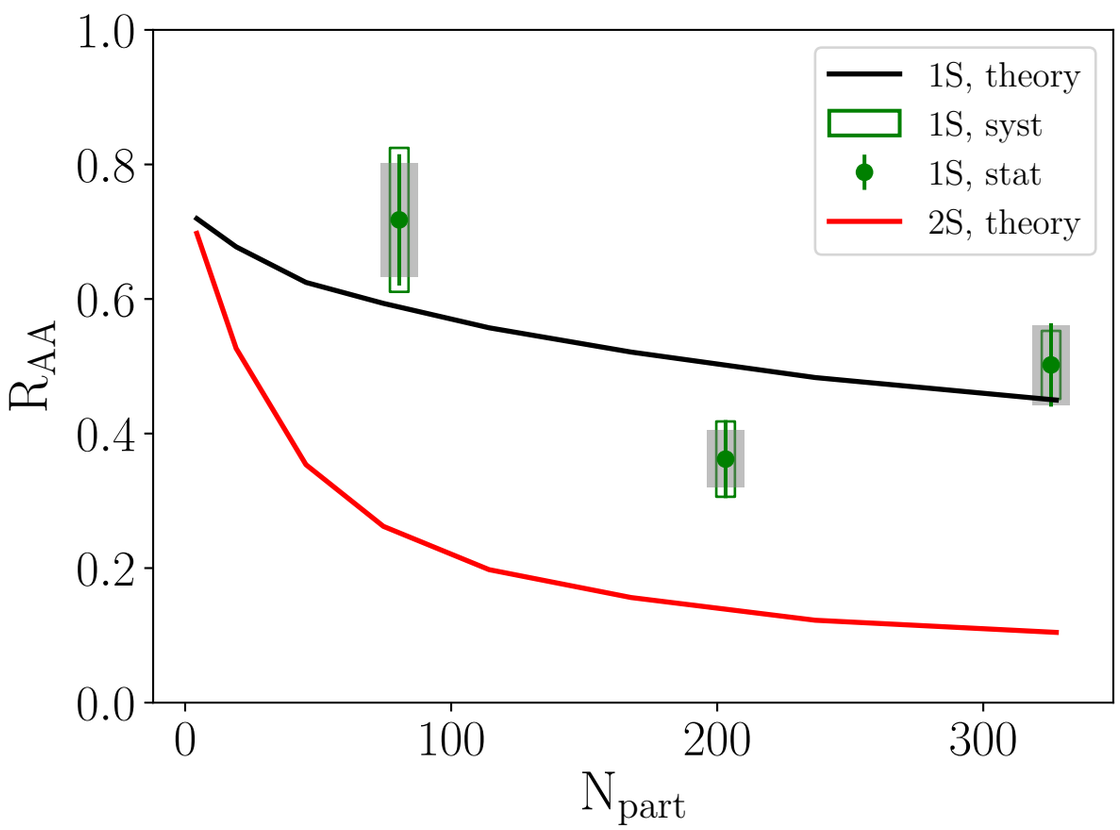


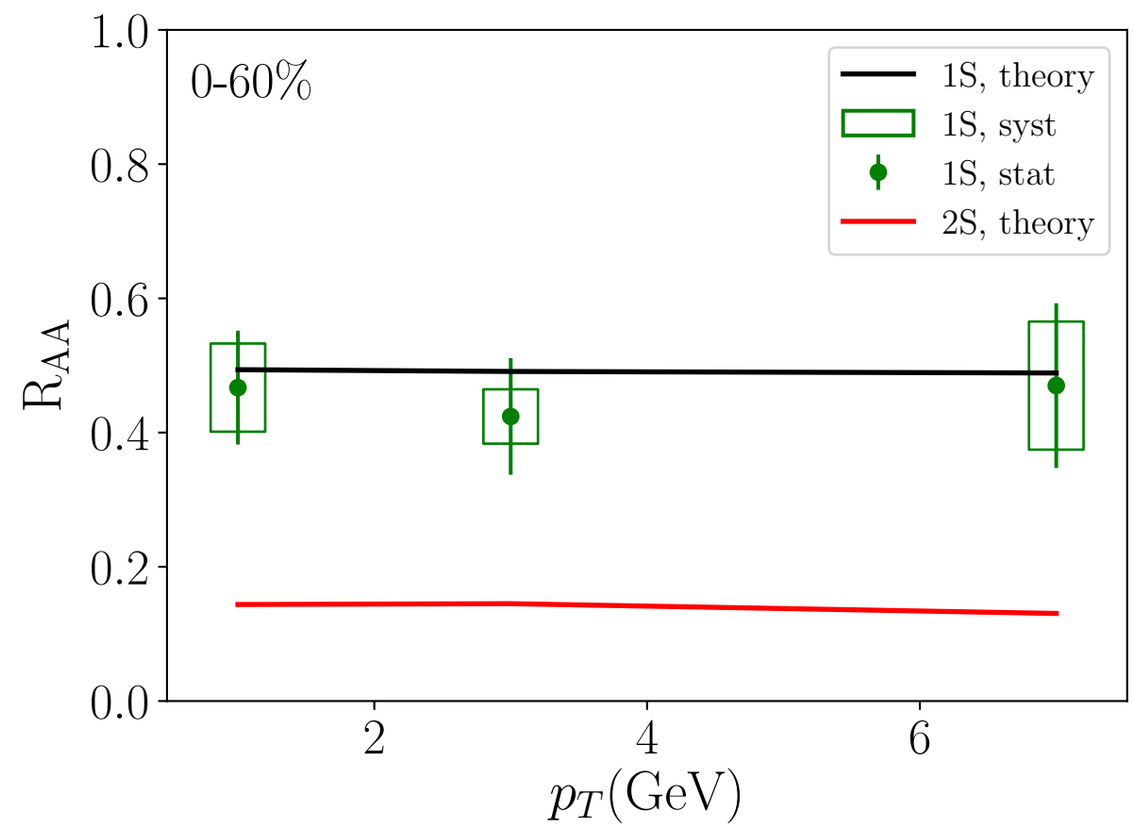


Upsilon in 200 GeV AuAu Collision

Use same set of parameters

Cold nuclear matter effect ~ 0.72





STAR measures 2S+3S; sPHENIX upgrades

STAR Talks at QM 17&18

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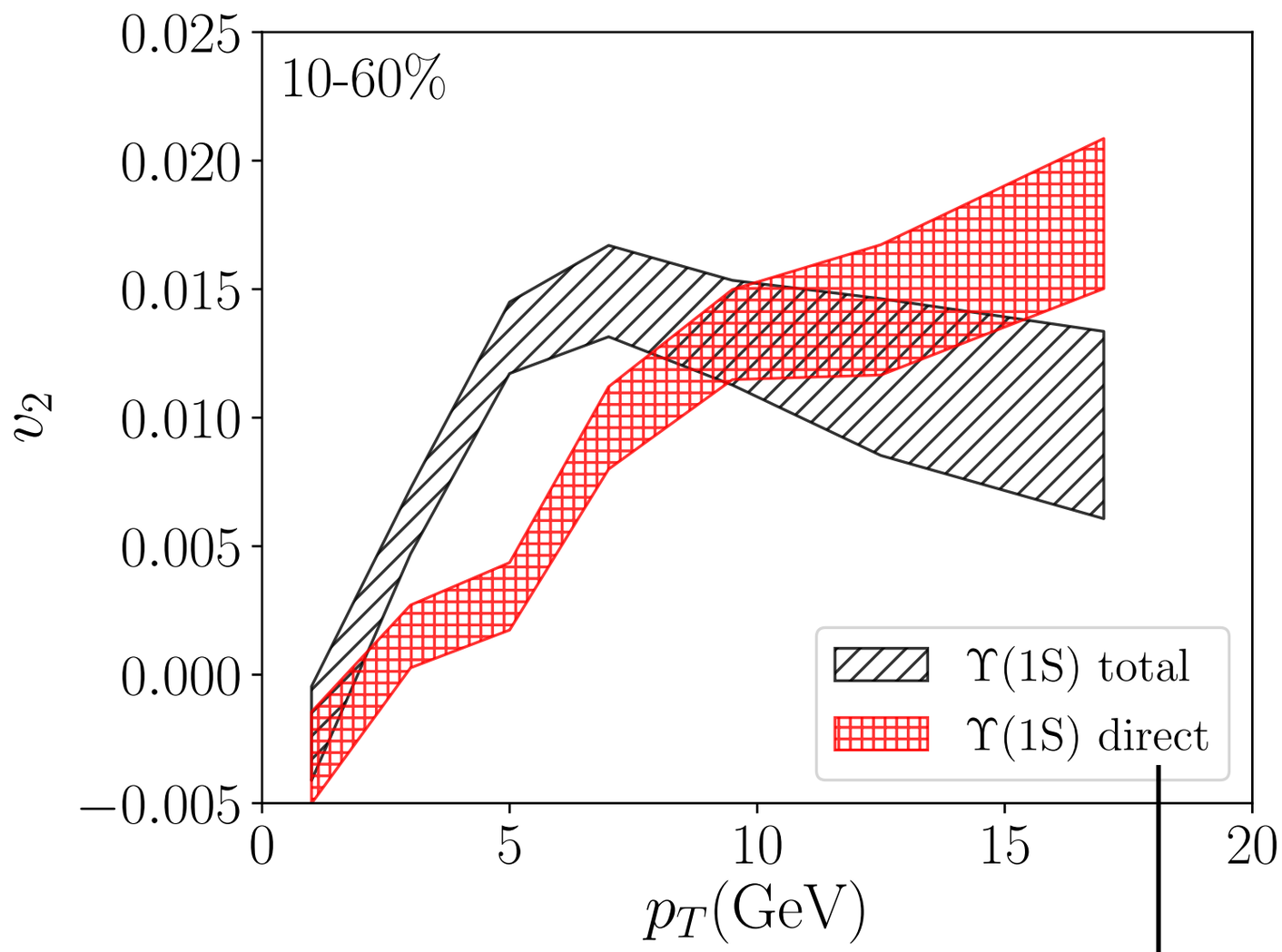
Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb



produced from initial hard collision

Develop azimuthal momentum anisotropy: dynamical evolution

Better understanding of quarkonium transport from v_2 measurements

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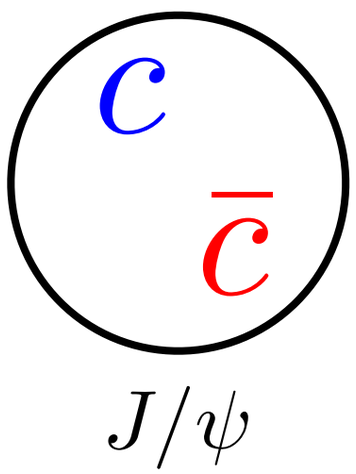
Doubly Charmed Baryon

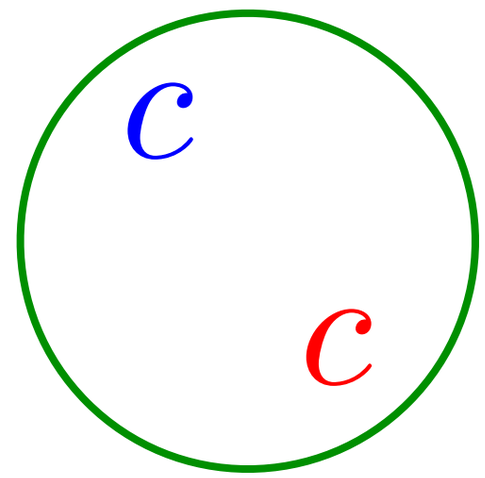
- LHCb observed a new baryon Ξ_{cc}^{++} (ccu): u bound around cc core

LHCb, Phys. Rev. Lett. 119, no.11,112001 (2017)

- Pair of heavy Q in anti-triplet forms bound state (diquark)



$Q\bar{Q}$ singlet
color neutral
exist in vacuum



QQ anti-triplet
colored
not exist in vacuum
exist in QGP

cc diquark (1S)

- Heavy diquark in QGP: dissociation, recombination (similar to quarkonium), carry color, energy loss different from quarkonium

- Hadronize into doubly charmed baryon

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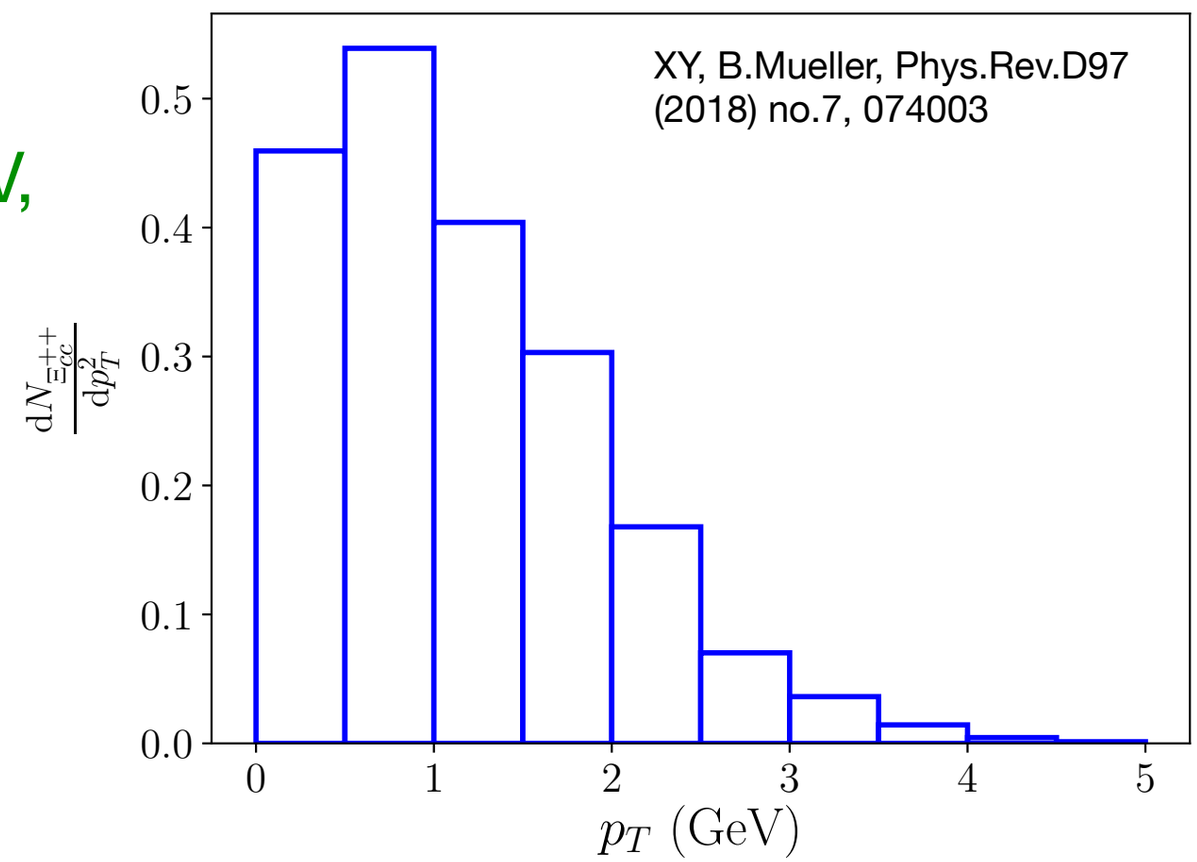
Doubly Charmed Baryon Production in Heavy Ion Collisions

Setup:
coupled Boltzmann for charm quark and diquark (add energy loss of diquark)
assume only charm quark produced initially, diquark comes from (re)combination

Predicted production rate of Ξ_{cc}^{++}
in 2760 GeV PbPb, -1<y<1, 0<pT<5 GeV,
0.02 per collision

With melting temperature = 250 MeV:
0.0125 per collision

Compare: c quark rate ~10 per collision



Production rate from initial hard collision probably small
Study recombination from measurements

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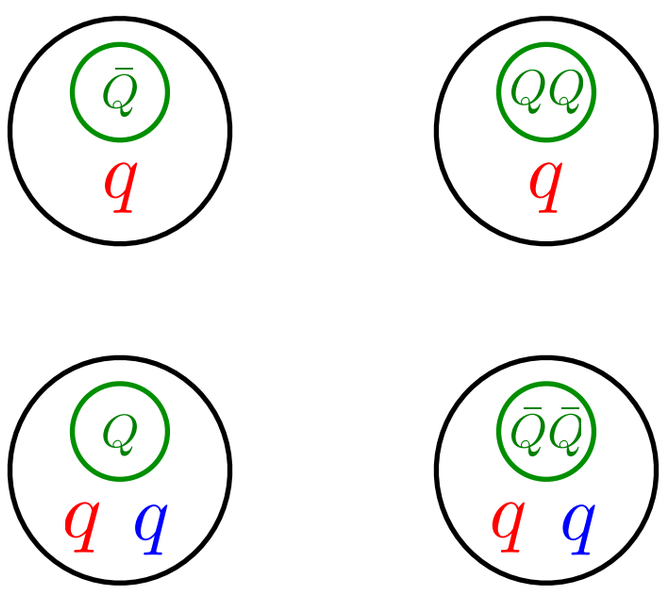
Doubly Heavy Tetraquark Production in Heavy-ion

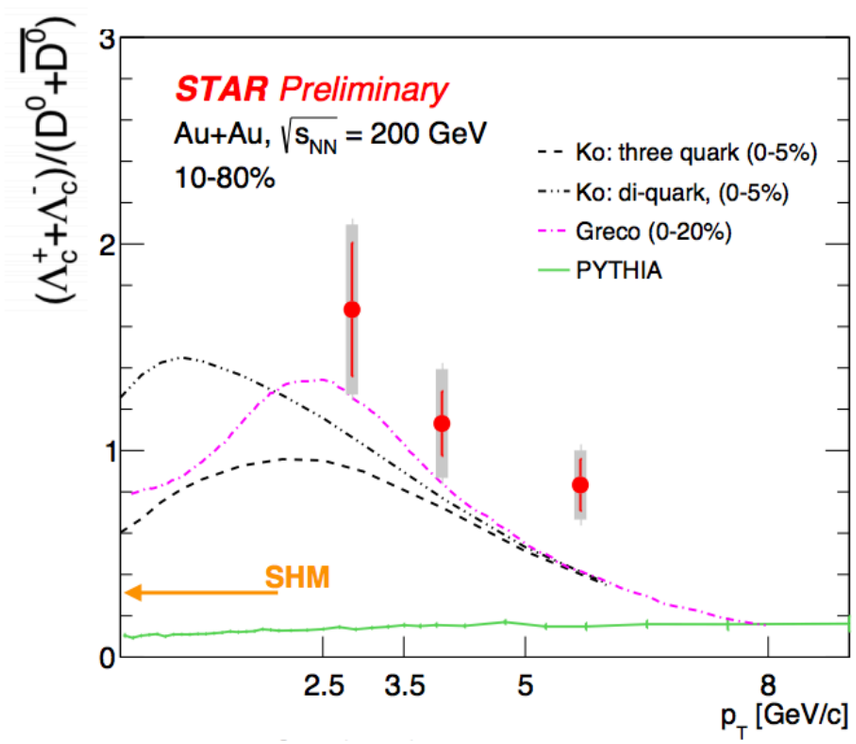
Same calculation can be extended to study doubly heavy tetraquark (**bound state**)
Only difference: at hadronization coalescence with two light quarks v.s. one

Hadronization of doubly heavy baryon similar to hadronization of singly heavy meson

Hadronization of doubly heavy tetraquark similar to hadronization of singly heavy baryon

Heavy quark diquark symmetry





enhancement of singly heavy baryon observed at STAR

expect enhancement of doubly heavy tetraquark in heavy-ion collisions

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Summary

- Describe both open and hidden heavy flavors: coupled Boltzmann equation
- Consistent dissociation and recombination from pNRQCD with realistic time-evolving HQ distributions
- Extract potential and melting temperature from data
- Future: include 1P 2P 3S states, temperature-dependent potential (extracted from data, compare with lattice), systematic extraction procedure (e.g. Bayesian)
- Heavy diquarks and doubly heavy baryons / tetraquarks

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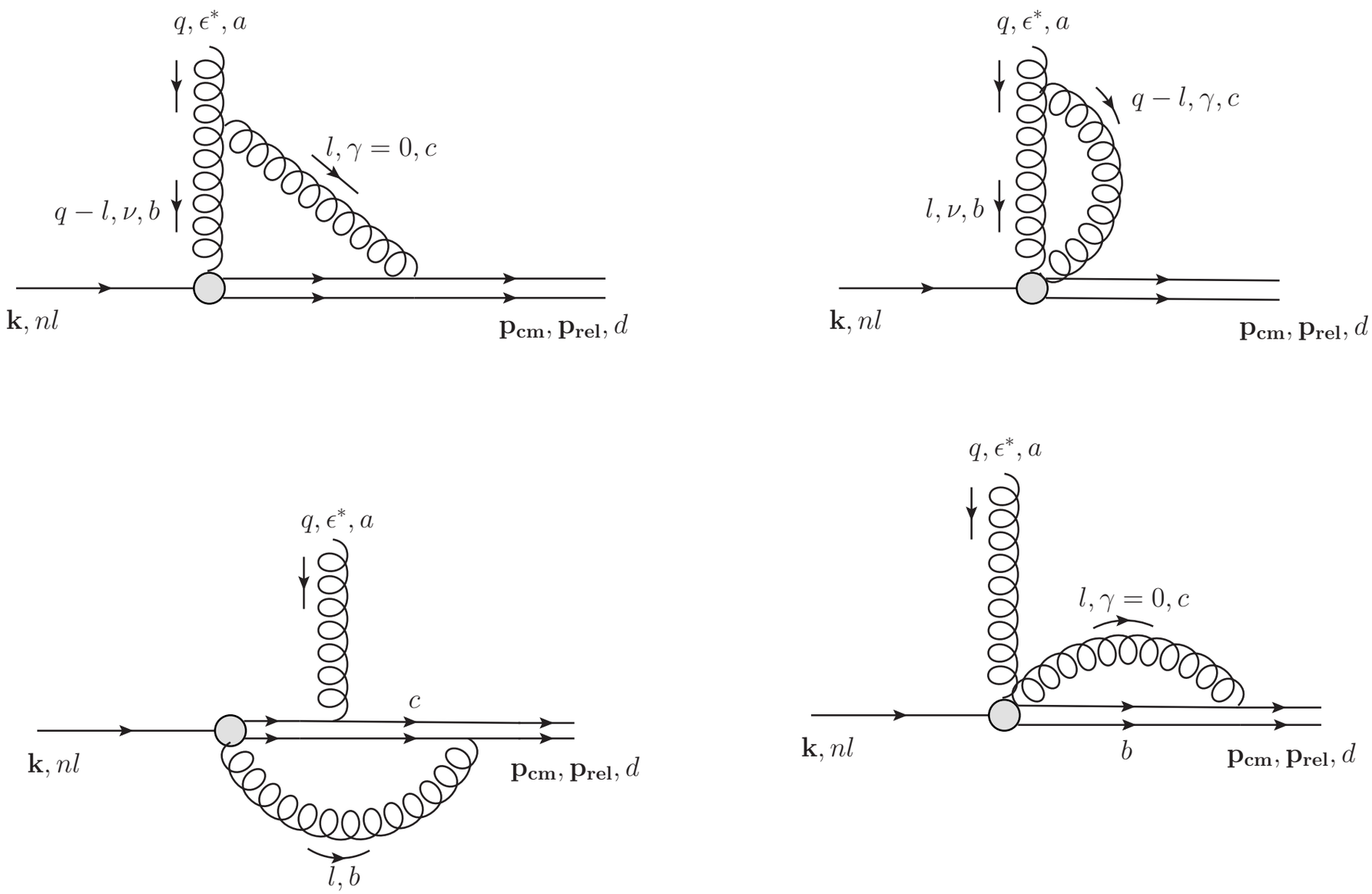
No Running of Dipole Vertex



Matching NRQCD and pNRQCD at scale ~ Mv, Wilson coefficient = 1
No running of the Wilson coefficient

Check Effects from Feed-down Contributions

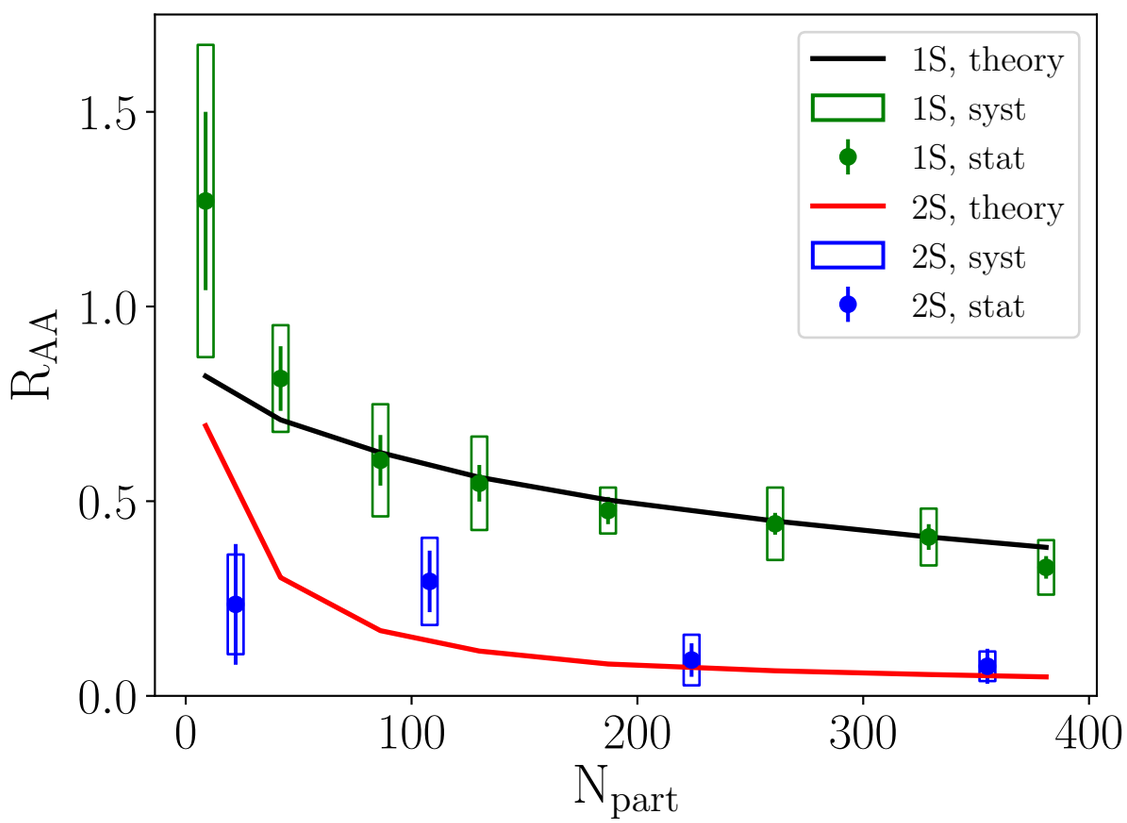
No 1P, 2P, 3S etc states in calculations, uncertainties?

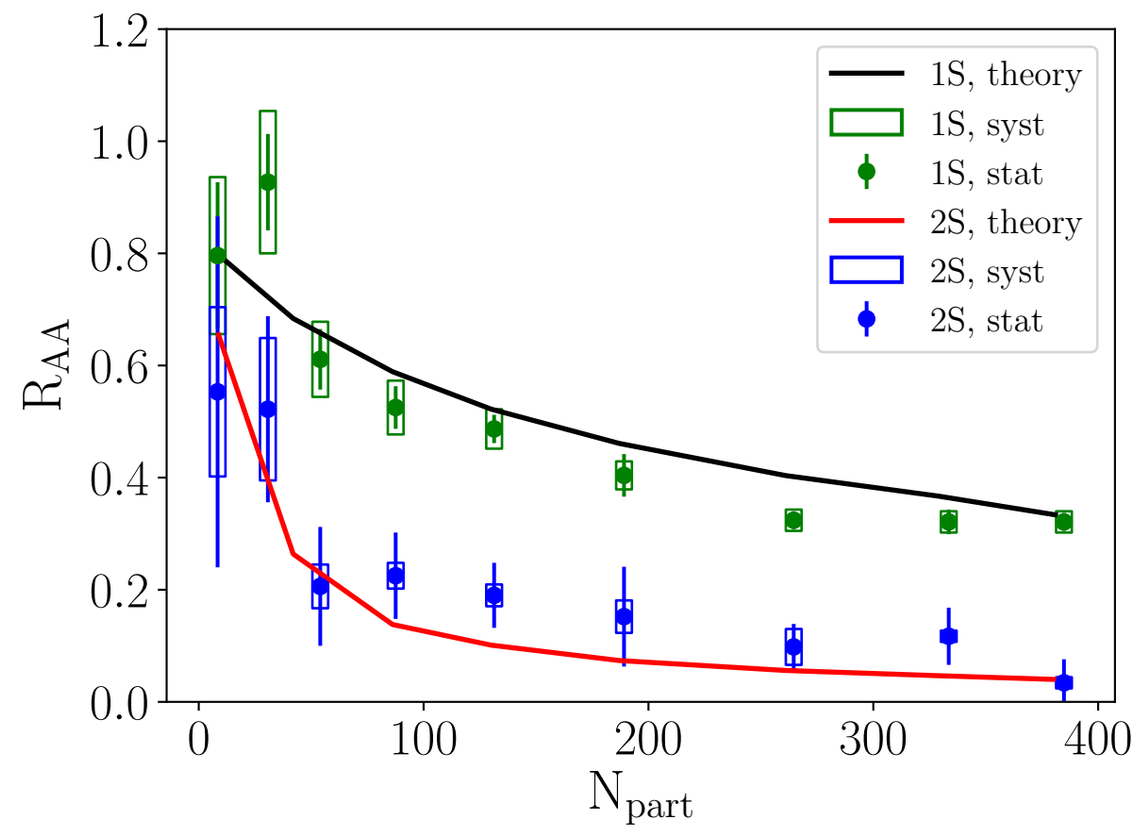
Change initial production ratio to 1S : 2S = 1:1 then ~20% 1S from feed-down vs ~6%

Results less sensitive to feed-down percentage than expected, WHY?

After 2S dissociates inside medium, it may recombine as 1S later.
Two competing factors: fewer feed-down v.s. in-medium recombination

Suppressed feed-down alone cannot explain Upsilon(1S) suppression
Upsilon(1S) dissociates inside QGP





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View from Open Quantum System

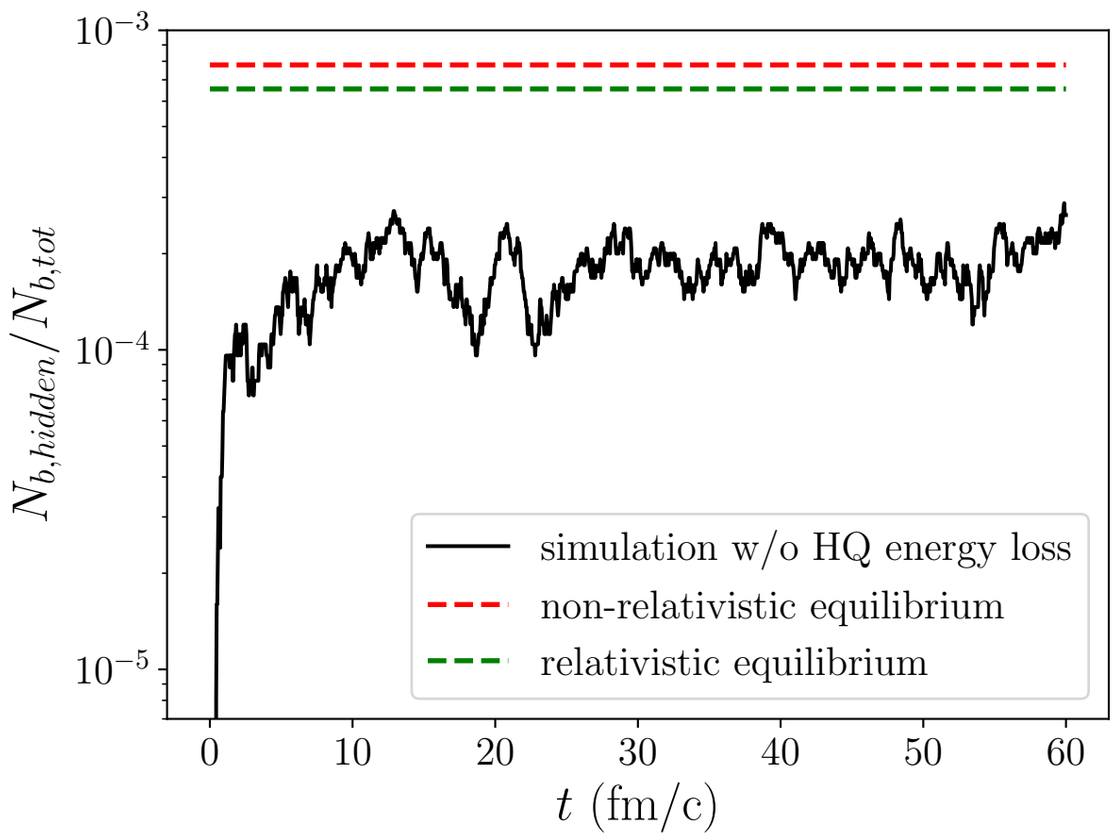
- Subsystem: heavy quark and quarkonium; environment: QGP
- Together evolve unitarily: von-Neumann equation $\frac{\partial \rho}{\partial t} = -i[H, \rho]$
- Trace out environment, Lindblad evolution equation of subsystem

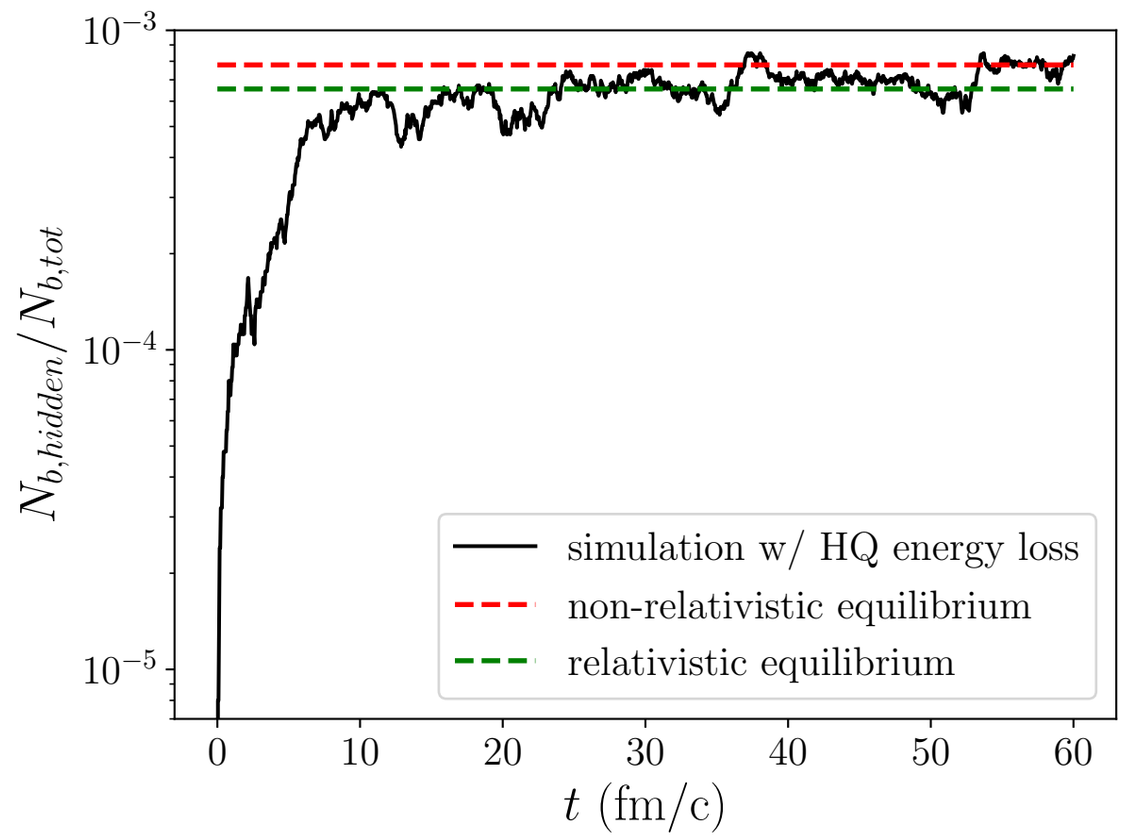
$$\rho_S \equiv \mathrm{Tr}_E \rho$$

$$\rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \Big(\underbrace{L_{ab}\rho_S(0)L^\dagger_{cd}}_{\text{recombination}} - \underbrace{\frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}}_{\text{dissociation}} \Big) - i \sum_{ab} \underbrace{\sigma_{ab}(t)[L_{ab}, \rho_S(0)]}_{\text{static screening correction of potential}}$$

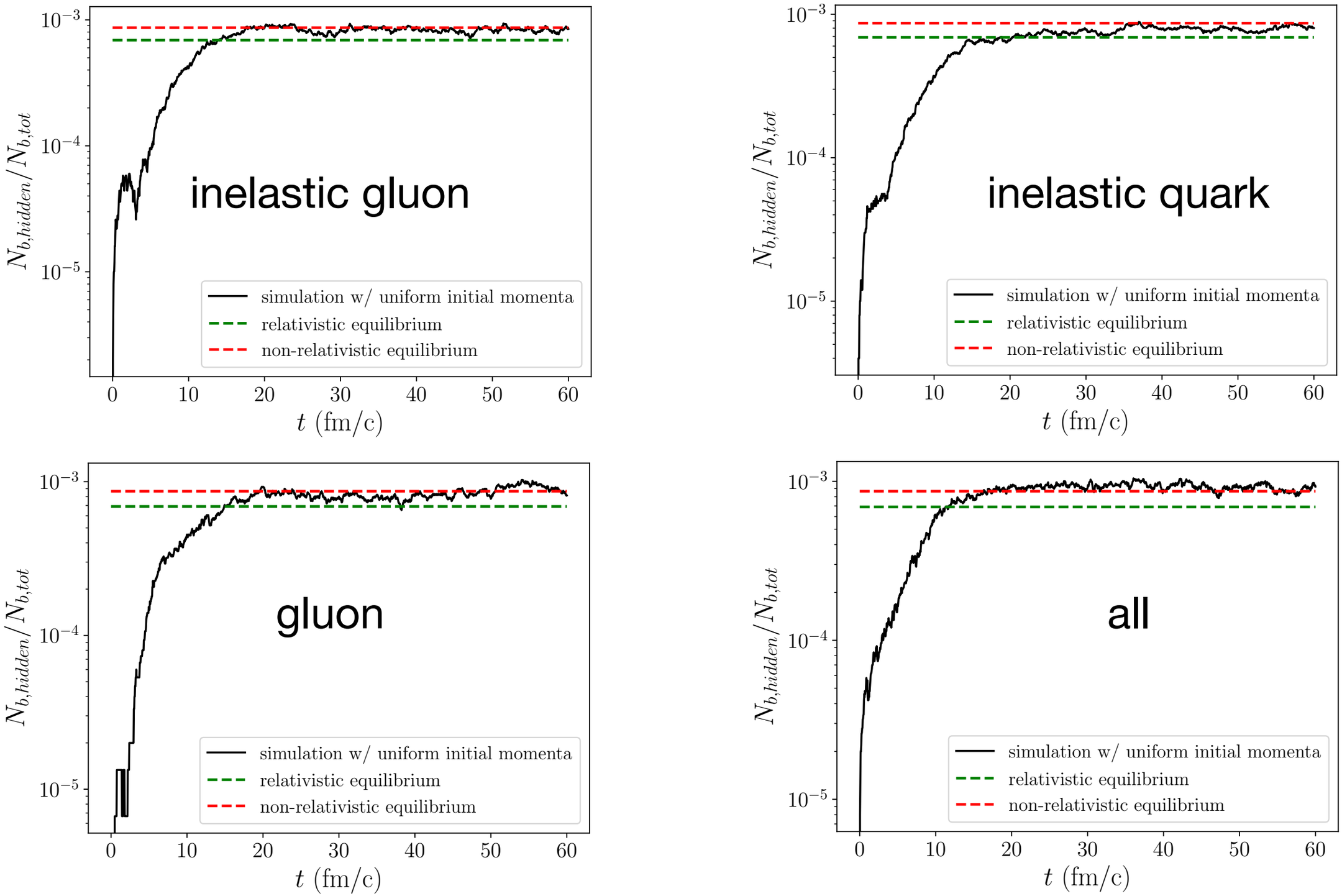
- Non-unitary, damping and trace conservation (dissociated quarkonium —> HQ)
- Time-irreversible (by monotonicity of relative entropy under partial trace)

Approach Equilibrium





Thermal Equilibrium



initial p uniformly sampled from 0-10 GeV

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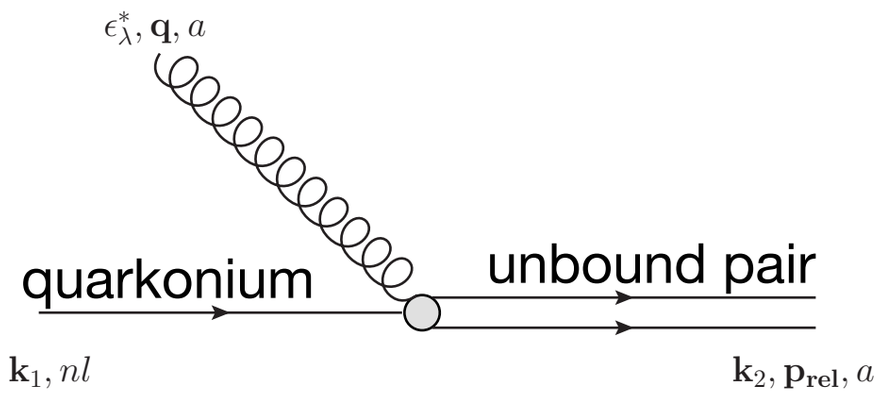
Scattering Amplitudes and Collision Terms

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = \boxed{+\mathcal{C}^{+}_{nl}} \boxed{-\mathcal{C}^{-}_{nl}}$$



gluon absorption/emission

$$\mathcal{T}^a = (2\pi)^4 \delta^3(\boldsymbol{q} + \boldsymbol{k}_1 - \boldsymbol{k}_2)\delta(q + E_{nl} - \frac{p^2_{\rm rel}}{M})\mathcal{M}^a$$

$$\mathcal{M}^a = -ig\sqrt{\frac{T_F}{N_c}} q \langle \psi_{nl} | \boldsymbol{\epsilon}^*_\lambda \cdot \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle$$

$$\overline{|\mathcal{M}^a|^2} = \sum_a |\mathcal{M}^a|^2$$

$$g_+ = \frac{3}{4}\frac{1}{d_8}\frac{d_8}{N_c^2} = \frac{1}{12}$$

$$g_- = 1$$

$$\mathcal{F}_+ \equiv g_+ \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3k_1}{(2\pi)^3}\frac{d^3q}{2q(2\pi)^3}(1 + n^{(q)}_B) f_b(\boldsymbol{x}, \boldsymbol{p}_1, t) f_{\bar{b}}(\boldsymbol{x}, \boldsymbol{p}_2, t)(2\pi)^4\delta^3(\boldsymbol{q} + \boldsymbol{k}_1 - \boldsymbol{k}_2)\delta(q + E_{nl} - \tfrac{p^2_{\rm rel}}{M})\overline{|\mathcal{M}^a|^2}$$

$$\mathcal{F}_- \equiv g_- \int \frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_1}{(2\pi)^3}\frac{d^3q}{2q(2\pi)^3} n^{(q)}_B f_{nl}(\boldsymbol{x}, \boldsymbol{k}_1, t)(2\pi)^4\delta^3(\boldsymbol{q} + \boldsymbol{k}_1 - \boldsymbol{k}_2)\delta(q + E_{nl} - \tfrac{p^2_{\rm rel}}{M})\overline{|\mathcal{M}^a|^2}$$

$$\mathcal{C}^{+}_{nl} = \left.\frac{\delta \mathcal{F}_+}{\delta \boldsymbol{k}_1}\right|_{\boldsymbol{k}_1 = \boldsymbol{p}}$$

$$\mathcal{C}^{-}_{nl} = \left.\frac{\delta \mathcal{F}_-}{\delta \boldsymbol{k}_1}\right|_{\boldsymbol{k}_1 = \boldsymbol{p}}$$

$$\frac{\delta}{\delta \boldsymbol{p}_i}\int \prod_{j=1}^{n} \frac{{\rm d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_n)\Big|_{\boldsymbol{p}_i = \boldsymbol{p}} \equiv \frac{\delta}{\delta a(\boldsymbol{p})}\int \prod_{j=1}^{n} \frac{{\rm d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_n) a(\boldsymbol{p}_i)$$

$$= \int \prod_{j=1, j\neq i}^{n} \frac{{\rm d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_{i-1}, \boldsymbol{p}, \boldsymbol{p}_{i+1}, \cdots, \boldsymbol{p}_n)$$

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Numerical Implementation

- Test particle Monte Carlo $f(\boldsymbol{x},\boldsymbol{p},t)=\sum_i \delta^3(\boldsymbol{x}-\boldsymbol{y}_i(t))\delta^3(\boldsymbol{p}-\boldsymbol{k}_i(t))$

- Each time step: consider diffusion, dissociation, recombination in particle's rest frame and boost back

- If specific process occurs, sample incoming medium particles and outgoing particles from integrands, conserving energy momentum

- Recombination term contains $f_Q(\boldsymbol{x},\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x},\boldsymbol{p}_2,t)$

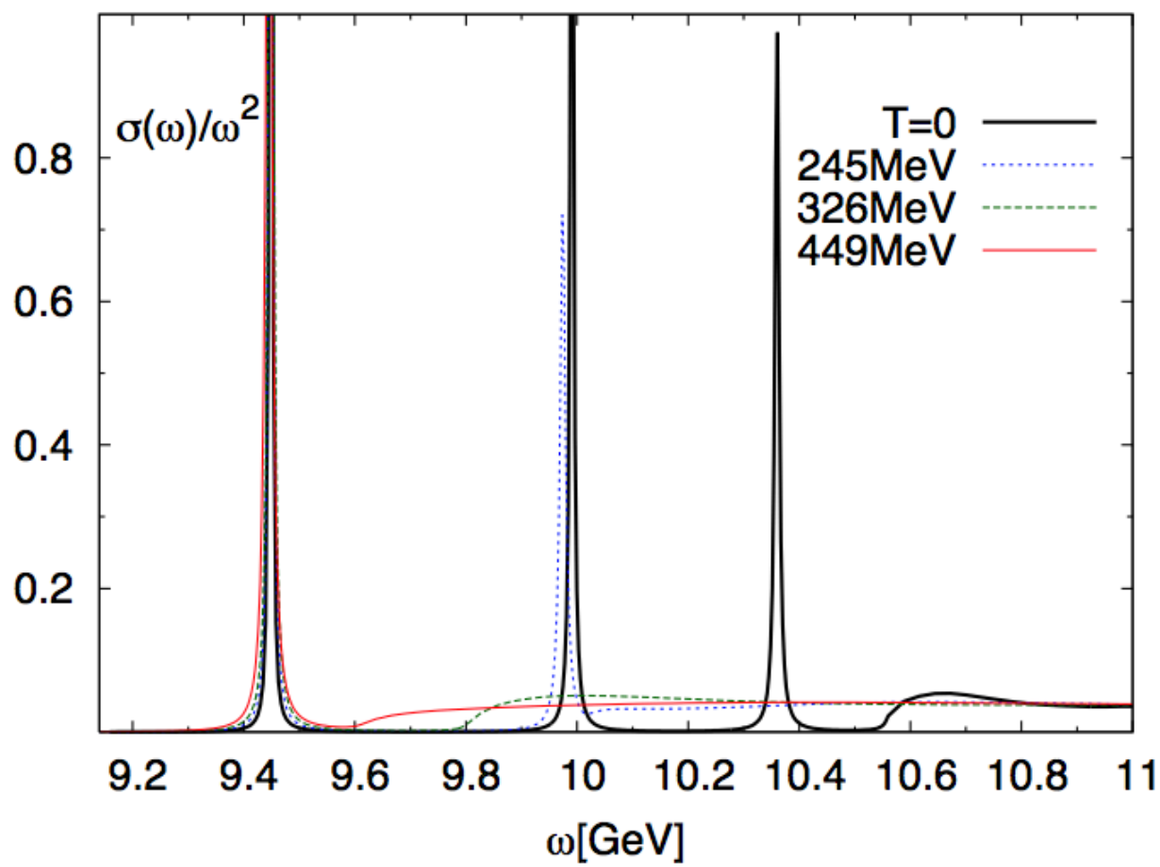
Two delta at same x ill-defined, almost never at same point

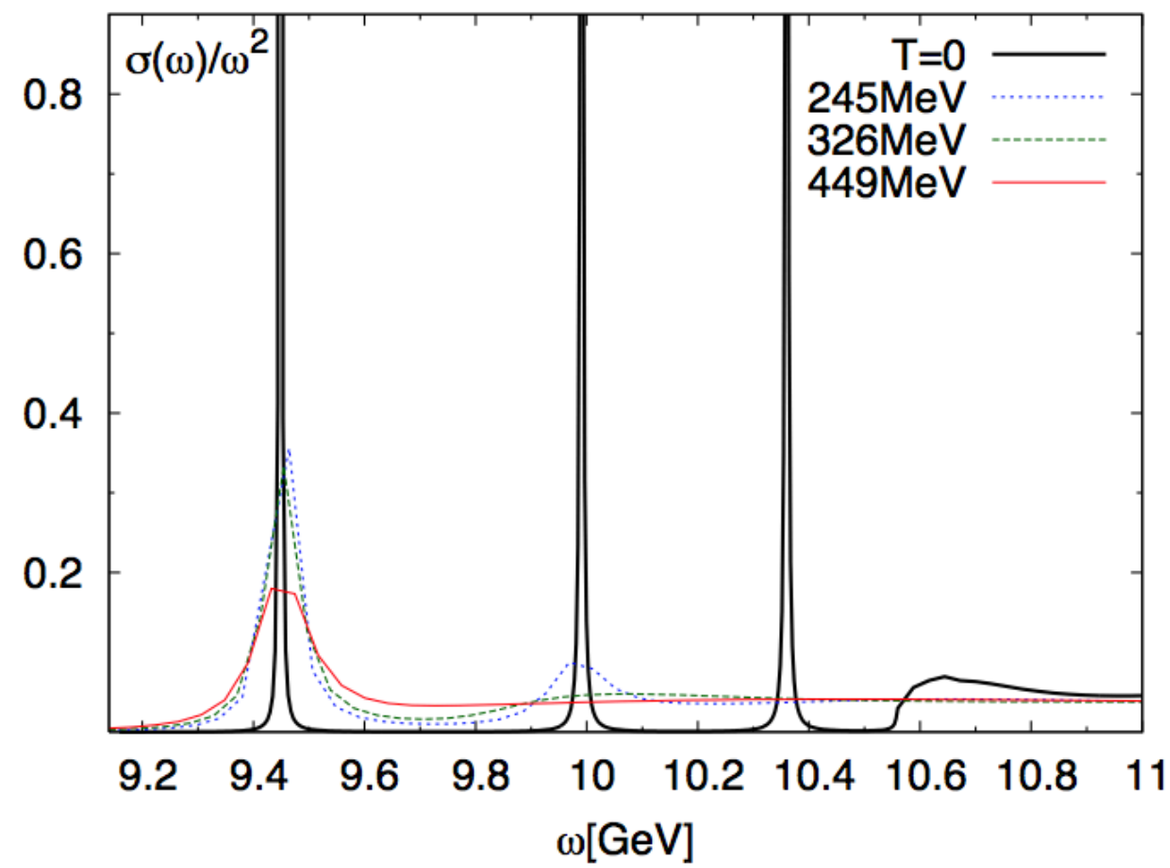
Enhance sampling for recombination, search pairs within a radius

$$f_Q(\boldsymbol{x},\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x},\boldsymbol{p}_2,t)\to\sum_{i,j}\frac{e^{-(\boldsymbol{y}_i-\boldsymbol{y}_j)^2/2a_B^2}}{(2\pi a_B^2)^{3/2}}\delta^3\left(\boldsymbol{x}-\frac{\boldsymbol{y}_i+\boldsymbol{y}_j}{2}\right)\delta^3(\boldsymbol{p}_1-\boldsymbol{k}_i)\delta^3(\boldsymbol{p}_2-\boldsymbol{k}_j)$$

Xiaojun Yao (Duke)

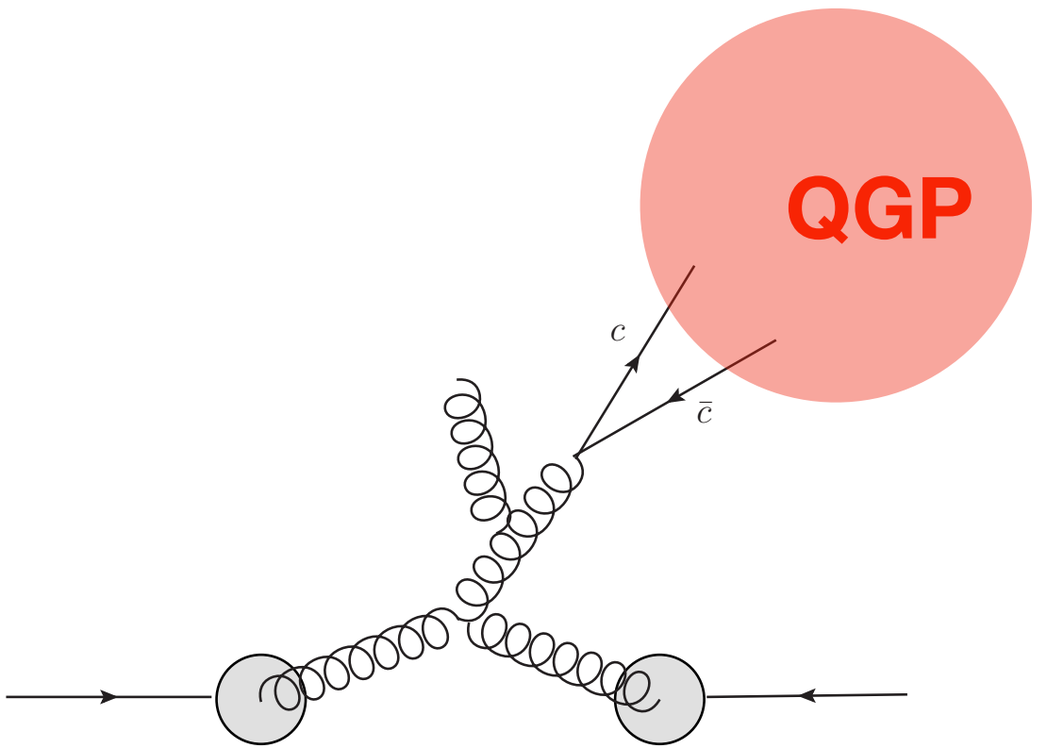
Backup: Imaginary Part More Important

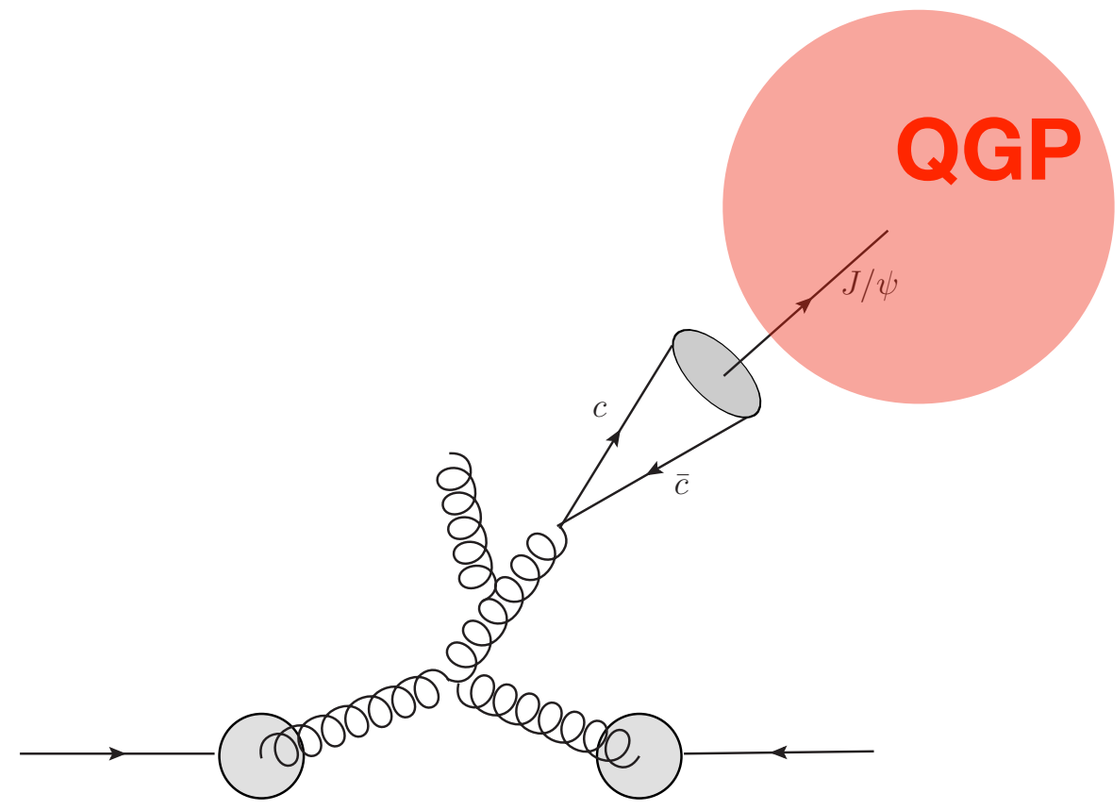




C. Miao, A. Mocsy, P. Petreczky arXiv:1012.4433

Backup: Initial Production





Initially no quarkonium enters QGP
quarkonium is formed (recombined)
inside QGP or later
(re)combination dominates

Initially quarkonium is generated
and enters QGP
suppressed due to screening
dissociation dominates