Quarkonium Production in Heavy Ion Collision: Coupled Boltzmann Transport Equations

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Introduction

- Debye (static) screening on heavy quark bound state, not enough explain quarkonium production suppression

- Production complicated by many factors:
  - Cold nuclear matter effect (CNM) initial production
  - Static screening (real part potential suppressed) v.s. dynamical screening (imaginary part potential, related to dissociation, thermal width)
  - In-medium evolution: dissociation v.s. recombination (sensitive to open HQ dynamics)
  - Feed-down, etc.

- Include all factors consistently:
  - Open quantum system (non-unitary, time irreversible dynamics from QCD)
  - Transport equations
Dynamical Evolution: Dissociation

1) not exist due to static screening
2) dissociate due to dynamical screening

Initial production of $b$ and $\bar{b}$ particles in the QGP medium expands and cools over time, leading to hadronization of $b$ and $\bar{b}$ into hadron gas.

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Dynamical Evolution: Recombination

melting temperature: above which a specific bound state
1) ill defined (thermal width too large)
2) not exists (potential not supports bound state)

in-medium formation
RL. Thews, M. Schroedter, J. Rafelski

initial production | QGP medium expands and cools | hadron gas

QGP medium expands and cools

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Coupled Boltzmann Equations

heavy quark
\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_Q(x, p, t) = -C_Q^+ + C_Q^- + C_Q
\]

anti-heavy quark
\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{\bar{Q}}(x, p, t) = -C_{\bar{Q}}^+ + C_{\bar{Q}}^- + C_{\bar{Q}}
\]

each quarkonium state
\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{nl}(x, p, t) = +C_{nl}^+ - C_{nl}^-
\]
Coupled Boltzmann Equations

heavy quark

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\]

each quarkonium state

\[ n_l = 1S, 2S, 1P \text{ etc.} \]

\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{nl}(x, p, t) = +C_{nl}^+ - C_{nl}^-
\]

phase space evolution of distribution function

recombination

dissociation

quarkonium gain

quarkonium loss

heavy quark loss

heavy quark gain

heavy Q energy loss

see talk by Weiyao Ke
Tu 14:40 pm

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Dissociation, Recombination, pNRQCD

\[ \mathcal{L}^{\text{pNRQCD}} = \int d^3 r \operatorname{Tr} \left( S^{\dagger} (i \partial_0 - H_s) S + O^{\dagger} (i D_0 - H_o) O + V_A (O^{\dagger} \mathbf{r} \cdot g \mathbf{E} + \text{h.c.}) + \frac{V_B}{2} O^{\dagger} \{ \mathbf{r} \cdot \mathbf{E}, O \} + \cdots \right) \]

- Separation of scales (bound state exists) \[ M \gg M v \gg M v^2, T, m_D \]
- Systematic expansion in \[ \frac{1}{M}, r \sim \frac{1}{M v} \]

\[ H_{s,o} = \frac{P_{c.m.}^2}{4 M} + \frac{p_{\text{rel}}^2}{M} + V_{s,o}^{(0)} + \frac{V_{s,o}^{(1)}}{M} + \frac{V_{s,o}^{(2)}}{M^2} + \cdots \]

\[ V_{s}^{(0)} = -C_F \frac{\tilde{\alpha}_s}{r}, \quad V_{o}^{(0)} = \frac{1}{2 N_C} \frac{\tilde{\alpha}_s}{r} \]

virial theorem

no imaginary potential

can be improve: lattice motivated potential

\[ \epsilon_1^a, q, a \]

quarkonium

unbound pair

\[ k_1, n l \]

\[ p_1, s_1, i \]

light quark

\[ p_2, s_2, j \]

inelastic scattering w/ light quark

\[ q_1, \epsilon_1^a, a \]

\[ q_2, \epsilon_2^b \]

in elastic scattering w/ gluon

\[ k_2, p_{\text{rel}}, a \]

\[ k_2, p_{\text{rel}}, c \]

\[ q \]

\[ \epsilon_1^a \]

\[ \epsilon_2^b \]

\[ k_1, n l \]

\[ k_2, p_{\text{rel}}, a \]

\[ k_1, n l \]

\[ k_2, p_{\text{rel}}, a \]
Approach Equilibrium

Setup:
QGP box w/ const T, 1S state and b quarks: total b flavor = 50 (fixed)
Initial momenta sampled from thermal or uniform distributions

Recombination from QCD effective field theory and real dynamics of HQ

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy loss necessary to drive kinetic equilibrium of quarkonium


Recombination from QCD effective field theory and real dynamics of HQ

Heavy quark energy loss necessary to drive kinetic equilibrium of quarkonium

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Collision Event Simulation

- Initial production:
  
  **PYTHIA 8.2: NRQCD factorization**

  **Nuclear PDF: EPS09 (cold nuclear matter effect)**
  Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

  **Trento, sample position, hydro. initial condition**

- Medium background: 2+1D viscous hydrodynamics (**calibrated**)


- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

- Effect of neglecting other states: **feed-down v.s. in-medium recombination**
Upsilon in 2760 GeV PbPb Collision

Fix $\alpha_s = 0.3$

Tune $T_{\text{melt}}(2S) = 210$ MeV

Tune $V_s = -C_F \frac{0.42}{r}$

Cold nuclear matter effect $\sim 0.87$ (PYTHIA + nPDF)


Use same set of parameters

Cold nuclear matter effect $\sim 0.85$ (PYTHIA + nPDF)
Upsilon in 200 GeV AuAu Collision

Use same set of parameters

Cold nuclear matter effect $\sim 0.72$

STAR measures 2S+3S; sPHENIX upgrades
Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

Better understanding of quarkonium transport from $v_2$ measurements

Develop azimuthal momentum anisotropy: dynamical evolution

produced from initial hard collision

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Doubly Charmed Baryon

- LHCb observed a new baryon $\Xi_{cc}^{++}$ (ccu): u bound around cc core
- Pair of heavy Q in anti-triplet forms bound state (diquark)
- Heavy diquark in QGP: dissociation, recombination (similar to quarkonium), carry color, energy loss different from quarkonium
- Hadronize into doubly charmed baryon
Doubly Charmed Baryon Production in Heavy Ion Collisions

Setup:
coupled Boltzmann for charm quark and diquark (add energy loss of diquark)
assume only charm quark produced initially, diquark comes from (re)combination

Predicted production rate of $\Xi^{++}_{cc}$
in 2760 GeV PbPb, -1<y<1, 0<pT<5 GeV,
0.02 per collision

With melting temperature = 250 MeV:
0.0125 per collision

Compare: c quark rate ~10 per collision

Production rate from initial hard collision probably small
Study recombination from measurements
Doubly Heavy Tetraquark Production in Heavy-ion

Same calculation can be extended to study doubly heavy tetraquark (bound state)
Only difference: at hadronization coalescence with two light quarks v.s. one

Hadronization of doubly heavy baryon similar to hadronization of singly heavy meson

Hadronization of doubly heavy tetraquark similar to hadronization of singly heavy baryon

Heavy quark diquark symmetry

enhancement of singly heavy baryon observed at STAR

expect enhancement of doubly heavy tetraquark in heavy-ion collisions
Summary

- Describe both open and hidden heavy flavors: coupled Boltzmann equation

- Consistent dissociation and recombination from pNRQCD with realistic time-evolving HQ distributions

- Extract potential and melting temperature from data

- Future: include 1P 2P 3S states, temperature-dependent potential (extracted from data, compare with lattice), systematic extraction procedure (e.g. Bayesian)

- Heavy diquarks and doubly heavy baryons / tetraquarks
No Running of Dipole Vertex

Matching NRQCD and pNRQCD at scale $\sim M_v$, Wilson coefficient = 1
No running of the Wilson coefficient
Check Effects from Feed-down Contributions

No 1P, 2P, 3S etc states in calculations, uncertainties?

Change initial production ratio to 1S : 2S = 1:1 then ~20% 1S from feed-down vs ~6%

Results less sensitive to feed-down percentage than expected, WHY?

After 2S dissociates inside medium, it may recombine as 1S later.
Two competing factors: fewer feed-down v.s. in-medium recombination

Suppressed feed-down alone cannot explain Upsilon(1S) suppression
Upsilon(1S) dissociates inside QGP

![Graphs showing R_AA vs N_part for 1S and 2S production and recombination]
View from Open Quantum System

- Subsystem: heavy quark and quarkonium; environment: QGP

- Together evolve unitarily: von-Neumann equation
  \[ \frac{\partial \rho}{\partial t} = -i[H, \rho] \]

- Trace out environment, Lindblad evolution equation of subsystem
  \[ \rho_S \equiv \text{Tr}_E \rho \]
  \[ \rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left( L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S(0) \} \right) - i \sum_{ab} \sigma_{ab}(t)[L_{ab}, \rho_S(0)] \]

- Non-unitary, damping and trace conservation (dissociated quarkonium \( \rightarrow \) HQ)

- Time-irreversible (by monotonicity of relative entropy under partial trace)
Approach Equilibrium

Simulation with HQ energy loss

Simulation without HQ energy loss

Non-relativistic equilibrium

Relativistic equilibrium
Thermal Equilibrium

initial p uniformly sampled from 0-10 GeV

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Scattering Amplitudes and Collision Terms

\[
\left( \frac{\partial}{\partial t} + \mathbf{x} \cdot \nabla \mathbf{x} \right) f_{nl}(\mathbf{x}, \mathbf{p}, t) = +C^+_{nl} - C^-_{nl}
\]

\[
\mathcal{T}^a = (2\pi)^4 \delta^3(q + \mathbf{k}_1 - \mathbf{k}_2) \delta(q + E_{nl} - \frac{p_{rel}^2}{M}) \mathcal{M}^a
\]

\[
\mathcal{M}^a = -i g \sqrt{\frac{T_F}{N_c}} q \langle \psi_{nl} | \epsilon^*_\lambda \cdot \mathbf{r} | \psi_{p_{rel}} \rangle
\]

\[
|\mathcal{M}^a|^2 = \sum_a |\mathcal{M}^a|^2
\]

\[
\mathcal{F}^+_a \equiv g_+ \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3k_1}{(2\pi)^3} \frac{d^3q}{2q(2\pi)^3} (1 + n_B^{(q)}) f_b(\mathbf{x}, \mathbf{p}_1, t) f_b(\mathbf{x}, \mathbf{p}_2, t) (2\pi)^4 \delta^3(q + \mathbf{k}_1 - \mathbf{k}_2) \delta(q + E_{nl} - \frac{p_{rel}^2}{M}) |\mathcal{M}^a|^2
\]

\[
\mathcal{F}^-_a \equiv g_- \int \frac{d^3p_{rel}^1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_1}{(2\pi)^3} \frac{d^3q}{2q(2\pi)^3} n_B^{(q)} f_{nl}(\mathbf{x}, \mathbf{k}_1, t) (2\pi)^4 \delta^3(q + \mathbf{k}_1 - \mathbf{k}_2) \delta(q + E_{nl} - \frac{p_{rel}^2}{M}) |\mathcal{M}^a|^2
\]

\[
C^+_{nl} = \frac{\delta \mathcal{F}^+_a}{\delta \mathbf{k}_1} \bigg|_{\mathbf{k}_1 = \mathbf{p}}
\]

\[
C^-_{nl} = \frac{\delta \mathcal{F}^-_a}{\delta \mathbf{k}_1} \bigg|_{\mathbf{k}_1 = \mathbf{p}}
\]

\[
\frac{\delta}{\delta \mathbf{p}_i} \int \prod_{j=1}^n \frac{d^3p_j}{(2\pi)^3} h(\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n) \bigg|_{\mathbf{p}_i = \mathbf{p}} \equiv \frac{\delta}{\delta a(p)} \int \prod_{j=1}^n \frac{d^3p_j}{(2\pi)^3} h(\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n) a(\mathbf{p}_i)
\]

\[
= \int \prod_{j=1, j \neq i}^n \frac{d^3p_j}{(2\pi)^3} h(\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{i-1}, \mathbf{p}, \mathbf{p}_{i+1}, \cdots, \mathbf{p}_n)
\]
Numerical Implementation

- Test particle Monte Carlo \( f(x, p, t) = \sum_i \delta^3(x - y_i(t))\delta^3(p - k_i(t)) \)

- Each time step: consider diffusion, dissociation, recombination in particle’s rest frame and boost back

- If specific process occurs, sample incoming medium particles and outgoing particles from integrands, conserving energy momentum

- Recombination term contains \( f_Q(x, p_1, t)f_{\bar{Q}}(x, p_2, t) \)

Two delta at same x ill-defined, almost never at same point

Enhance sampling for recombination, search pairs within a radius

\[
f_Q(x, p_1, t)f_{\bar{Q}}(x, p_2, t) \rightarrow \sum_{i,j} e^{-\frac{(y_i - y_j)^2}{2a_B^2}} \frac{1}{(2\pi a_B^2)^{3/2}} \delta^3\left(x - \frac{y_i + y_j}{2}\right) \delta^3(p_1 - k_i)\delta^3(p_2 - k_j)
\]
Backup: Imaginary Part More Important

C. Miao, A. Mocsy, P. Petreczky arXiv:1012.4433
Backup: Initial Production

Initially no quarkonium enters QGP; quarkonium is formed (recombined) inside QGP or later. (re)combination dominates.

Initially quarkonium is generated and enters QGP; suppressed due to screening, dissociation dominates.