

HEAVY FLAVOUR ENERGY LOSS FROM ADS/CFT: A NOVEL DIFFUSION COEFFICIENT DERIVATION AND ITS PREDICTIONS

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Special thanks: R. W. Moerman, R. Morad, and N. Moodley

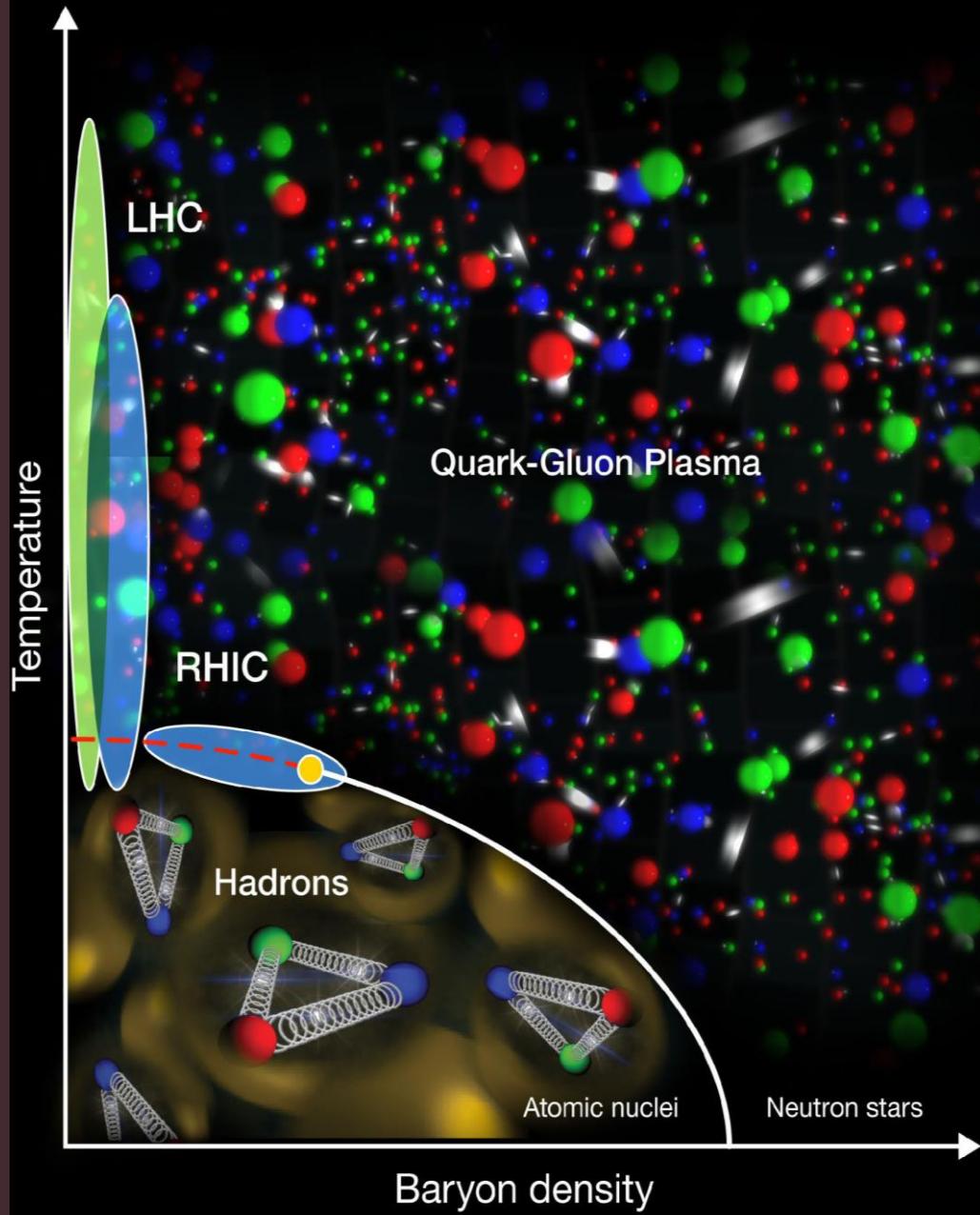
R. W. Moerman and W. A. Horowitz, arXiv:1605.09285

RH and W. A. Horowitz, arXiv:1703.05845

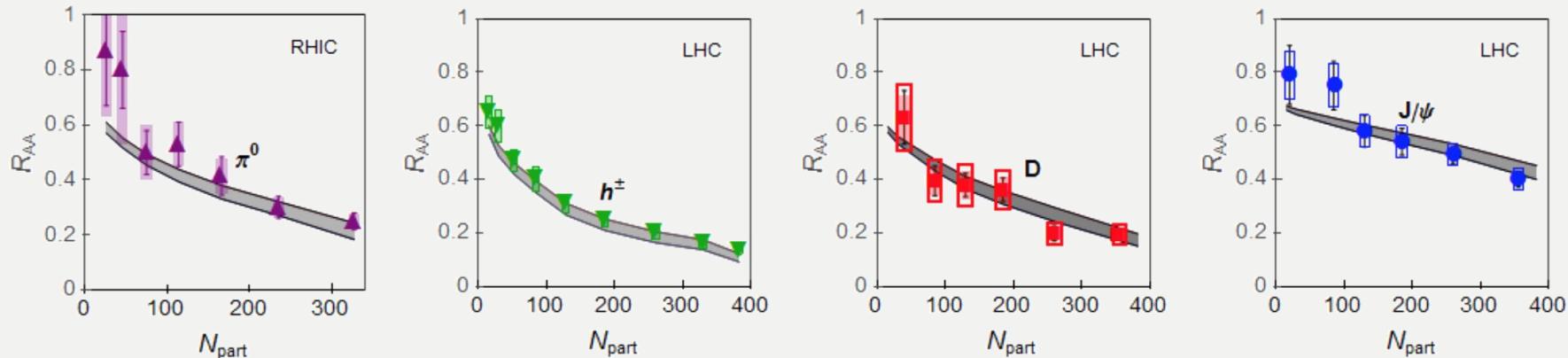


**National
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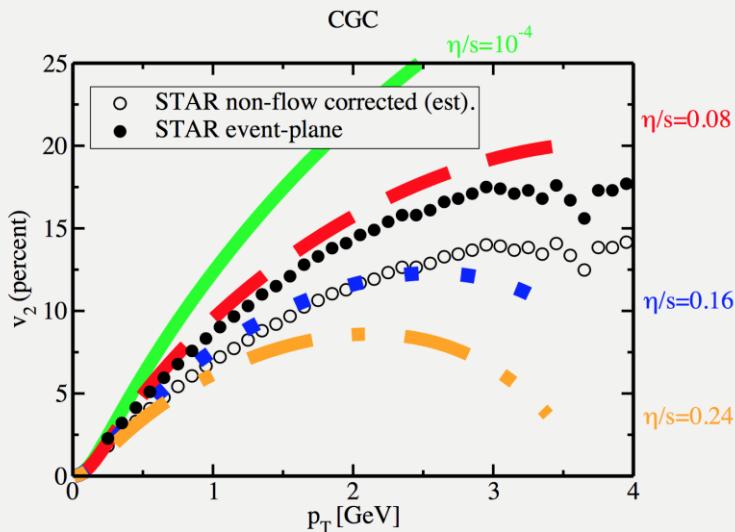
- Quark-Gluon Plasma first probe of the emergent hot many-body dynamics of QCD
- Many of its basic properties still unknown after 15 years



pQCD in the QGP



M. Djordjevic, PLB737 (2014)

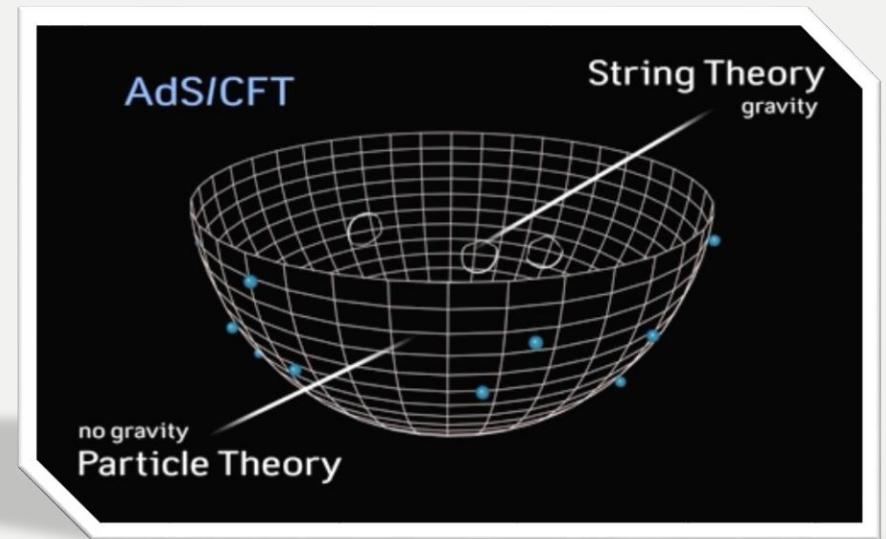


However, a naïve application of pQCD yields $\frac{\eta}{s} \sim 1$

M. Luzum, PRC78 (2009)

AdS/CFT

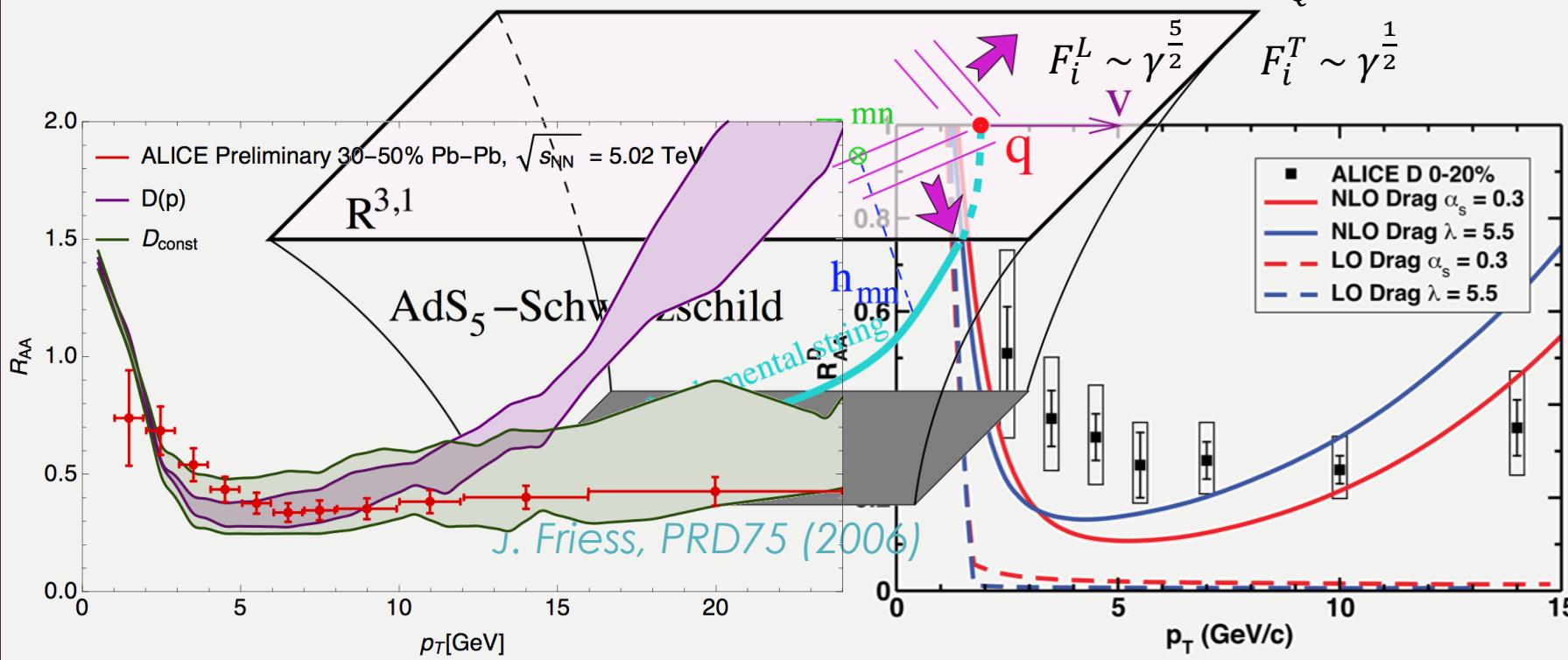
- $3+1 \mathcal{N}=4$ SYM \Leftrightarrow Type IIB on $AdS_5 \times S^5$
- Universal lower bound $\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$ – agrees with data from particle colliders



NLO drag in AdS/CFT

$$\frac{dp_i}{dt} = -\mu p_i + F_i^L + F_i^T$$

$$\mu = \frac{\pi\sqrt{\lambda} T^2}{2M_Q}$$

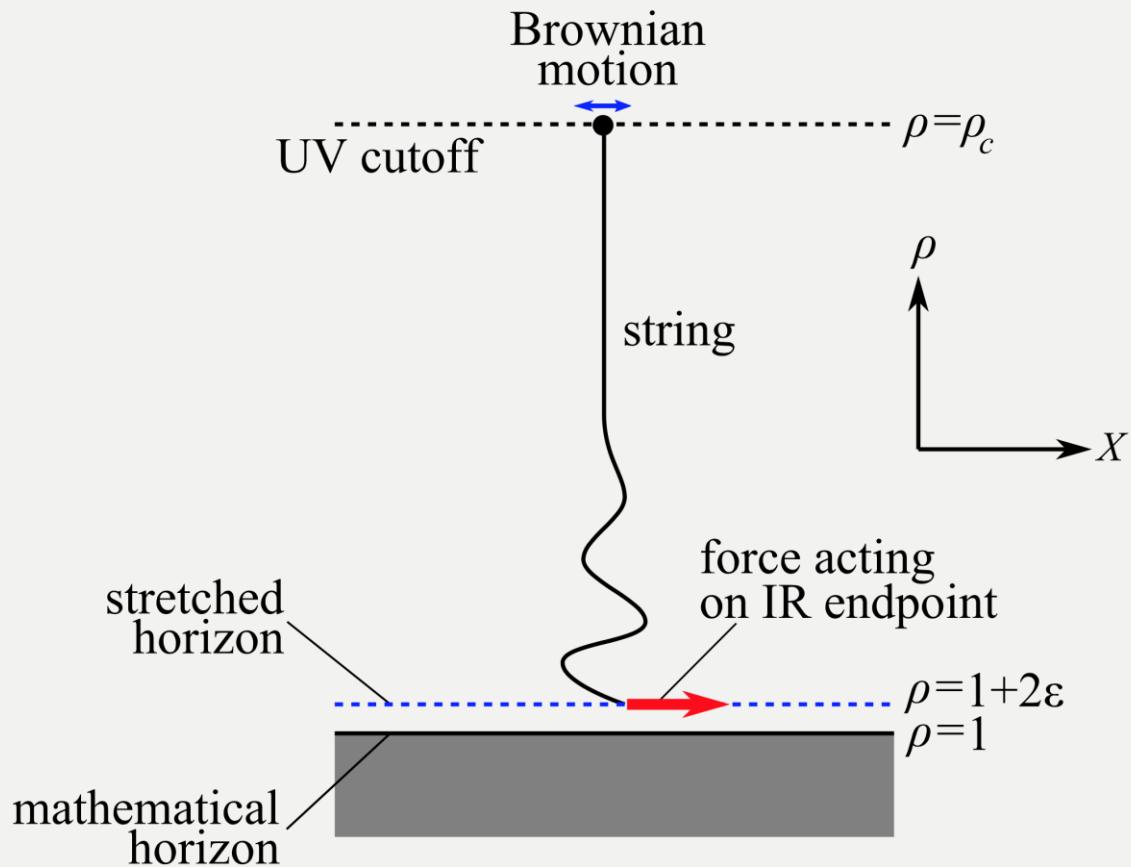


W. A Horowitz, PRD91 (2015)

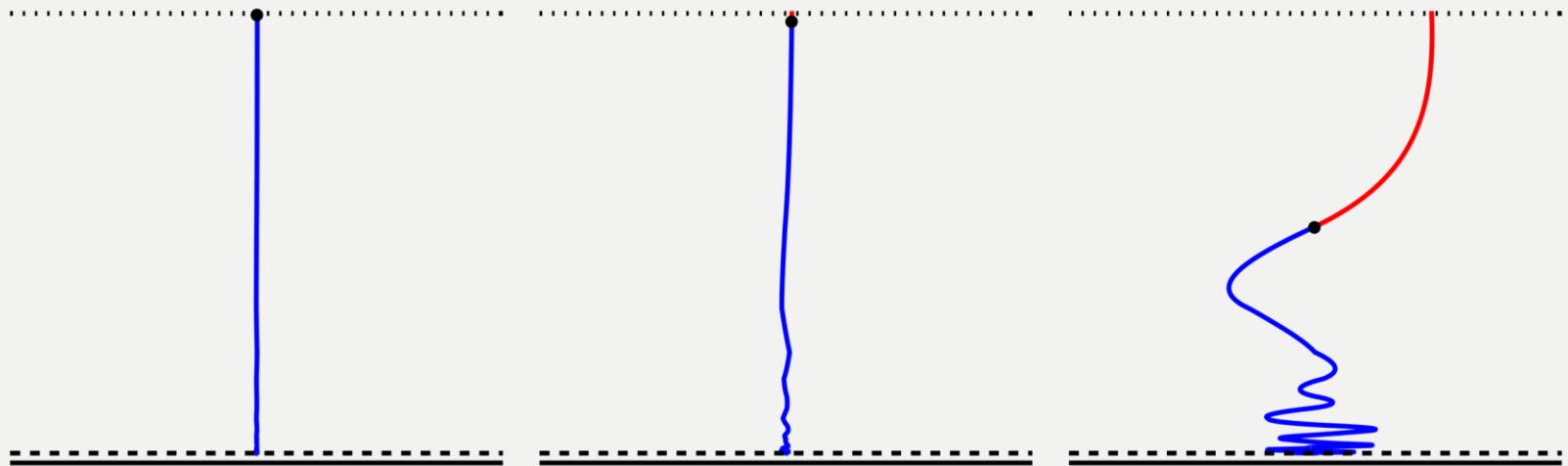
Derivation outline

- Find leading order classical solution of analogue of offshell $\nu = 0$ light quark in AdS_3
- Quantize transverse fluctuations on limp noodle
- Semi-classical approximation: Take mode expansion and populate according to Bose statistics
- Compute correlators and extend to arbitrary d (in particular, d=5)
- Motivate validity for $\nu \neq 0$

Setup



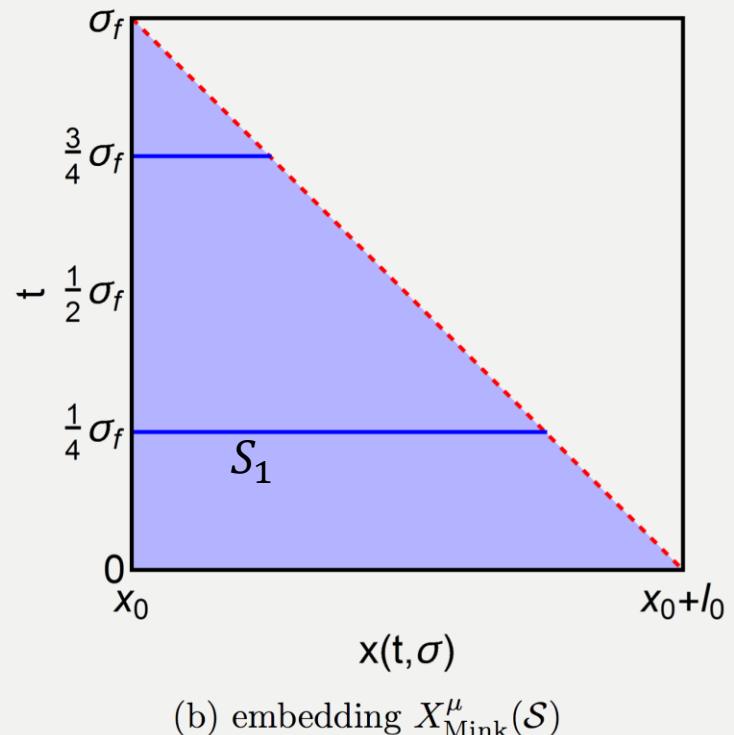
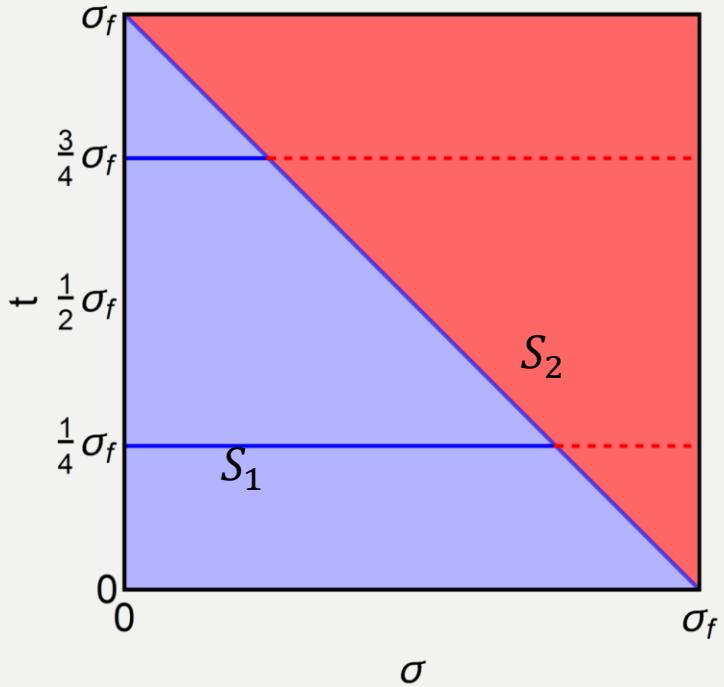
Setup



Red + Blue: heavy flavour solution

Blue: light flavour propagating along heavy flavour solution

Setup



$$S_1(a) = \{(t, \sigma) \in S \mid \sigma \in [0, \sigma_f - at]\}$$

$$S_2(a) = \{(t, \sigma) \in S \mid \sigma \in (\sigma_f - at, \sigma_f]\}$$

$$X_{\text{Mink}}^\mu(t, \sigma) = (t, x_0 + \begin{cases} \sigma, & \text{if } (t, \sigma) \in S_1 \\ \sigma_f - at, & \text{if } (t, \sigma) \in S_2 \end{cases}, 0)$$

LO classical solution

Metric:

$$ds_d^2 = \frac{r^2}{L^2} (-h(r; d)dt^2 + d\mathbf{x}_{d-2}^2) + \frac{L^2}{r^2} \frac{dr^2}{h(r; d)}, \quad h(r; d) := 1 - \left(\frac{r_H}{r}\right)^{d-1}$$

Tortoise coordinate:

$$r_*(d) := L^2 \int \frac{dr}{r^2 h(r; d)} = -\frac{L^2}{r} {}_2F_1 \left(1, \frac{1}{d-1}; \frac{d}{d-1}; \left(\frac{r_H}{r}\right)^{d-1} \right)$$

$$r_*(d) \rightarrow -\infty \text{ as } r \rightarrow r_H$$

Cannot be inverted for
 $d > 3$
-> work in AdS_3 for
analytical solution that is
valid in entire spacetime

LO classical solution

Metric:

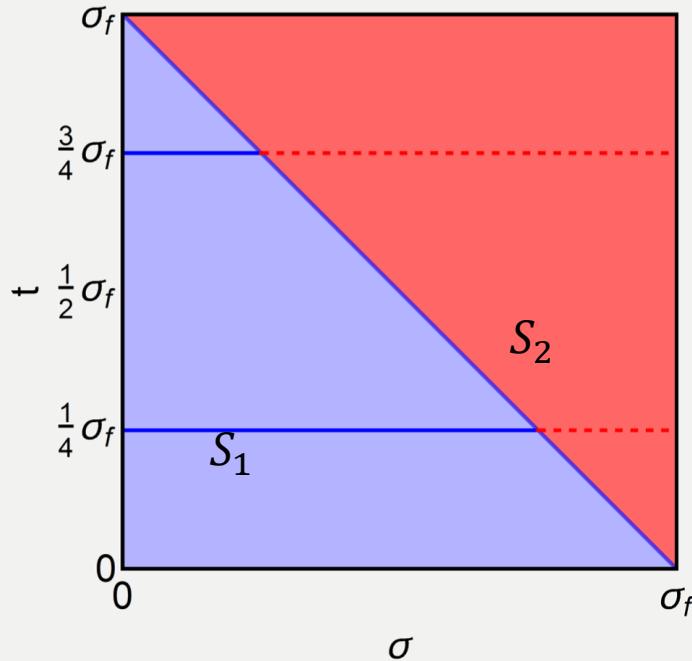
$$ds_d^2 = \frac{r^2}{L^2} (-h(r; d)dt^2 + dx_{d-2}^2) + \frac{L^2}{r^2} \frac{dr^2}{h(r; d)}, \quad h(r; d) := 1 - \left(\frac{r_H}{r}\right)^{d-1}$$

AdS_3 :

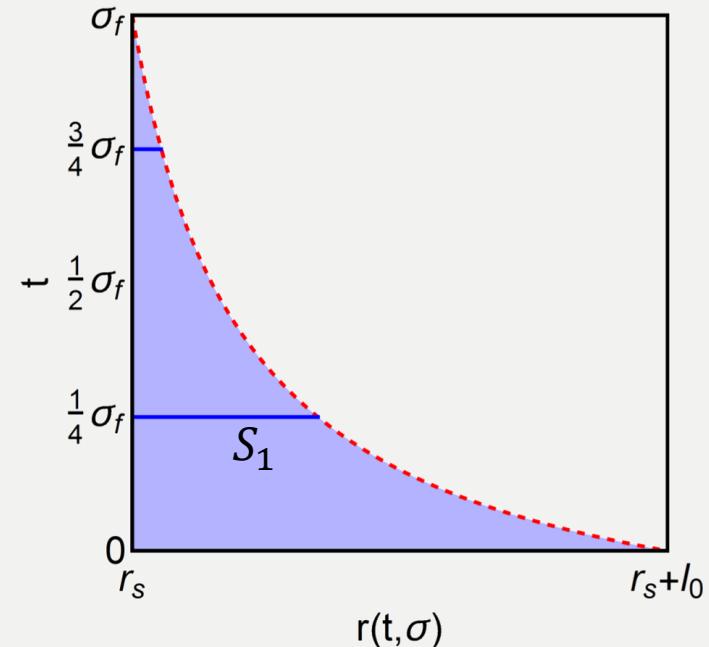
$$\begin{aligned} r_* &:= \frac{L^2}{r_H} \coth^{-1} \left(-\frac{r}{r_H} \right) \\ r &:= -r_H \coth \left(\frac{r_H r_*}{L^2} \right), \end{aligned}$$

$$ds^2 = \frac{r_H^2}{L^2} \operatorname{csch}^2 \left(\frac{r_H r_*}{L^2} \right) (-dt^2 + dr_*^2) + \frac{r_H^2}{L^2} \coth^2 \left(\frac{r_H r_*}{L^2} \right) dx^2$$

LO classical solution



(a) world-sheet \mathcal{S}



(b) embedding $X_{AdS_3}^\mu(\mathcal{S})$

$$S_1(a) = \{(t, \sigma) \in S | \sigma \in [0, \sigma_f - at]\}$$

$$S_2(a) = \{(t, \sigma) \in S | \sigma \in (\sigma_f - at, \sigma_f]\}$$

$$X_{AdS_3}^\mu \Big|_{S_1(a)} (t, \sigma) = (t, -r_H \coth\left(\frac{r_H(r_{s*} + \sigma)}{L^2}\right), 0)$$

$$r_{s*} = \frac{L^2}{r_H} \coth^{-1}\left(-\frac{r_s}{r_H}\right) \quad \sigma_f = \frac{L^2}{r_H} \coth^{-1}\left(-\frac{r_s + l_0}{r_H}\right) - r_{s*}$$

Quantization of transverse fluctuations

Expand leading order Nambu-Goto action with quadratic terms:

$$S_{\text{NG}} = \int_{\mathcal{M}} d^2\sigma \mathcal{L}_{\text{NG}}|_{X_0^\mu} + S_{\text{NG}}^{(2)} + \mathcal{O}\left((X^I)^3\right),$$
$$S_{\text{NG}}^{(2)} := \frac{1}{2} \int_{\mathcal{M}} d^2\sigma \left. \frac{\partial^2 \mathcal{L}_{\text{NG}}}{\partial(\partial_a X^I) \partial(\partial_b X^J)} \right|_{X_0^\mu} \partial_a X^I \partial_b X^J$$

$$X_{AdS_3}^\mu|_{\mathcal{S}_1(a)}(t, \sigma; a) = X_0^\mu(t, \sigma; a) + \delta^{\mu 2} X(t, \sigma)$$
$$= \left(t, -r_H \coth \left(\frac{r_H(r_{s*} + \sigma)}{L^2} \right), X(t, \sigma) \right)^\mu$$

Quantization of transverse fluctuations

$$\begin{aligned}
X_{AdS_3}^\mu|_{\mathcal{S}_1(a)}(t, \sigma; a) &= X_0^\mu(t, \sigma; a) + \delta^{\mu 2} X(t, \sigma) \\
&= \left(t, -r_H \coth \left(\frac{r_H(r_{s*} + \sigma)}{L^2} \right), X(t, \sigma) \right)^\mu
\end{aligned}$$

EOM and boundary conditions:

$$\begin{aligned}
0 &= -\partial_t^2 X(t, \sigma) + \frac{1}{\coth^2 \left(\frac{r_H}{L^2} (r_{s*} + \sigma) \right)} \partial_\sigma \left(\coth^2 \left(\frac{r_H}{L^2} (r_{s*} + \sigma) \right) \partial_\sigma X(t, \sigma) \right) \\
&= -\partial_t^2 X(t, r) + \frac{r^2 - r_H^2}{L^4 r^2} \partial_r \left(r^2 (r^2 - r_H^2) \partial_r X(t, r) \right),
\end{aligned}$$

$$\left[\sqrt{-g} g^{\sigma b} G_{IJ} \right] \Big|_{X_0^\mu} \partial_a X^I \delta X^J \Big|_{\sigma=0}^{\sigma=\sigma_f} = 0$$

Quantization of transverse fluctuations

$$X(t, r) = \int_0^\infty \frac{d\omega}{2\pi} A_\omega [f_\omega(r) e^{-i\omega t} a_\omega + f_\omega^*(r) e^{i\omega t} a_\omega^*]$$

Promote Fourier coefficients to operators:

$$\hat{X}(t, \sigma) := \int_0^\infty \frac{d\omega}{2\pi} A_\omega [f_\omega(\sigma) e^{-i\omega t} \hat{a}_\omega + f_\omega^*(\sigma) e^{i\omega t} \hat{a}_\omega^\dagger]$$

$$[\hat{X}, \hat{P}] = i\hbar \longrightarrow [\hat{a}_\omega, \hat{a}_{\omega'}^\dagger]_\Sigma = 2\pi\delta(\omega - \omega'), \quad \longrightarrow \quad A_\omega := \frac{L}{r_H} \sqrt{\frac{\pi\alpha'}{\omega}}$$

$$[\hat{a}_\omega, \hat{a}_{\omega'}]_\Sigma = 0 = [\hat{a}_\omega^\dagger, \hat{a}_{\omega'}^\dagger]_\Sigma \quad \longrightarrow \quad = \frac{\beta}{2\sqrt{\pi\omega}\lambda^{1/4}}$$

Semi-classical approximation

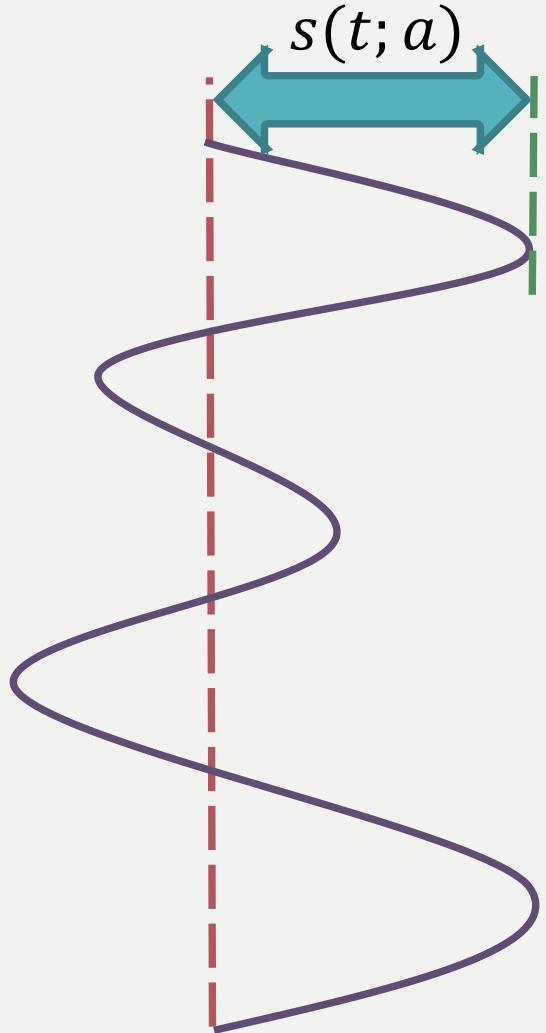
Transverse fluctuations originate from Hawking radiation of black brane

Distributed according to Bose-Einstein statistics:

$$\langle \hat{a}_\omega^\dagger \hat{a}_{\omega'} \rangle = \frac{2\pi\delta(\omega - \omega')}{e^{\beta\omega} - 1}$$

$$s^2(t; a) := \langle : (\hat{X}_{\text{End}}(t; a) - \hat{X}_{\text{End}}(0; a))^2 : \rangle$$

$$= \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \int_0^\infty \frac{d\omega}{\omega} \frac{1}{e^{\beta\omega} - 1} |f_\omega(\sigma_f - at) - f_\omega(\sigma_f)e^{i\omega t}|^2$$



Limit values

Small virtuality ($l_0 \ll r_H$):

$$s_{\text{small}}^2(t; a) = \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \ln \left(\frac{2a\beta^3 \sinh^2 \left(\frac{\pi(a+1)t}{\beta} \right) \sinh^2 \left(\frac{\pi(a-1)t}{\beta} \right) \operatorname{csch} \left(\frac{2\pi at}{\beta} \right)}{\pi^3 (a^2 - 1)^2 t^3} \right)$$

Ballistic at early times:

$$s_{\text{small}}^2(t; a) \xrightarrow{t \ll \beta} \frac{t^2}{6\sqrt{\lambda}} + \mathcal{O}((t/\beta)^4)$$

Diffusive at late times:

$$s_{\text{small}}^2(t; a) \xrightarrow{\beta \ll t} \frac{\beta t}{\pi\sqrt{\lambda}} \left(1 - \frac{a}{2} \right) + \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \left\{ \begin{array}{ll} 4 \ln \left(\frac{\beta}{2\pi t} \right) & , \text{if } a = 0 \\ \ln \left(\frac{a\beta^3}{4\pi^3(a^2-1)^2 t^3} \right) & , \text{if } 0 < a < 1 \\ \ln \left(\frac{\beta}{4\pi t} \right) & , \text{if } a = 1 \end{array} \right\} + \mathcal{O}(1)$$

Limit values

Arbitrary virtuality:

$$s^2(t; a) \Big|_{t \ll \beta} = \frac{t^2}{6\tilde{r}_0^4\sqrt{\lambda}} \left(\tilde{r}_0^2 - 6(\tilde{r}_0^2 - 1) \left(-2\gamma_E - \pi \cot\left(\frac{\pi}{\tilde{r}_0}\right) + H_{-1/\tilde{r}_0} + H_{1/\tilde{r}_0} + 2\ln(\tilde{r}_0) \right) \right)$$

$$s^2(t; a) \Big|_{t \gg \beta} = s_{\text{small}}^2(t; a)$$

Important result for extension to arbitrary dimensions

Extend from AdS_3 to AdS_d

Near black hole horizon:

$$ds_d = \frac{r_H^2}{L^2} (d-1) \exp\left(\frac{r_H^2}{L^2} (d-1)\varepsilon_*\right) (-dt^2 + d\varepsilon_*^2) + \frac{r_H^2}{L^2} d\mathbf{x}_{d-2}^2$$

$$s_{\text{small}}^2(t; a, d) = \frac{1}{\sqrt{\lambda}} \left(\frac{(d-1)\beta}{4\pi} \right)^2 \ln \left(\frac{2a\beta^3 \sinh^2\left(\frac{\pi(a+1)t}{\beta}\right) \sinh^2\left(\frac{\pi(a-1)t}{\beta}\right) \operatorname{csch}\left(\frac{2\pi at}{\beta}\right)}{\pi^3 (a^2 - 1)^2 t^3} \right)$$

Late time:

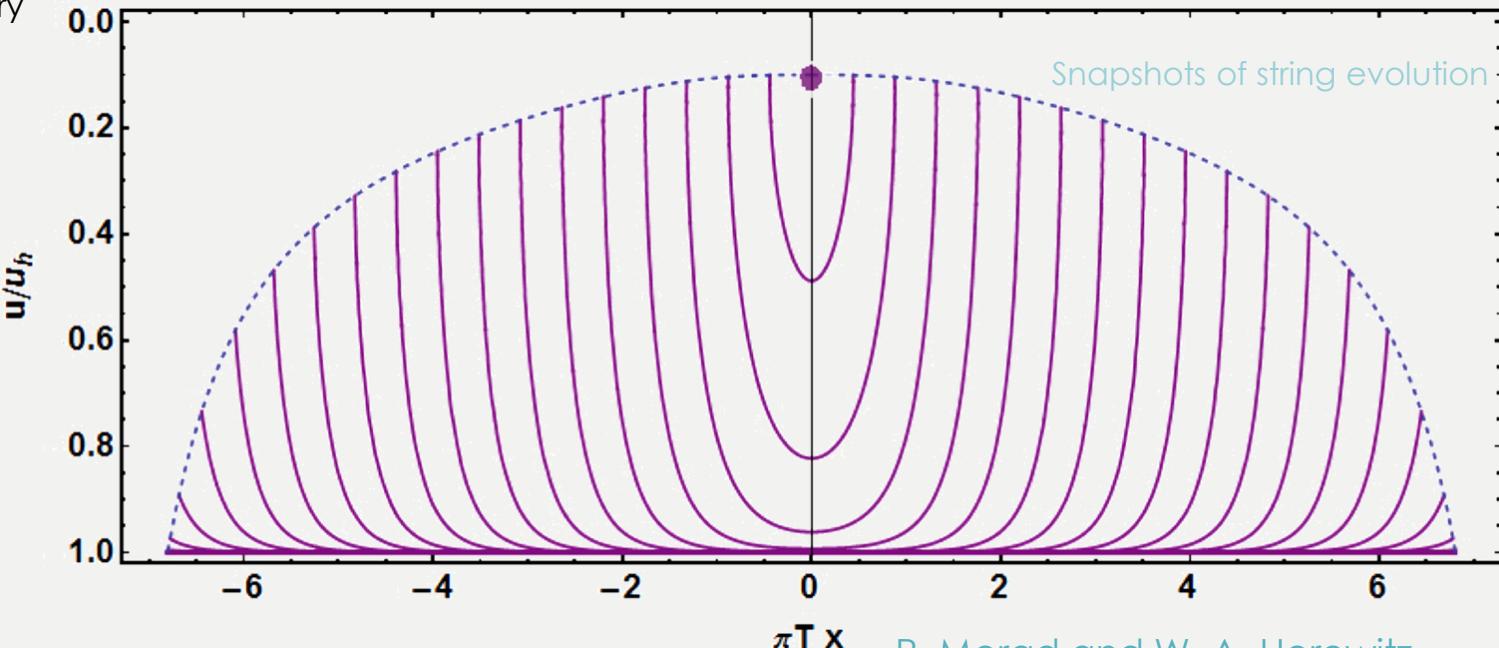
$$s^2(t; a, d) \Big|_{t \gg \beta} = \frac{(d-1)^2 \beta}{16\pi\sqrt{\lambda}} \left(1 - \frac{a}{2}\right) t = 2D(a, d)t$$

$$D(a, d) := \frac{(d-1)^2 \beta}{8\pi\sqrt{\lambda}} \left(1 - \frac{a}{2}\right)$$

$$\nu \neq 0$$

Minkowski Boundary

\uparrow 5th Dimension
 \downarrow BH Horizon



R. Morad and W. A. Horowitz,
JHEP 1411 (2014) 017 [1409.7545]

$$\hat{q} = \frac{4T^2}{vD} = \frac{2\pi\sqrt{\lambda}T^3}{v(1 - \frac{a}{2})}$$

average transverse momentum squared transferred from the plasma to the probe per unit distance travelled

Comparison with other methods

Heavy flavour ($\alpha = 0$):

$$\hat{q} = \frac{2\pi\sqrt{\lambda}T^3}{v}$$

$$\hat{q}_{Gubser} = \frac{2\pi\sqrt{\lambda}T^3}{v}\sqrt{\gamma}$$

Light flavour ($\alpha = 1, v \sim 1$):

$$\hat{q} = 4\pi\sqrt{\lambda}T^3 \cong 12.5\sqrt{\lambda}T^3$$

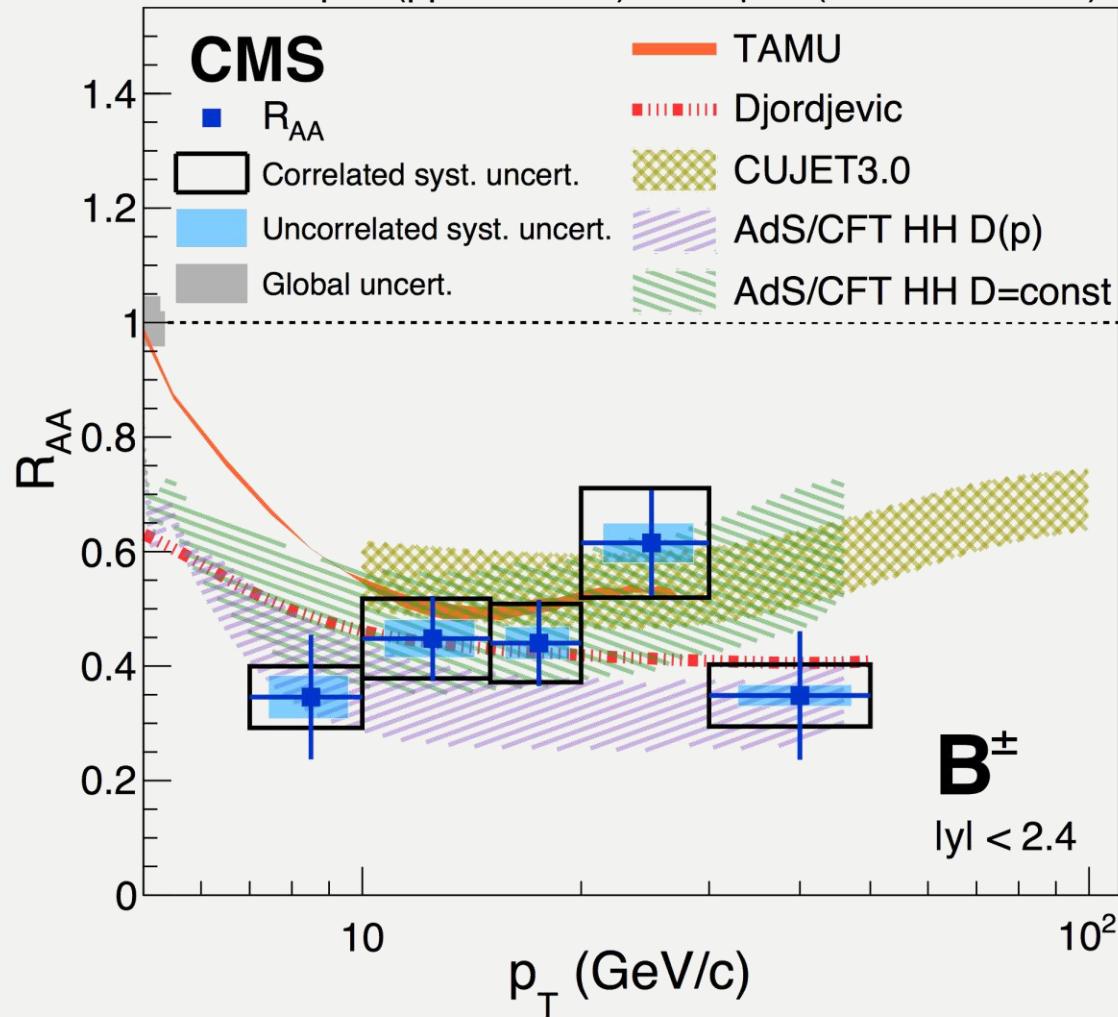
$$\hat{q}_{LRW} = \frac{\pi^{\frac{3}{2}}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{\lambda}T^3\gamma \cong 7.5\sqrt{\lambda}T^3$$

Predictions for heavy flavor

- Performed using two 't Hooft coupling constants
 $\lambda_1 = 5.5$ and $\lambda_2 = 12\pi\alpha_s \approx 11.3, \alpha_s = 0.3$
 $(E_{QCD} = E_{SYM})$ $(T_{QCD} = T_{SYM})$
- aMC@NLO matched to Herwig++
- 2+1d VISHNU hydro model, assuming boost invariance
- 5.02TeV R_{AA}^D and R_{AA}^B

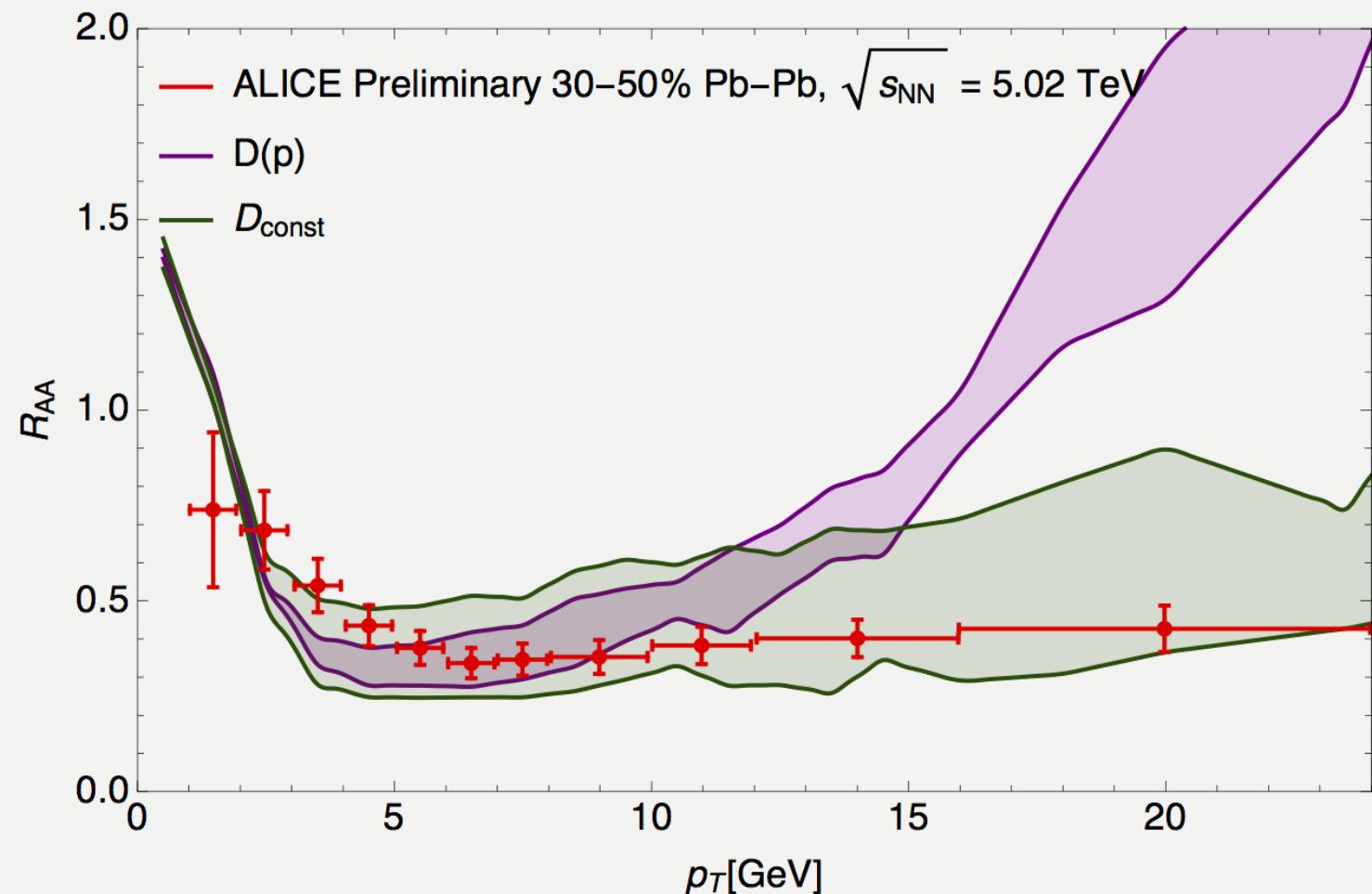
$$R_{AA}^B$$

28.0 pb^{-1} (pp 5.02 TeV) + $351 \mu\text{b}^{-1}$ (PbPb 5.02 TeV)



The CMS Collaboration, arXiv:1705.04727

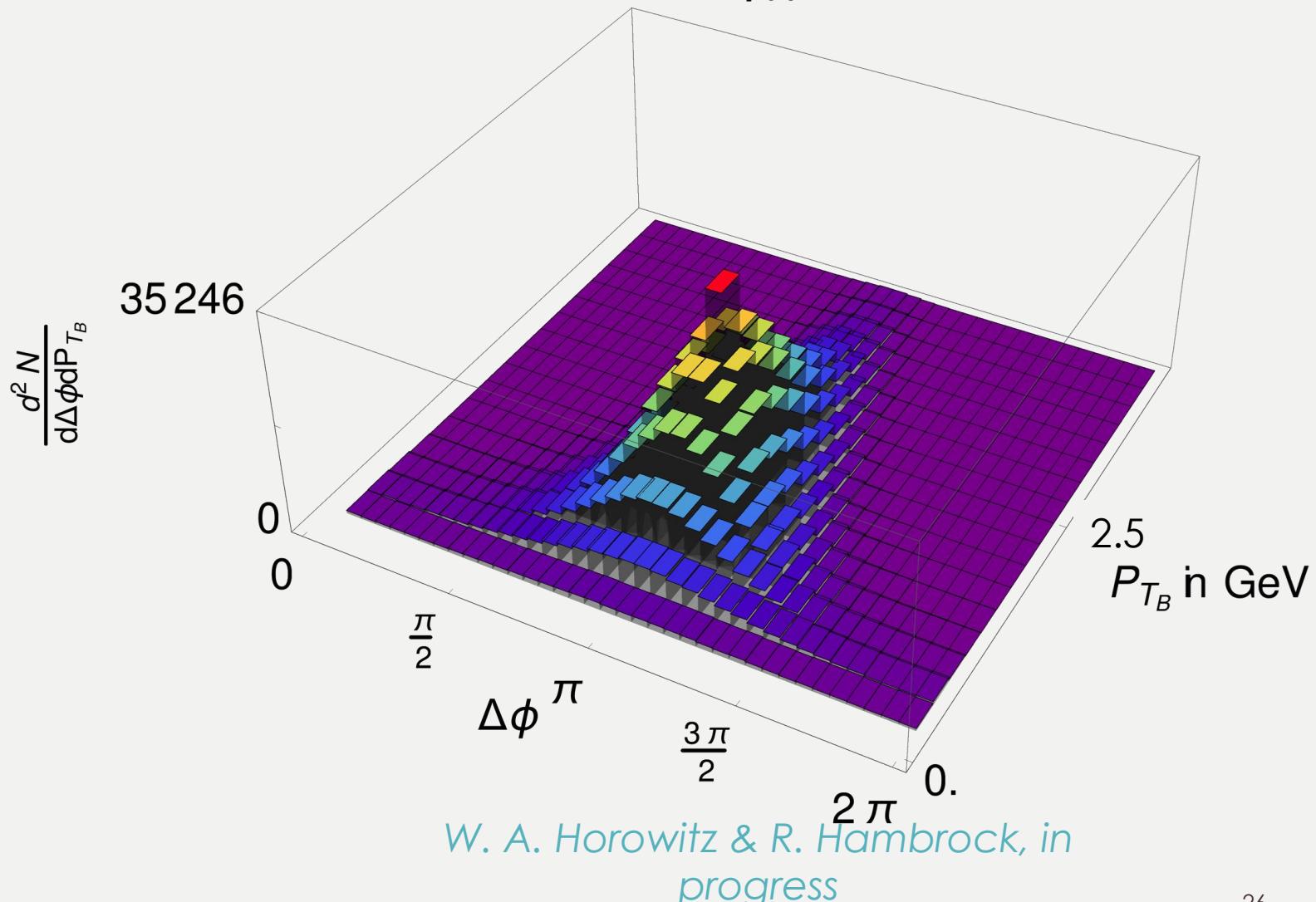
$$R_{AA}^D$$



LO Production $b\bar{b}$ (FONLL)

$2.25 \text{ GeV} < P_{T_A} \leq 2.75 \text{ GeV}$

$0.83\pi \leq \phi_A \leq 1\pi$

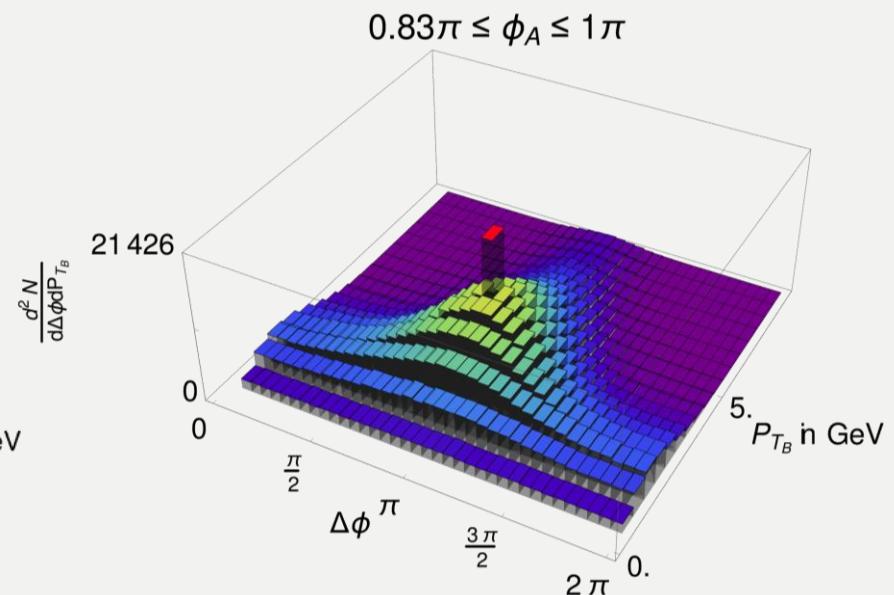
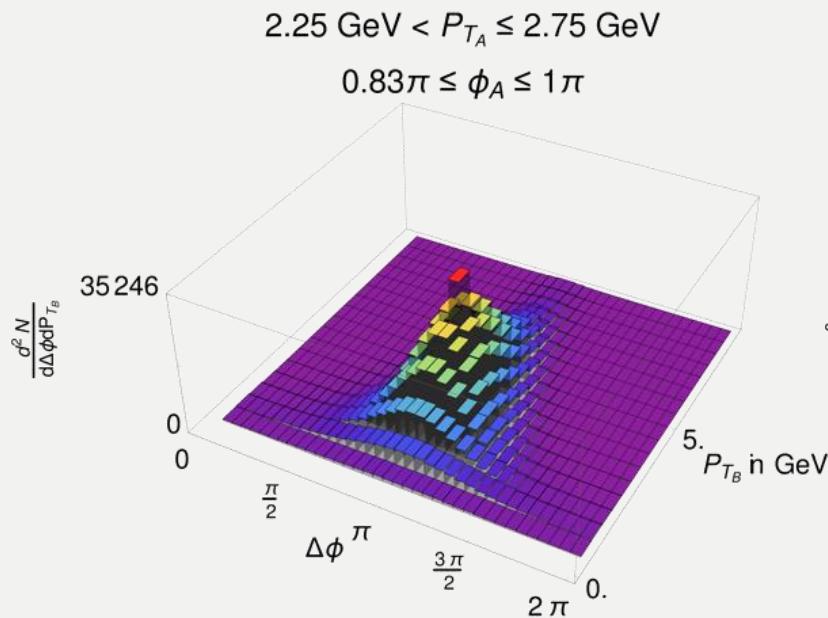


Low P_T $b\bar{b}$

$2.25 \text{ GeV} < P_{T_A} \leq 2.75 \text{ GeV}$

$$\lambda_1 = 5.5$$

$$\alpha_s = 0.3$$

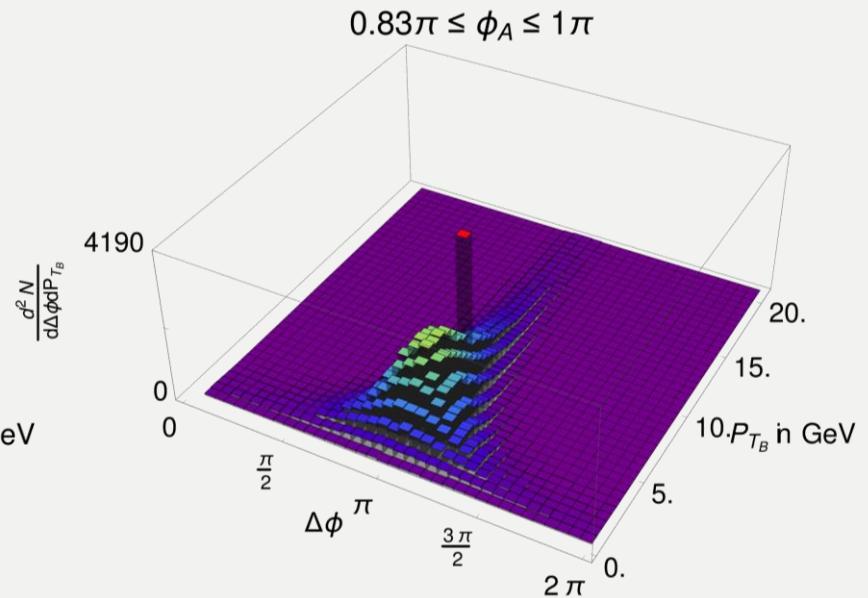
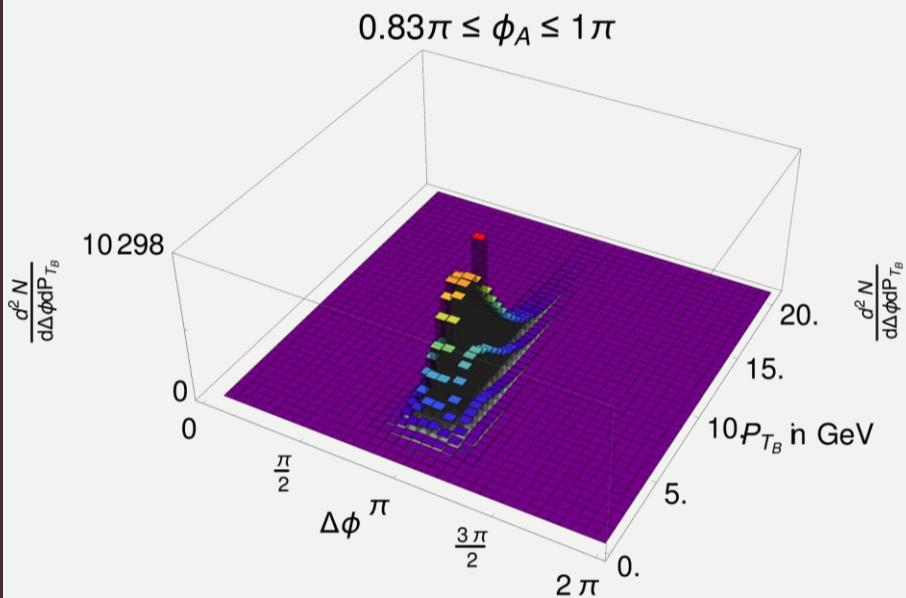


Medium P_T $b\bar{b}$

$6.25 GeV < P_{TA} \leq 6.75 GeV$

$$\lambda_1 = 5.5$$

$$\alpha_s = 0.3$$

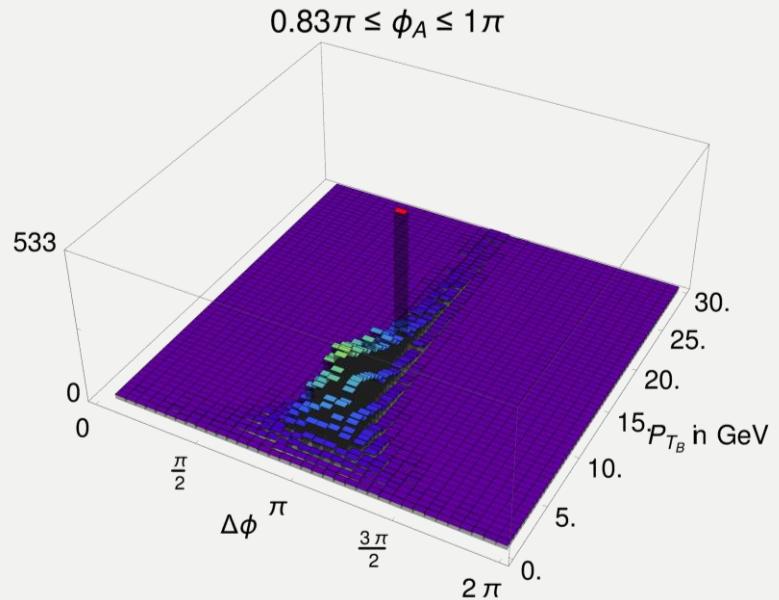
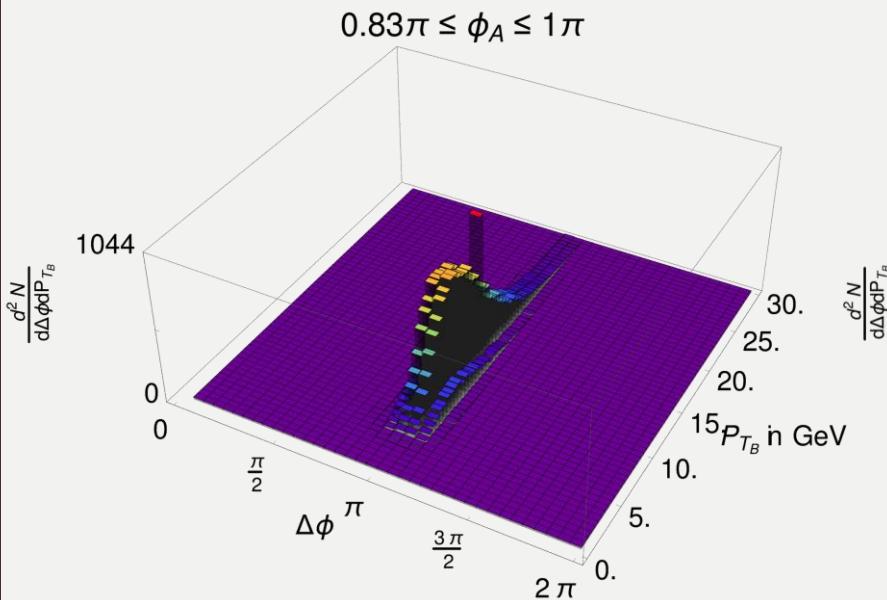


High P_T $b\bar{b}$

$12.25 GeV < P_{T_A} \leq 12.75 GeV$

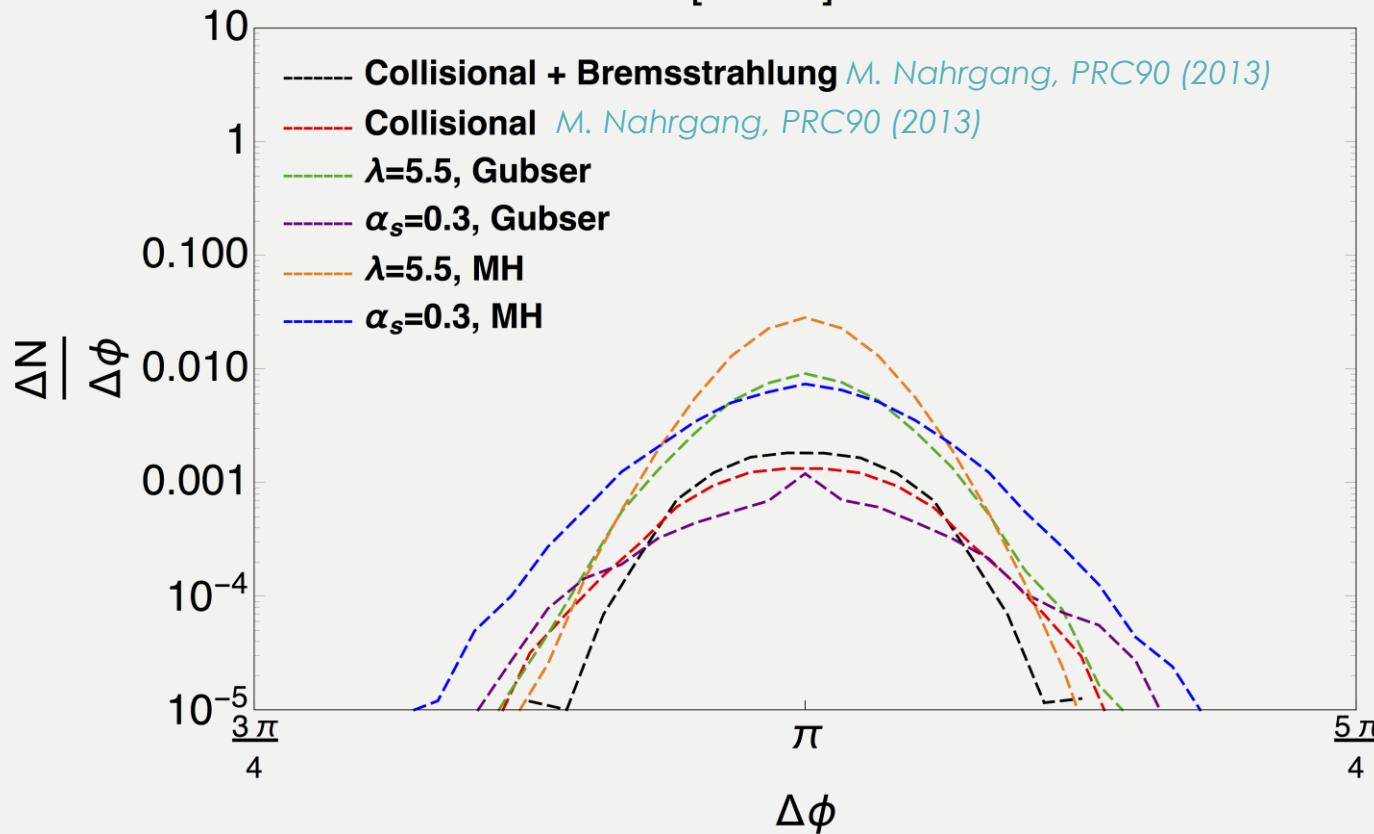
$$\lambda_1 = 5.5$$

$$\alpha_s = 0.3$$



LO production $b\bar{b}$ azimuthal correlations

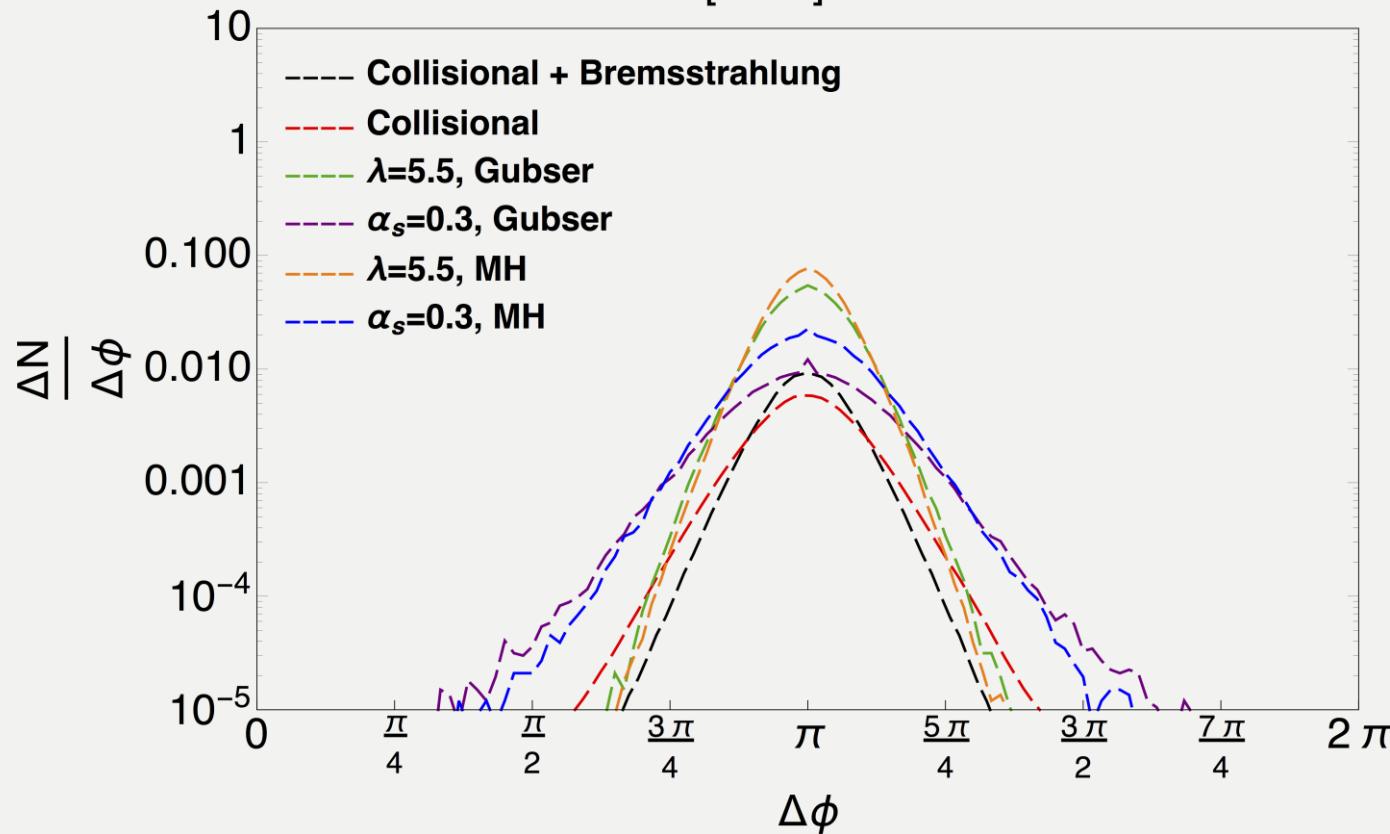
$\sqrt{s} = 2.76 TeV$ with 40-60% centrality
[10–20]GeV



RH and W. A. Horowitz, arXiv:1703.05845

LO production $b\bar{b}$ azimuthal correlations

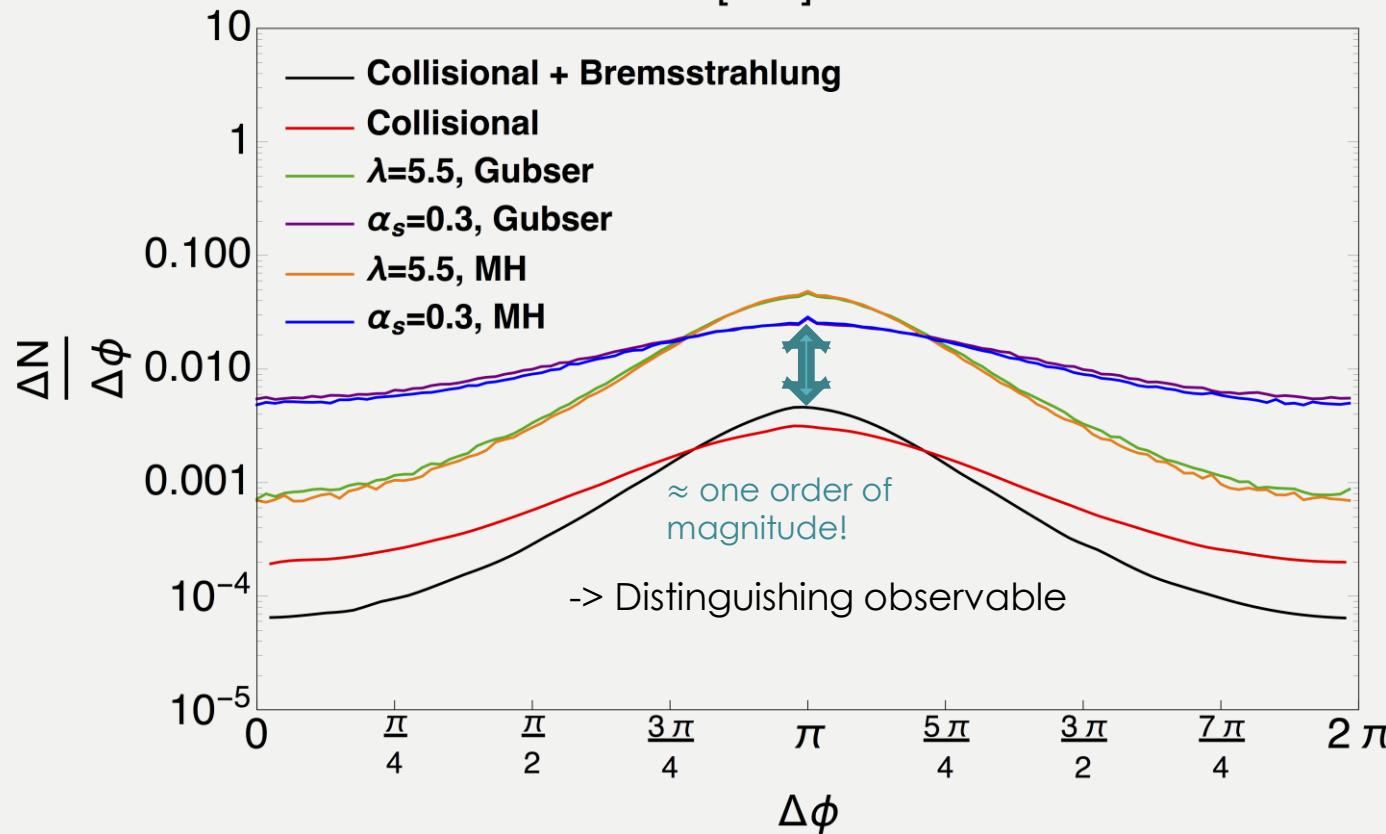
$\sqrt{s} = 2.76 TeV$ with 40-60% centrality
[4–10] GeV



RH and W. A. Horowitz, arXiv:1703.05845

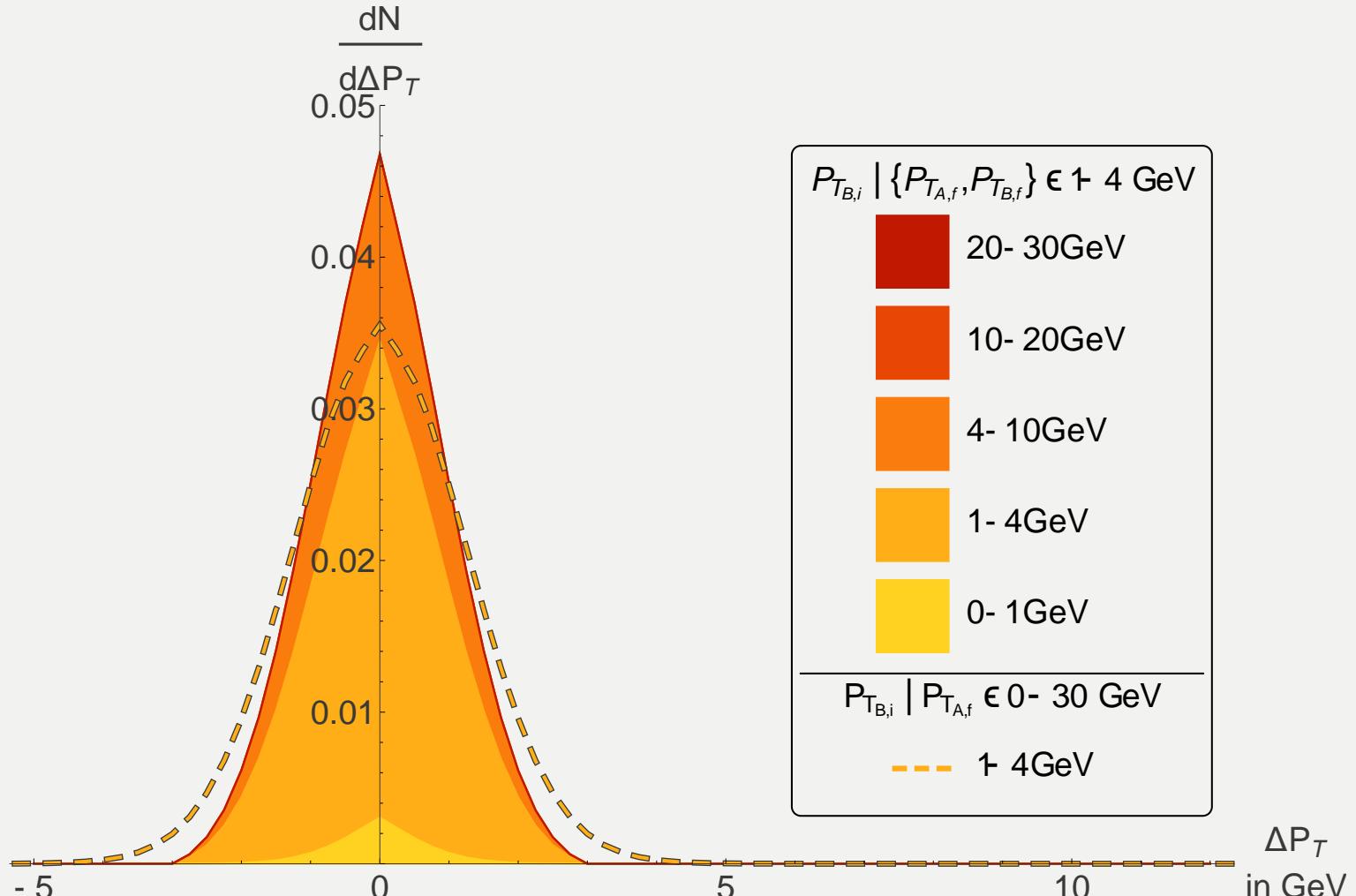
LO production $b\bar{b}$ azimuthal correlations

$\sqrt{s} = 2.76 TeV$ with 40-60% centrality
[1–4]GeV



RH and W. A. Horowitz, arXiv:1703.05845

LO production $b\bar{b}$ momentum correlations



RH and W. A. Horowitz, arXiv:1703.05845

Conclusion

Theory:

- Smooth transition from heavy to light flavour energy loss with diffusion coefficient conjectured to be independent of momentum

Phenomenology:

- Strong and weak coupling in qualitative agreement on nuclear modification factor
- Qualitatively indistinguishable azimuthal correlations
- **BUT** large qualitative discrepancy in momentum correlations of $b\bar{b}$ pairs in a weak- or strong coupled QGP
- Initial momentum correlations show difference in low pT momentum correlations is due to smaller momentum fluctuations for low pT pairs in a strongly coupled plasma

Future

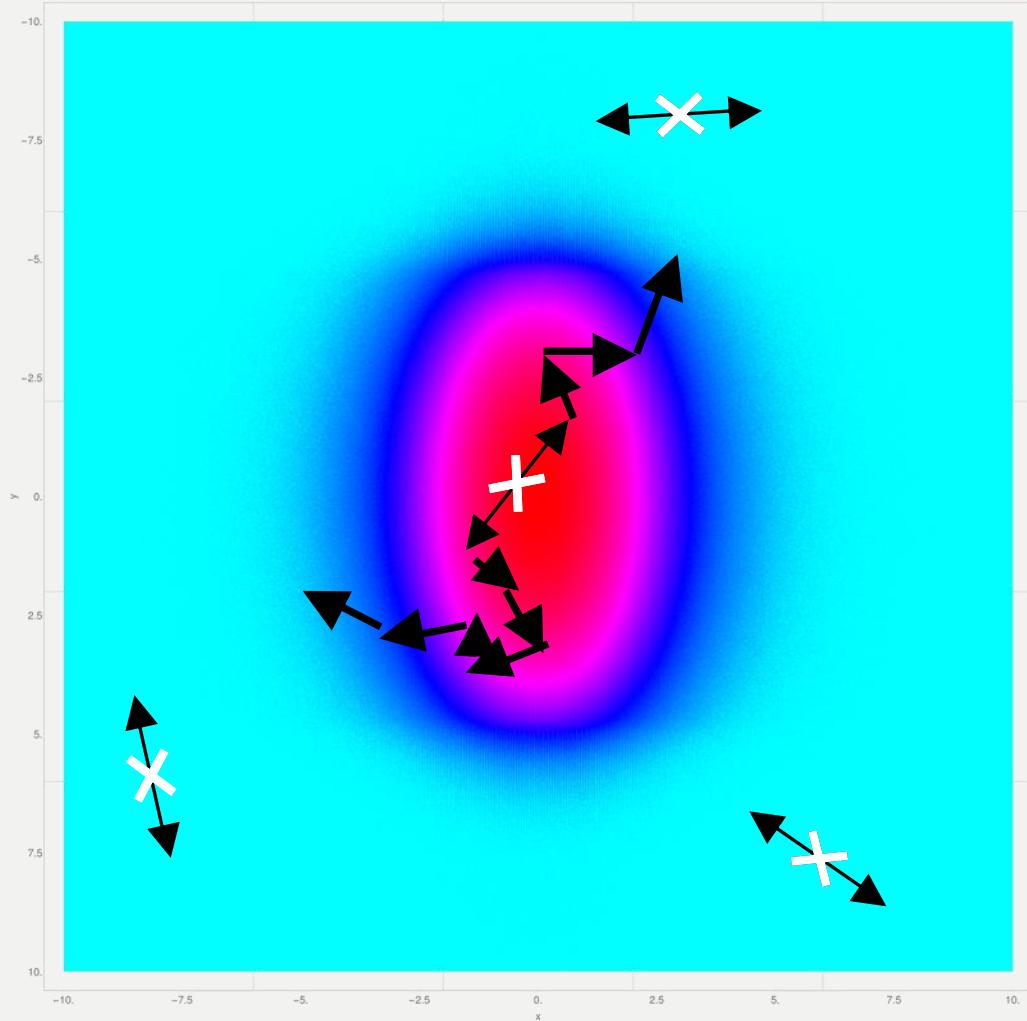
- Back up conjecture with further numerical investigation
- Observables limited in pT due to statistics. Migrate to Pythia for weighted production spectrum

These might be interesting

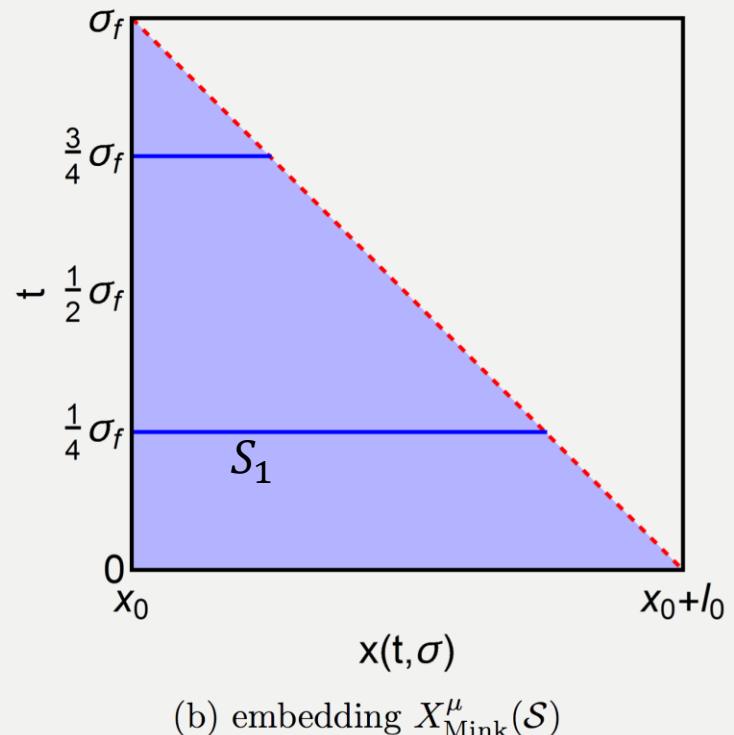
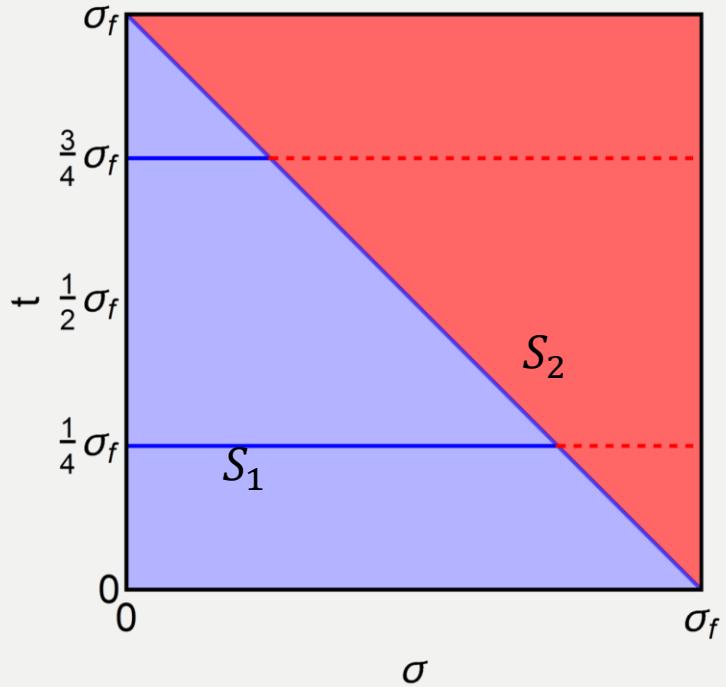
- Initial momentum correlations with only trigger particle momentum cut-offs
- Spike in correlations
- More illustrative 2D correlations
- Azimuthal correlations for higher pT classes
- Momentum correlation

Initial Glauber Distribution

Production points of $b\bar{b}$ pairs



Setup



$$S_1(a) = \{(t, \sigma) \in S | \sigma \in [0, \sigma_f - at]\}$$

$$S_2(a) = \{(t, \sigma) \in S | \sigma \in (\sigma_f - at, \sigma_f]\}$$

$$X_{\text{Mink}}^\mu(t, \sigma) = (t, x_0 + \begin{cases} \sigma, & \text{if } (t, \sigma) \in S_1 \\ \sigma_f - at, & \text{if } (t, \sigma) \in S_2 \end{cases}, 0)$$

Quantization of transverse fluctuations

Separable Ansatz:

$$X(t, r) = f_\omega(r)e^{-i\omega t}$$

Plug into EOM:

$$f_\omega^{(\pm)}(r) = \frac{1}{1 \pm i\nu} \frac{r \pm ir_H\nu}{r} e^{\pm i\omega(r_{s*} + \sigma)}, \quad \nu := \frac{L^2\omega}{r_H}$$



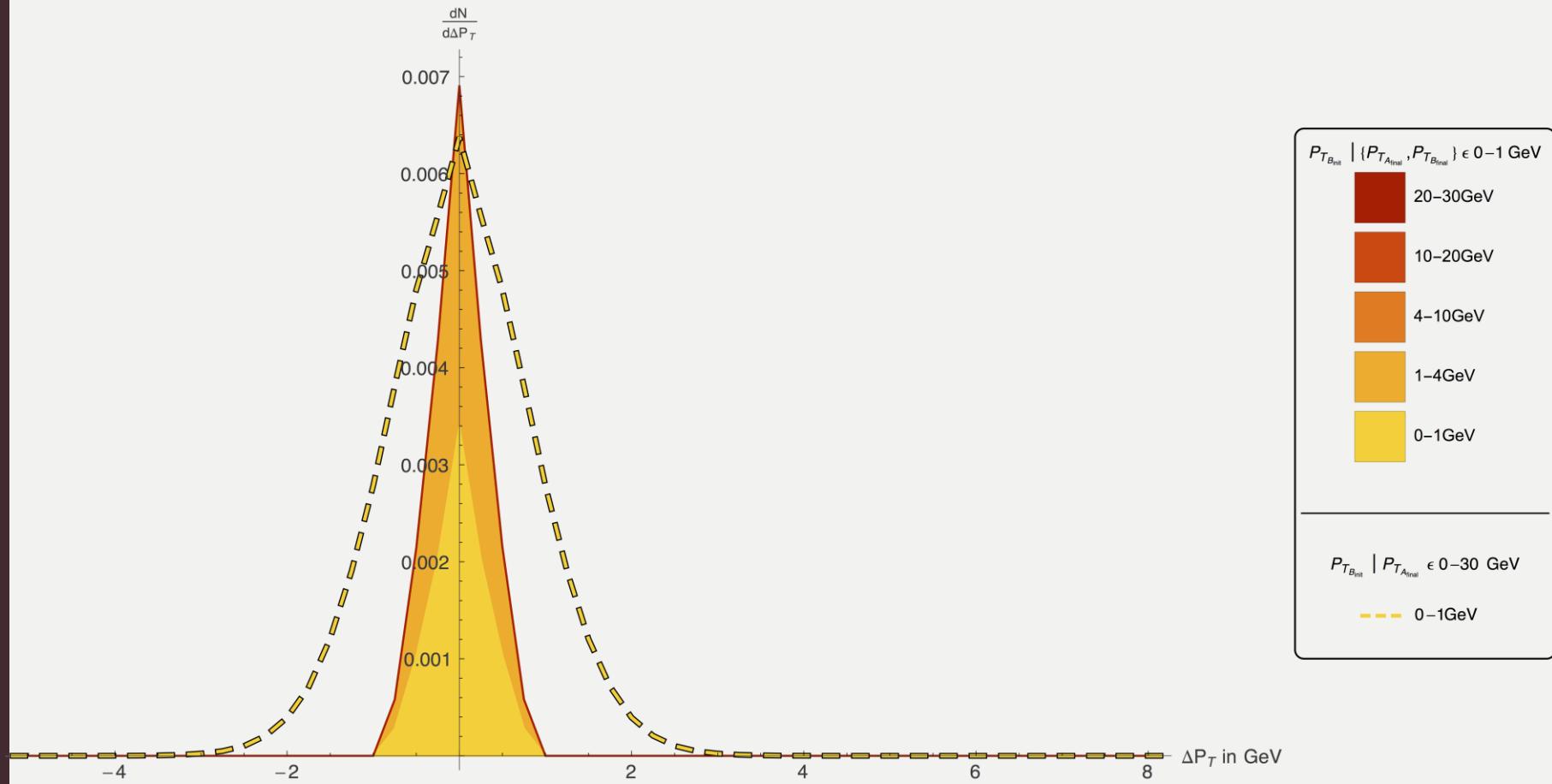
$$f_\omega^{(\pm)}(r) \xrightarrow{r \rightarrow r_H} e^{\pm i\omega(r_{s*} + \sigma)}$$

Apply Neumann BCs:

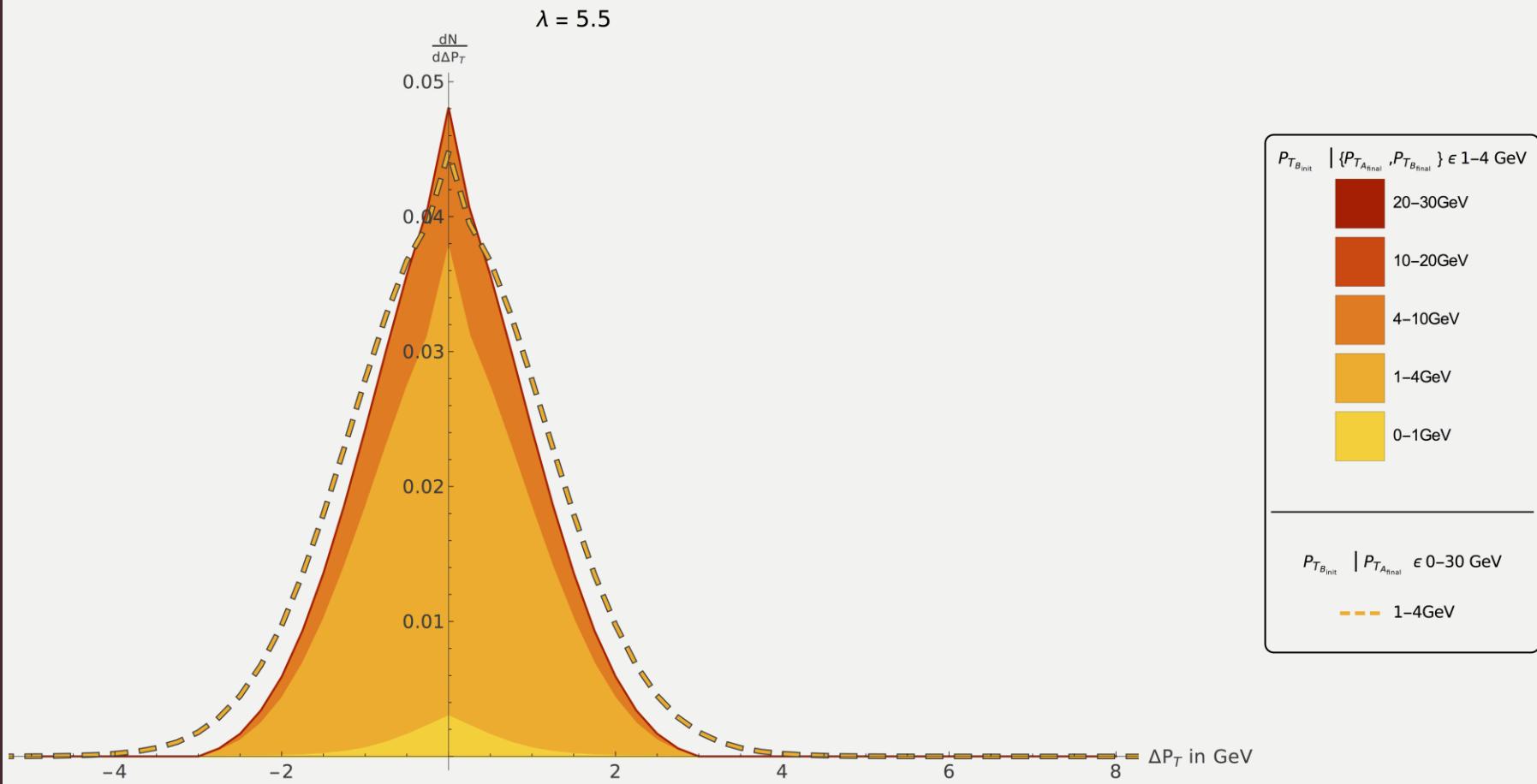
$$X(t, r) = \int_0^\infty \frac{d\omega}{2\pi} A_\omega [f_\omega(r)e^{-i\omega t} a_\omega + f_\omega^*(r)e^{i\omega t} a_\omega^*]$$

(still need to fix normalization)

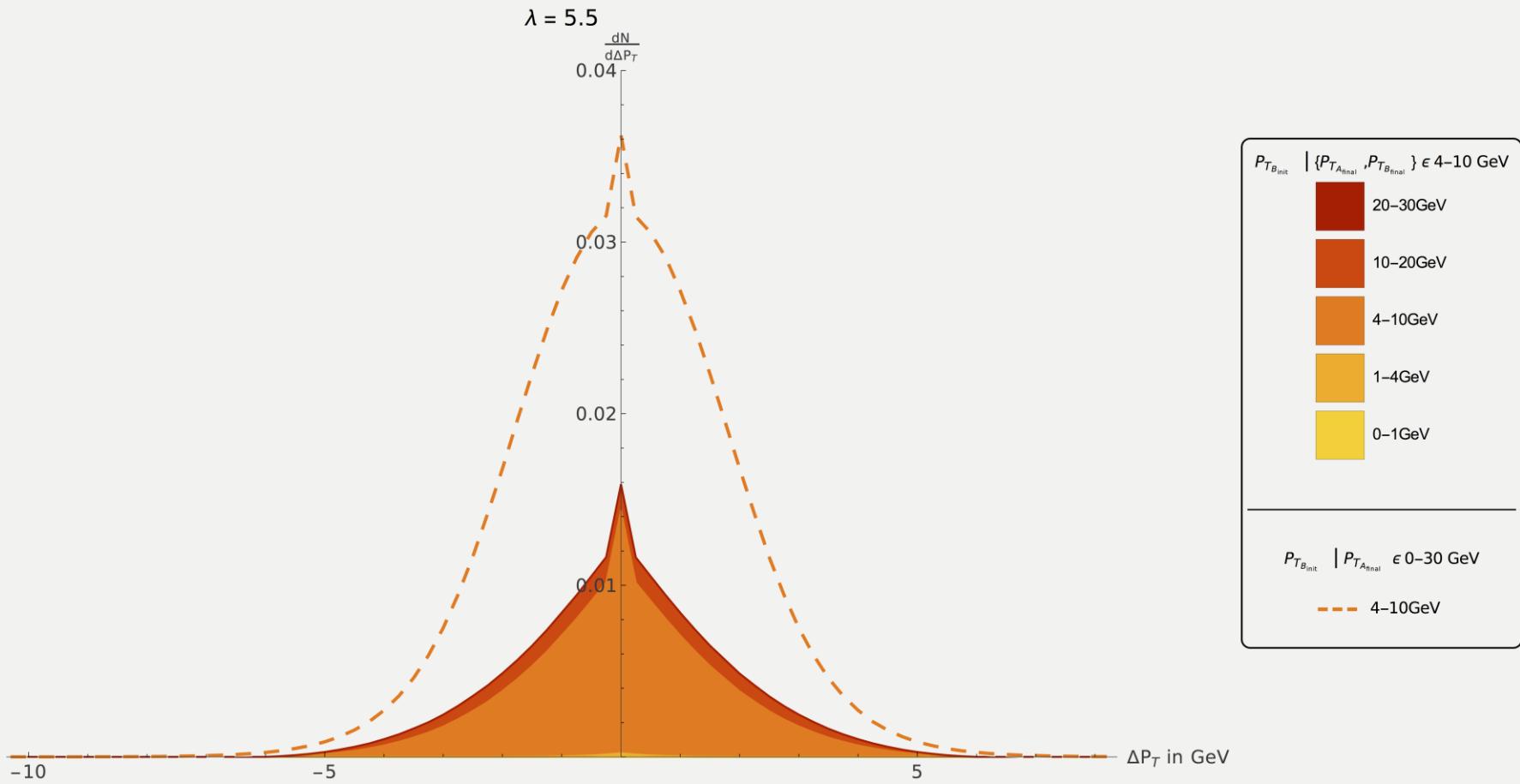
LO initial momentum correlations



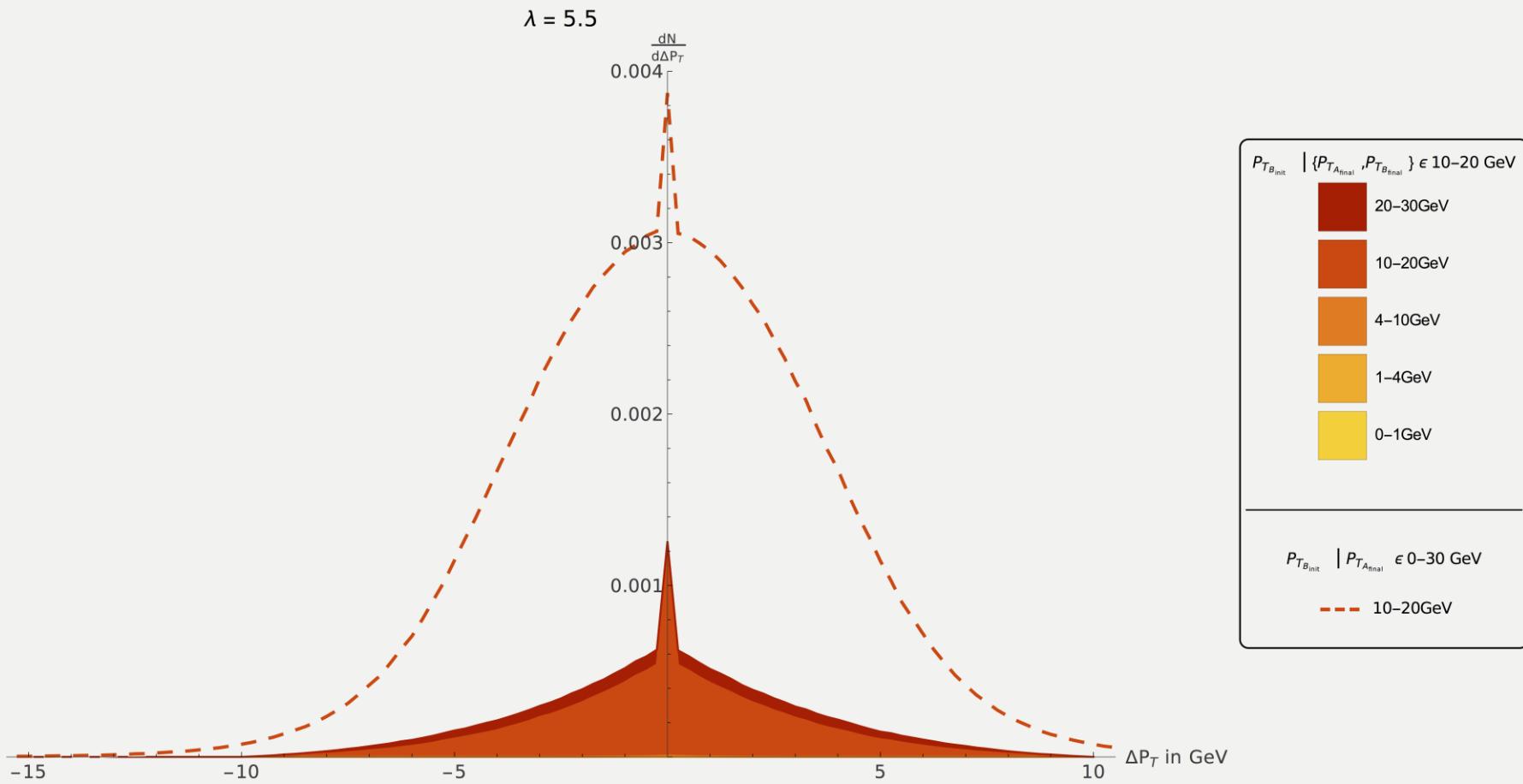
LO initial momentum correlations



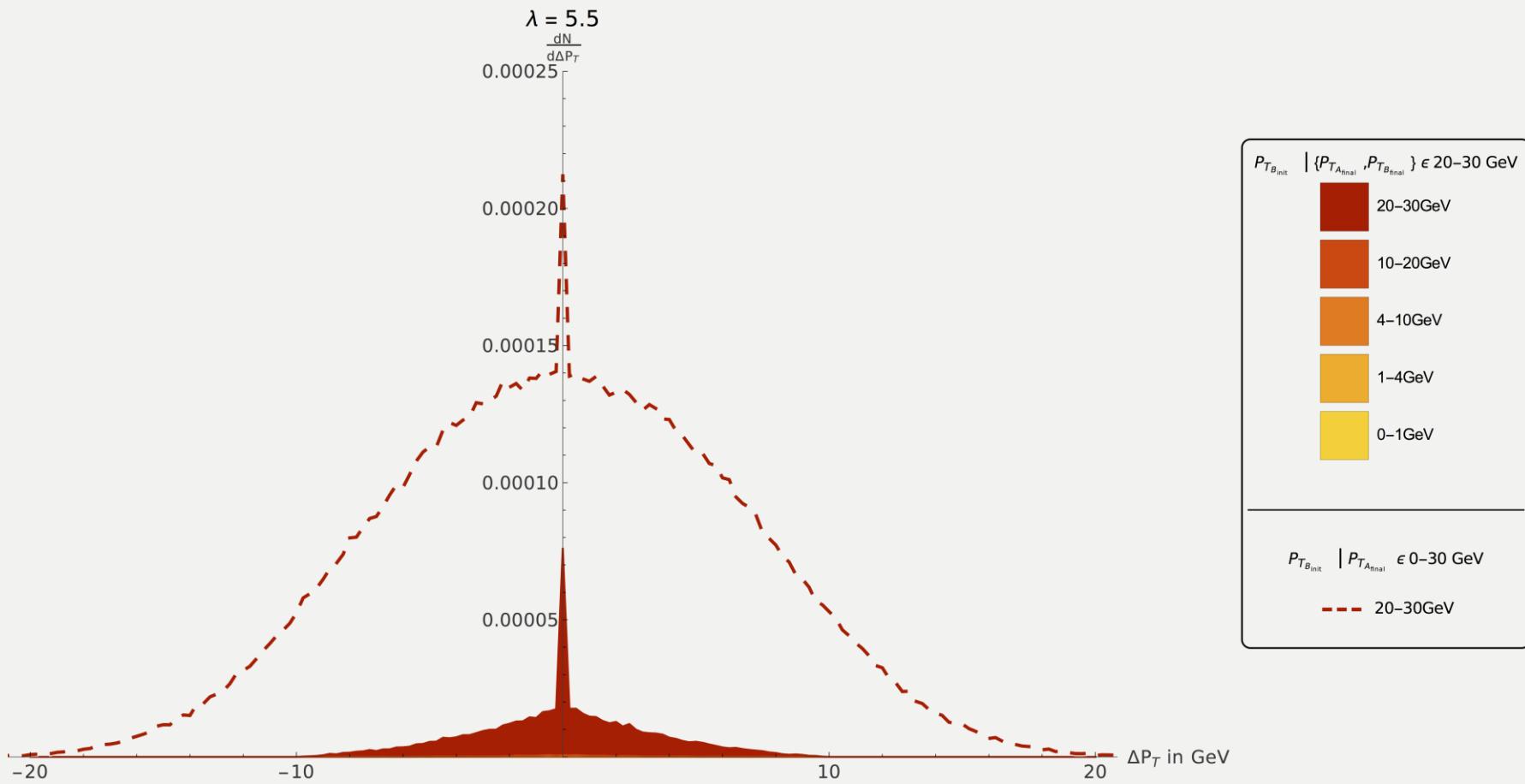
LO initial momentum correlations



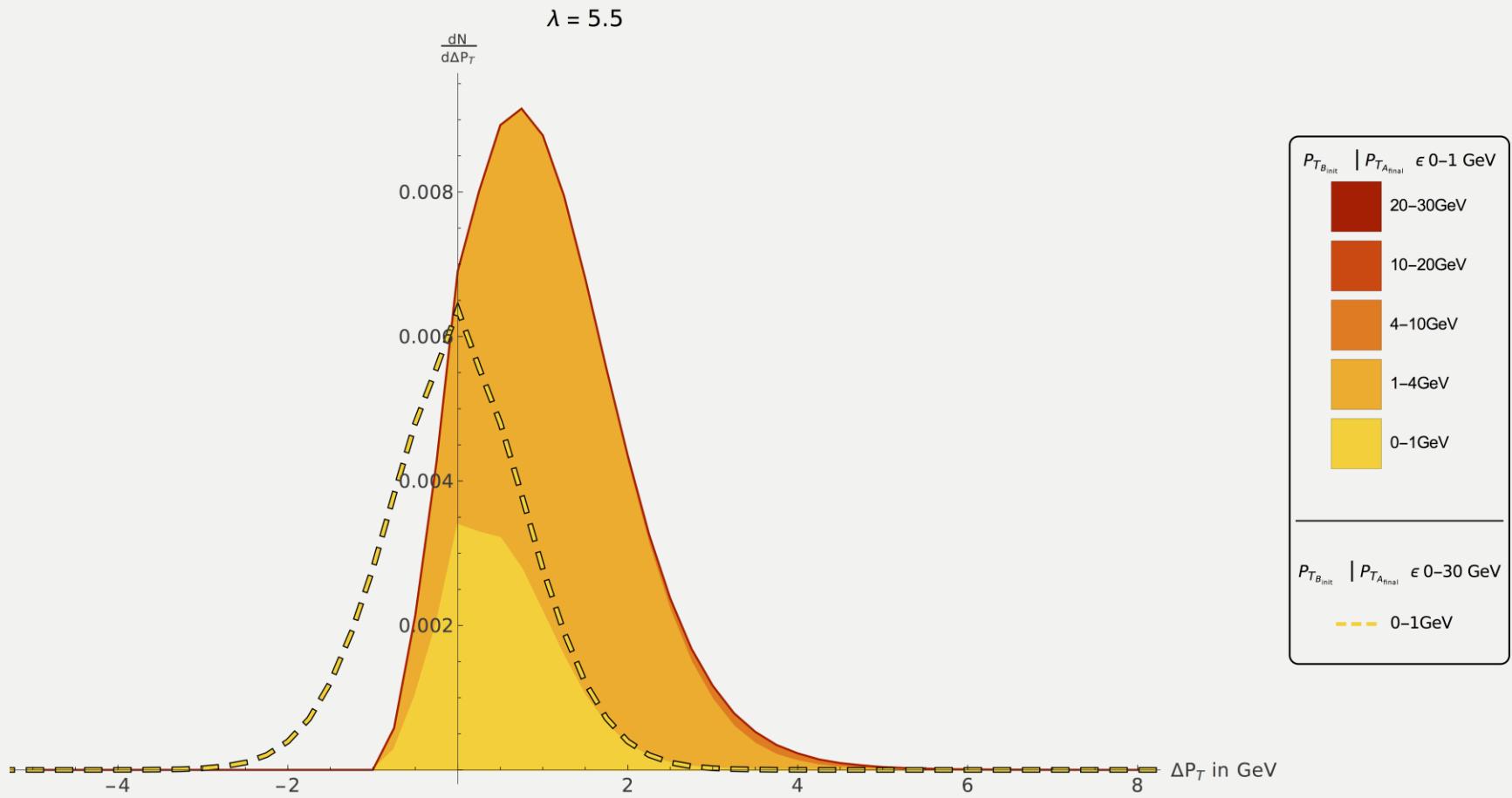
LO initial momentum correlations



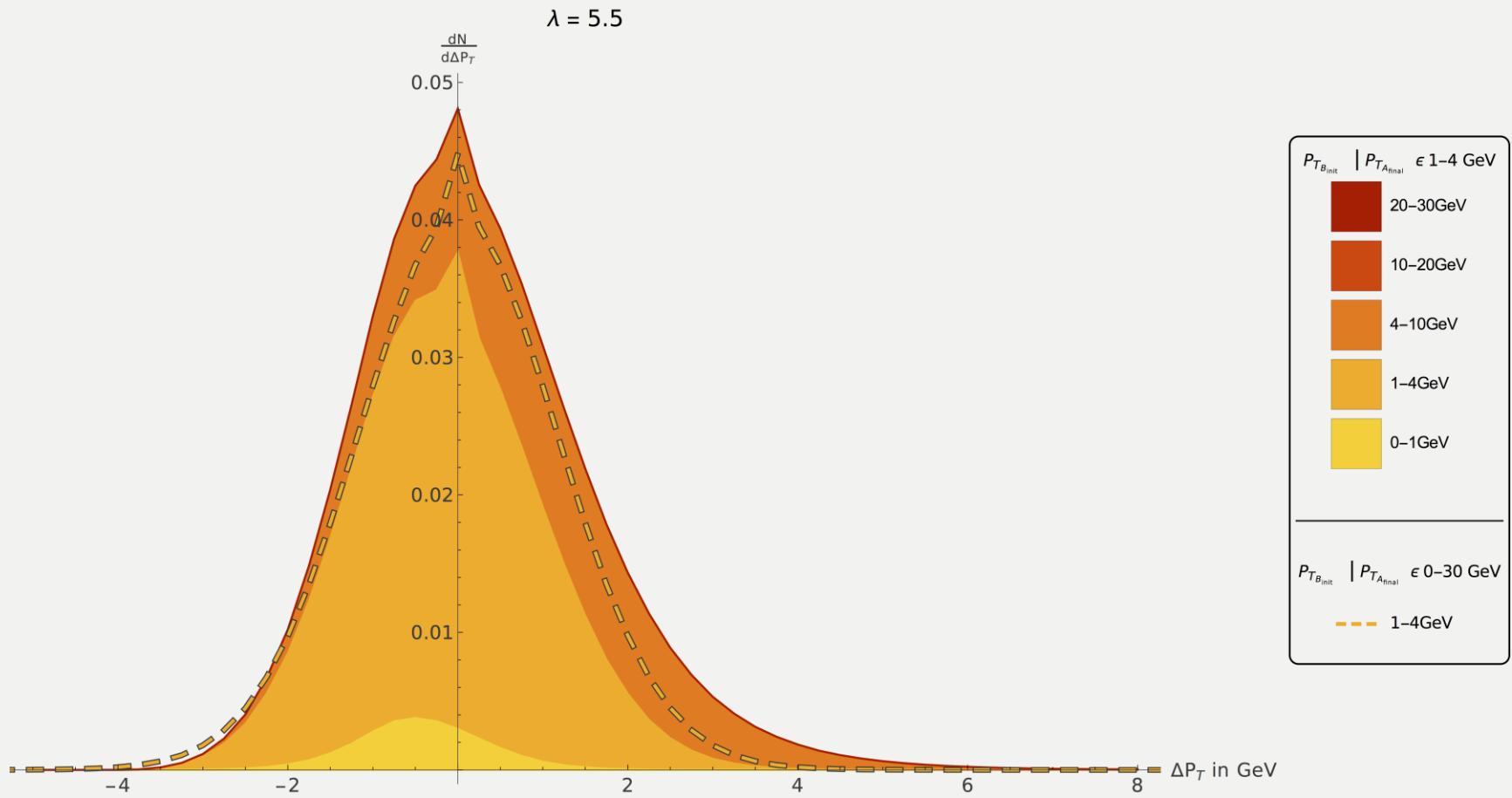
LO initial momentum correlations



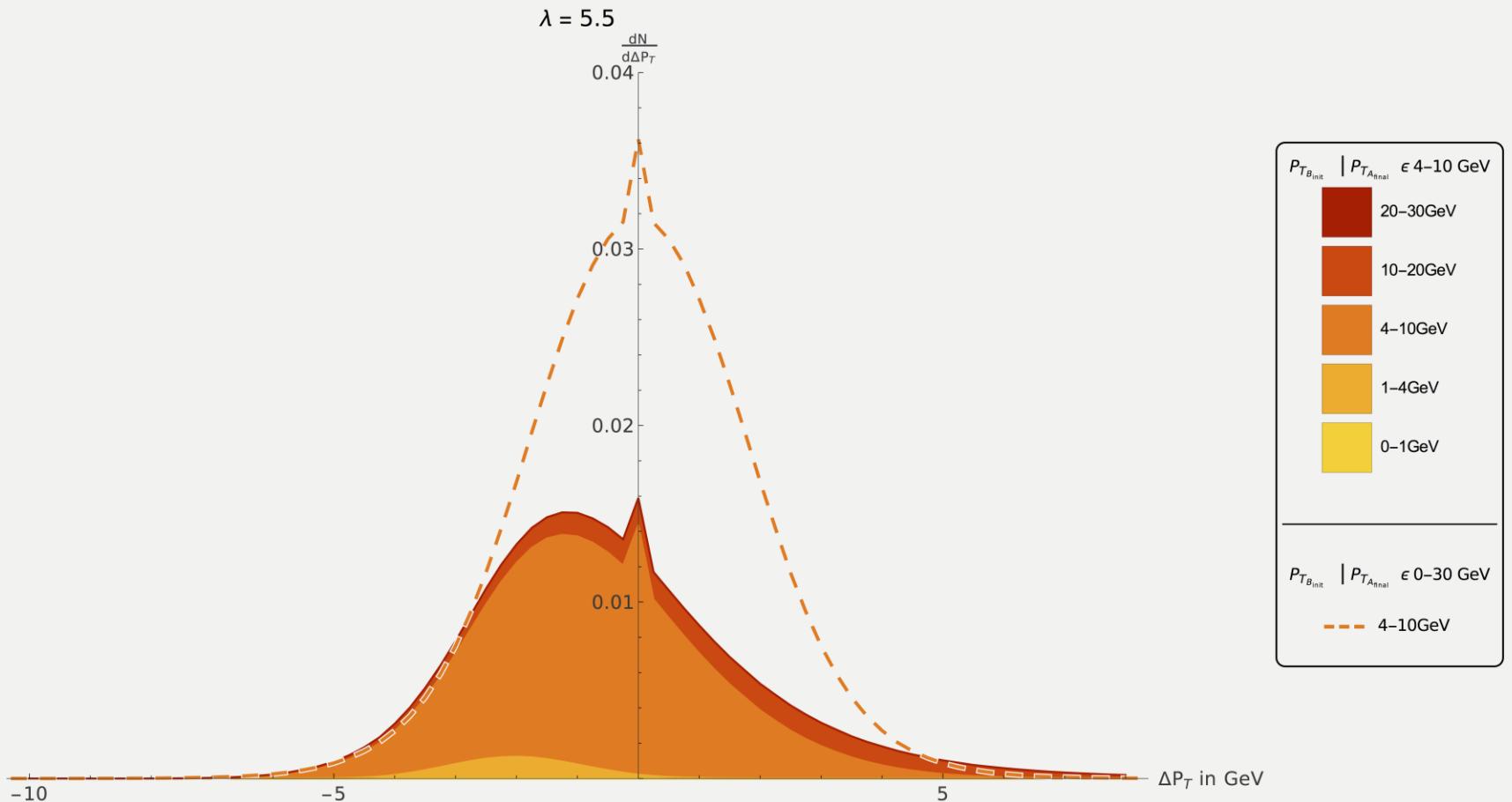
LO initial momentum correlations – single cut-off



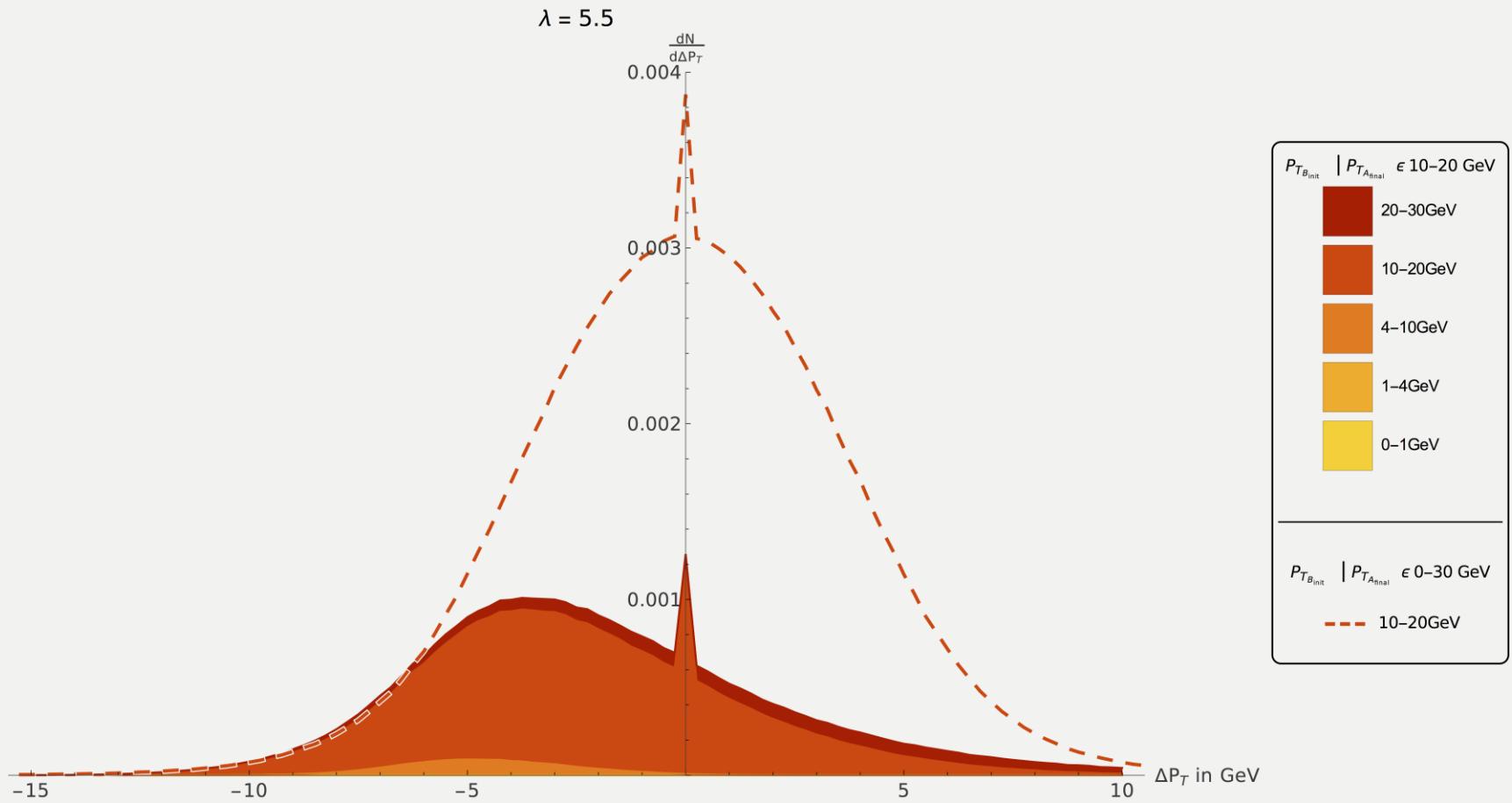
LO initial momentum correlations – single cut-off



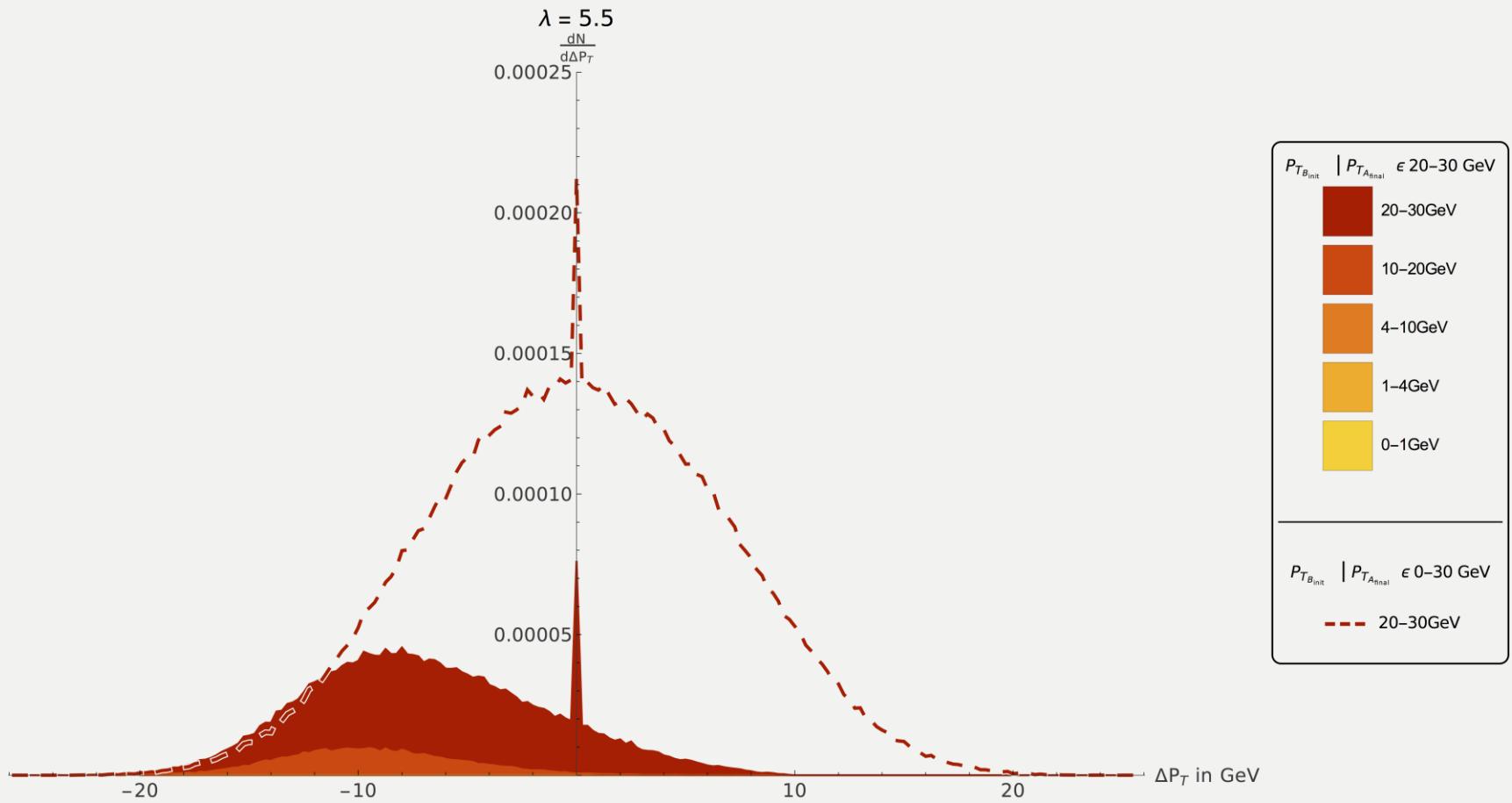
LO initial momentum correlations – single cut-off



LO initial momentum correlations – single cut-off

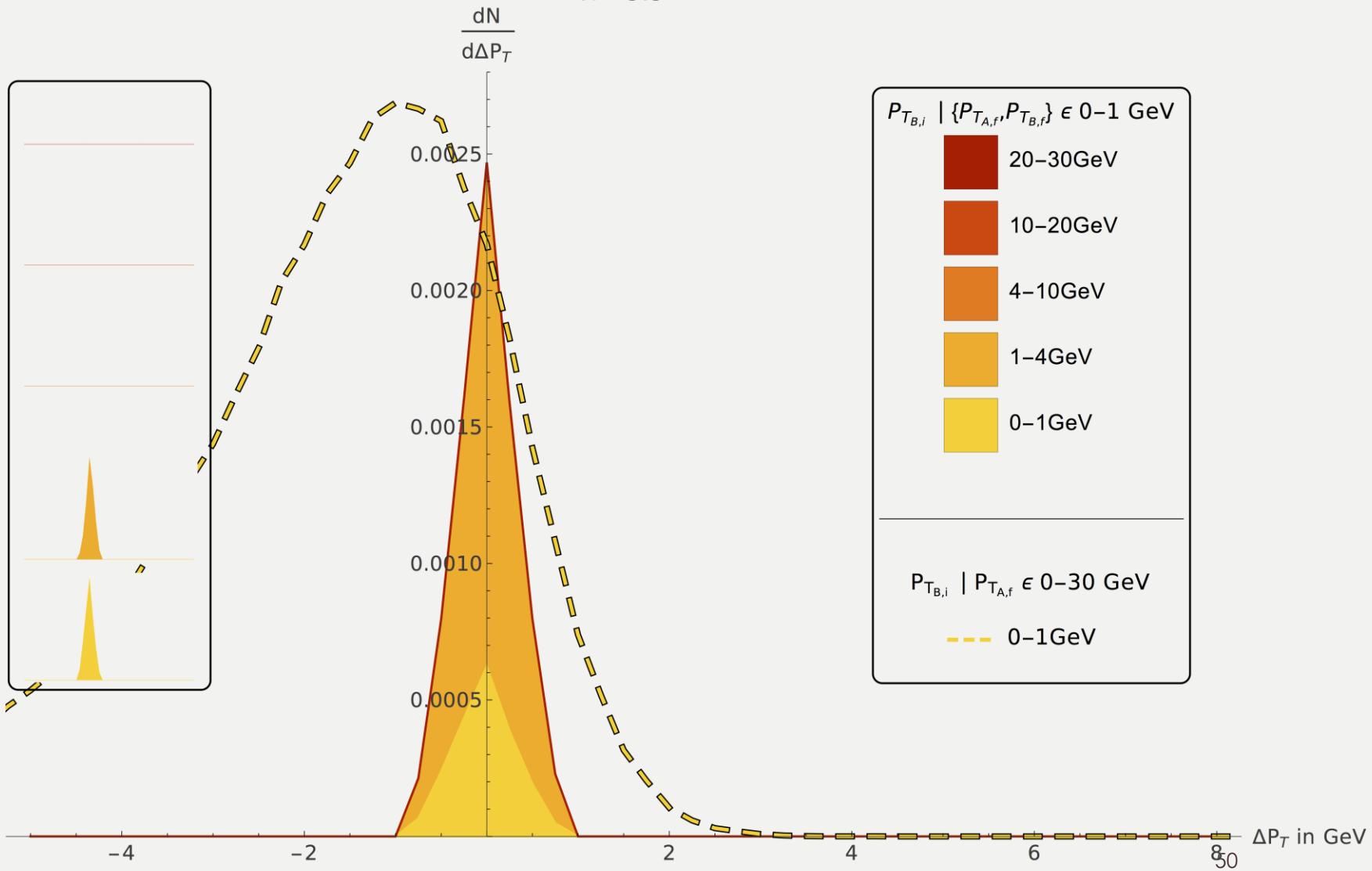


LO initial momentum correlations – single cut-off

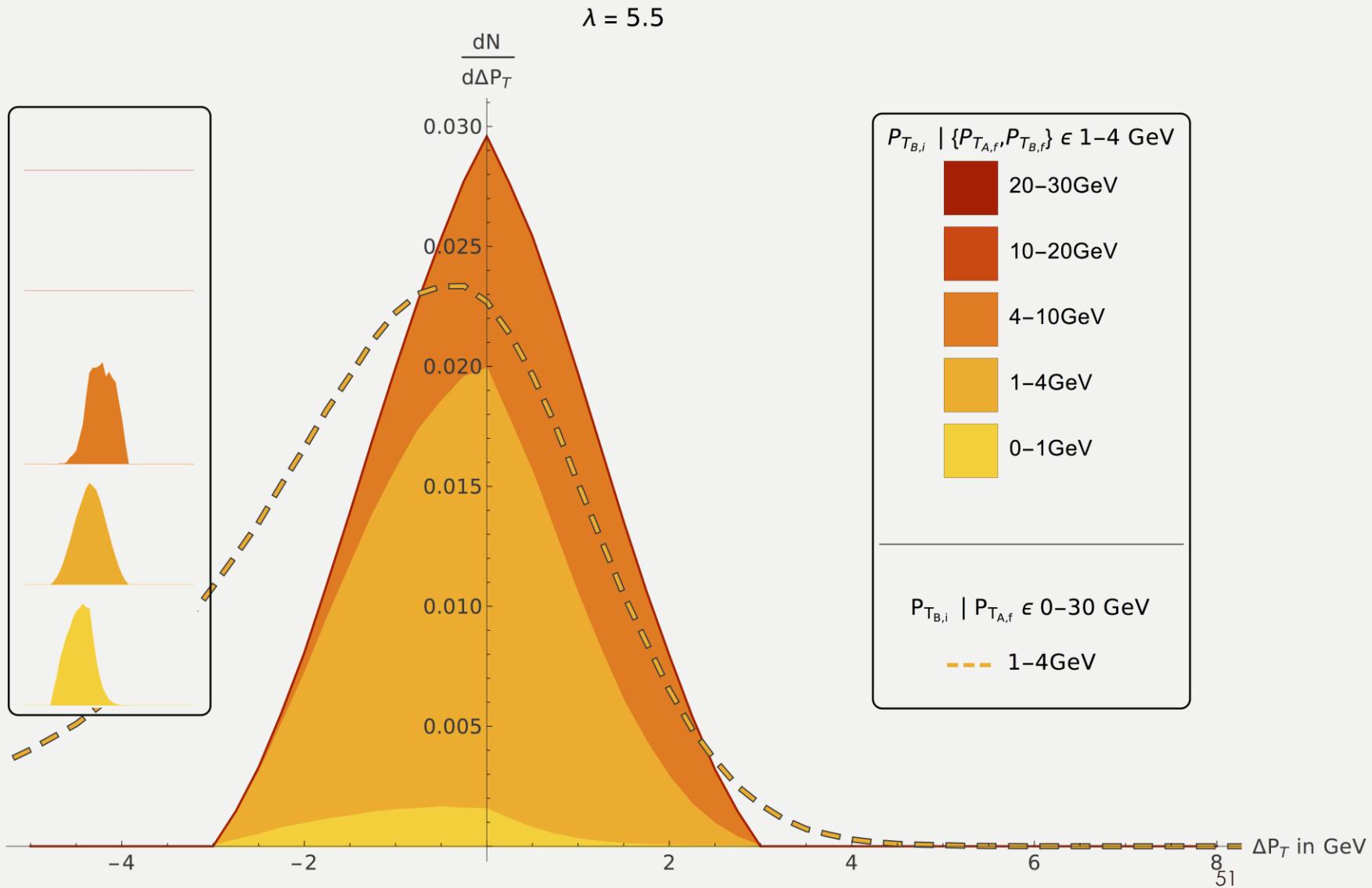


NLO initial momentum correlations

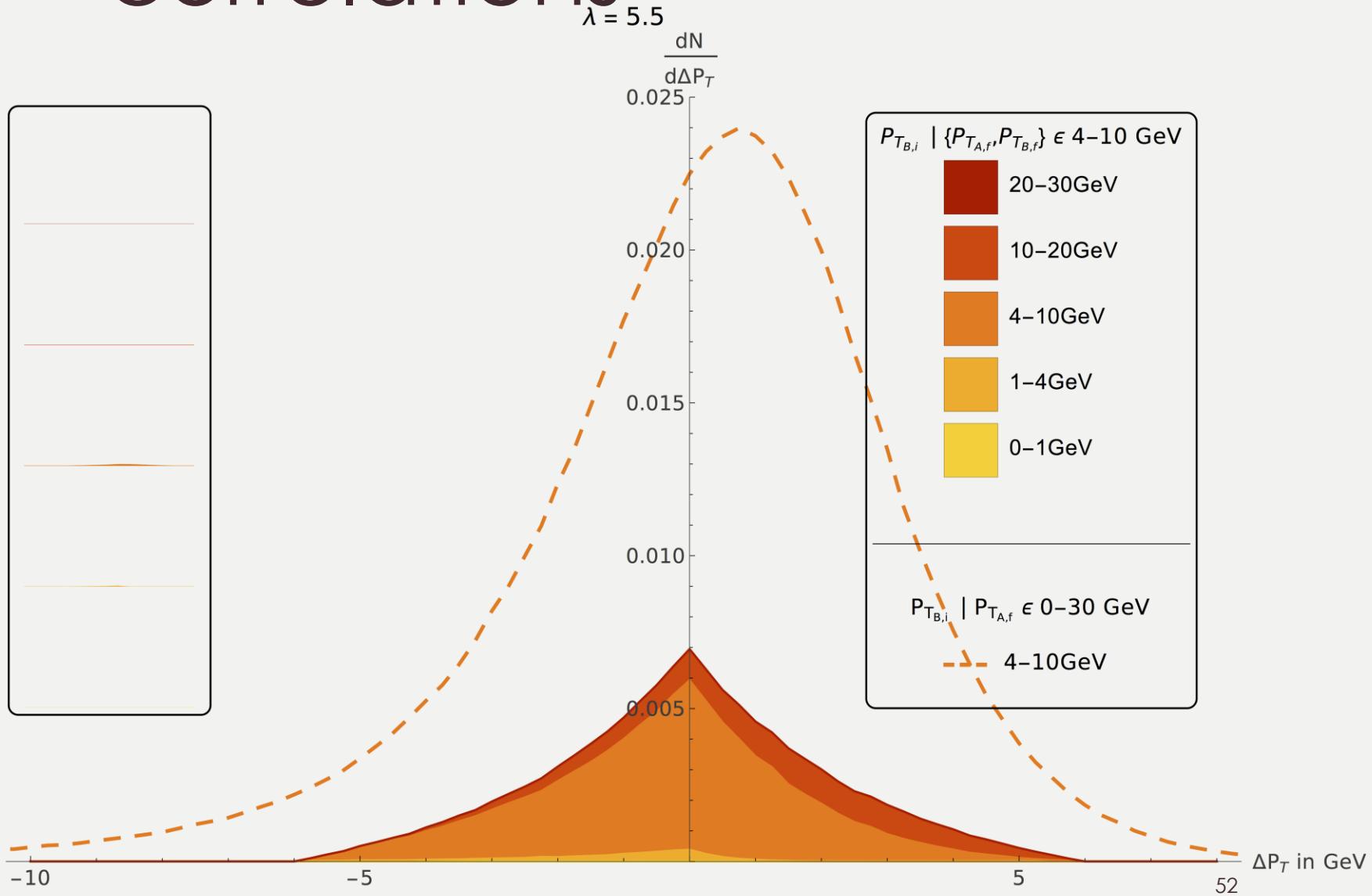
$\lambda = 5.5$



NLO initial momentum correlations

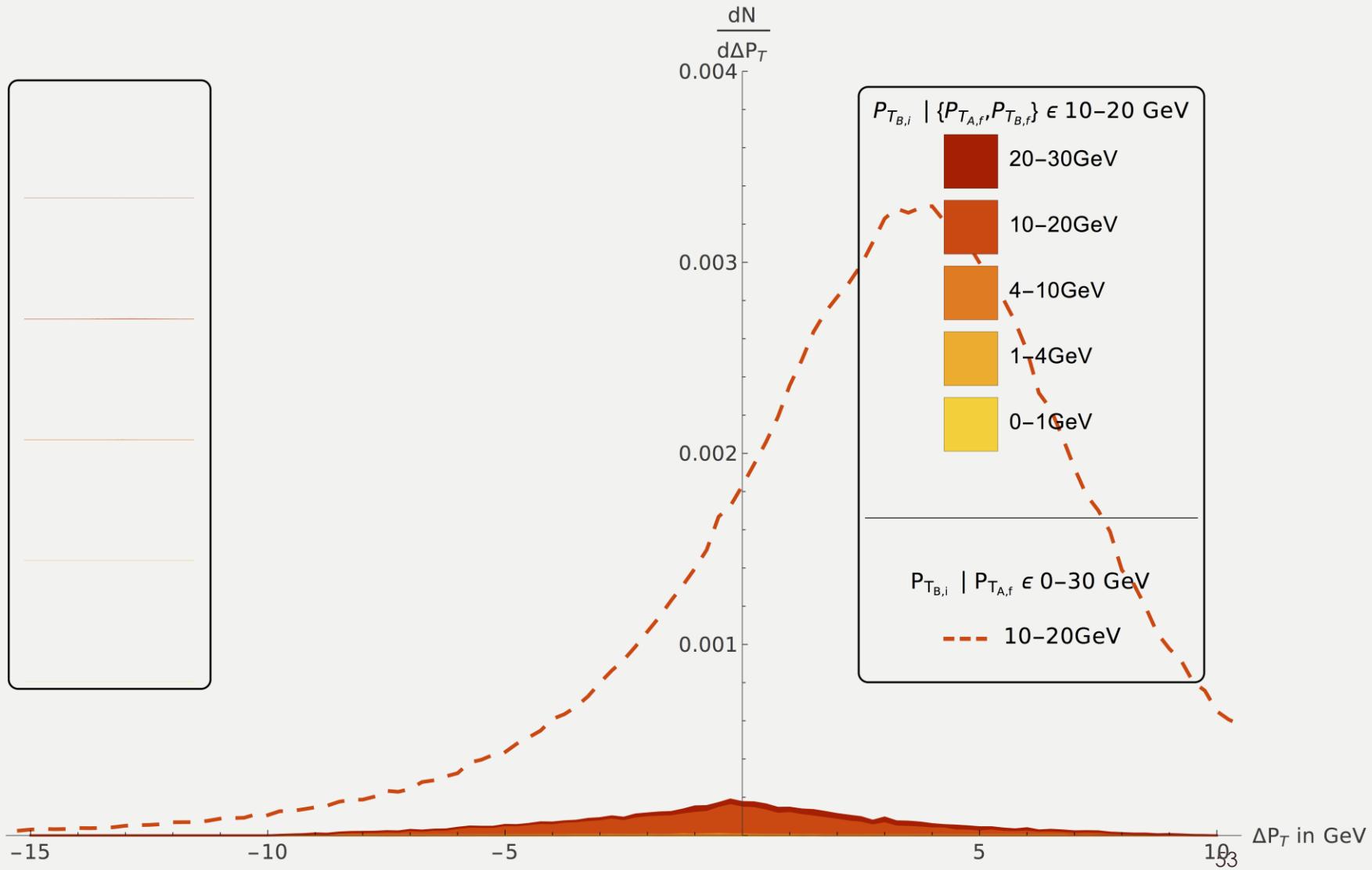


NLO initial momentum correlations

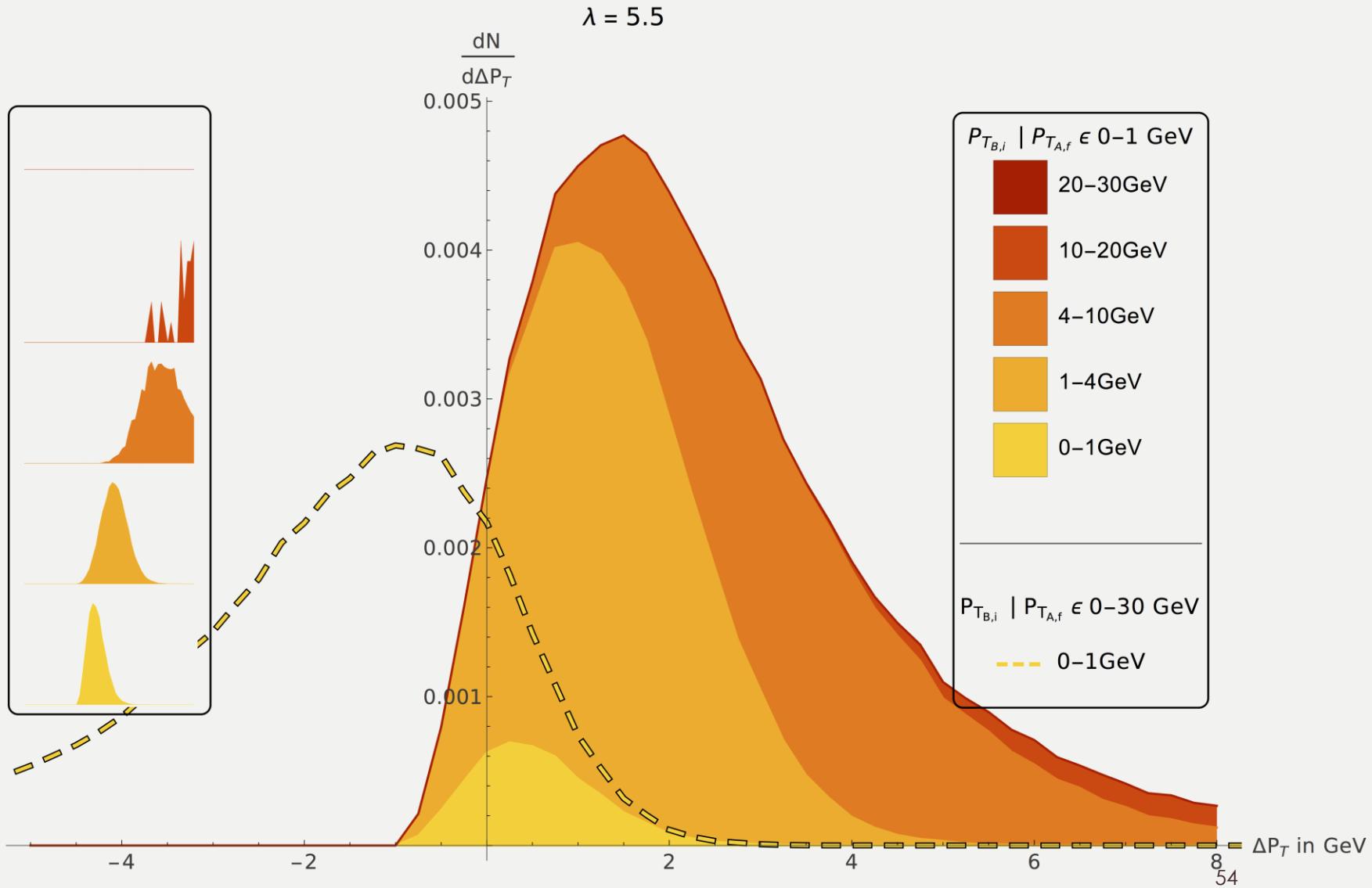


NLO initial momentum correlations

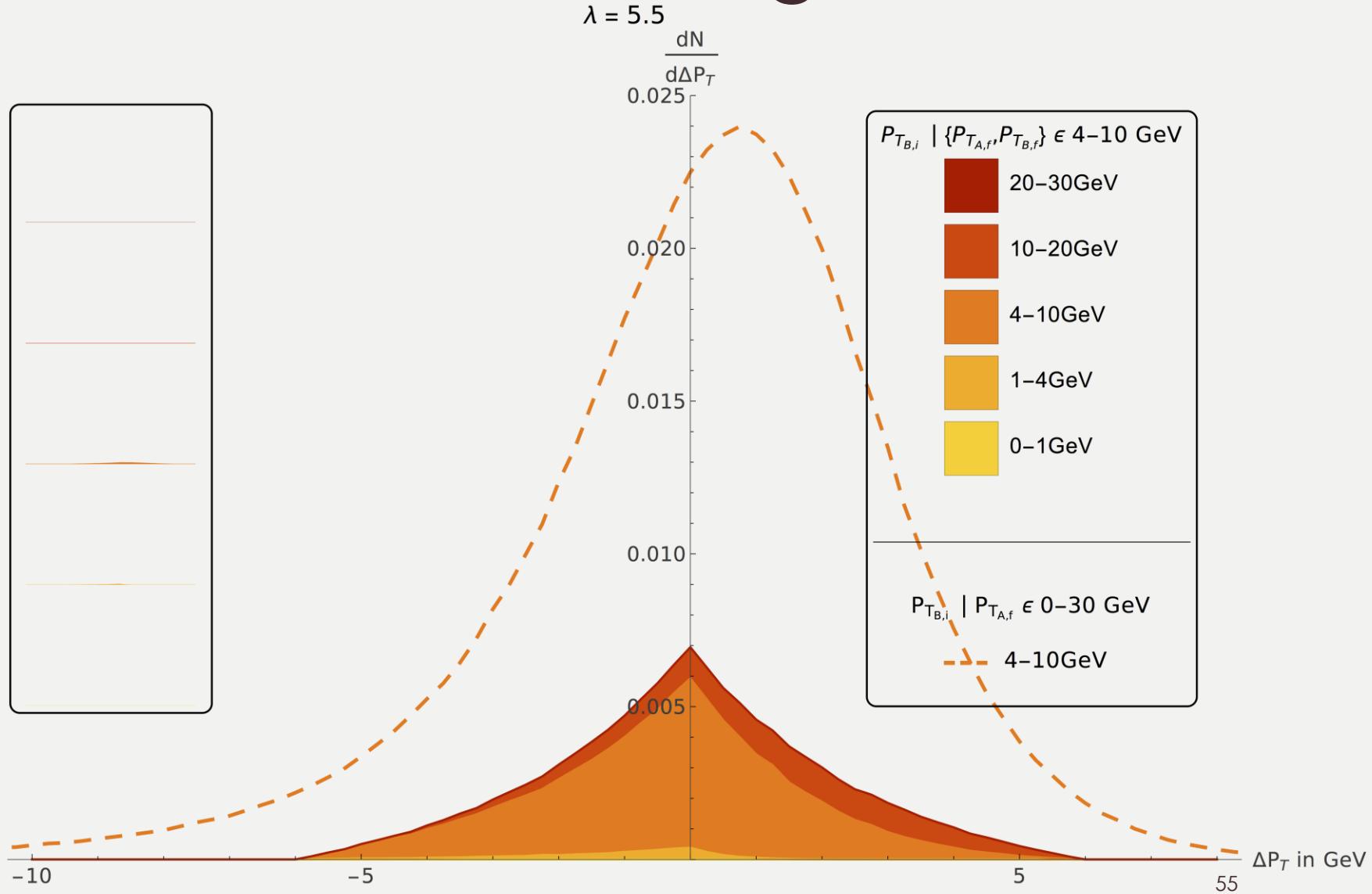
$\lambda = 5.5$



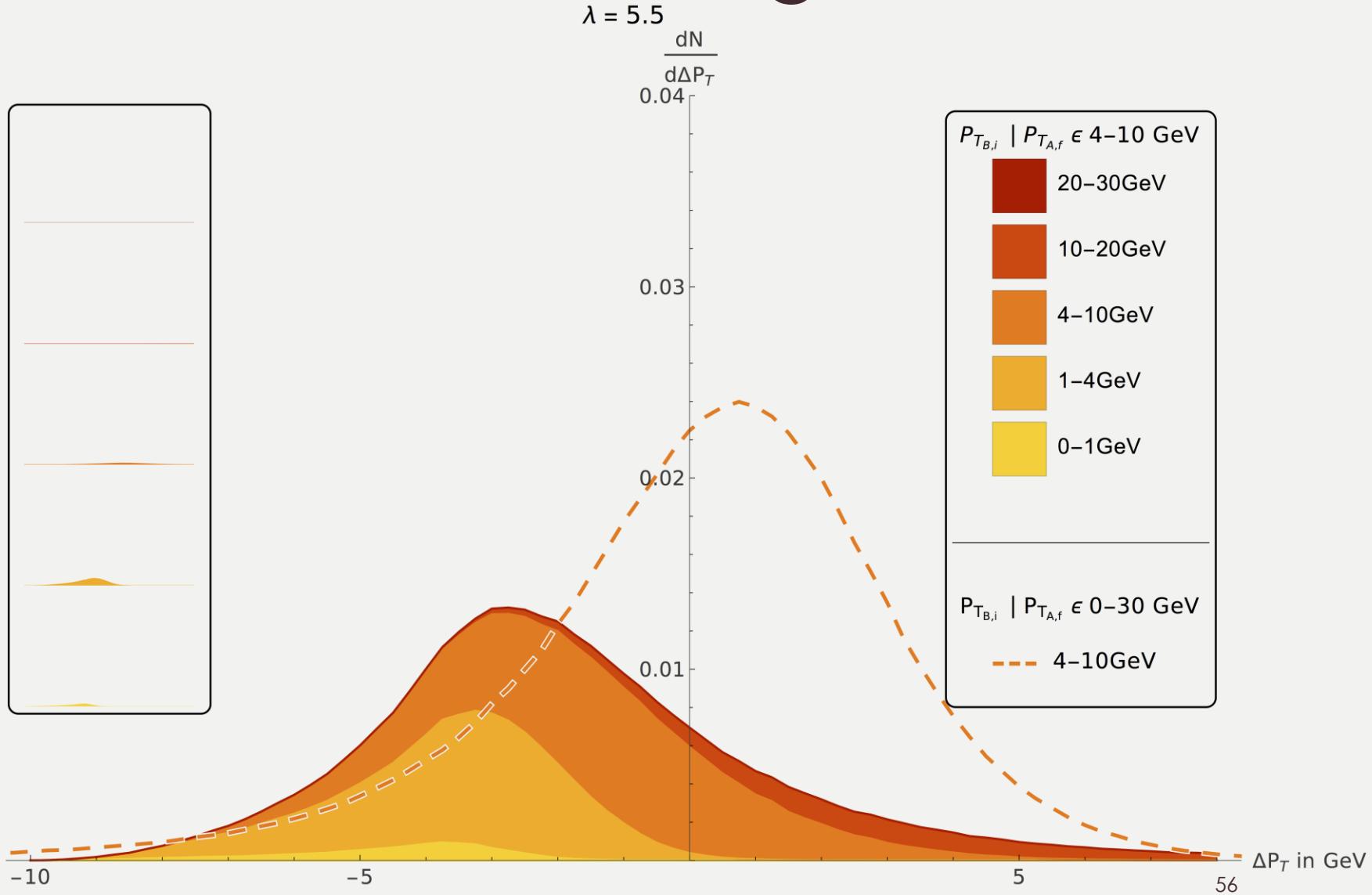
NLO initial momentum correlations – single cut-off



NLO initial momentum correlations – single cut-off

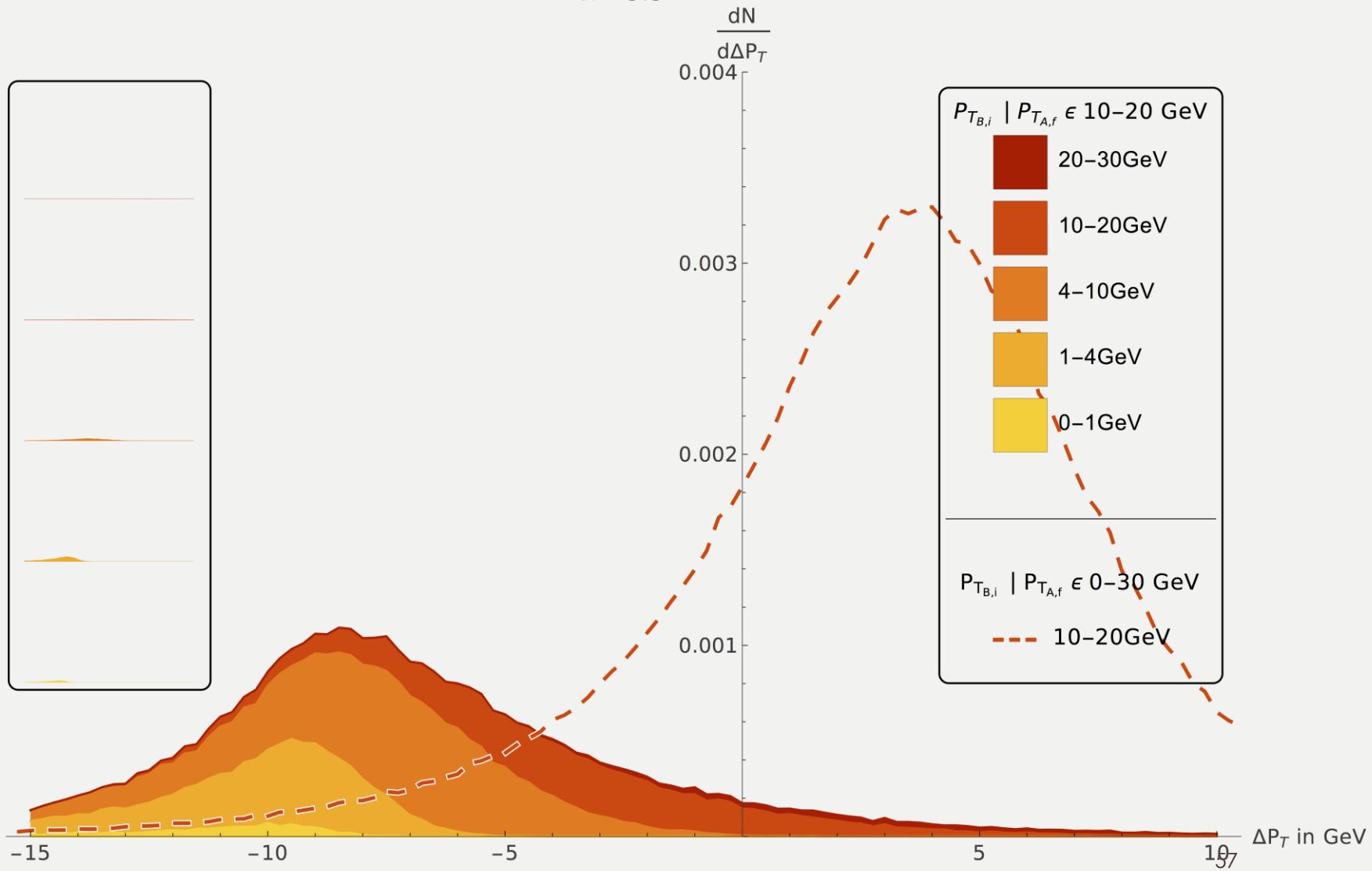


NLO initial momentum correlations – single cut-off

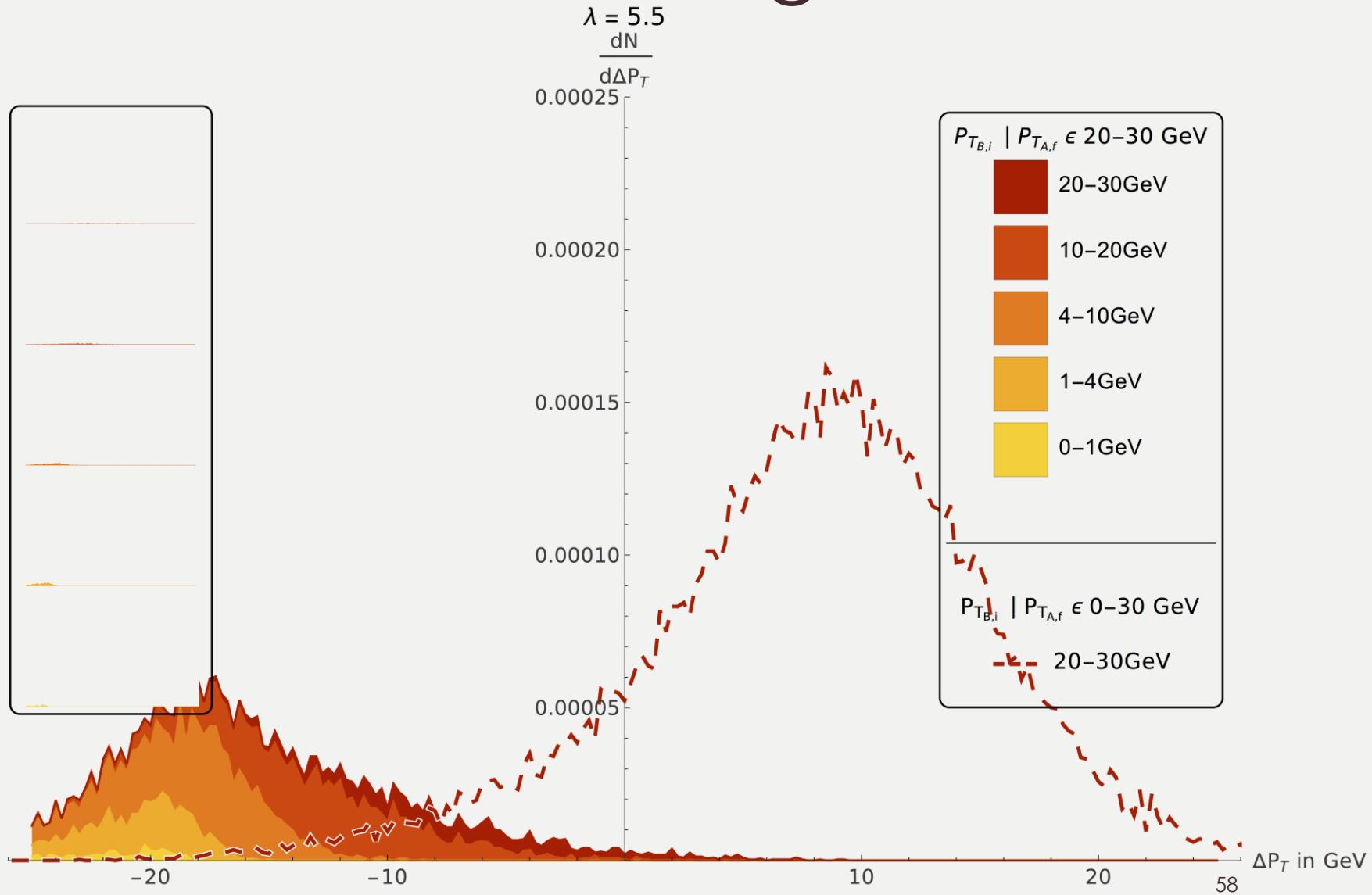


NLO initial momentum correlations – single cut-off

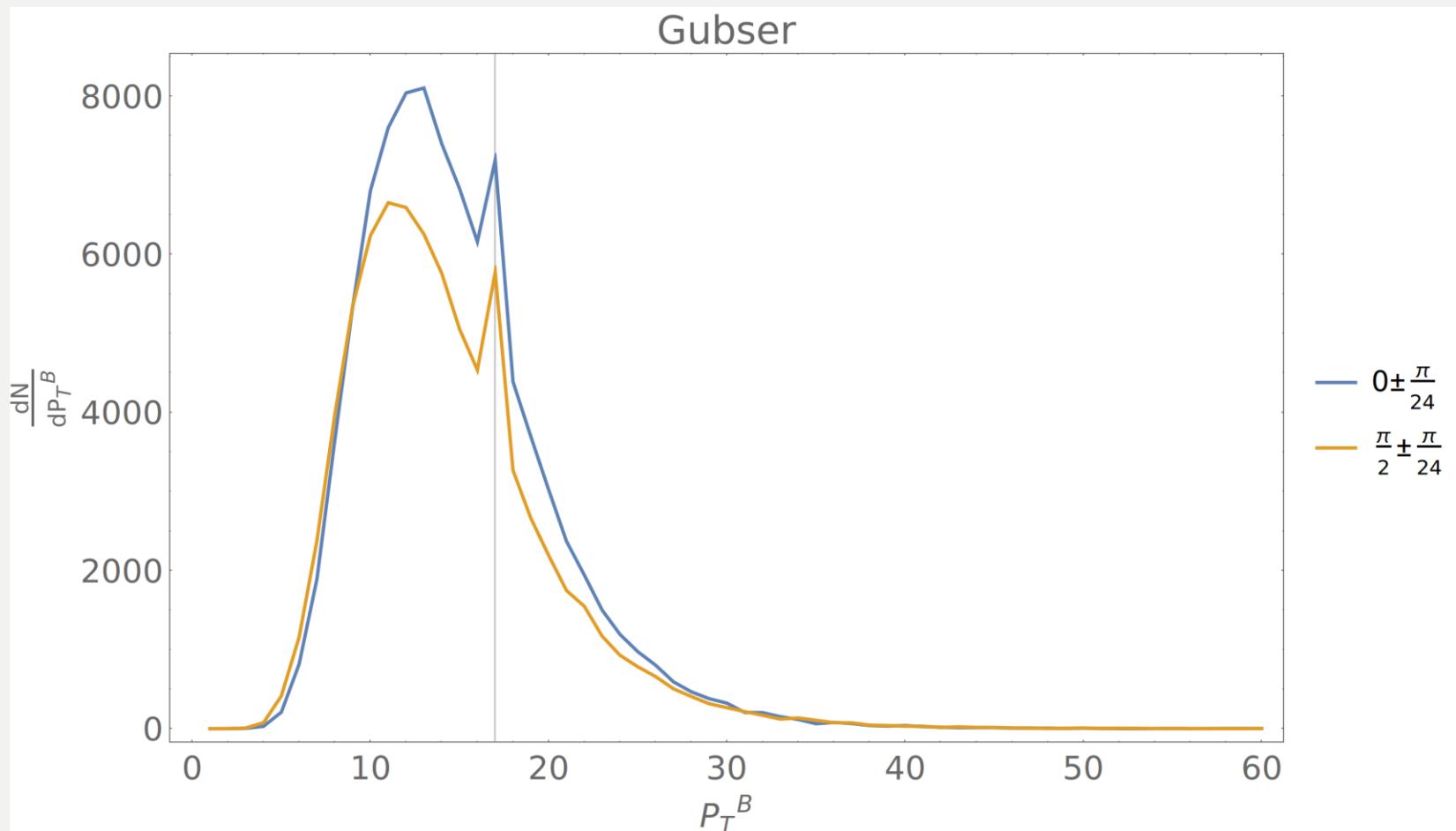
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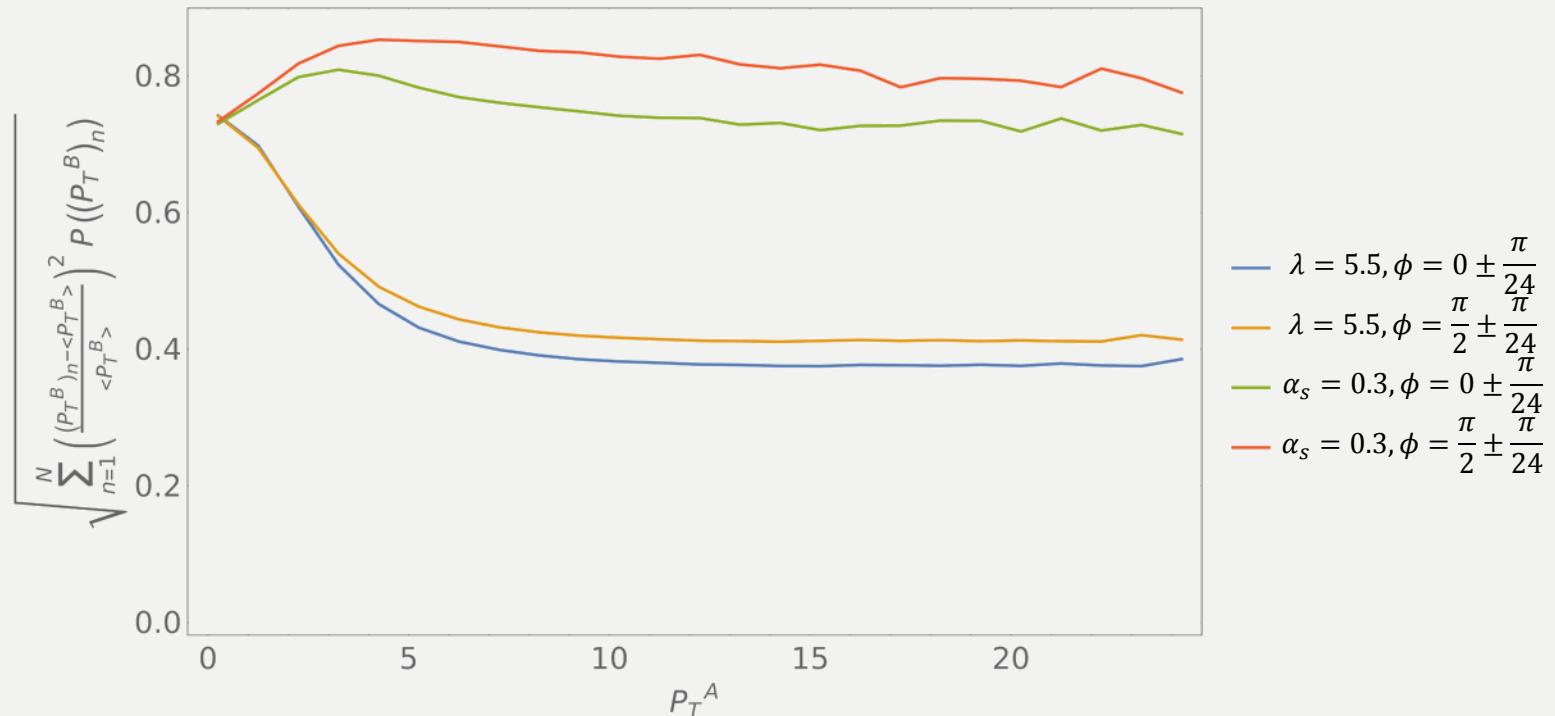
NLO initial momentum correlations – single cut-off



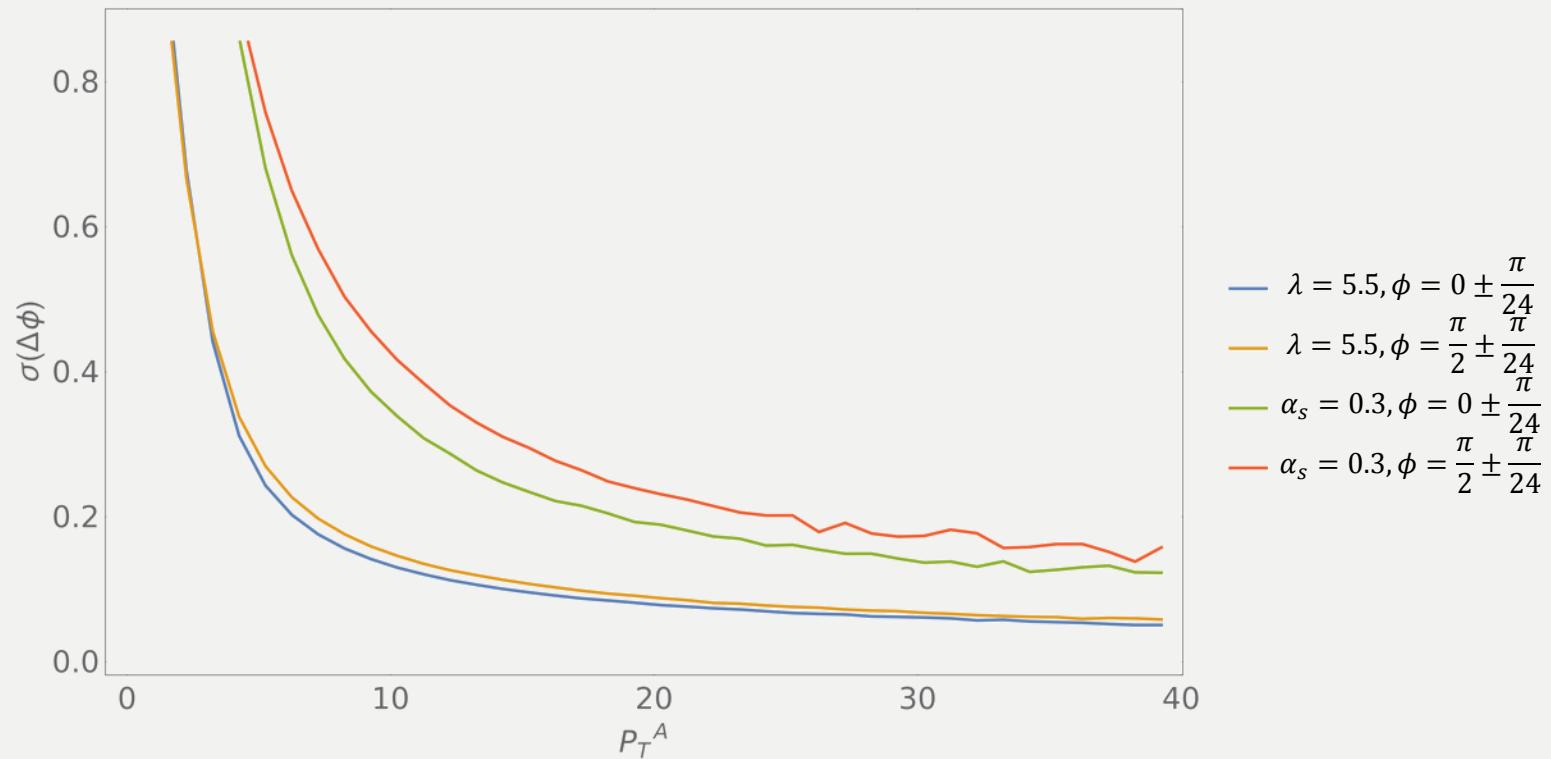
Momentum correlation



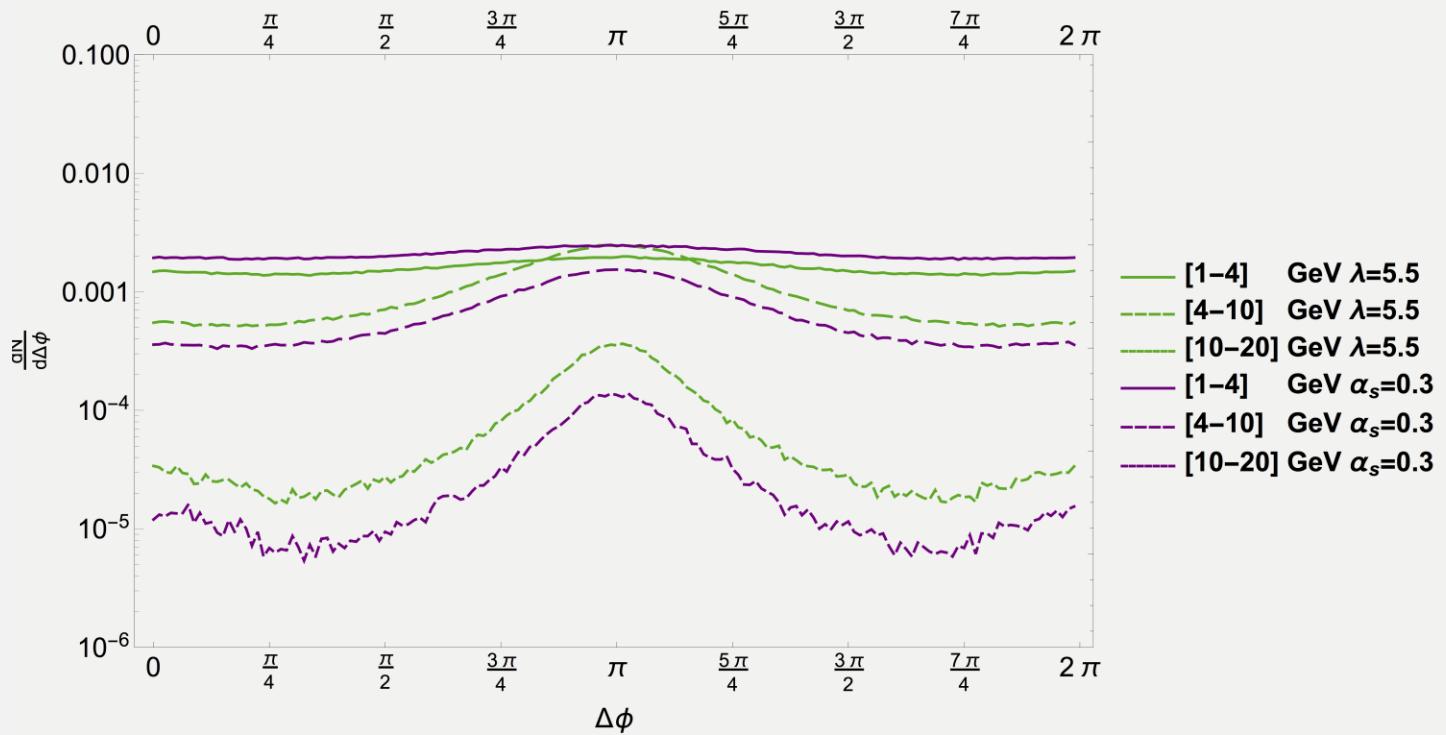
Relative Momentum Variance



Azimuthal Variance



NLO production azimuthal correlations



W. Horowitz & R. Hambrock, in progress