Probing spectral properties of the QGP with real-time lattice simulations

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Talk based on:

KB, Aleksi Kurkela, Tuomas Lappi, Jarkko Peuron,
PRD 98, 014006 (2018) ***(arXiv:1804.01966)***

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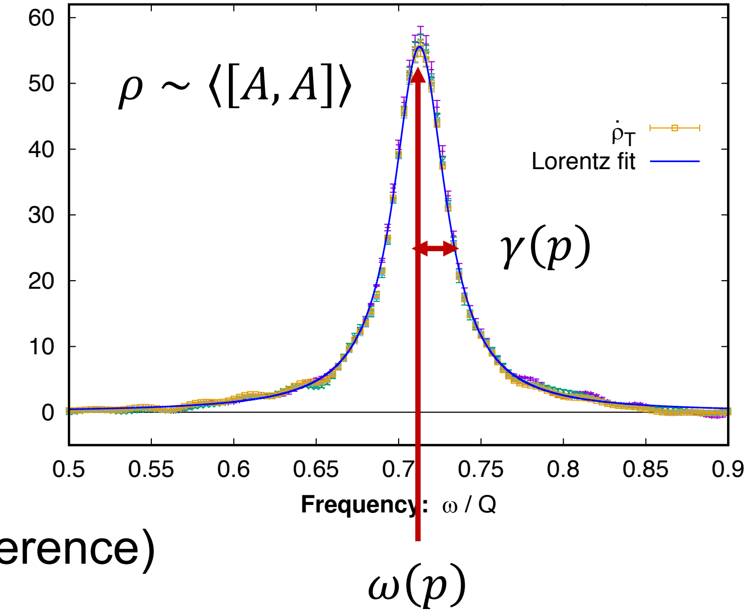
Motivation

A typical (single-particle) spectral function

- $\rho(\omega, p)$ includes *all possible excitations*
  (ω: frequency, p: momentum)
- *Quasiparticles* emerge as Lorentz peaks

$$\sim \frac{1}{\left(\omega - \omega(p)\right)^2 + \gamma^2(p)}$$

- *Dispersion* $\omega(p)$ is energy of "on-shell" particles
- *Damping rate* $\gamma(p)$ is inverse of their life time
- Unstable modes emerge in $\rho(\Delta t, p)$ (Δt: time difference)
- More complicated structures can also be found
  (cuts, extra poles, etc.)



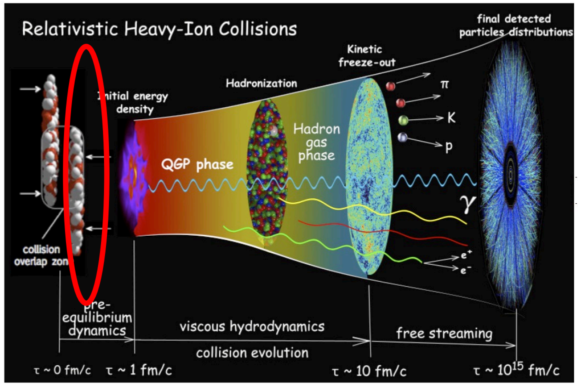
Motivation: Unsolved problems

- $\rho(\omega, p)$ of QCD (and other theories) far from equilibrium

- Initial stages in ultra-relativistic heavy-ion collisions (URHICs) for $g \ll 1$:
  - Quasiparticles, instabilities?
  - Transition 2+1D Glasma $\Rightarrow$ 3+1D kinetic theory

    not fully understood!

    E.g., no dynamical transition in descriptions like KoMPoST.
    Kurkela et al., 1805.01604, 1805.00961



Little Bang by P. Sorensen and C. Shen

- Once ab-initio understanding complete

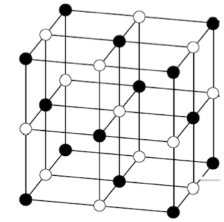
  $\Rightarrow$ extract $\hat{q}$, κ, transport coefficients at initial stages!

  Crucial to understand jet quenching, heavy-quark diffusion. See talks by Apolinario and Cao.

- What is $\rho(\omega, p)$ in high-T thermal equilibrium?

New method to extract the spectral function

Classical-statistical lattice simulations



- Large occupancies $f(p \sim \Lambda) \gg 1$, weak coupling, real time
  $\Rightarrow$ *Classical* approximation for *field dynamics* applicable

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005)

- *SU*(N_c) *gauge theory* in temporal $A_0 = 0$ gauge
- Here we will use $N_c = 2$, 3D spatial lattice N_s^3, lattice spacing a_s
- Initialization with "quantum" initial conditions (IC):

$$\langle |A(t_0, \boldsymbol{p})|^2 \rangle \sim \frac{f(t_0,p)}{p}, \qquad \langle |E(t_0, \boldsymbol{p})|^2 \rangle \sim p\, f(t_0,p)$$

- Fields: $A_i(t_0, \boldsymbol{p}) \to U_i(t_0, \boldsymbol{x}) \simeq e^{i g a_s A_i(t_0, \boldsymbol{x})}$, $E_i(t_0, \boldsymbol{p}) \to E_i(t_0, \boldsymbol{x})$, restore Gauss law
- Evolve classical EOMs in time $D_\mu F^{\mu\nu} = 0$, but in U_i, E_i fields

Literature: *E.g., Berges, KB, Schlichting, Venugopalan; Kurkela, Moore ; KB, Kurkela, Lappi, Peuron; …*

Remark: Gauge-invariant IC also possible: start with (classical) thermal system, gauge cool until hard scale $\Lambda \ll 1/a_s$

New method to extract the spectral function

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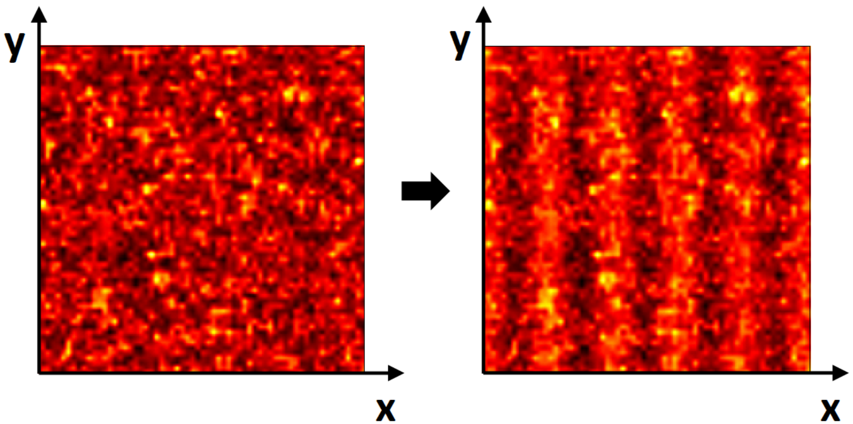
<u>Combine</u>: linear response, classical simulations

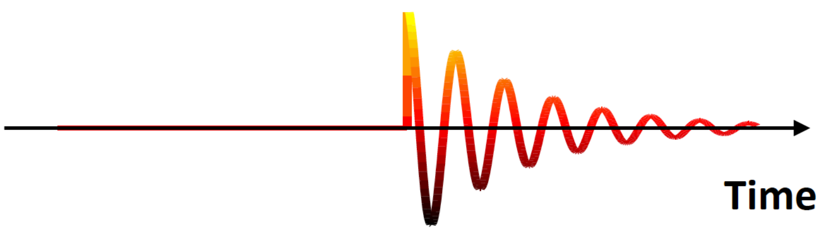
Perturbation



- Classical field simulations for background
- Source j at time t', Coulomb gauge $\partial_j A_j = 0$
- Split $A_j(t,\boldsymbol{x}) \mapsto A_j(t,\boldsymbol{x}) + a_j(t,\boldsymbol{x})$
- Response in linear fluctuations a_j for $t > t'$:
  Solve (linear) EOM for a_j

Kurkela, Lappi, Peuron,
EUJC 76 (2016) 688

Response

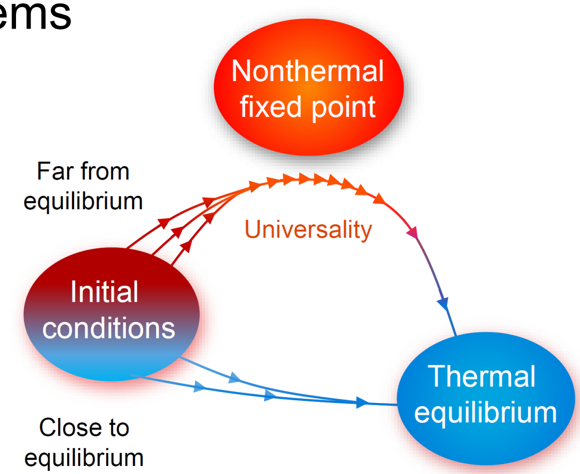


$\Delta t = t - t'$

- $\langle a_j(t,\boldsymbol{p})\rangle = \int dt' G_{R,jk}(t,t',\boldsymbol{p})\, j^k(t',\boldsymbol{p})$,
  obtain ret. propagator $G_{R,jk}$ from response
- Spectral function: $G_{R,jk} = \theta(t-t')\, \rho_{jk}$
- Distinguish polarizations (trans., long.)

First application: a universal attractor (NTFP) ...

Far-from-equilibrium overoccupied $f \sim 1/g^2$ systems approach a NTFP



⇨
- ✓ Details of initial conditions lost
- ✓ Time scale independence
- ✓ Universal self-similar dynamics

$$f(t,p) = t^{\alpha} f_S(t^{\beta} p)$$

Micha, Tkachev (2004); Berges, Rothkopf, Schmidt (2008);
Berges, KB, Schlichting, Venugopalan (2015); ...

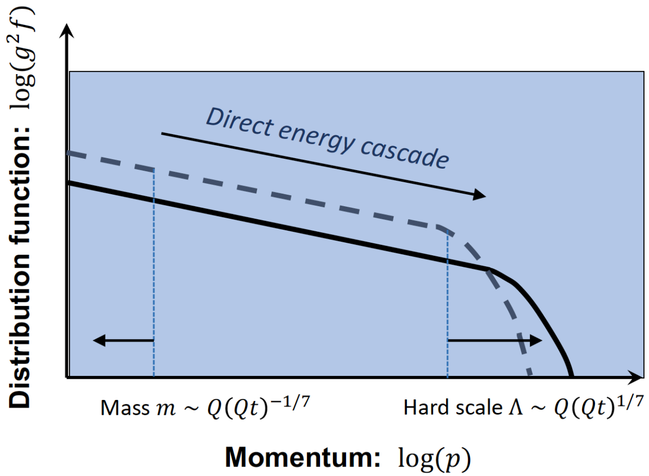
… in isotropic non-Abelian plasmas

Universal
$\alpha = -4/7$
$\beta = -1/7$

Scale separation grows

$$m/\Lambda \sim (Qt)^{-2/7} \ll 1$$

Q: constant scale where energy dominated initially



Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014); …

First application: Comparison to HTL

Hard-thermal loop theory

Braaten, Pisarski (1990); Blaizot, Iancu (2002)

Perturbative formalism, usually applied to thermal equilibrium, high $T \gg \omega, p$

Applicable out of equilibrium when:

- Hard scale Λ, soft modes m (mass), scale separation $m/\Lambda \ll 1$

  $\Rightarrow$ HTL can be applied to attractor in non-Abelian plasmas!

- Details of $f(t,p)$ "hidden" in few parameters, e.g.:

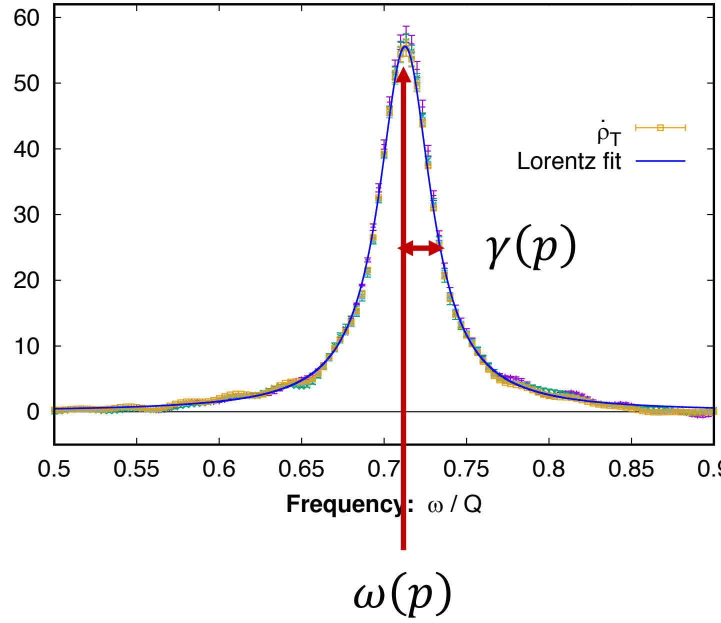
$$m^2 \approx 2N_c \int \frac{d^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p}, \qquad g^2 T_* \approx \frac{\int d^3p \, (g^2 f)^2}{2\int d^3p \, (g^2 f)/p}, \qquad \ldots$$

Opportunity for thermal equilibrium:

Compute $\rho(\omega, p)$ non-perturbatively with new method, compare it to HTL

First application

Reminder: typical spectral function



First application

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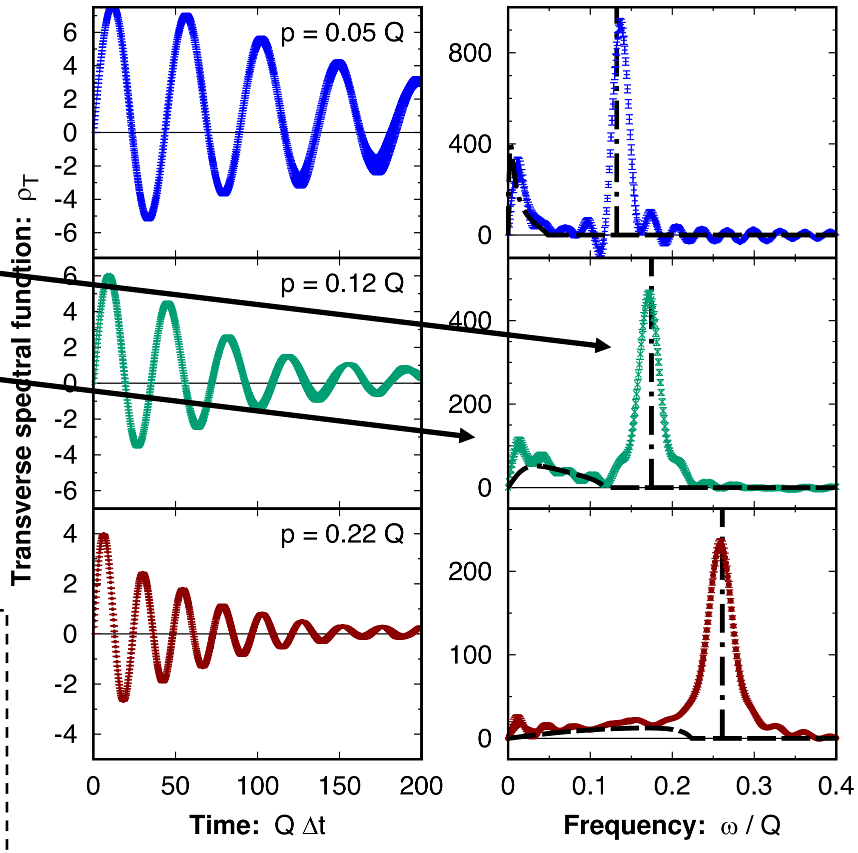
Transverse spectral function ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks:
  existence of quasi-particles
- for $|\omega| \leq p$: *Landau cut*
- black dashed lines:
  Hard-thermal Loop (HTL) at LO

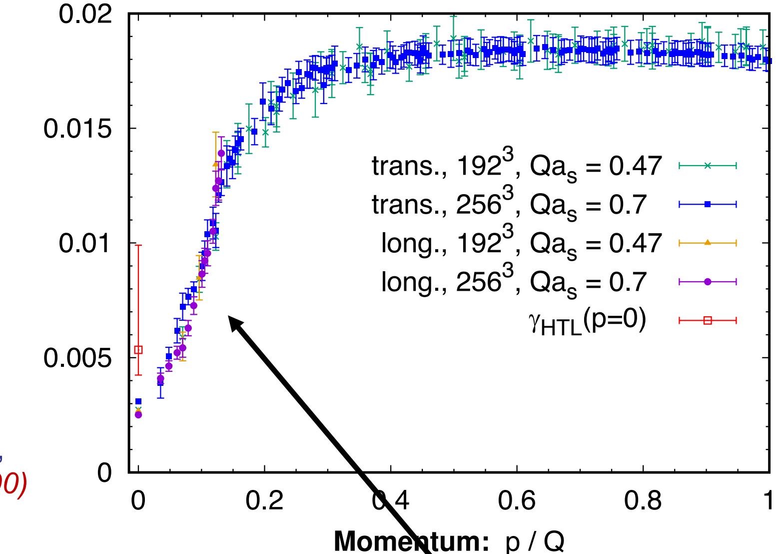


- ✓ Good agreement with HTL at LO!
- ✓ System dominated by quasiparticles with narrow width (beyond HTL at LO)!

First application

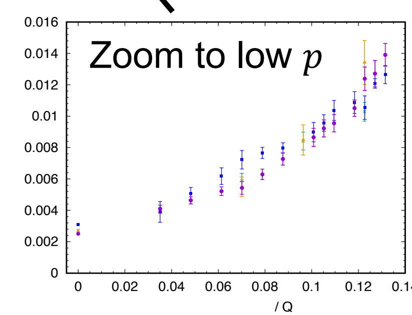
Extracted damping rates $\gamma_{T,L}(p)$

$\gamma_{T,L}(p)$ / Q



- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$ Braaten, Pisarski, *PRD 42, 2156 (1990)*

$\gamma_{\mathrm{HTL}}(p \to \infty)$: not included here, too large uncertainties due to necessary estimation of magnetic scale Pisarski, *PRD 47, 5589 (1992)*



- "Isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$ while dispersions different $\omega_T > \omega_L$ (Backup)

First application

Check: Is damping rate indeed subleading?

- Observe: (indication, but low statistics)

  $\gamma(p) \sim Q(Qt)^{-3/7}$

  as function of p/m

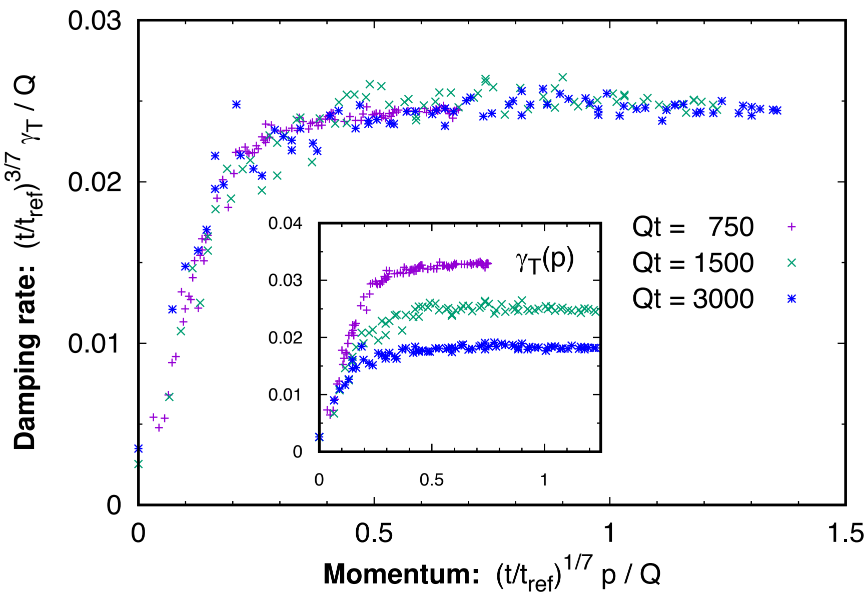
- Reminder:

  $m/\Lambda \sim (Qt)^{-2/7}$

- Conclusion:

  $\gamma(p)/\Lambda \sim (m/\Lambda)^2$

- This shows $\gamma(p)/m \xrightarrow{t\to\infty} 0$, quasi-particle peaks become Delta functions



- *Late-time limit* corresponds to *LO HTL*, damping rates are effects beyond LO

First application

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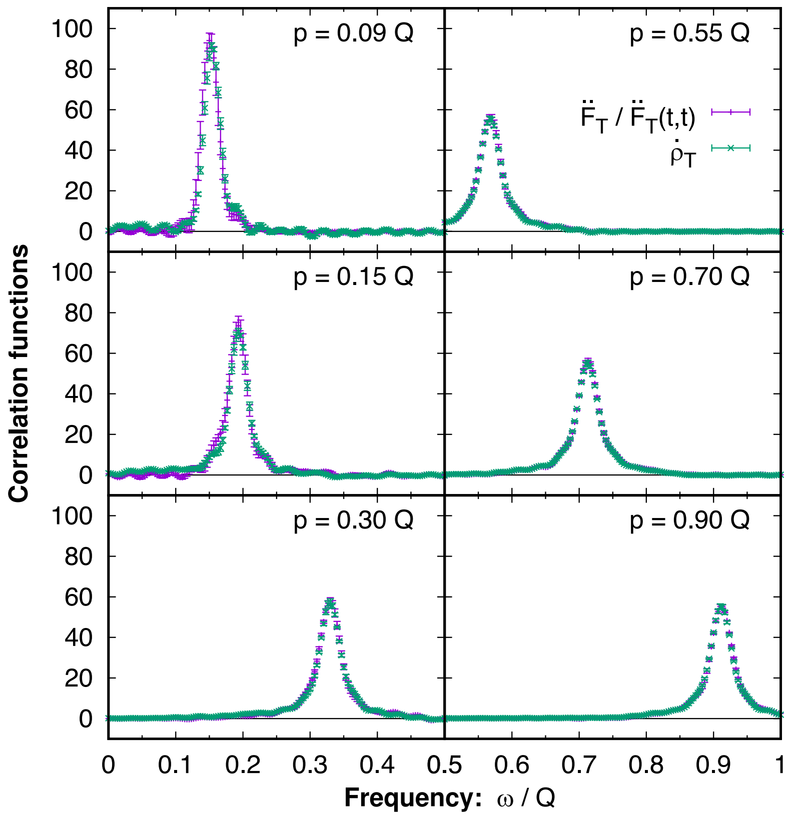
Statistical correlation function F



<u>(Classical) definition</u>:

$$\ddot{F}^{jk}(t, \Delta t, \boldsymbol{p}) = \langle E^j(t, \boldsymbol{p})\, E^{*,k}(t', \boldsymbol{p}) \rangle$$

(As always: $\Delta t = t - t'$, Fourier transform to ω)

<u>(Thermal) fluctuation-dissipation relation</u>:

$$\ddot{F}_T(t, \omega, p) \;/\; T = \dot{\rho}_T(t, \omega, p)$$

Used to estimate thermal spectral function

Example: G. Aarts, *PLB 518, 315 (2001)*

We *extract $\ddot{F}$, $\dot{\rho}$ independently*, we observe a <u>generalized fluctuation-dissipation relation:</u>

$$\frac{\ddot{F}_T(t, \omega, p)}{\ddot{F}_T(t, \Delta t = 0, p)} = \dot{\rho}_T(t, \omega, p)$$

<u>Remarks</u>: $\dot{\rho}_T = \partial_t \rho_T$, $\dot{\rho}_T(t, \Delta t = 0, p) = 1$

Conclusion: One method to rule them all

✓ What is $\rho(\omega, p)$ in high-T thermal equilibrium? $\Rightarrow$ **this work**, $\gamma_{T,L}(p)$ determined!

- Completing ab-initio understanding of initial stages of URHICs for $g \ll 1$

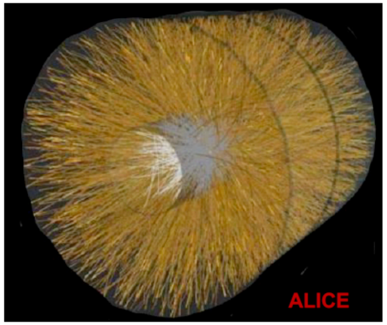
  $\Rightarrow$ New method can distinguish between plasma and vacuum numerically, will be applied to transition 2+1D classical state $\Rightarrow$ 3+1D kinetic theory

- Extract $\hat{q}$, κ, transport coefficients at initial stages

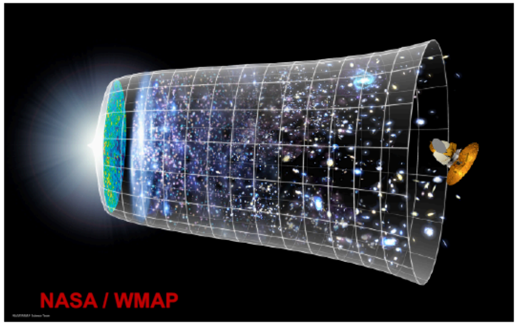
  $\Rightarrow$ we started for far-from-equilibrium states, *in progress*

- $\rho(\omega, p)$ in physical systems far from equilibrium $\Rightarrow$ first results, *in progress*

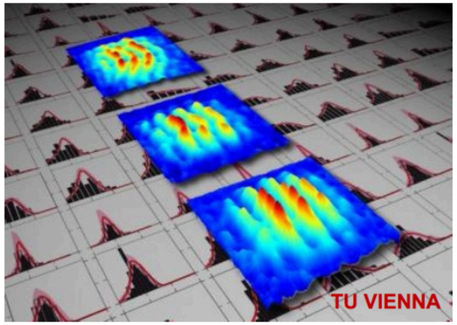
Heavy-ion collisions

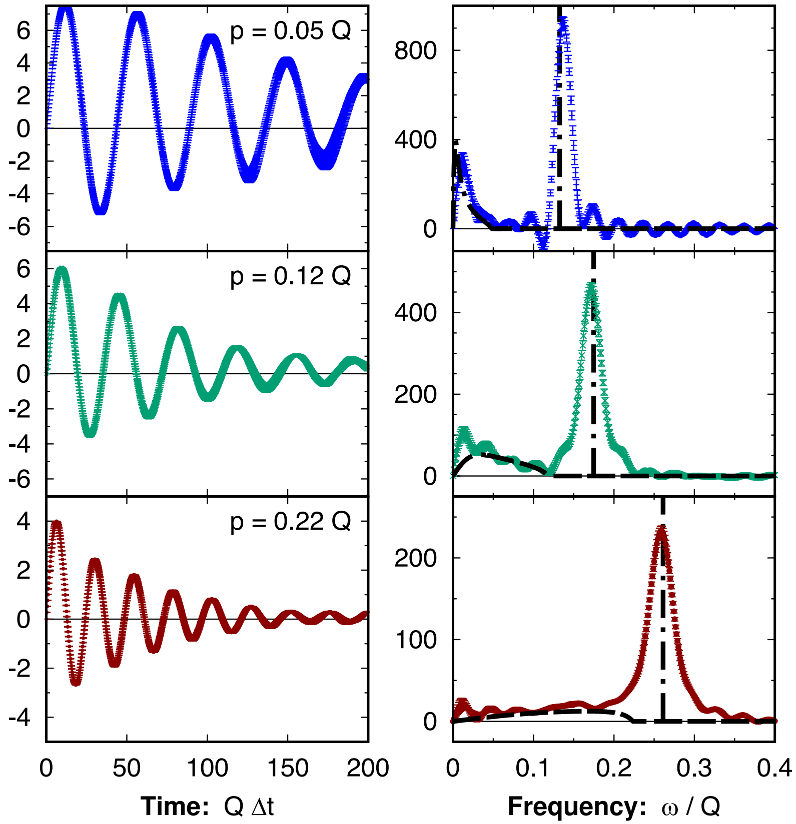


Early Universe



Ultra-cold atoms





Thank you for your attention!

BACKUP SLIDES

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New method

Computation of (single-particle) spectral function ρ

Linear response theory:

- Linear reponse $\langle a_j(t,\boldsymbol{p})\rangle = \int dt' G_{R,jk}(t,t',\boldsymbol{p})\, j^k(t',\boldsymbol{p})$ to perturbation j

Our numerical approach:

- Split gauge field $A_j(t,\boldsymbol{x}) \mapsto A_j(t,\boldsymbol{x}) + a_j(t,\boldsymbol{x})$, solve *newly developed equations* for $a_j(t,\boldsymbol{x})$ (also: restore Gauss law)

Kurkela, Lappi, Peuron,
EUJC 76 (2016) 688

- Perturb system at $t = t_{\mathrm{pert}}$ in Coulomb gauge $\partial_j A_j = 0$ with a source

$$\langle\, j^k_{0,a}(\boldsymbol{p})\, (j^l_{0,b}(\boldsymbol{q}))^*\rangle_j = \delta_{a,b}\; V \delta_{\boldsymbol{p},\boldsymbol{q}}\, v_k(\boldsymbol{p})\, v_l^*(\boldsymbol{q})$$

Polarization
Transverse: $\boldsymbol{v} \perp \boldsymbol{p}$,
Longitudinal: $\boldsymbol{v} = \boldsymbol{p}$

- Obtain $G_{R,kl}(t,t_{\mathrm{pert}},\boldsymbol{p}) = \frac{1}{(N^2-1)V}\langle\, \langle a^b_k(t,\boldsymbol{p})\rangle\, (j^b_{0,l}(\boldsymbol{p}))^*\rangle_j$

- *Spectral function*: use relation $G_{R,jk} = \theta(t - t_{\mathrm{pert}})\, \rho_{jk}$ $\quad$ ($\rho \simeq i\langle [A,A]\rangle$)

New method to extract the spectral function

What is gauge invariant / covariant?

- *Equations of motion* of background (BG) field and of linearized fluctuations
- *Initial conditions for the BG* can, in principle, be set gauge-invariantly (starting with classical thermal system $T \gg 1/a_s$, gauge cool until $\Lambda \ll 1/a_s$)
- *Correlation function* $\langle \langle a_k^a(t,\boldsymbol{x}) \rangle \, U_0^{ab}(t, t_{\text{pert}}, \boldsymbol{x}) \, j_{0,l}^b(\boldsymbol{x}) \rangle_j$, where $U_0(t, t_{\text{pert}}, \boldsymbol{x})$ is a Wilson line, i.e., product of U_0 links. In our framework:
  - $A_0 = 0 \;\Rightarrow\; U_0^{ab} = \delta_{ab}$
  - Use source only for one momentum $\boldsymbol{p}$

$\Rightarrow$ $\boxed{\langle \langle a_k^a(t,\boldsymbol{x}) \rangle \, U_0^{ab}(t, t_{\text{pert}}, \boldsymbol{x}) \, j_{0,l}^b(\boldsymbol{x}) \rangle_j \propto G_{R,kl}\big(t, t_{\text{pert}}, \boldsymbol{p}\big)}$, a *gauge-inv. observ.*!

Remark: But it corresponds to G_R only in temporal gauge

- Gauge-dependent: The choice of the source $j_{0,l}^b(\boldsymbol{x}) \propto \sqrt{V}\, v_l(\boldsymbol{p}) \cos \boldsymbol{p}\boldsymbol{x}$

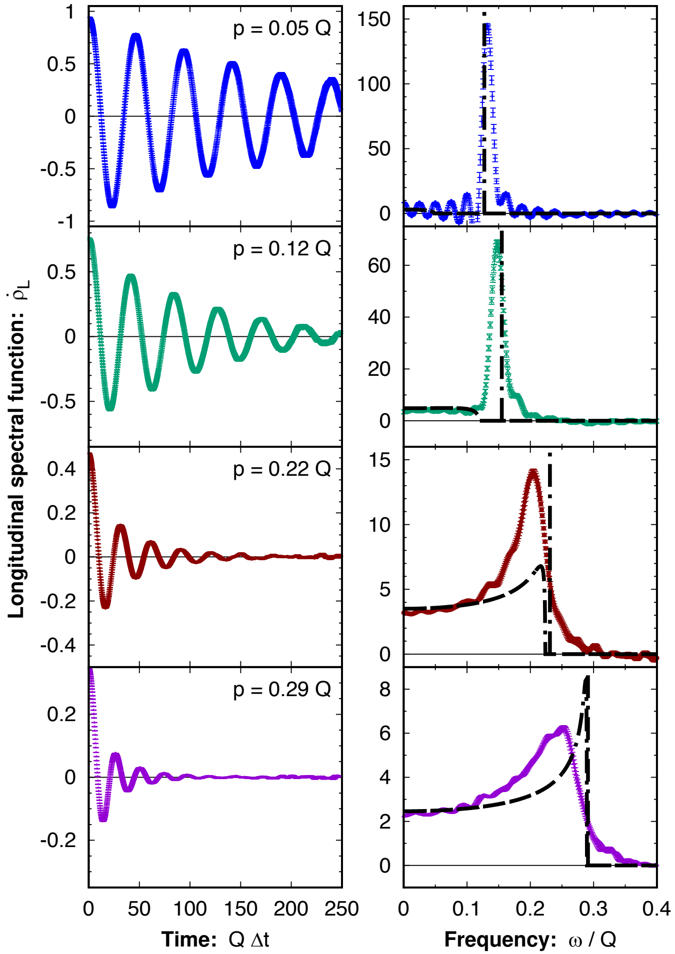
First application

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Longitudinal spectral function ρ_L

$\dot{\rho}_L(\simeq \omega\rho_L)$ as function of $\Delta t = t - t'$ (left) or ω (right) at late time $t_{\mathrm{pert}} \gg \Delta t$

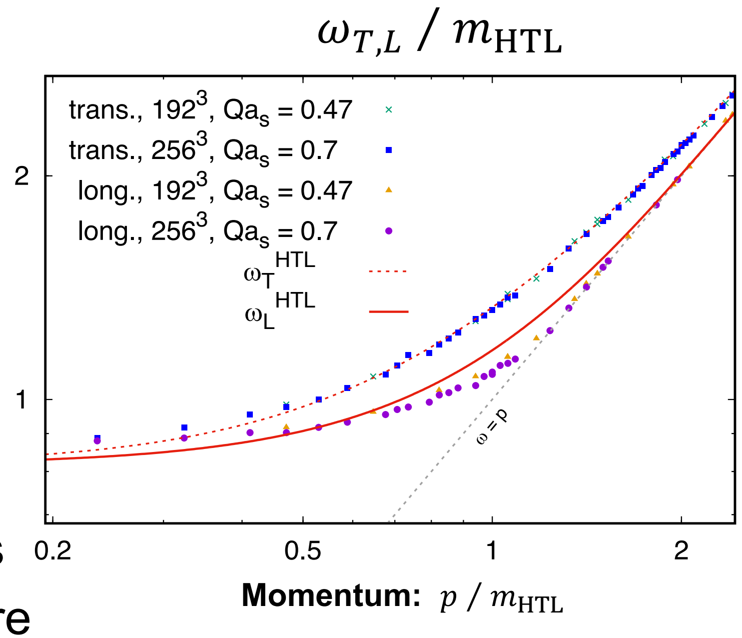
- Similar as for ρ_T, *existence of quasi-particles* with $\omega_L(p)$ and $\gamma_L(p)$ …
- … but for $p \gtrsim m \approx 0.15\, Q$:
  - Quasiparticle peak suppressed exponentially
  - Landau cut dominates oscillations
  - And smeared around light cone



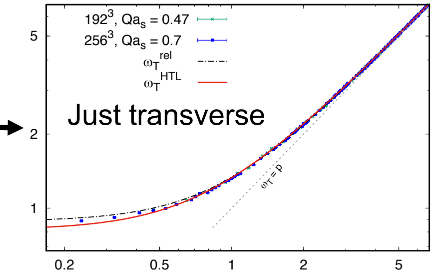
First application

Extracted dispersion relations $\omega_{T,L}(p)$

- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- Deviations at small p, for finite m/Λ?
- "$\omega_L(p)$" deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure



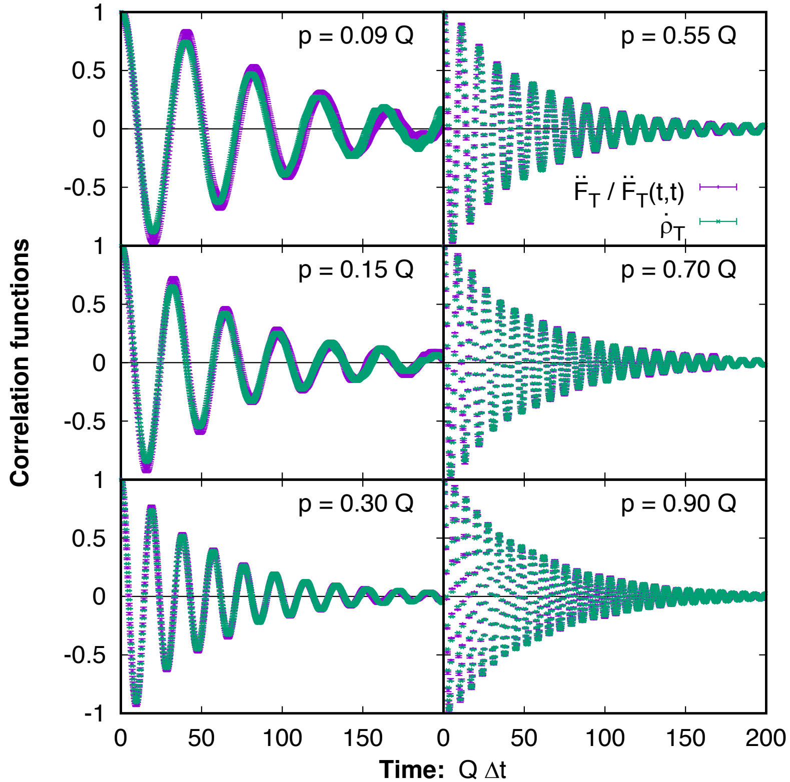
Remark: $\omega_T(p)$ also compatible with $\omega_T^{\mathrm{rel}} = \sqrt{m_\infty^2 + p^2}$ →

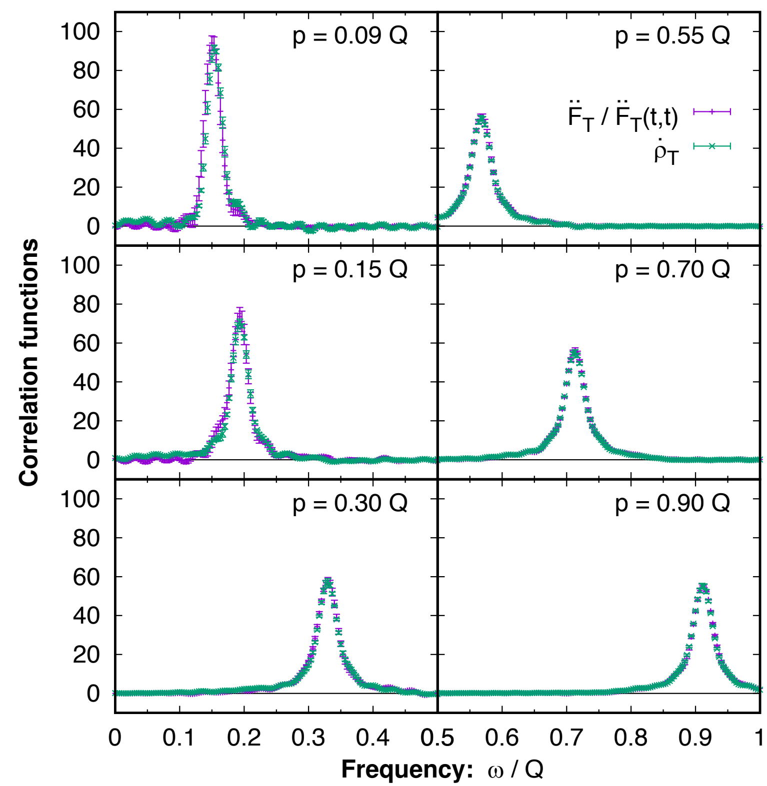


First application

Observation: $\frac{\ddot{F}_T(t,\Delta t,p)}{\dot{\rho}_T(t,\Delta t,p)} = \frac{\ddot{F}_T(t,\omega,p)}{\dot{\rho}_T(t,\omega,p)} = \ddot{F}_T(t,\Delta t = 0, p)$

$\ddot{F}_T$ and $\dot{\rho}_T$ have same functional form in Δt or ω!





<u>Remark</u>: Similarly for longitudinal but with larger statistical error for $\ddot{F}_L$ in our simulations