Jets in a non-equilibrium quark-gluon plasma

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- Non-equilibrium QGP has exponentially growing chromomagnetic fields.

  [e.g. Mrowczynski, Schenke, Strickland 2016]

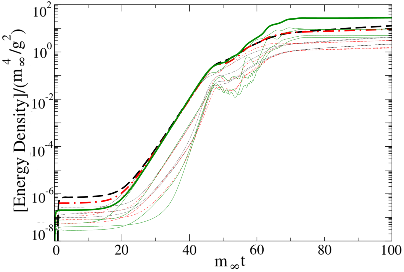
- Strong fields should have a big impact on jet-medium interaction.

- Instabilities come from anisotropic distributions, e.g.

$$f(\mathbf{p}) = f_{\mathrm{eq}}(\sqrt{p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}/\Lambda)$$

- Get poles in upper plane of $G_{\mathrm{ret}}(P)$.

  [Romatschke, Strickland 2003]



Adapted from Rebhan, Romatschke, Strickland, 2005

Introduction

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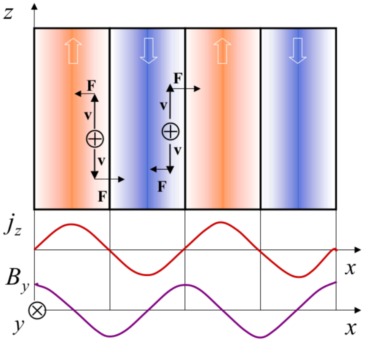

[Mrowczynski 2005]

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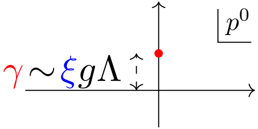


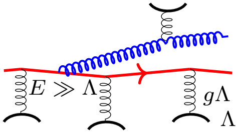
$\sim \quad e^{\gamma t}$

- Get poles in upper plane of $G_{\text{ret}}(P)$.

  [Romatschke, Strickland 2003]

Introduction

- Consider medium-induced radiation in unstable medium.
  - Include LPM effect.

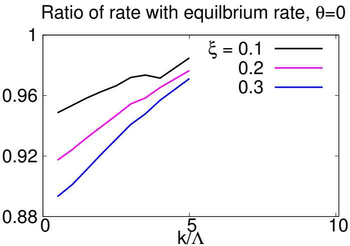


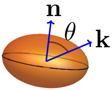
- Earlier work on probes in unstable plasma:
  - Classical case: [Carrington, Mrowczynski, Schenke, 2017]
  - In quantum case got divergent rate of interaction with medium. [e.g. Baier, Mehtar-Tani 2008]
- First consistent work on probes in unstable, quantum QGP.
  - Solves puzzle of divergences.
  - Instabilities and jets described microscopically.
- Extend AMY framework to unstable medium. [Arnold, Moore, Yaffe 2002]

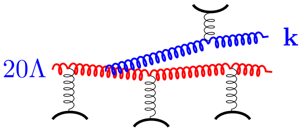
Our earlier work

[Hauksson, Jeon, Gale, 1807.07138, to appear in NPA]

- Focuses on corrections to soft mediators.
- Cut instabilities away.
- Emission of low energy gluons affected the most.
- Changes directionality of jets.



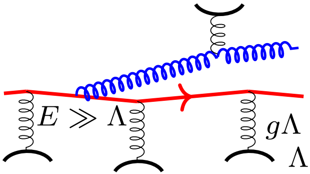




Setup

- Three scales ($g \ll 1$):

	Hard	Soft	Instability
P	Λ	$g\Lambda$	$\xi g\Lambda$
P^2	$g^2\Lambda^2$		



- Follow time evolution starting at time $T = 0$.
- Calculation is valid for time

$$\frac{1}{g^2\Lambda} \ll T \ll \frac{1}{\xi g\Lambda} \log g^{-2}$$

Initial excitations decayed

$e^{\xi g\Lambda T} \ll 1/g^2$

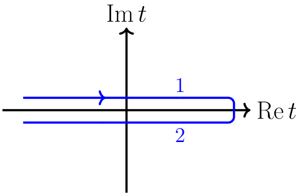
  - At late times instability field stops growing. [Kurkela, Moore 2011]

- Small anisotropy but strong fields: $\xi \ll g$.

Setup

- Use the real time formalism:

$$\phi_r = \frac{1}{2}\left(\phi_1 + \phi_2\right), \quad \phi_a = \phi_1 - \phi_2$$



- Specifying ρ_0 gives density of hard particles:

$$S^0_{rr}(P) \sim \left(\tfrac{1}{2} - f_q(\mathbf{p})\right)\delta(P^2).$$

- No soft gluons in initial state.

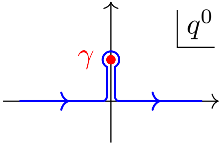
- Describe soft fluctuations and instability simultaneously:

  - $G_{ra} = G_{\mathrm{ret}}$: dispersion relations

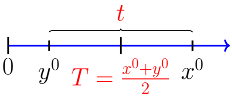
  - G_{rr}: particle density

- Retarded correlator has a modified contour:

$$G_{\rm ret}(Q) = G^{\rm reg}_{\rm ret}(Q) + \frac{D}{q^0 - i\gamma}$$



- $A = A_{\rm fluct} + A_{\rm inst}$
  - fluct + fluct: $G^{\rm reg}_{rr}(P) = G^{\rm reg}_{\rm ret}\,\Pi_{aa}\,G^{\rm reg}_{\rm adv}$
  - fluct + inst: ≈ 0
  - inst + inst:



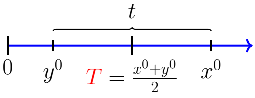
$$G^{\rm inst}_{rr}(x^0, y^0; \mathbf{q}) = \left(D\,\Pi_{aa} D^\dagger\right)\ \tfrac{1}{2\gamma}\left[e^{2\gamma T} - e^{\gamma|t|}\right]$$

- Get exponential growth.

Instabilities

- Express some results in frequency domain:

$$g(t,T) = \int d\omega\, e^{-i\omega t}\, g(\omega; T) \quad \Leftrightarrow \quad g(\omega; T) = \int_{-2T}^{2T} dt\, e^{i\omega t}\, g(t,T)$$



- Approximations:
  - $T \gg 1/g^2\Lambda \implies$ Drop e^{-aT} with $a \gtrsim g^2\Lambda$
  - Time scale of LPM is $1/g^2\Lambda \implies$ Drop e^{iaT} with $a \gtrsim g\Lambda$
  - Only include ξ corrections with exponential growth.

- Correlator defined by

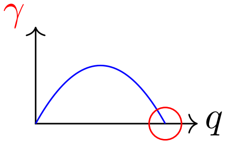
$$G_{\rm ret} = G^0_{\rm ret} + \int_0 \int_0 G^0_{\rm ret}\, \Pi_{\rm ret}\, G_{\rm ret}; \qquad G_{rr} = \int_0 \int_0 G_{\rm ret}\, \Pi_{aa}\, G_{\rm adv} + \ldots$$

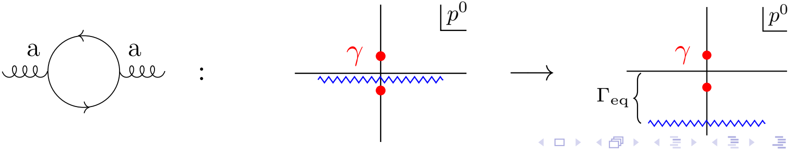
Instabilities

- $G_{rr} = G_{rr}^{\rm eq} + G_{rr}^{\rm inst}$,

$$G_{rr}^{\rm inst}(x^0, y^0) = \left(D\,\Pi_{aa} D^\dagger\right)\ \frac{1}{2\gamma}\left[e^{2\gamma T} - e^{\gamma|t|}\right]$$

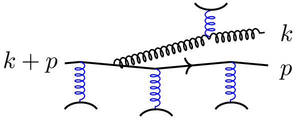
- No divergence in rate when $\gamma \to 0$.
- Puzzle of divergences solved through:
  - Following time evolution gives new term, $e^{2\gamma T}$.
  - Include decay width of emitters in Π_{aa}.





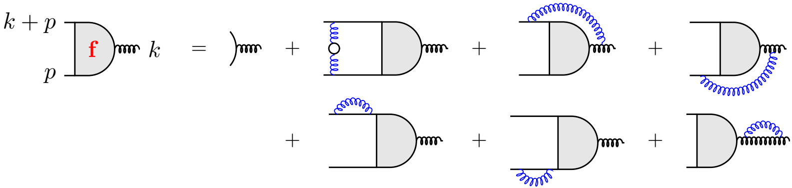
Jets in an unstable plasma

- Send a jet particle through unstable plasma.
- It radiates gluons collinearly with rate



$$\frac{dR}{d^3k} \sim \int_{\mathbf{h}_\perp} \mathbf{h}_\perp \cdot \mathrm{Re}\, \mathbf{f}$$

- Go beyond AMY to include instabilities



Jets in an unstable plasma

- Equation describing broadening: $(\delta E = E_p + E_k - E_{p+k})$

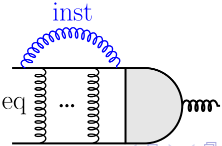
$$2\mathbf{h} = i\delta E\, \mathbf{f}(\mathbf{h}) + \int_{\mathbf{q}_\perp} \mathcal{C}_{\rm eq}(\mathbf{q}_\perp)\left[\mathbf{f}(\mathbf{h}) - \mathbf{f}(\mathbf{h} - k\mathbf{q}_\perp)\right] + \ldots$$

$$+ \frac{i}{\delta E + 2i\Gamma_{\rm eq}}\, \mathbf{F}\cdot\nabla_{\mathbf{h}}\, \mathbf{f}(\mathbf{h}) + \ldots + \mathcal{O}\left(\partial_T \mathbf{f}\right)$$

- Instabilities appear as a force term

$$\mathbf{F} \sim g^2\, \hat{K}_\mu \hat{K}_\nu \int d^3q\, \left(D\,\Pi_{aa} D^\dagger\right)^{\mu\nu} \gamma^{-1}\left[e^{2\gamma T} - 1\right] \mathbf{q}_\perp$$

- Particles deflected in plasma.
- To go beyond leading log need



Conclusion

- Weibel instabilities could have big impact on probes of QGP.
- Have a general description of unstable plasma using non-equilibrium QFT.
  - No divergences in rate.
- Jet particles broaden in strong chromomagnetic field.
- Further developments:
  - Derive full leading order.
  - Solve equations describing LPM radiation.
  - Implications for phenomenology.

Frequency domain

$$g(t) = \int d\omega\; e^{-i\omega t}\; g(\omega) \quad \Leftrightarrow \quad g(\omega) = \int_b^a dt\; e^{i\omega t}\; g(t)$$

For $a < t < b$

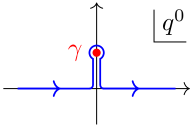
$$\int d\omega\; e^{-i\omega t} \int_b^a dt'\; e^{i\omega t'}\; g(t') = g(t)$$

$$\int_b^a dt\; e^{i\omega t} \int d\omega'\; e^{-i\omega' t}\; g(\omega') = \int d\omega' \frac{1}{i(\omega-\omega')} \left[e^{i(\omega-\omega')b} - e^{i(\omega-\omega')a} \right] g(\omega') = g(\omega)$$

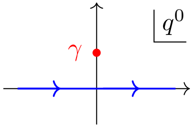
- Use that
  - $g(\omega)$ has no poles
  - No pole when $\omega' \to \omega$ so can take principal value.
  - $g(\omega) \to e^{-a \operatorname{Im} \omega}$ when $\operatorname{Im} \omega \to \infty$
  - $g(\omega) \to e^{-b \operatorname{Im} \omega}$ when $\operatorname{Im} \omega \to -\infty$

Different form of functions in frequency domain

$$G_{\rm ret}(P) = G_{\rm ret}^{\rm reg}(P) + \frac{B}{p^0 - i\gamma}$$



$$G_{\rm ret}(P) = G_{\rm ret}^{\rm reg}(P) + \frac{B}{p^0 - i\gamma}\left(1 - e^{(ip^0+\gamma)T}\right)$$



Earlier suggestions for Weibel instabilities

- Weibel instabilities in anisotropic plasma not a problem for some calculations:
  - Heavy quark energy loss [Romatschke, Strickland 2005]

$$\left(\frac{dW}{dt}\right)_{\text{soft}} = \text{Re} \int d^3\mathbf{x}\ \mathbf{J}_{\text{ext}} \cdot \mathbf{E}_{\text{ind}}$$

  - Only dependence on G_{ret}, not G_{rr}.
- Some suggestions:
  - Could be cured by some higher order corrections in g: $\lim_{\omega\to 0} \Pi \sim ag^3$ [Romatschke 2006]
    - Full calculation in real-time formalism would give divergent contribution.
  - Subtract instability poles [Carrington, Rebhan 2008, 2009]
    - Adequate at early times but misses strong classical fields at later times.