Jet fragmentation in a QCD medium: Universal q/g ratio and wave turbulence

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## Jets/High-p<sub>T</sub> probes

Description of hard probes in HIC involves at least three different processes



Will follow focus on the evolution of the hard probe inside the medium (neglecting vacuum like radiation)

Develop (semi-) analytic insights into jet/medium interaction

New things to look at with Monte-Carlo generators for HIC (JEWEL, Martini, CoLBT, Hybrid...)

#### Jet-Medium interactions

Describe jet fragments as collection of highly energetic on-shell quarks/gluons (E>>T) propagating in a thermal QGP

=> Elastic & Inelastic interactions with the medium can modify properties



=> Evolution is dominated by radiative branching up to scales E~T

(c.f. Arnold, Moore, Yaffe (LO); Ghiglieri, Moore, Teaney (NLO)

### Medium induced radiation

Splitting of the hard on-shell parton in a (thermal) medium is induced by elastic interactions with the medium



 $t_{form} \sim \sqrt{\frac{z(1-z)E}{\hat{a}}}$ 

Since scatterings with small momentum transfer occur frequently inside the medium it is important to consider multiple scatterings and interference effects

=> Coherence effects lead to suppression (LPM) of radiative emission

Emission rate is controlled by formation time

$$\Gamma_{\rm fi}^{\rm split}(E, zE, (1-z)E) = \sqrt{\frac{\alpha^2 \hat{\bar{q}}}{\pi^2 E}} K_{\rm fi}(z)$$

Will consider leading log E/T where splitting kernels  $K_{\rm fi}$  are

$$\mathcal{K}_{\rm gg}(z) = \frac{1}{2} 2C_A \frac{[1-z(1-z)]^2}{z(1-z)} \sqrt{\frac{(1-z)C_A + z^2C_A}{z(1-z)}} \qquad \mathcal{K}_{\rm qg}(z) = \frac{1}{2} 2N_f T_R \Big( z^2 + (1-z)^2 \Big) \sqrt{\frac{C_F - z(1-z)C_A}{z(1-z)}}$$

(c.f. Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov; Wiedemann; Arnold, Moore, Yaffe)

### Multiple branchings

Since emissions can occur anywhere inside the medium, need to consider successive branchings



Will keep track of distribution of fragments in terms of in-medium fragmentation function

$$D_i(x,\tau) \equiv x \frac{\mathrm{d}N_i}{\mathrm{d}x}$$

which measures distribution of i=q,g fragments after some evolution in the medium

Subsequent splittings are independent of each other and quasi-instantaneous ( $t_{form} << L$ )

=> Effective kinetic equation for in-medium FF

#### In-Medium fragmentation

Decomposing into flavor singlet/ non-singlet:

$$\begin{split} D_{\rm S} &\equiv \sum_{i=1}^{N_f} (D_{{\rm q}_i} + D_{\bar{{\rm q}}_i}) & \text{Defining scaled} \\ & \text{time variable:} & \tau = \sqrt{\frac{\alpha^2 \hat{\bar{q}}}{\pi^2 E}} t \\ D_{\rm NS}^{(i)} &\equiv D_{{\rm q}_i} - D_{\bar{{\rm q}}_i}, \end{split}$$

Kinetic equations for in-medium fragmentation function:

Ma

$$\begin{aligned} \frac{\partial}{\partial \tau} D_{\rm g}\left(x,\tau\right) &= \int_{0}^{1} dz \,\mathcal{K}_{\rm gg}(z) \left[ \sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_{\rm g}(x) \right] - \int_{0}^{1} dz \,\mathcal{K}_{\rm qg}(z) \frac{z}{\sqrt{x}} \,D_{\rm g}\left(x\right) \\ &+ \int_{0}^{1} dz \mathcal{K}_{\rm gq}(z) \sqrt{\frac{z}{x}} \,D_{\rm S}\left(\frac{x}{z}\right), \end{aligned}$$
$$\begin{aligned} \frac{\partial}{\partial \tau} D_{\rm S}\left(x,\tau\right) &= \int_{0}^{1} dz \,\mathcal{K}_{\rm qq}(z) \left[ \sqrt{\frac{z}{x}} D_{\rm S}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{\rm S}(x) \right] + \int_{0}^{1} dz \,\mathcal{K}_{\rm qg}(z) \sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right) \\ \frac{\partial}{\partial \tau} D_{\rm NS}^{(i)}\left(x,\tau\right) &= \int_{0}^{1} dz \,\mathcal{K}_{\rm qq}(z) \left[ \sqrt{\frac{z}{x}} D_{\rm NS}^{(i)}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{\rm NS}^{(i)}(x) \right] \end{aligned}$$

where  $\sqrt{z/x}$  and  $1/\sqrt{x}$  factors follow from shorter formation time for successive branchings (c.f. Arnold, Moore, Yaffe)

### In-Medium fragmentation

Solution to leading order in  $\tau$ , i.e. for a single splitting:



However the range of validity of single splitting approximation also limited in x:

small x: Emission rates at small x enhanced by  $1/\sqrt{x}$  due to shorter formation time Consider e.g. probability for extra g->qq splitting  $\mathcal{P}_{qg}^{split} = \frac{\tau}{\sqrt{x}} \int_0^1 dz \, \mathcal{K}_{qg}(z) \overset{N_f=3}{\simeq} 3.54 \frac{\tau}{\sqrt{x}}$ 

=>Dynamically generated scale  $x_c \sim \tau^2$  below which probability for subsequent splitting is O(1) for any  $\tau$ >0

### In-Medium fragmentation

Numerical solution of coupled evolution equations for gluon jet ( $\tau < <1$ )



Emergence of dynamically generated scale  $x_{C}$  clearly visible in numerical solution of evolution equations

#### Stationary solution

Since splitting rates become of O(1) for x < x<sub>c</sub>, expect solution to become insensitive to initial conditions and instead approach fixed point of kinetic equation

Stationary non-equilibrium solution:

$$D_{\rm g}(x) = rac{G}{\sqrt{x}}, \quad D_{\rm S}(x) = rac{2N_f Q}{\sqrt{x}}$$

$$\begin{aligned} \frac{\partial}{\partial \tau} D_{\mathrm{g}}\left(x,\tau\right) &= \int_{0}^{1} dz \, \mathcal{K}_{\mathrm{gg}}(z) \left[ \sqrt{\frac{z}{x}} D_{\mathrm{g}}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_{\mathrm{g}}(x) \right] - \int_{0}^{1} \mathrm{d}z \, K_{\mathrm{qg}}(z) \frac{z}{\sqrt{x}} \, D_{\mathrm{g}}\left(x\right) \\ &+ \int_{0}^{1} \mathrm{d}z K_{\mathrm{gq}}(z) \sqrt{\frac{z}{x}} \, D_{\mathrm{S}}\left(\frac{x}{z}\right), \end{aligned}$$

Chemistry of fragments fixed by balance of g->qq and q->gq processes

$$\frac{Q}{G} = \frac{\int_0^1 dz \ z \ \mathcal{K}_{qg}(z)}{2N_f \int_0^1 dz \ z \ \mathcal{K}_{gq}(z)} \approx 0.07$$

Existence of solution does not rely on detailed form of K(z) but only on the fact that emission rates behave as  $1/\sqrt{E}$  due to formation time

(c.f. Blaizot, Mehtar-Tani; Blaizot, Mehtar-Tani, lancu)

### Energy cascade

Solution is analogous to Kolmogorov-Zhakarov spectrum in weak wave turbulence

Even though the solution is stationary

 $\partial_{\tau} D_{\mathrm{g}}(x) = \partial_{\tau} D_{\mathrm{S}}(x) = 0$ 

it is associated with a finite scale invariant energy flux

$$\dot{\epsilon}(x_0) = \int_{x_0}^{\infty} dx \, \left[\partial_{\tau} D_{\mathrm{g}}(x) + \partial_{\tau} D_S(x)\right]$$



with flux constants

$$\gamma_{\rm g} = \int_0^1 \mathrm{d}z z \left( \mathcal{K}_{\rm gg}(z) + \mathcal{K}_{\rm qg}(z) \right) \log \left(\frac{1}{z}\right) \approx 25.78 + 0.177 \, N_f \, .$$
$$\gamma_{\rm q} = 2N_f \int_0^1 \mathrm{d}z z \left( K_{\rm qq}(z) + K_{\rm gq}(z) \right) \log \left(\frac{1}{z}\right) \approx 23.19 N_f$$





#### Energy cascade

Scale invariant energy flux associated with energy transfer from  $x \sim 1$  to  $x \sim T/E$  where it is absorbed by medium



-> analogous to Richardson cascade in wave-turbulence

Energy loss rate is dominated by g->gg. Contributions from q->qg and g->qq to energy loss give 16% (1.6%) corrections for  $N_f=3$ 

(c.f. Blaizot, Mehtar-Tani; Blaizot, Mehtar-Tani, lancu)



### In-Medium fragmentation of g-jet

Numerical solution of coupled evolution equations for gluon jet



Enhanced splitting rates at small x lead to approach a nonequilibrium steady state, characterized by  $D(x) \sim 1/\sqrt{x}$  behavior of gluon & quark distribution at small x

### In-Medium fragmentation of g-jet

Numerical solution of coupled evolution equations for gluon jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

#### In-medium jet chemistry

Balance of the g->qq and q->qg processes at the non-equilibrium steady state (x < <1) uniquely determines chemistry



Universal Kolmogorov ratio realized approximately over a substantial range of momentum fractions x and evolution times  $\tau$ 

### Energy loss

Single emission off the original hard parton create a  $1/\sqrt{x}$  gluon spectrum at small x with amplitude

 $G \simeq \tau C_A^{1/2} C_R$ 

Multiple emissions off soft fragments create turbulent energy flux

 $\dot{\epsilon} = -\gamma_g G$ 

all the way to T/E where energy is absorbed by the thermal QGP

Energy loss rate at early times given by



$$\frac{1}{E} \frac{dE}{d\tau} \bigg|_{\tau \ll 1} \approx -\gamma_g C_A^{1/2} C_R \tau$$

(c.f Baier, Mueller, Schiff, Son; Blaizot, Iancu, Mehtar-Tani)

#### Energy loss of quark & gluon jets

Energy loss at early times follows expected behavior from turbulence analysis



Breakdown of Casimir scaling of energy loss beyond very early times as chemistry of fragments is strongly modified

#### Jet chemistry



#### medium filtering:

Since large x gluons loose energy faster than large x quarks, the large x distribution at late times is always dominated by quark d.o.f.

#### <u>quark jet:</u>

Energy always dominated by valence quarks

#### gluon jet:

Energy dominated by sea quarks once jet has lost ~80% of total energy

#### Phenomenological implications

So far connection to experiment is loose, but assuming factorization

 $d\sigma_{\gamma,h/jet} \sim f_a \times f_b \times H_{ab \to \gamma c} \times D_{c \to d}^{\text{Medium}} \times D_{d->h/jet}^{\text{Vacuum}}$ 

can speculate on phenomenological consequences of our findings

Universal q/g ratio for small x fragments & medium filtering

=> strangeness enhancement at small x (respectively large x)

Could be observable by looking at

- ratios of identified particles (e.g. K/ $\pi$  or  $\Lambda/\pi$ ) inside jets
- ratios of identified particles (e.g. K/π or Λ/π) in backward hemisphere of high-p<sub>T</sub> trigger particle

in A+A collisions relative to p+p reference

Ideally one would have an estimate of energy loss calibrate  $(x,\tau)$  e.g. by photon tagging

Detailed predictions will require to combine medium evolution with initial production and hadronization

### Conclusion & Outlook

In-Medium fragmentation of jets is governed by turbulent cascade, associated with scale independent energy flux from energy scales of jet (p~E) all the way to the energy scale of the medium (p~T)

=> allows for analytic predictions of interesting features

$$\frac{D_S(x)}{D_G(x)} = \frac{\int_0^1 dz \ z \ K_{\rm qg}(z)}{\int_0^1 dz \ z \ K_{\rm gq}(z)} \approx 0.07 \times 2N_f$$



Interesting phenomenological consequences for jet chemistry => could be interesting to extend analysis to heavy (c,b) flavors

Ultimately, would like to extend study to include all processes relevant at lower scales  $x \sim T/E$  to obtain a more complete picture of jet fragmentation in medium & address chemical equilibration of the QGP at early times

# Backup

#### Energy cascade

Naive scaling solution requires  $D(x) \sim 1/\sqrt{x}$  for all  $x (0,\infty)$  to satisfy stationarity condition

-> necessary to provide infinite energy reservoir to realize stationarity

However even if we limit the support of the distribution to the physical range (0,1), the energy flux becomes scale invariant for x<<1

 $\dot{\epsilon}(x_0 \ll 1) \simeq -\gamma_g G - \gamma_q Q$ 

=> locality of interactions implies that scaling solution can be realized within an inertial range of momenta T/E << x << 1 of free-cascade



#### Driven turbulence



**Figure 5**. Stationary solutions for the distributions  $D_{g/S/NS}(x)$  in a driven cascade. Solid curves correspond to quark forcing ( $F_g = 0$ ,  $F_S = 5$ ,  $F_{NS} = 5$ ) whereas dashed curves show the results for gluon forcing ( $F_g = 5$ ,  $F_S = 0$ ,  $F_{NS} = 0$ ). Horizontal gray lines show the KZ solution, which is realized to high accruacy in an inertial range of energy fractions  $x \leq 0.02$ .

Splitting probability divergent for soft splittings min(z, 1-z) << 1

$$\mathcal{P}_{\rm gg}^{\rm split}\Big(z > z_{\rm min}\Big) = \frac{\tau}{\sqrt{x}} \int_{z_{\rm min}}^{1-z_{\rm min}} \mathrm{d}z \ \mathcal{K}_{\rm gg}(z) \ \simeq \frac{4C_A^{3/2}}{\sqrt{z_{\rm min}}} \frac{\tau}{\sqrt{x}}$$

Soft splittings do not appreciable change the momentum of the emitter, cancellations occur between gain and loss terms regulate divergence

soft splittings:  

$$\frac{\partial}{\partial \tau} D_{g}(x,\tau) \Big|_{z_{\min} \ll 1} = \frac{C_{A}^{3/2}}{\sqrt{x}} \sqrt{z_{\min}} \left[ 2x \frac{\partial}{\partial x} D_{g}(x) - D_{g}(x) \right]$$

quasi-democratic splittings: (z~1/2)

$$\mathcal{P}_{gg}^{\text{split}}\left(z > z_{\min}\right)\Big|_{z_{\min} \sim 1/2} \simeq \mathcal{K}_{gg}(1/2)(1 - 2z_{\min})\frac{\tau}{\sqrt{x}},$$

=> Quasi-democratic ( $z \sim 1/2$ ) splittings dominate

=>Dynamically generated scale  $x_c \sim \tau^2$  parametrically the same for all splitting processes

### In-Medium fragmentation of q-jet

#### Numerical solution of coupled evolution equations for quark jet



Enhanced splitting rates at small x lead to approach a nonequilibrium steady state, characterized by  $D(x) \sim 1/\sqrt{x}$  behavior of gluon & quark distribution at small x

#### In-Medium fragmentation of -qjet

#### Numerical solution of coupled evolution equations for quark jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

#### Valence flavor cascade

Quark jets also carry a conserved valence flavor (u,d,s) which gives rise to a non-vanishing NS distribution



Energy cascade: gluon + flavor singlet quark channel

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D_{g/S}(x) \sim 1/\sqrt{x}
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Valence particle number cascade:

flavor non-singlet quark channel

 $D_{NS}(x) \sim \sqrt{x}$