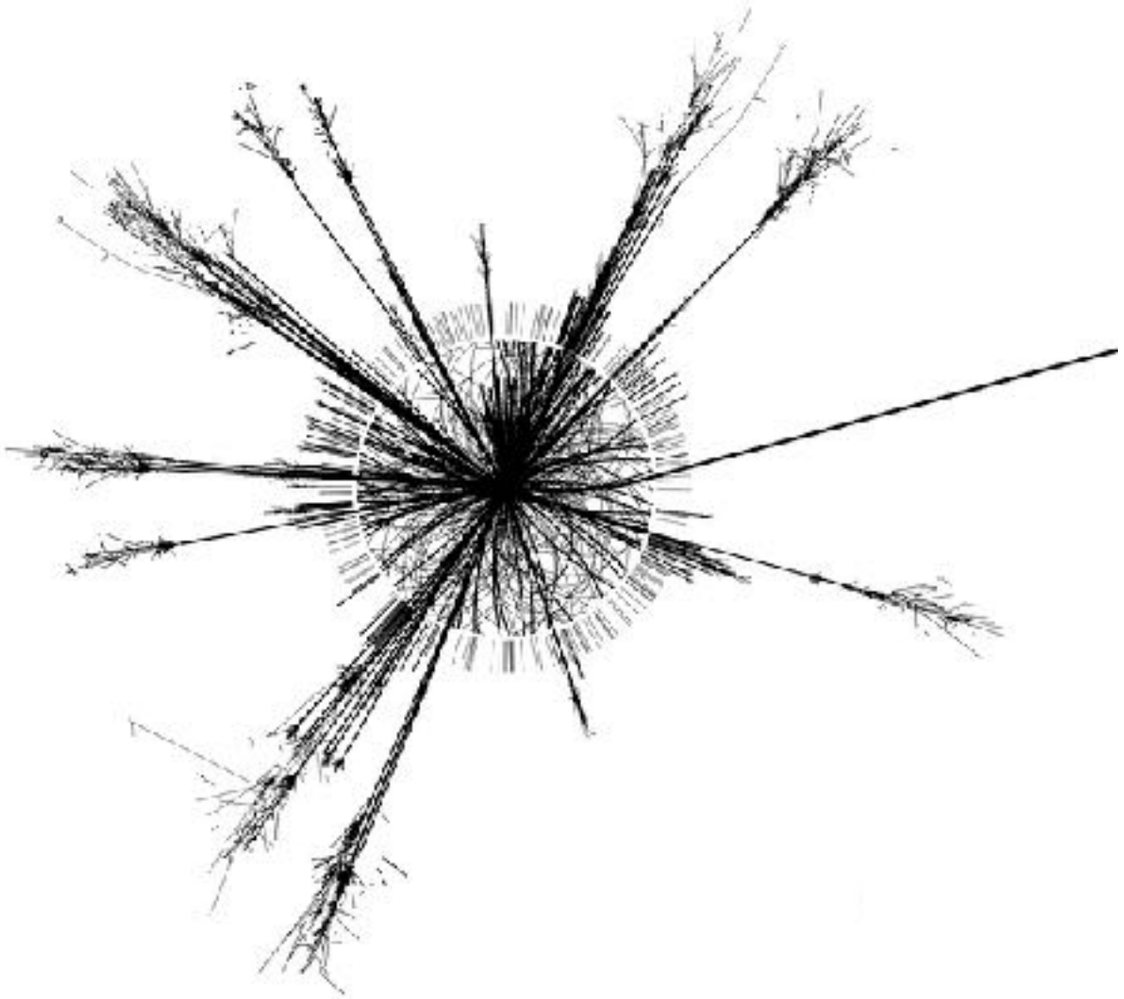
Hard probes 2018



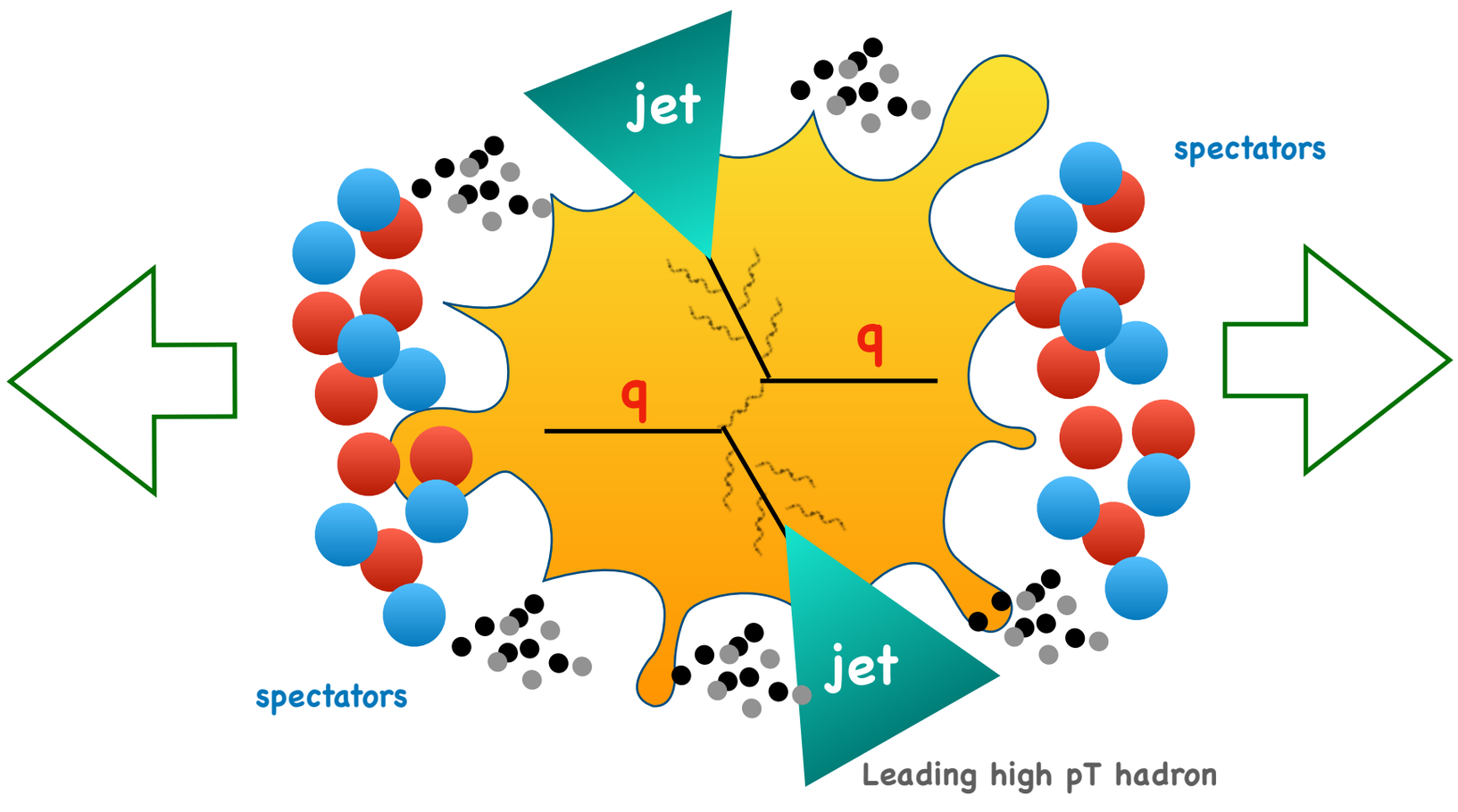
# Dynamical quenching weights in expanding medium

Souvik Priyam Adhya, Charles University, Prague
[in collaboration with Konrad Tywoniuk and Carlos Salgado]

# Jet formation

- High energy partons, resulting from an initial hard scattering, will create a high energy collimated spray of particles →JETS.

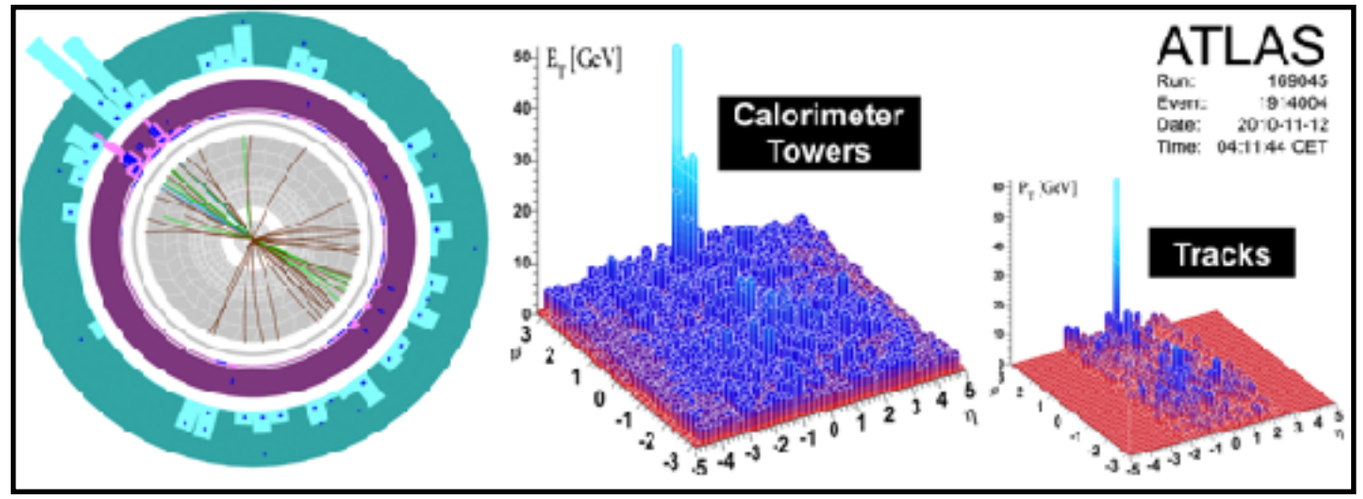
- Partons traveling through a dense color medium are expected to lose energy via medium induced gluon radiation, "jet quenching", and the magnitude of the energy loss depends on the gluon density of the medium.

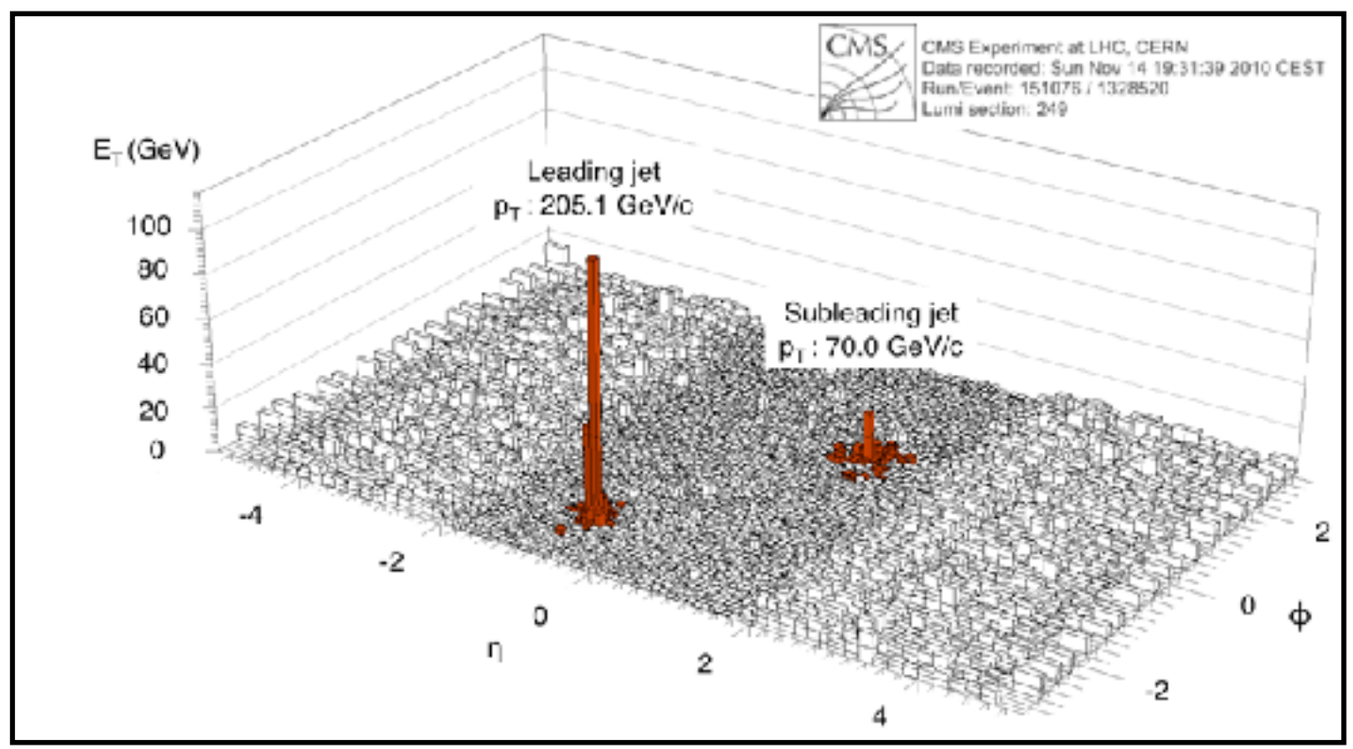


# Experimental signatures from LHC

Questions we need to ask ?

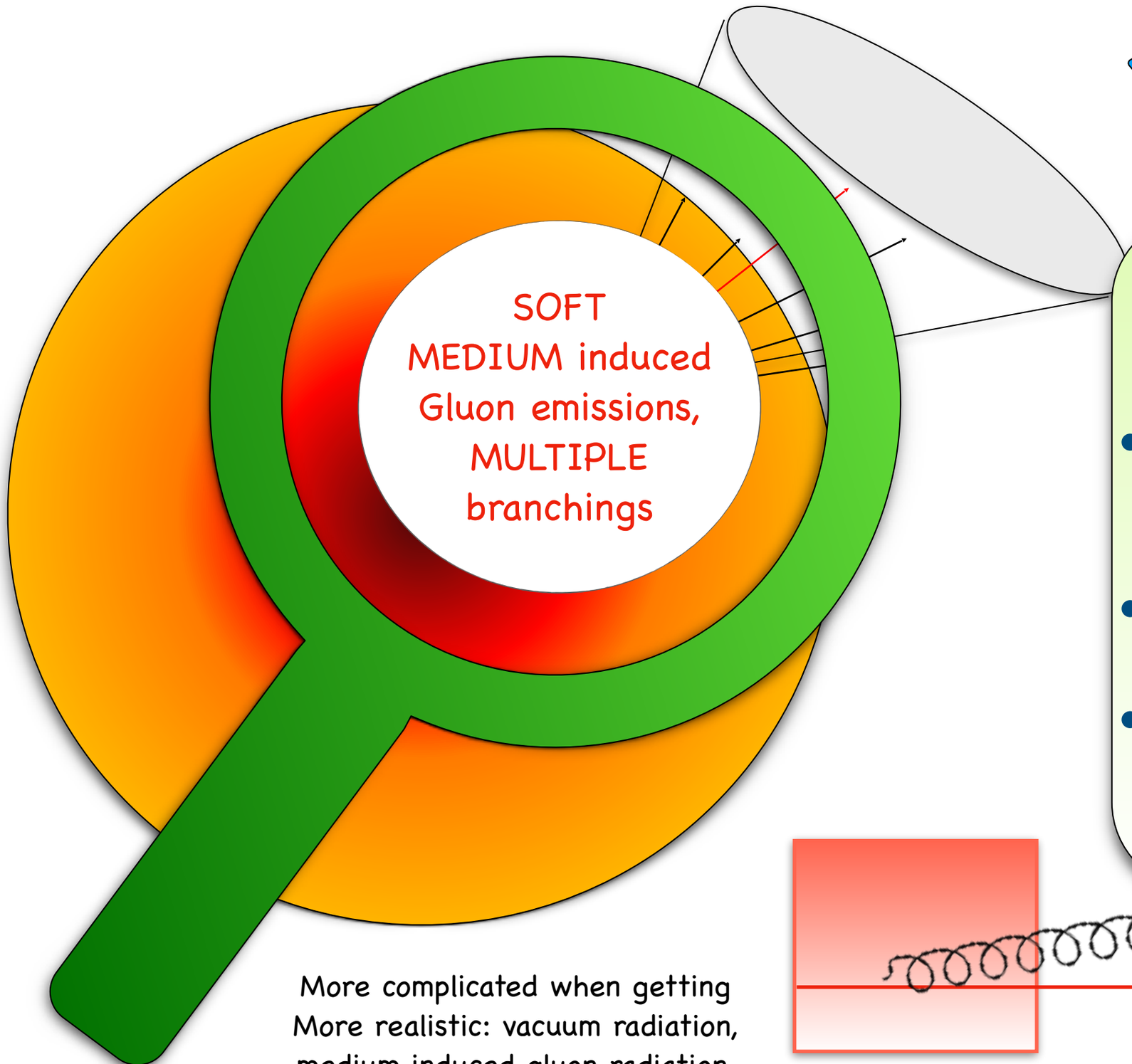
- Di-jet asymmetry; a strong imbalance in energy between 2 back to back jets.
- Missing energy..
- Linked to transportation of a majority of jet energy by soft particles towards large angles.
- LHC data calls for comprehensive analysis of Jet shape for which effects of MULTIPLE BRANCHINGS AT LARGE ANGLES are important.
- MC event generators: JEWEL, Q-PYTHIA, MARTINI (realistic modelling)...missing something?

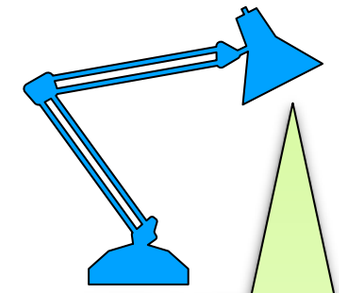



Reference: G. Aad et. al. (ATLAS collaboration), Phys. Rev. Lett. 105, 252303 (2010).
G. Aad et. al. (ATLAS collaboration), Phys. Lett. B 774, 379 (2017)
S. Chattrachyan et. al. (CMS collaboration), Phys. Rev. C 84, 024906 (2011).
S. Chattrachyan et. al. (CMS collaboration), Phys. Lett. B 712, 176 (2012).

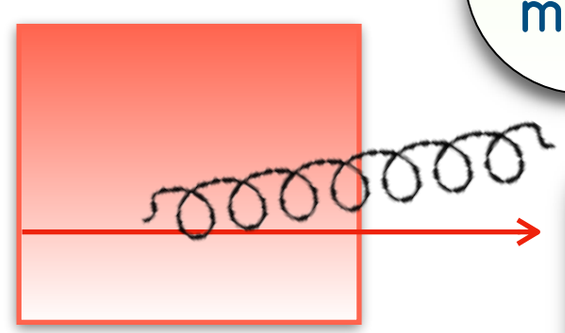
# Let us frame the problem ...



- Controls energy transport at large angles w.r.t leading partons.
- Offspring gluons carry away sizable amount of energy of parent gluons.
- Efficient way of energy transfer to medium.
- Repeated application of single gluon emission kernel to mimic multiple gluon emission

More complicated when getting
More realistic: vacuum radiation, medium induced gluon radiation, expanding medium..



Q) Is parton splitting in vacuum same as in medium (expanding) ?

# Medium induced gluon radiation

Inclusive energy distribution of an in-medium produced parton :

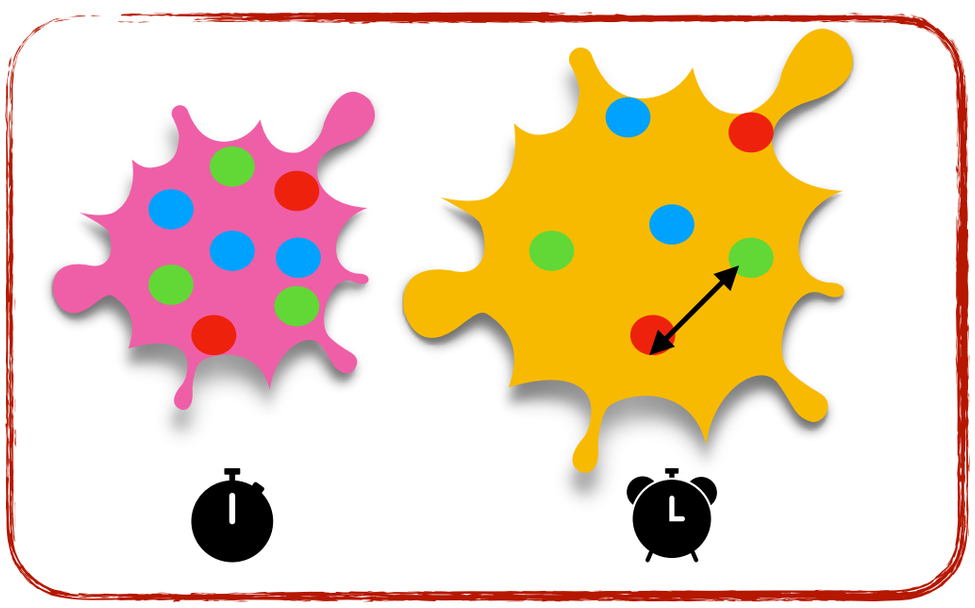
$\chi \to \infty$ **or** $R \to \infty$ **"unconstrained"**
$\chi \to 1$ **"constrained" kinematics**

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2\mathrm{Re} \int_{\xi_0}^{\infty} dy_l \int_{y_l}^{\infty} d\bar{y}_l \int d\mathbf{u} \int_0^{\chi\omega} d\mathbf{k}_\perp \, e^{-i\mathbf{k}_\perp \cdot \mathbf{u}} \, e^{-\frac{1}{2}\int_{\bar{y}_l}^{\infty} d\xi \, n(\xi)\, \sigma(\mathbf{u})} \frac{\partial}{\partial \mathbf{y}} \cdot$$
$$\frac{\partial}{\partial \mathbf{u}} \int_{\mathbf{y}=0}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r} \exp\left[ i \int_{y_l}^{\bar{y}_l} d\xi \frac{\omega}{2} \left( \dot{\mathbf{r}}^2 - \frac{n(\xi)\sigma(\mathbf{r})}{i\,\omega} \right) \right]$$

Reference: U. A. Weidemann; Nucl. Phys. A 690, 731 (2001)

$n(\xi)\sigma(r) \sim (1/2)\hat{q}(\xi) r^2$

$$\hat{q}(\xi) = \hat{q}_0 \left(\frac{\xi_0}{\xi}\right)^{\alpha} \qquad \bar{\hat{q}} = \frac{2}{L^2} \int_{\xi_0}^{L+\xi_0} d\xi (\xi - \xi_0) \hat{q}(\xi)$$




**Multiple soft scattering approximation**

**Dipole approximation**
**For an expanding medium**
**BDMPS-Z**

# Introduction of expanding medium

- The path integral in the dipole approximation for an expanding medium can be written as (identical as 2D harmonic oscillator with time dependent imaginary frequency):

$$\mathcal{K}(\mathbf{r}_1, y_1; \mathbf{r}_2, y_2|\omega) = \int_{\mathbf{y}=0}^{\mathbf{u}} \mathcal{D}\mathbf{r} \exp\left[i\frac{\omega}{2}\int_{y_1}^{y_2} d\xi \left(\dot{\mathbf{r}}^2 - \frac{\Omega_\alpha^2(\xi_0)}{\xi^\alpha}\mathbf{r}^2\right)\right]$$

- Time dependent transport co-efficient :

$$\frac{\Omega_\alpha^2(\xi_0)}{\xi^\alpha} = \frac{\hat{q}(\xi)}{i\,2\,\omega} = -i\frac{\hat{q}_0}{2\omega}\left(\frac{\xi_0}{\xi}\right)^\alpha$$

$\alpha$ =0 (static)
$\alpha$ > 0 (expanding)
**$\nu$=1/(2- $\alpha$)**

- Medium undergoes cooling from $T_0$ to T while parton propagates in space time. Hydrodynamical model; Bjorken expansion in assuming longitudinal expansion:

$$(T_0/T)^3=(\tau_0/\tau)^\alpha$$

$$\mathcal{K}(\mathbf{r}_1, y_1; \mathbf{r}_2, y_2|\omega) = \frac{i\,\omega}{2\pi D(y_1, y_2)} \exp\left[-\frac{-i\omega}{2\,D(y_1, y_2)}\left(c_1\mathbf{r}_1^2 + c_2\mathbf{r}2^2 - 2\mathbf{r}_1\cdot\mathbf{r}_2\right)\right]$$

- The path integral can be solved to obtain (where $D(\xi,\xi')$ is the fluctuation determinant and c1, c2 are dependent on $\xi$ and $\xi'$).



C.A. Salgado and U. A. Weidemann, Phys. Rev. D 68, 014008 (2003)

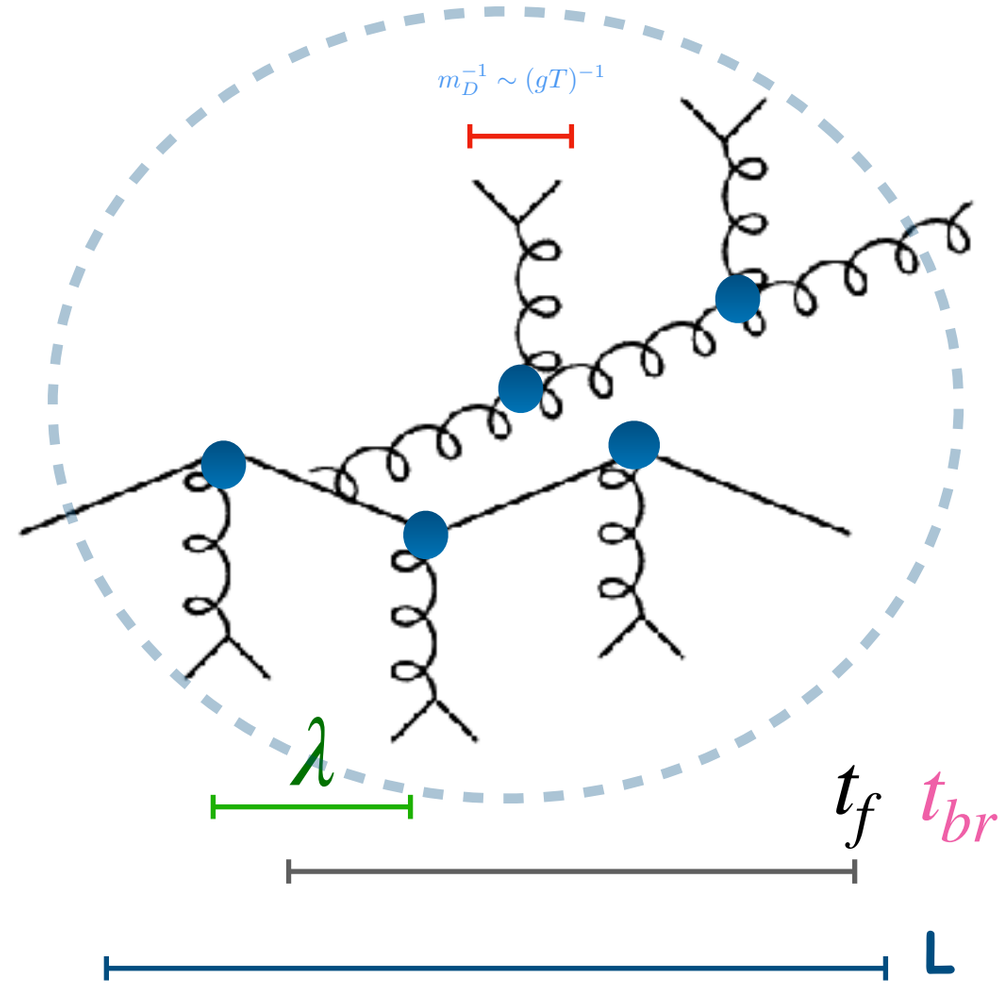
# In medium radiation: setting our scales

- Particles undergo diffusion in medium : Radiative process.
- Typical formation time of radiated gluon is ($t_f$).
- During this timescale, the gluon undergoes multiple kicks which causes an increase in its transverse momentum.
- The typical time for induced emission is the branching time ($t_{br}$) .

$$\left.\begin{aligned} \langle \boldsymbol{k}^2 \rangle &= \hat{q} t_{\rm f} \\ t_{\rm f} &\sim \frac{\omega}{\langle \boldsymbol{k}^2 \rangle} \end{aligned}\right\} \quad \begin{aligned} k_{\rm br}^2 &= \sqrt{\hat{q}\omega} \\ t_{\rm br} &= \sqrt{\frac{\omega}{\hat{q}}} \end{aligned}$$

Scattering points act coherently to radiate one gluon in limit

$$t_{br} >> \lambda; \qquad N_{coh} = \frac{t_{br}}{\lambda}$$




BDMPS-Z spectrum

$$\lim_{R\to\infty} \omega \frac{dI^{(med)}}{d\omega} = \frac{2\alpha_s C_R}{\pi} \ln \left| \cos\left[ (1+i)\sqrt{\frac{\omega_c}{2\omega}} \right] \right|$$

$$\lim_{R\to\infty} \omega \frac{dI}{d\omega} \simeq \frac{2\alpha_s C_R}{\pi} \sqrt{\frac{\omega_c}{2\,\omega}} \quad ; \quad \omega < \omega_c$$

$$\lim_{R\to\infty} \omega \frac{dI}{d\omega} \simeq \frac{2\alpha_s C_R}{\pi} \frac{1}{12} \left(\frac{\omega_c}{\omega}\right)^2 \quad ; \quad \omega > \omega_c$$

Energy loss of gluons

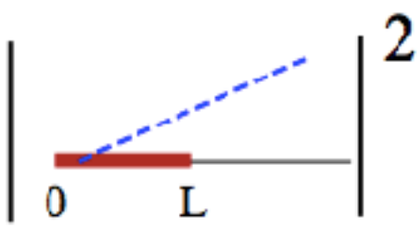
$$\Delta E = \int_0^\infty {\rm d}\omega\, \omega \frac{{\rm d}I}{{\rm d}\omega} = 2\bar{\alpha}\hat{q}L^2$$

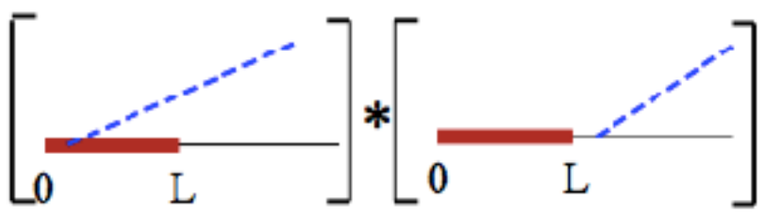
$$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

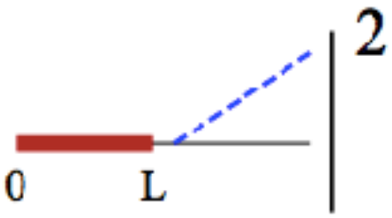
Gluon splitting rate is proportional to L

# Introduction of expanding medium

- Contributions to gluon radiation spectrum radiation :

$I_4 =$ 


$I_5 = 2\,\mathrm{Re}$ 

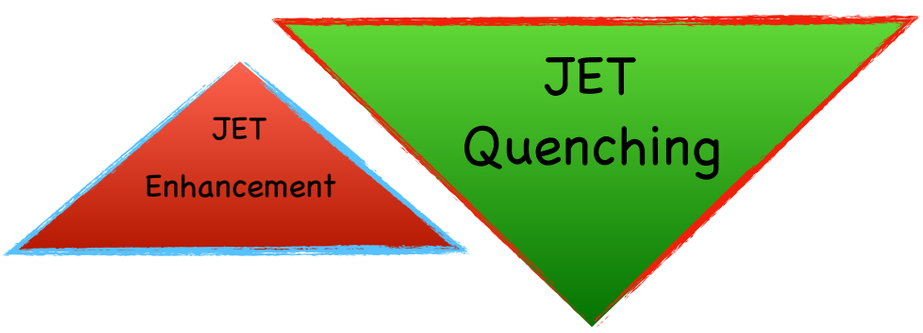

$I_6 =$ 


Direct production term for gluon emission inside medium

Destructive interference term between gluon emission in and beyond medium

Hard vacuum radiation term; IR divergent; needs to be subtracted

➡ Total radiation spectra is +ve; WHY?




$$\frac{dI^{(\mathrm{tot})}}{d\omega} = \frac{\alpha_s}{\pi^2} C_F([I_4 + I_5] + I_6) = \frac{dI^{(\mathrm{med})}}{d\omega} + \frac{dI^{(\mathrm{vac})}}{d\omega}$$

$I_6 = 1/k_\perp^2$

$I_4$, $I_5$ collinear safe

U. A. Weidemann; Nucl. Phys. A 690, 731 (2001)

# The limiting case of R->∞

- Now, we do need to calculate the forms of the integrals.

$$I_4 = \frac{1}{4\omega^2}\, 2\mathrm{Re} \int_{\xi_0}^{L+\xi_0} dy_l \int_{y_l}^{L+\xi_0} d\bar{y}_l \left( \frac{-4A_4^2 \bar{D}_4}{(\bar{D}_4 - iA_4B_4)^2} + \frac{iA_4^3 B_4\, \mathbf{k}_\perp^2}{(\bar{D}_4 - iA_4B_4)^3} \right) \exp\left[ -\frac{\mathbf{k}_\perp^2}{4\,(\bar{D}_4 - i\,A_4\,B_4)} \right]$$

$$I_5 = \frac{1}{\omega}\, \mathrm{Re} \int_{\xi_0}^{L+\xi_0} dy_l\, \frac{-i}{B_5^2}\, \exp\left[ -i \frac{\mathbf{k}_\perp^2}{4\,A_5\,B_5} \right]$$

C.A. Salgado and U. A. Weidemann, Phys. Rev. D 68, 014008 (2003)

- In the limiting case of R->∞, we obtain,

$$I_4 = -2\frac{\alpha_s C_F}{\pi} Re[\int_{\xi_0}^{L+\xi_0} \int_{y_l}^{L+\xi_0} \frac{1}{D(\bar{y}_l, y_l)^2} - \frac{1}{(\bar{y}_l - y_l)^2}] dy_l d\bar{y}_l$$

$$I_5 = -2\frac{\alpha_s C_F}{\pi} Re[\int_{\xi_0}^{L+\xi_0} \frac{-1}{D(\bar{y}_l, y_l) c_1(\bar{y}_l, y_l)} - \frac{1}{(L + \xi_0 - y_l)}] dy_l$$

The maximal value of the transport coefficient is reached at time of the highest density of the system which is the formation time ($\xi_0$)

where,

$$\omega_c = \frac{1}{2}\bar{\hat{q}} L^2$$

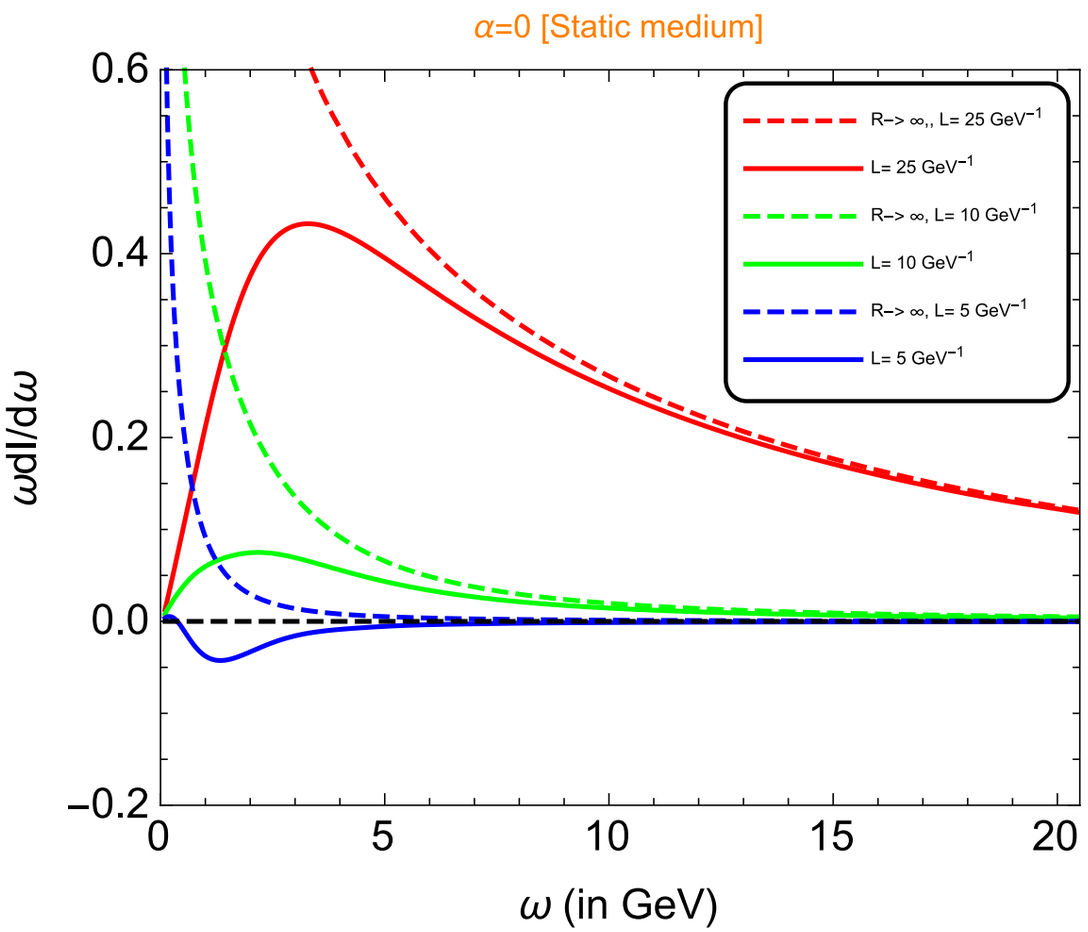
Transport coefficient

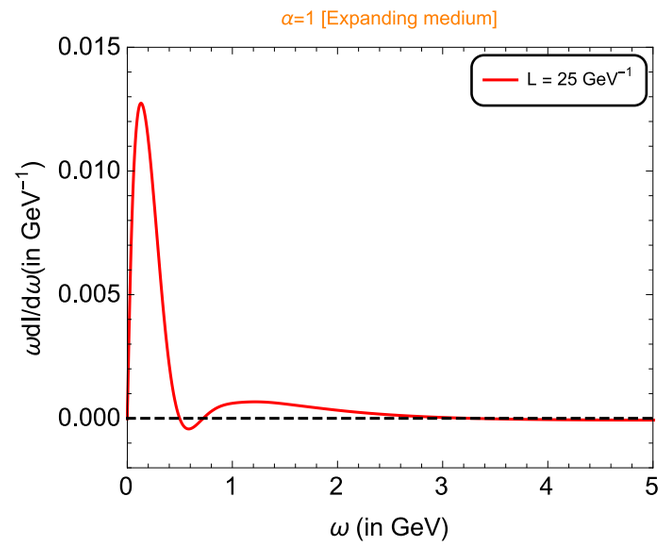
$$\hat{q}(\xi) = \hat{q}_0 (\frac{\xi_0}{\xi})^\alpha$$

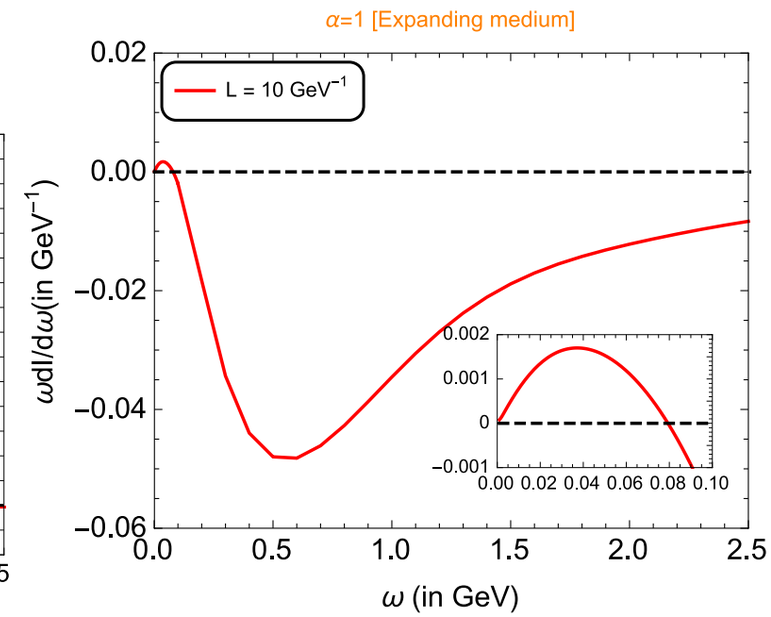
Equivalent static transport coefficient

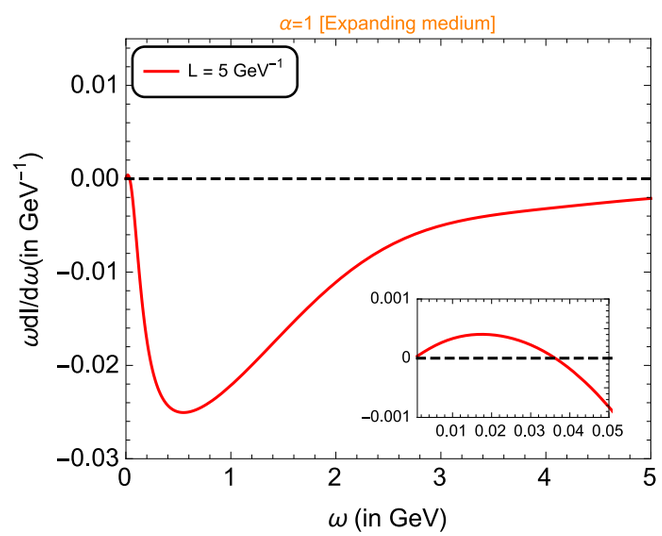
$$\bar{\hat{q}} = \frac{2}{L^2} \int_{\xi_0}^{L+\xi_0} d\xi (\xi - \xi_0) \hat{q}(\xi)$$

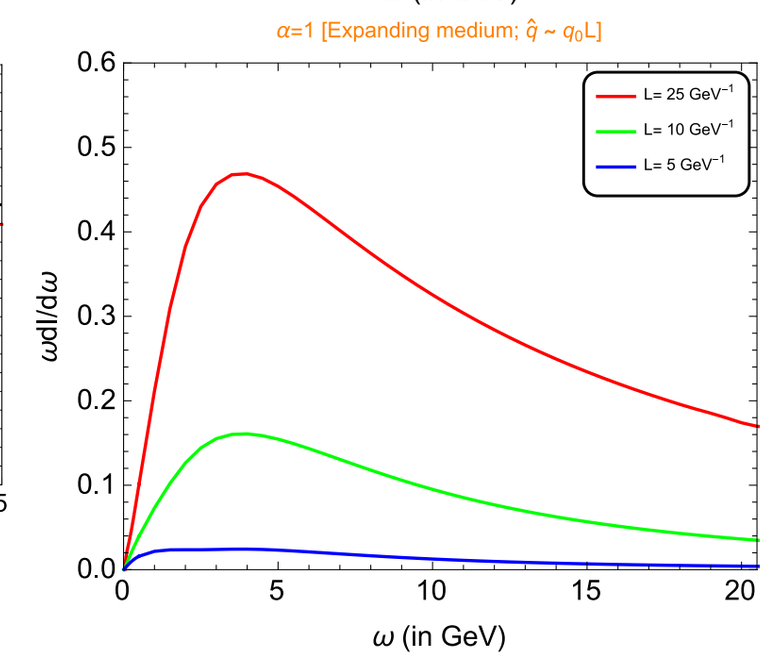
# Gluon spectra for static and expanding medium



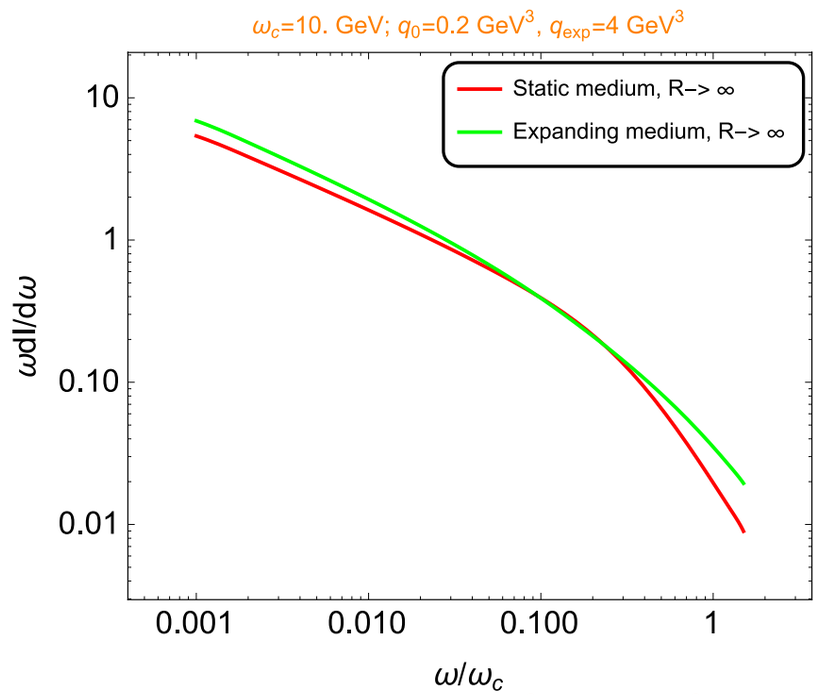

Reference: C.A. Salgado and U. A. Weidemann, Phys. Rev. D 68, 014008 (2003)

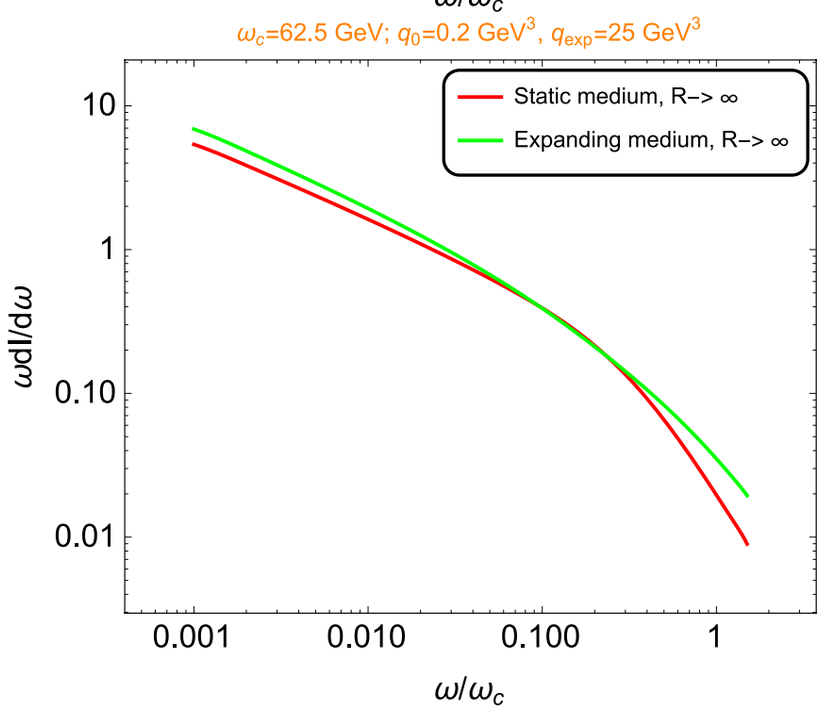
U. A. Weidemann; Nucl. Phys. A 690, 731 (2001)

When plotted with q^ as the expanding case (not scaled up), we observe the spectra to be negative for the expanding case at L = 1 and 2 fm
For scaled spectra, we observe the scaling behavior for expanding case

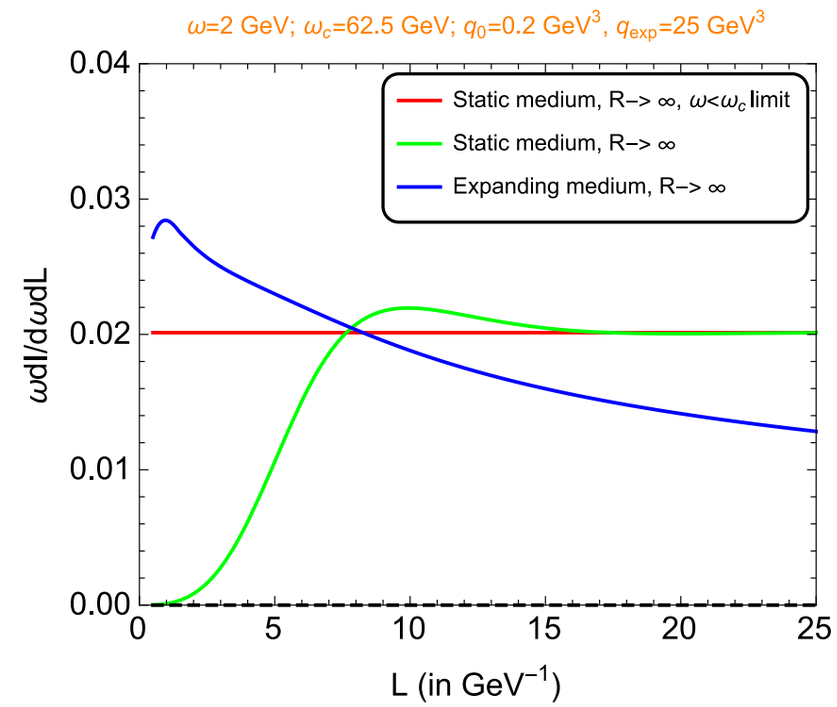
# Scaling behaviour of the spectrum; gluon splitting rate

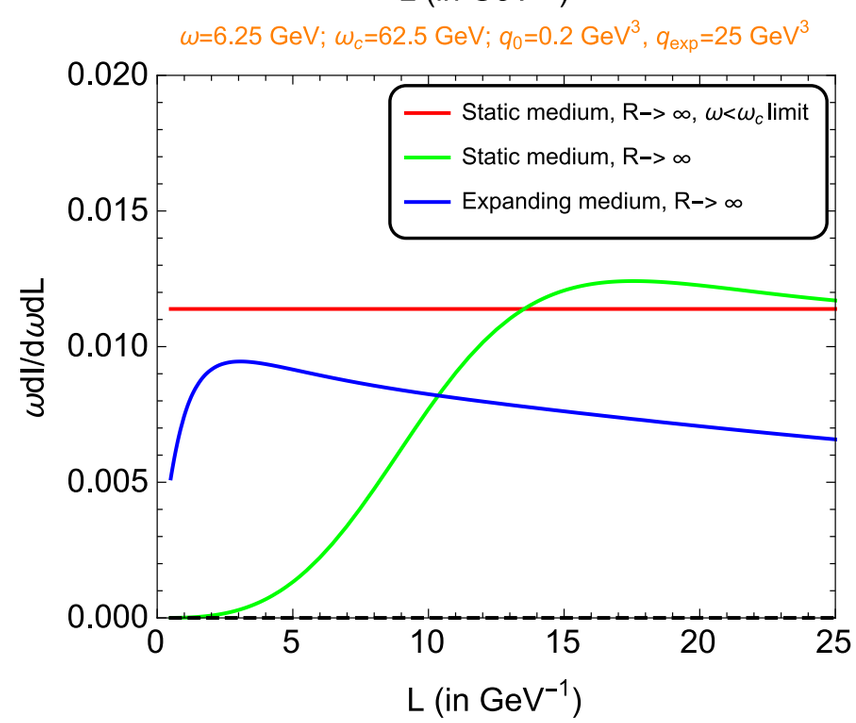
Spectra shows a scaling;
Just increase q^ from static to expanding
at same order of w_c



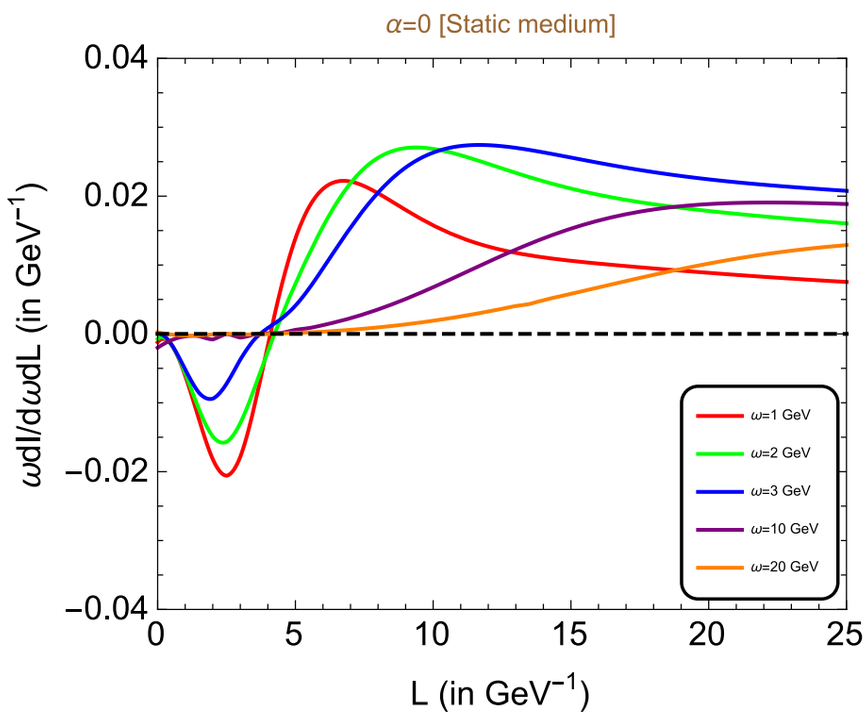



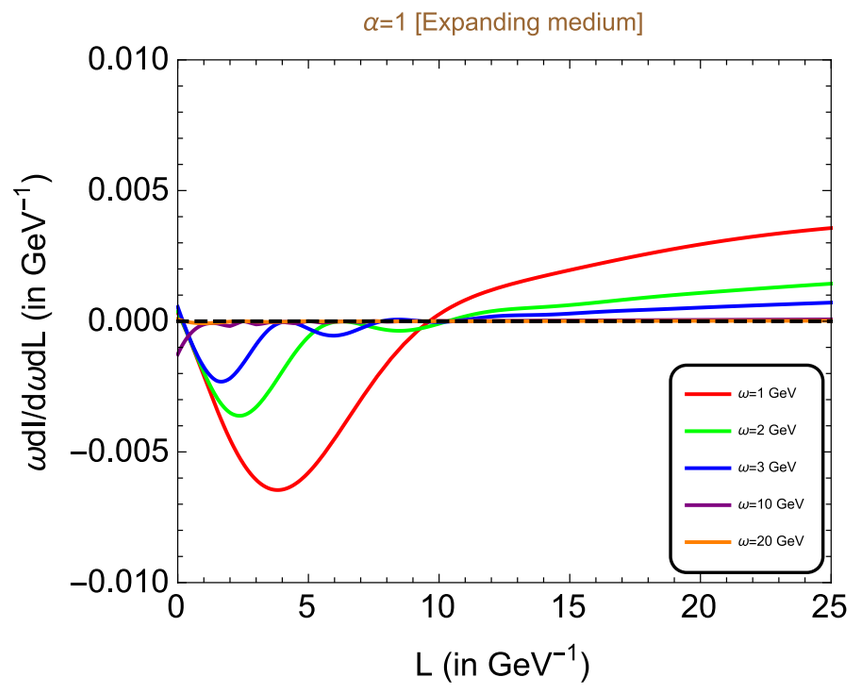

The splitting rate for R-> ∞ case



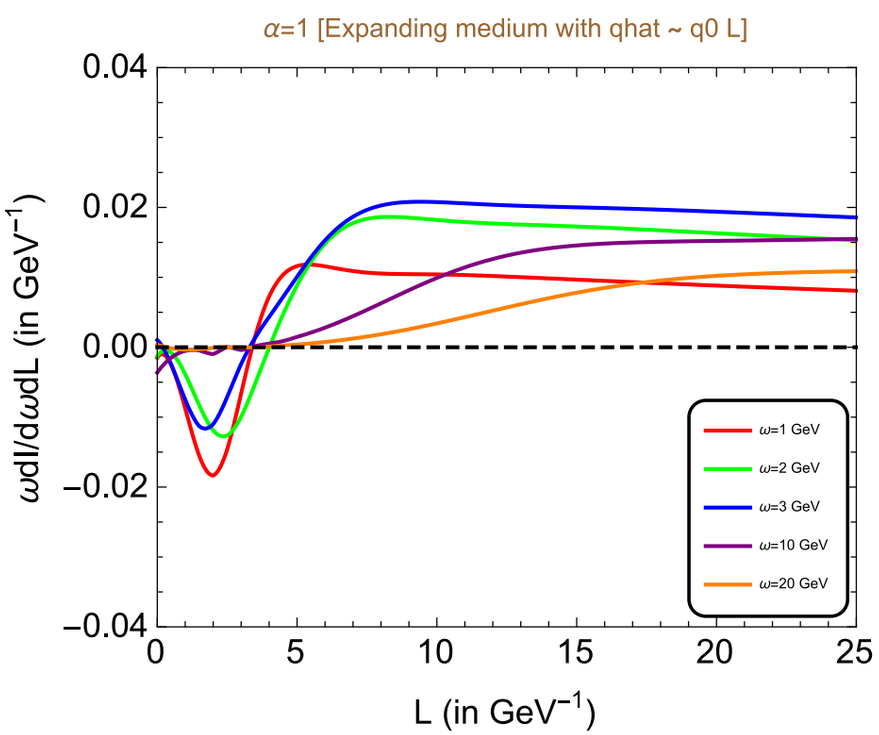



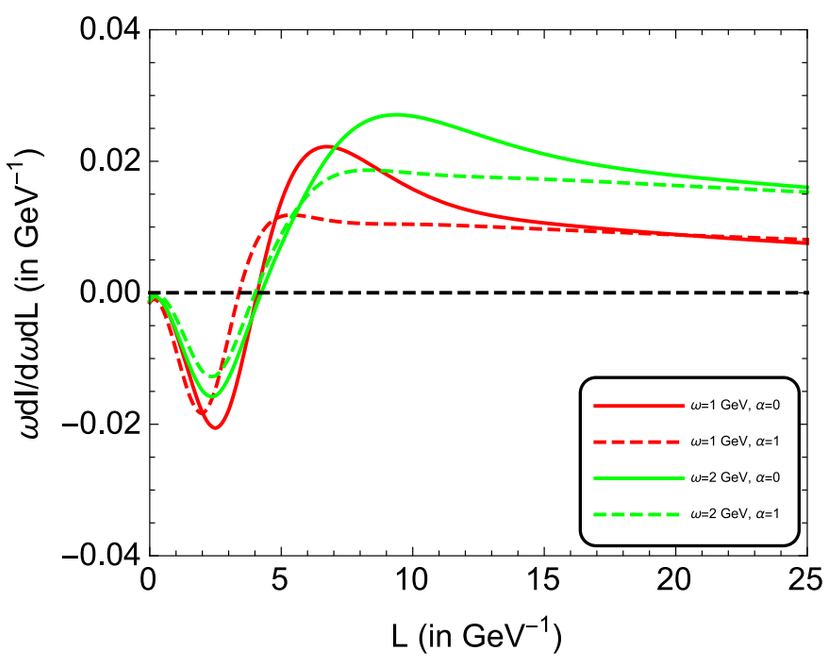
# Splitting rate: Static vs Bjorken expanding medium

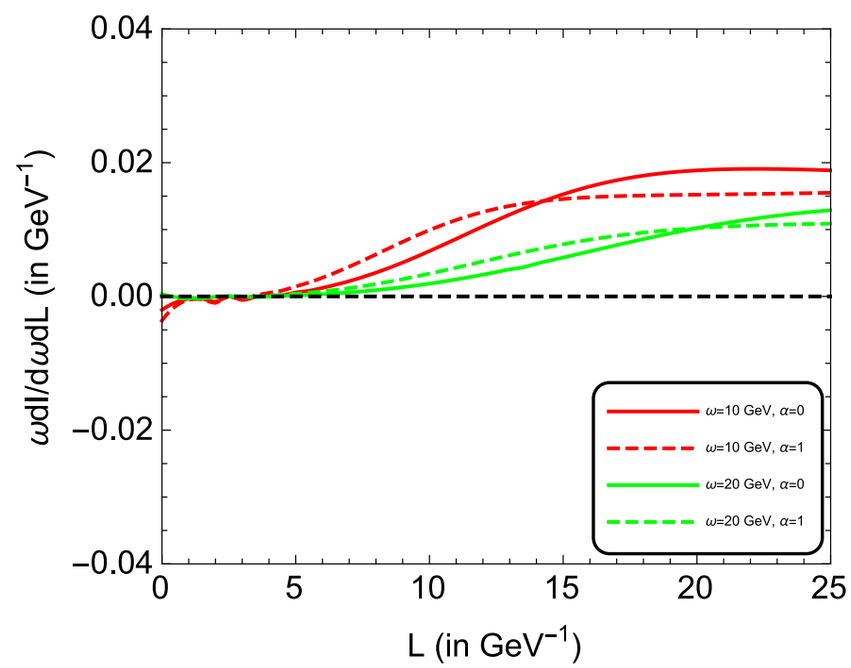


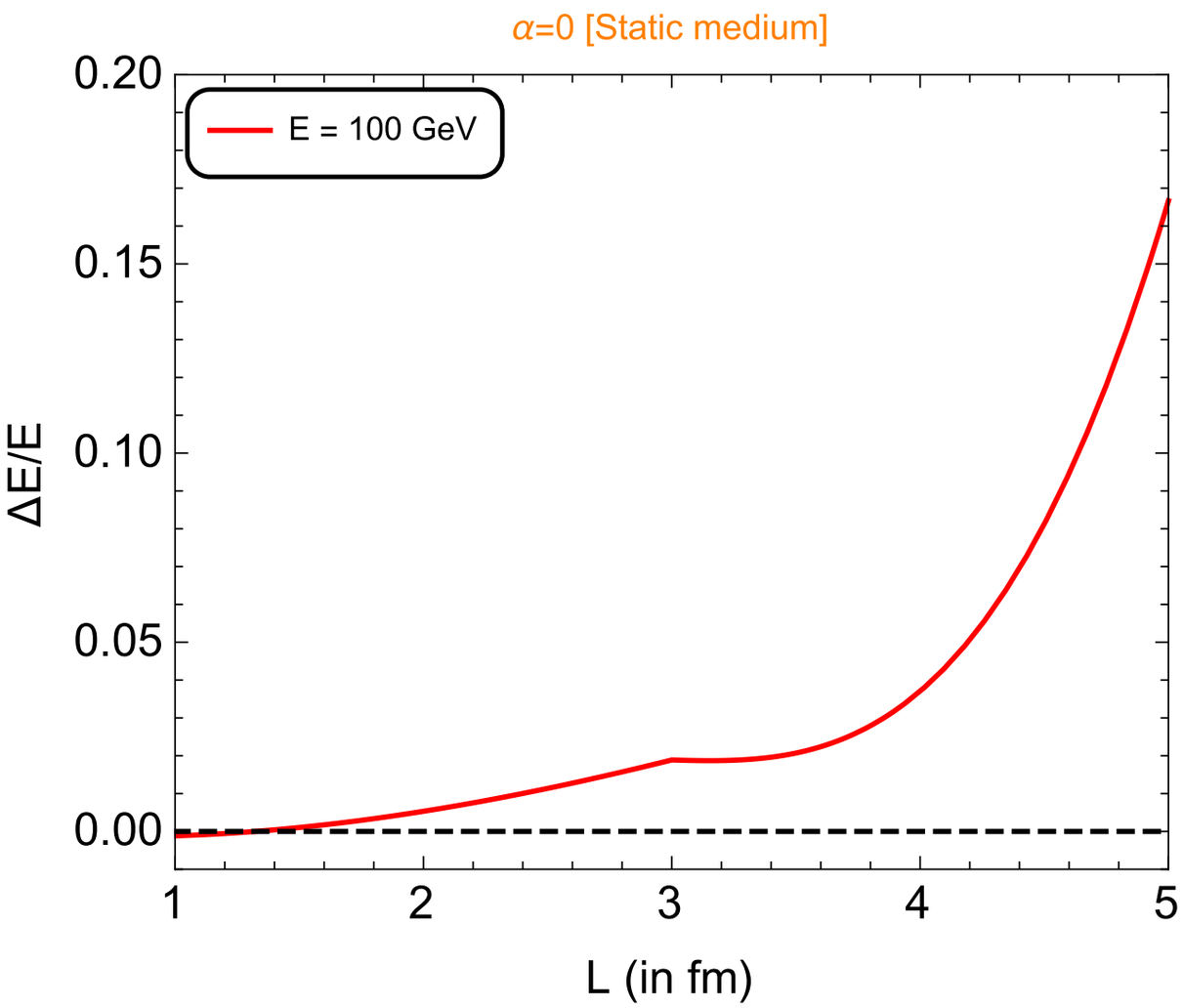

A comparison of the gluon splitting rates in a static medium with that of an expanding medium ; without any scaling (left)
With scaling (bottom)
in q^

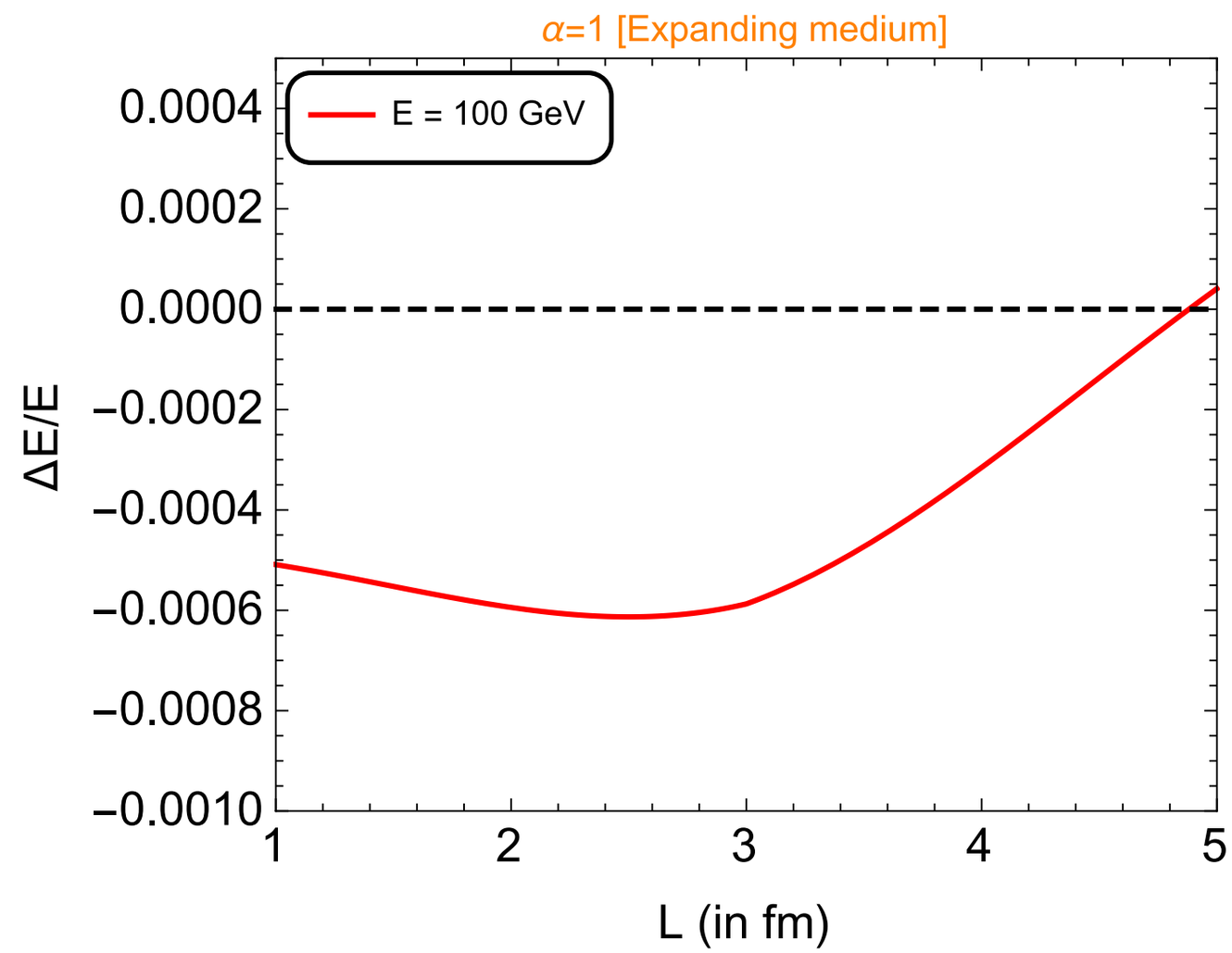


# Energy loss scenario




Recall that in a large static limit we expect to get energy loss of gluons :

$$\Delta E = \int_0^\infty \mathrm{d}\omega\, \omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = 2\bar{\alpha}\hat{q}L^2$$

A comparison of the energy loss in a static medium with that of an expanding medium ; without any scaling.
Note: The energy loss for the expanding medium is negative ==> Reason is small q^

Energy loss ==> insight into Quenching weights

U. A. Weidemann; Nucl. Phys. A 690, 731 (2001)

# A peek into next steps ... Summary and discussions

- Understanding scaling properties and positive/negative contributions to the spectrum and the rate.

- Re-summing multiple medium-induced radiation using

  ==> rate equation for expanding medium with the kinematical constraints

  where , multiple-gluon emission is calculated by repeating the single-gluon emission kernel as needed.

- Going beyond energy loss: medium-modified intra-jet and out-of-jet distribution of particles.

- Generalizing to more complicated medium models (implementation of hot spots)

  ==> Can we develop analytical insights for a rate that depends on local medium properties ?

Thank you !

Acknowledgements:
I would like to thank Martin Spousta & Najmul Haque
for nice discussions and suggestions.

S. P. Adhya, C. A. Salgado & K. Tywoniuk (in preparation)

" Beauty is in the simplicity..."

Thank you