IGFAE
Instituto Galego de Física de Altas Enerxías

USC
UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

Hard Probes 2018
Aix-Les-Bains, France, October 4th 2018

Multi gluon correlations in the Color Glass Condensate: quantum interference in proton-nucleus collisions

Tolga Altinoluk (NCBJ), Néstor Armesto (Santiago de Compostela), Alexander Kovner (UConn) and Michael Lublinsky (BGU)
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Unión Europea
Fondo Europeo de Desarrollo Regional
"Una manera de hacer Europa"

EXCELENCIA MARÍA DE MAEZTU

GOBIERNO DE ESPANA
MINISTERIO DE ECONOMIA Y COMPETITIVIDAD

XUNTA DE GALICIA

Contents:

Eur. Phys. J. C78 (2018) 702, arXiv:1805.07739 [hep-ph]

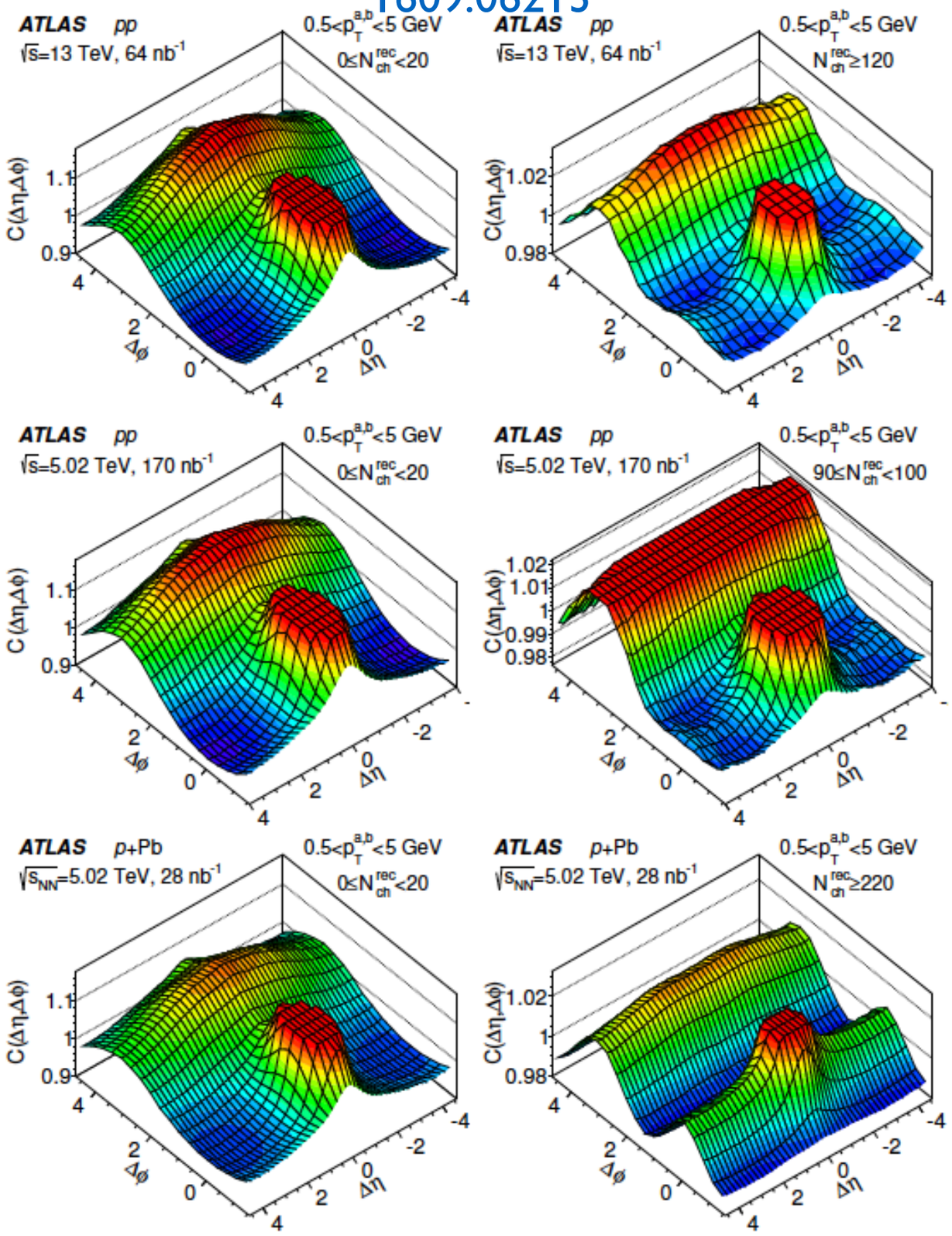
See the following talk by Mark Mace.

The ridge:

- Two-particle correlations in pp and pPb at the LHC show features that in AA are attributed to final state interactions describable by viscous relativistic hydrodynamics and interpreted as a signal of equilibration.
- EKT and AdS/CFT: hydro works even for large momentum anisotropies.
- What about a non-hydro initial-state explanation?

(anyway long range rapidity correlations must come from the very early times…).

1609.06213



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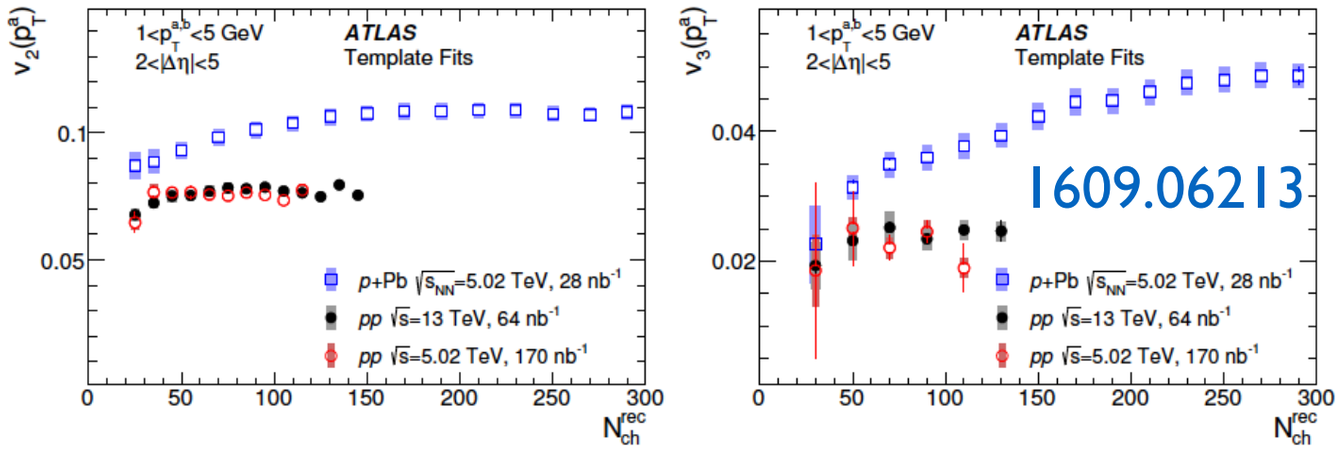
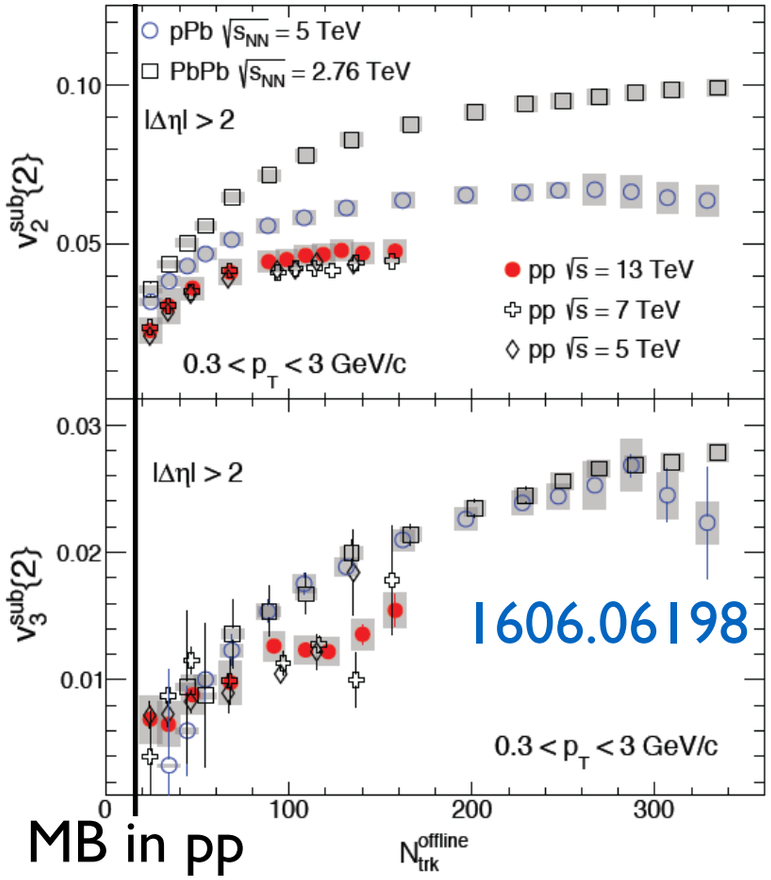
$$\frac{dN}{d\phi} = \left\langle \frac{dN}{d\phi} \right\rangle \left(1 + \sum_n 2v_n \cos\left[n\left(\phi - \Psi_n\right) \right] \right) \quad Y^{\mathrm{ridge}}(\Delta\phi) = G\left(1 + \sum_{n=2}^{\infty} 2v_{n,n} \cos\left(n\Delta\phi\right) \right)$$

$$v_{n,n}(p_{\mathrm{T}}^a, p_{\mathrm{T}}^b) = v_n(p_{\mathrm{T}}^a) v_n(p_{\mathrm{T}}^b)$$





CGC explanations:

- Several explanations in the CGC, that use/assume that:
  - ➜ the final state carries the imprint of initial-state correlations,
  - ➜ the CGC wave function is rapidity invariant over $Y \propto 1/\alpha_s$,
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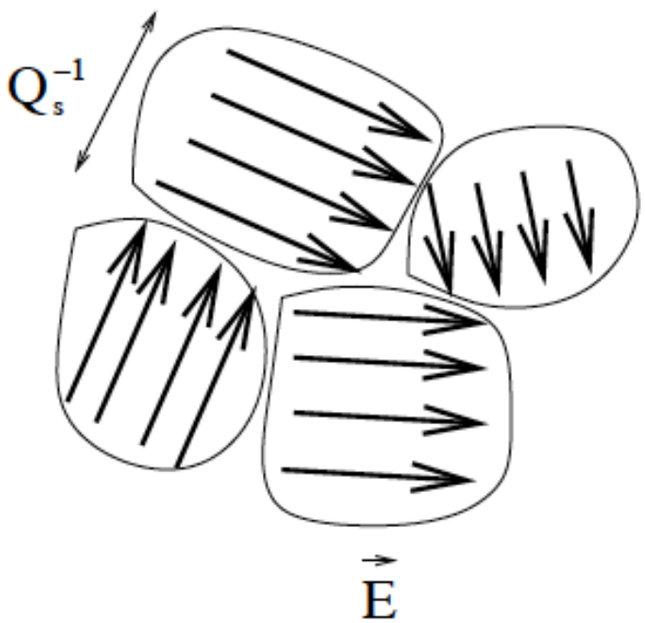
- Local anisotropy of target fields (Kovner-Lublinsky, Dumitru-McLerran-Skokov).



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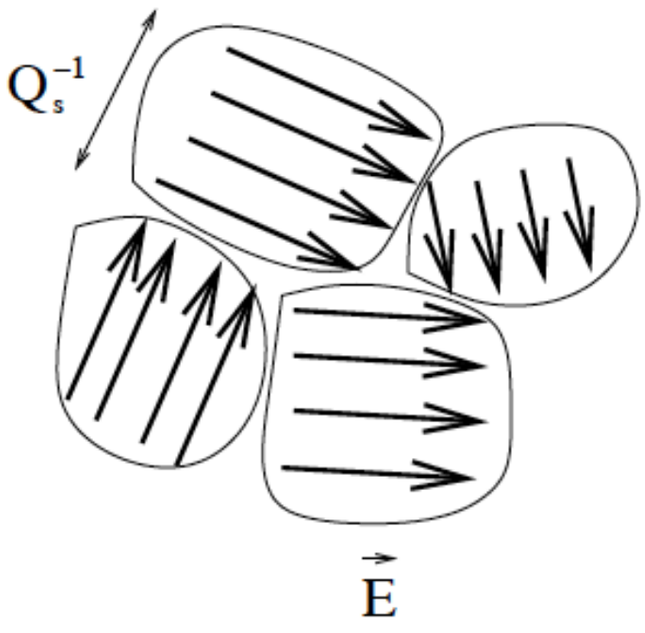

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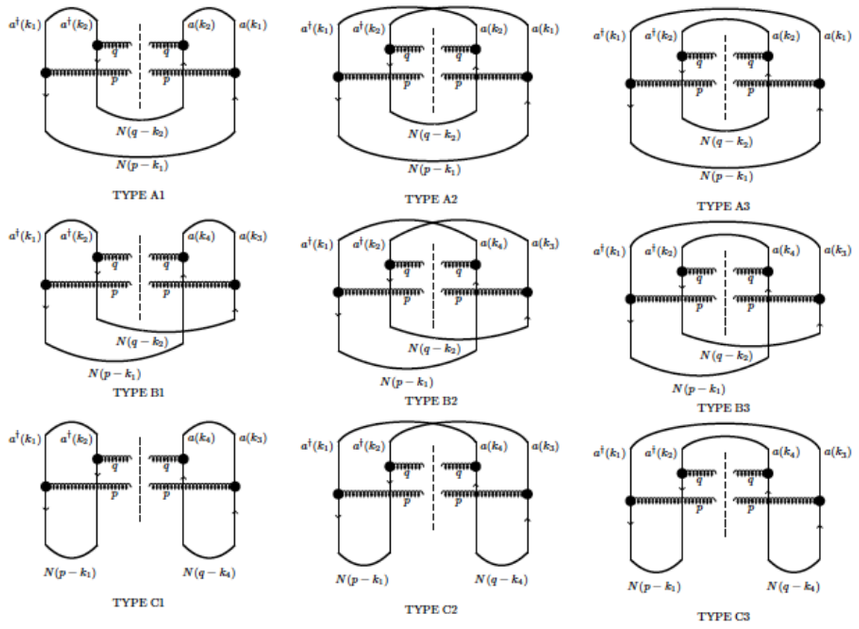

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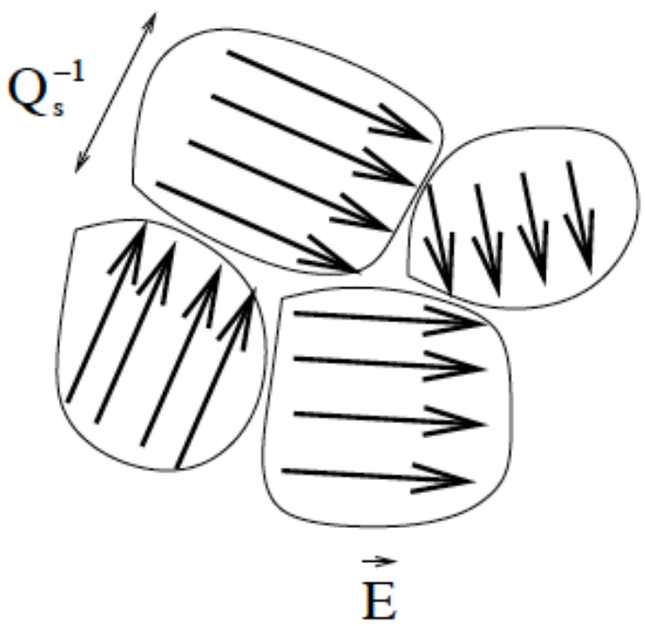

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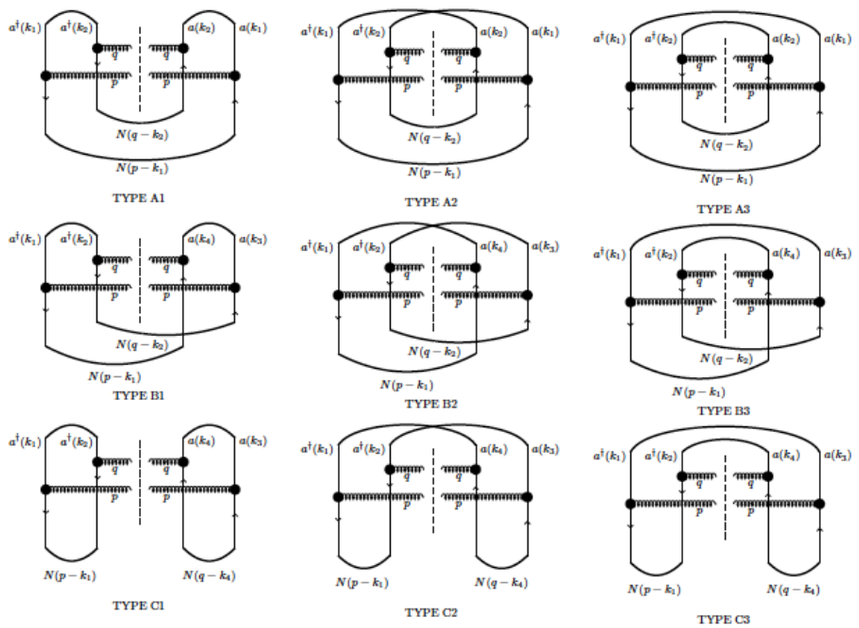
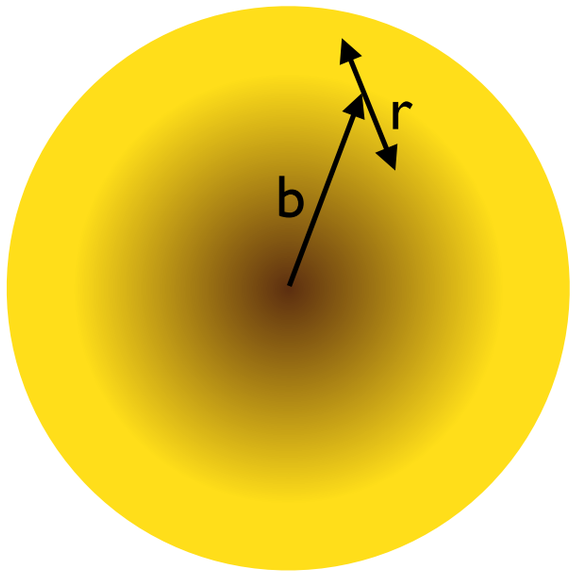


- Spatial variation of partonic density (Levin-Rezaeian-Iancu).



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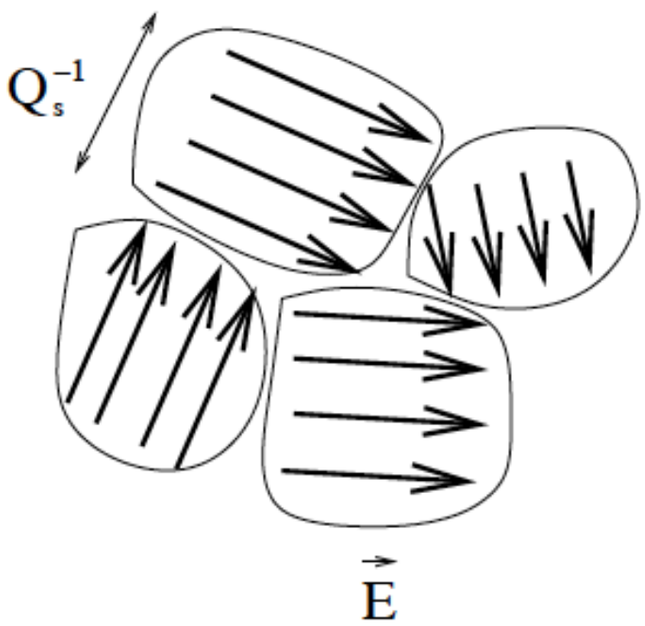

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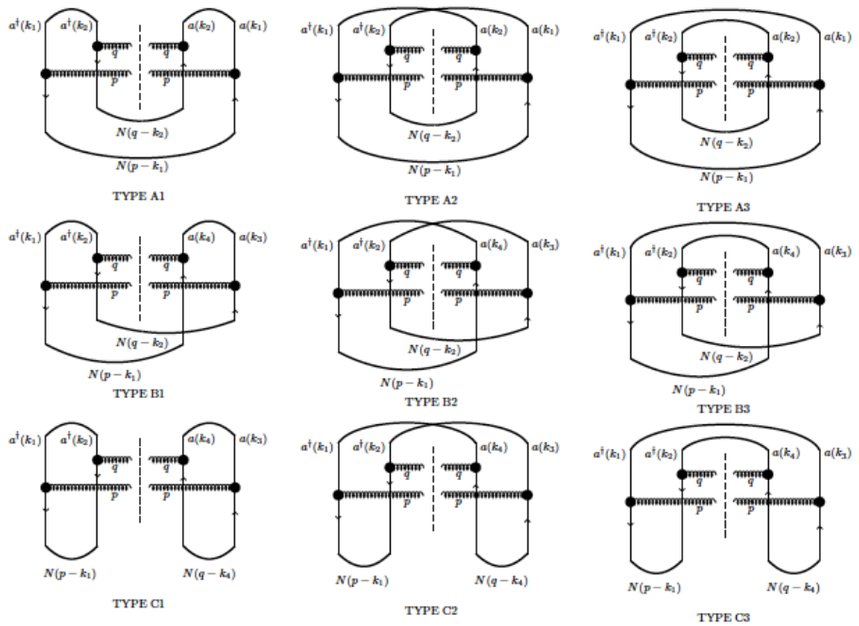

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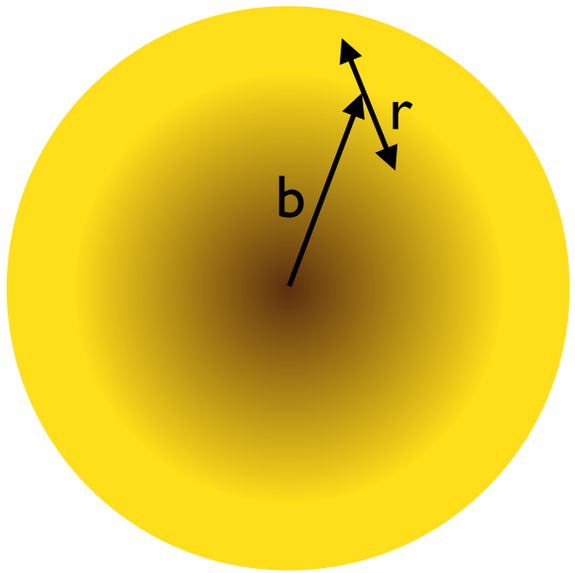
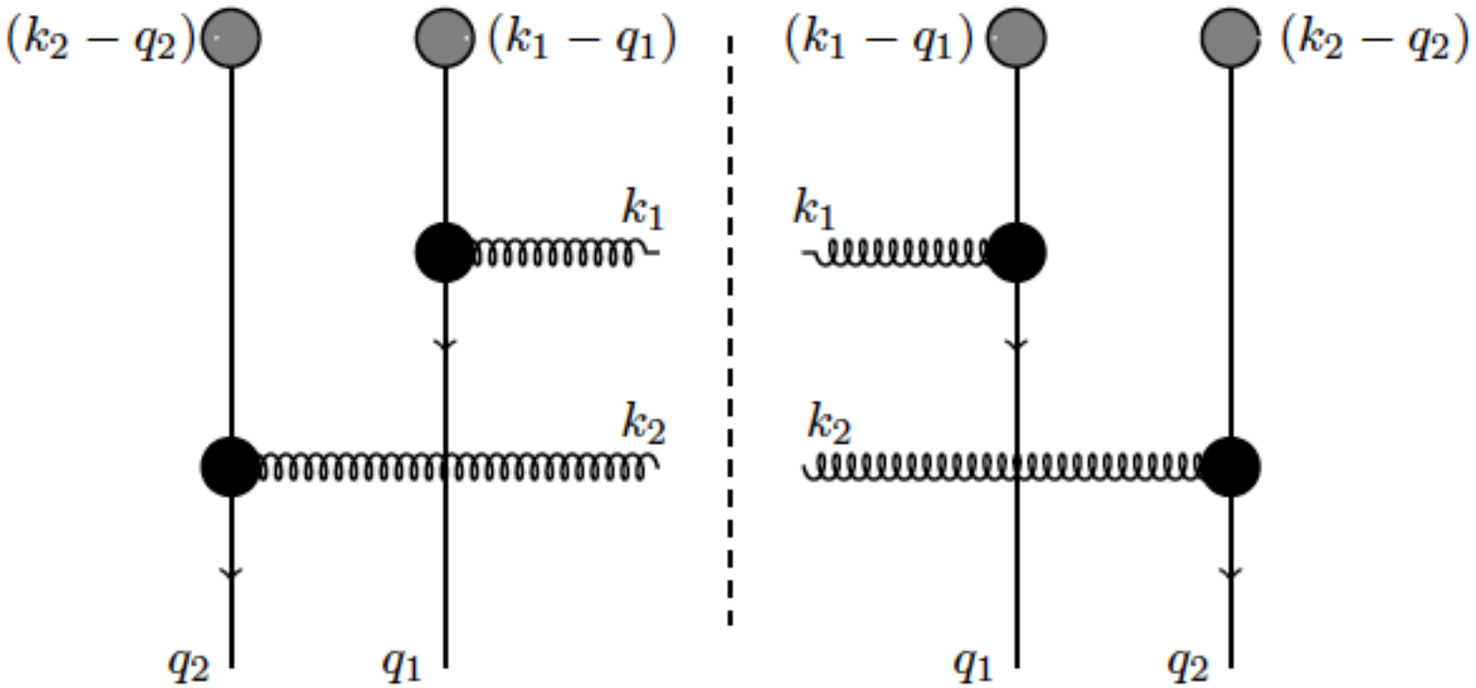


- Also interference (Blok-Jakel-Strikman-Wiedemann).

Correlations in glasma graphs:

- Two physical effects in correlations in the glasma graph approach:



Projectile: sources ρ

Black blob: rescatterings

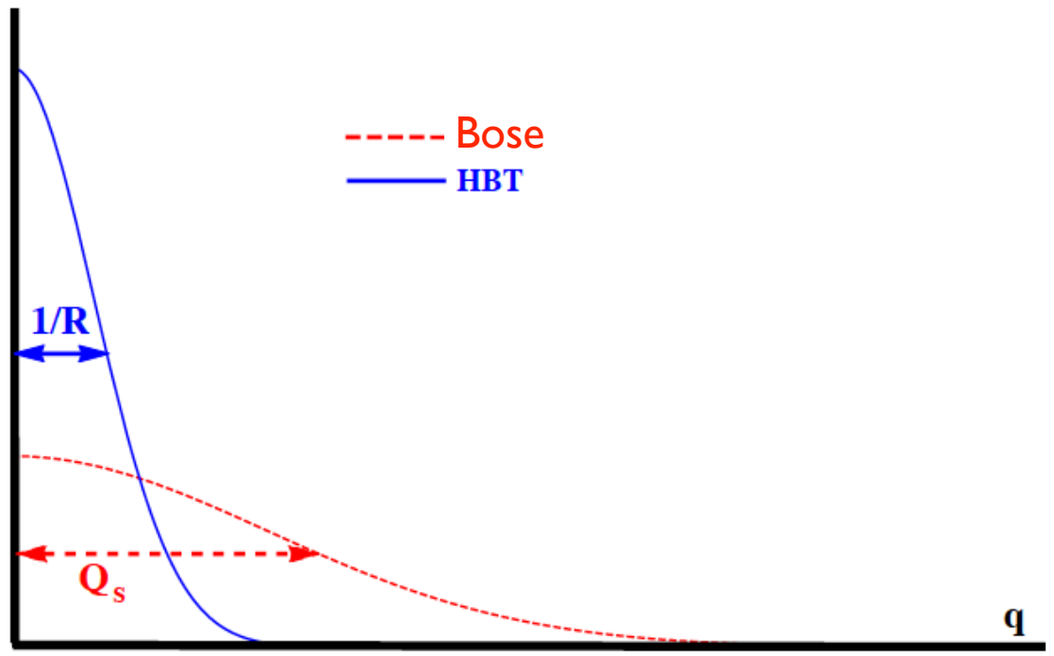
Target: colour field

1) Bose enhancement of gluons in the projectile wave function (1503.07126).

$$\propto \delta^{(2)}[k_1 - q_1 - (k_2 - q_2)] + \delta^{(2)}[k_1 - q_1 + (k_2 - q_2)]$$

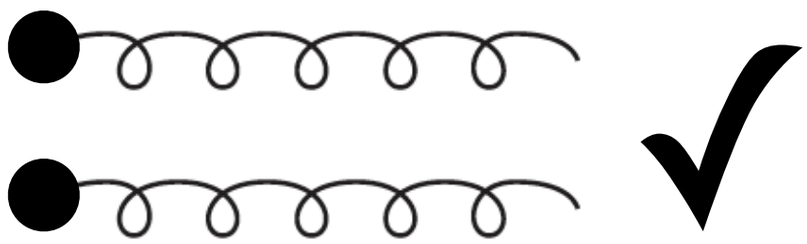
2) HBT of gluons separated in rapidity (1509.03223, 1212.1195,1310.6701).

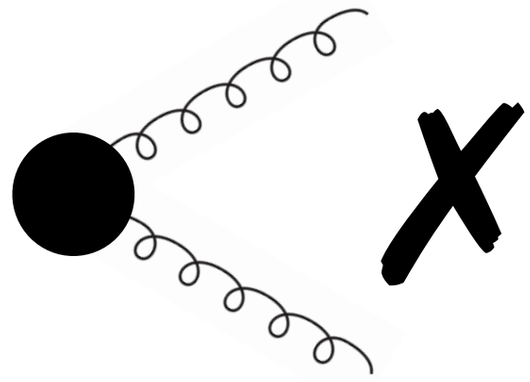
$$\propto \delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2)$$



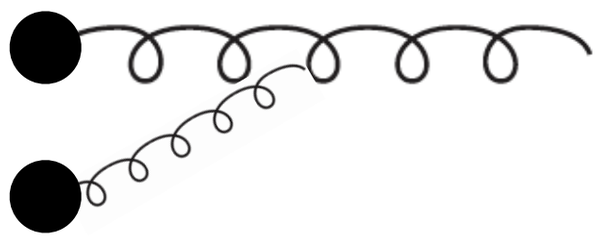
Open problems:

- CGC calculations for the central rapidity region resum terms in which each source emits one gluon, $\mathcal{O}(g\rho)$





➜ Odd harmonics require additional terms (1611.09870, 1612.07790, 1802.08166, see Mark Mace's talk), $\mathcal{O}(g^2\rho)$



- Glasma graph calculations are valid for a dilute target (pp) and usually performed for two particles (up to 4 in 1409.6347, 1712.055571):
  - ➜ Extension to dilute-dense (pA) numerically (1509.03499, 1705.00745, 1706.06260) or analytically (1804.02910, 1808.04896): **this work**.
  - ➜ Three gluons in pA: **this work**.
- Correlations are subleading in N_c in the MV model: new ones including anisotropies (Dumitru-Skokov).

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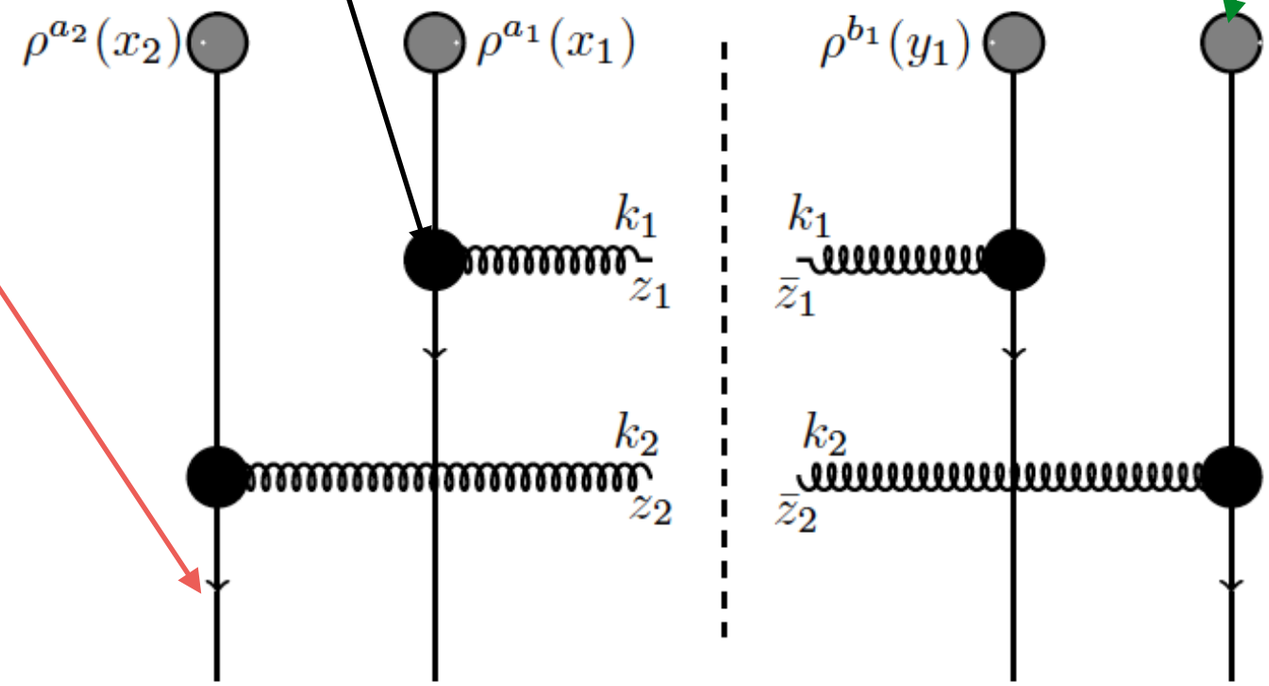
Double inclusive cross section:

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2 (4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)}$$

$$\times \int_{x_1 x_2 y_1 y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)\left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$

$$\times\left\langle \left[U(z_1)-U(x_1)\right]^{a_1 c}\left[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\right]^{cb_1}\left[U(z_2)-U(x_2)\right]^{a_2 d}\left[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\right]^{db_2}\right\rangle_T$$



$$A^i(x-y) = -\frac{1}{2\pi}\frac{(x-y)_i}{(x-y)^2} = \int \frac{d^2k}{(2\pi)^2} e^{-ik\cdot(x-y)}\frac{k^i}{k^2}$$

$$U_{\vec{x}} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^+ \mathcal{A}^-(x^+, x^- = 0, \vec{x})\right]$$

Weizsäcker-Williams field

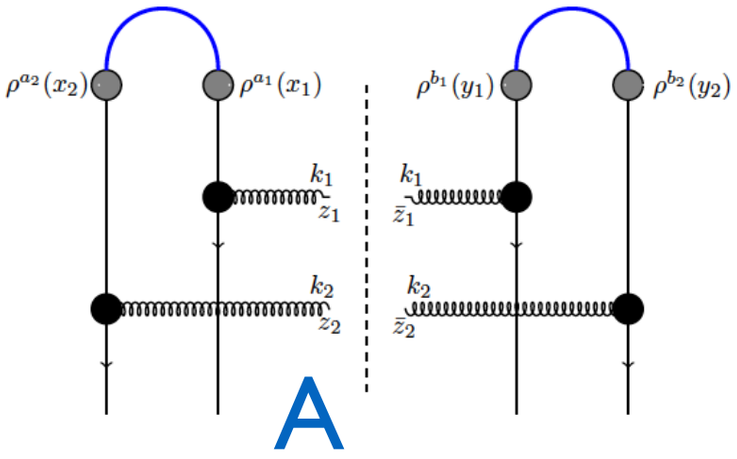
Adjoint Wilson line: eikonal rescattering

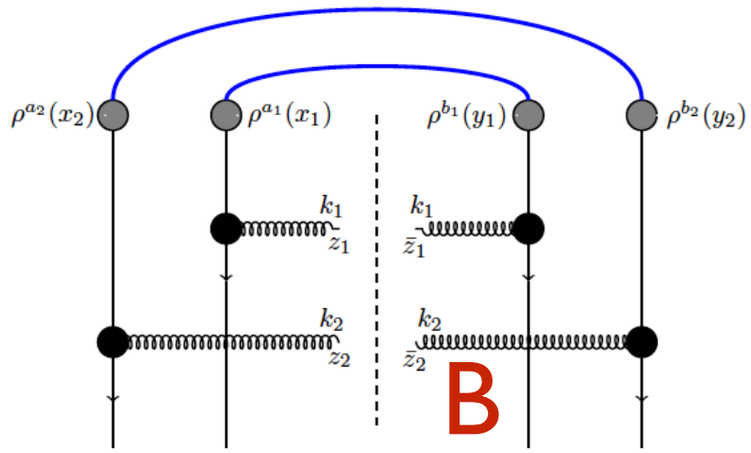
Projectile averaging:

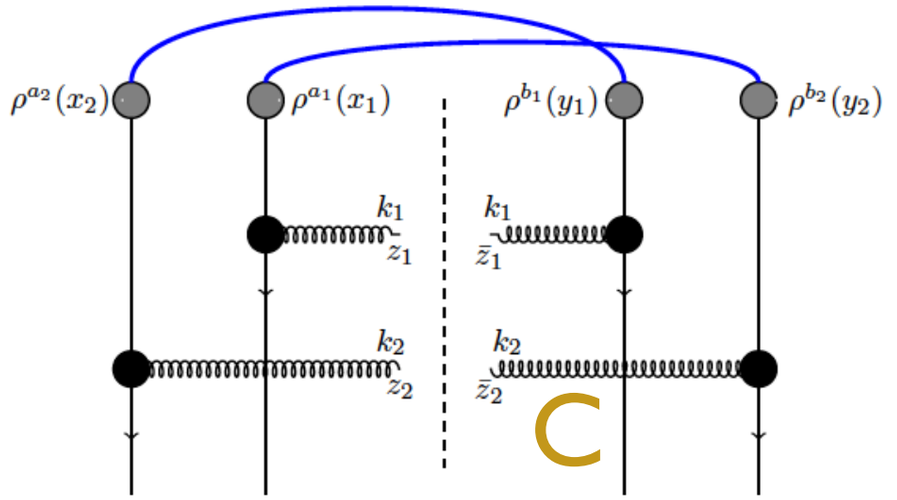
- Averages of colour charges contracted pair-wise (MV model):

$$\langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$$

$$\left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P = \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\right\rangle_P \left\langle \rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$
$$+\left\langle \rho^{a_1}(x_1)\rho^{b_1}(y_1)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_2}(y_2)\right\rangle_P + \left\langle \rho^{a_1}(x_1)\rho^{b_2}(y_2)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_1}(y_1)\right\rangle_P$$







$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$

$$\times\left\{\mu^2(x_1,x_2)\,\mu^2(y_1,y_2)\left\langle \mathrm{tr}\left\{[U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)][U(\bar{z}_2)-U(y_2)][U^\dagger(z_2)-U^\dagger(x_2)]\right\}\right\rangle_T\right.$$

$$+\mu^2(x_1,y_1)\,\mu^2(x_2,y_2)\left\langle \mathrm{tr}\left\{[U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)]\right\}\mathrm{tr}\left\{[U(z_2)-U(x_2)][U^\dagger(\bar{z}_2)-U^\dagger(y_2)]\right\}\right\rangle_T$$

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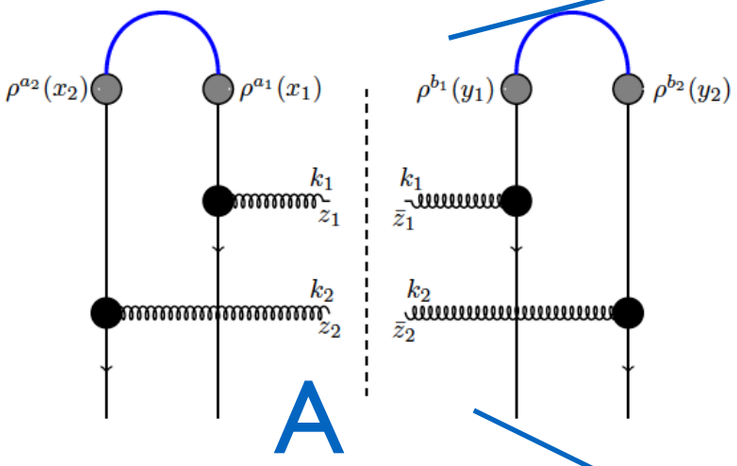

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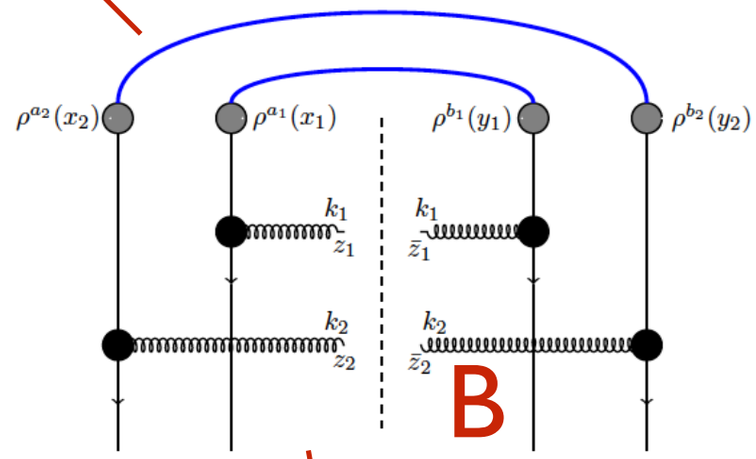

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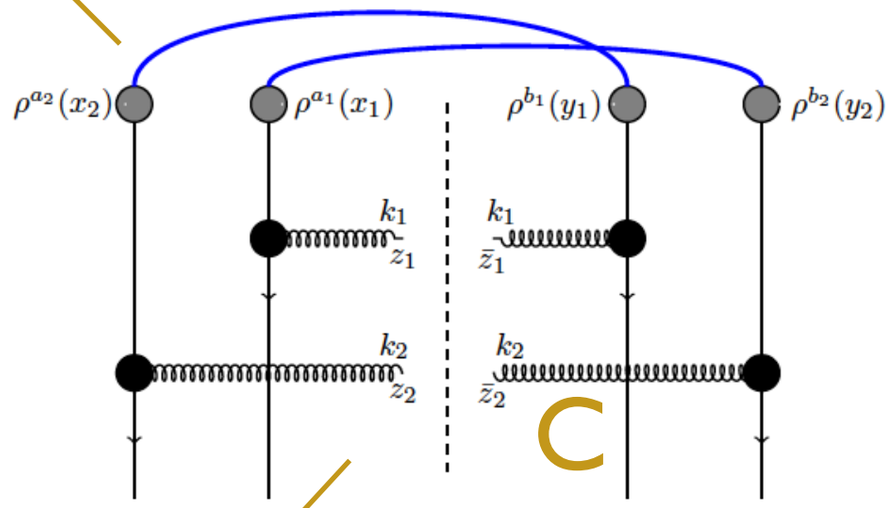

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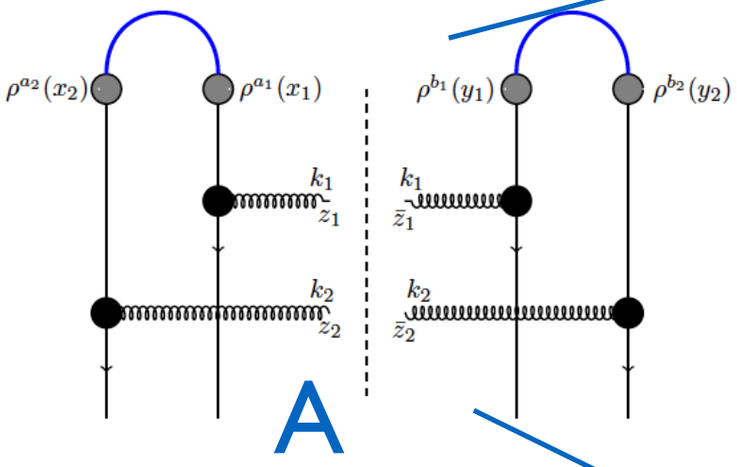
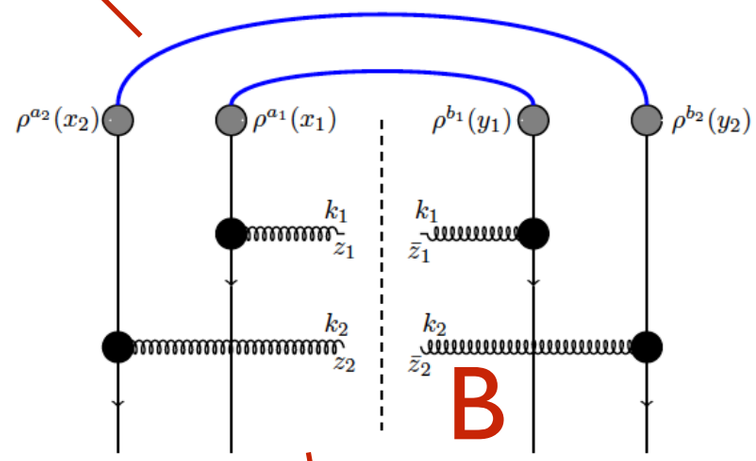
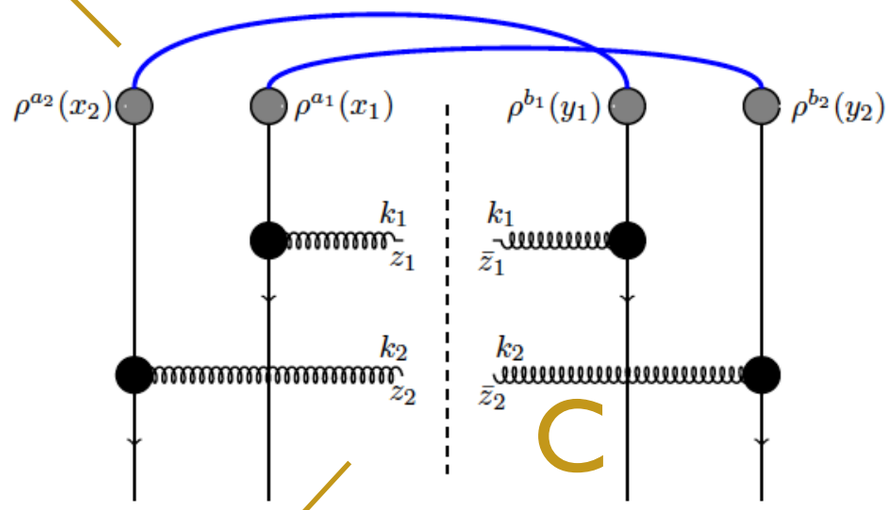


A



B



C

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$

$$\times \Big\{ \mu^2(x_1,x_2)\,\mu^2(y_1,y_2) \Big\langle \mathrm{tr}\Big\{[U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)][U(\bar{z}_2)-U(y_2)][U^\dagger(z_2)-U^\dagger(x_2)]\Big\}\Big\rangle_T \quad Q$$

$$+\mu^2(x_1,y_1)\,\mu^2(x_2,y_2) \Big\langle \mathrm{tr}\Big\{[U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)]\Big\}\,\mathrm{tr}\Big\{[U(z_2)-U(x_2)][U^\dagger(\bar{z}_2)-U^\dagger(y_2)]\Big\}\Big\rangle_T$$

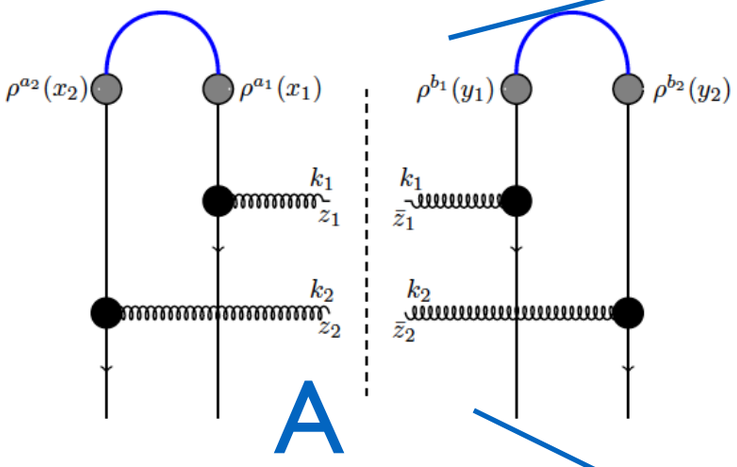
$$+\mu^2(x_1,y_2)\,\mu^2(x_2,y_1) \Big\langle \mathrm{tr}\Big\{[U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)][U(z_2)-U(x_2)][U^\dagger(\bar{z}_2)-U^\dagger(y_2)]\Big\}\Big\rangle_T \Big\} \quad Q$$

Projectile averaging:

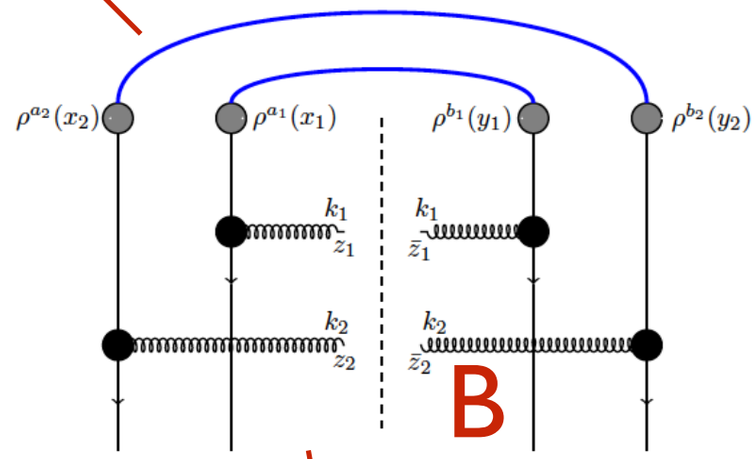
- Averages of colour charges contracted pair-wise (MV model):

$$\langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$$

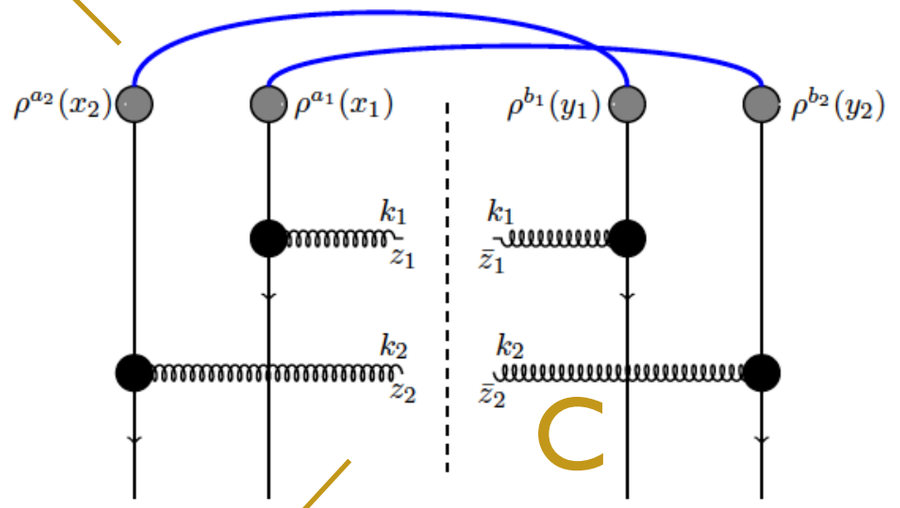
$$\left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P = \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\right\rangle_P \left\langle \rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$
$$+\left\langle \rho^{a_1}(x_1)\rho^{b_1}(y_1)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_2}(y_2)\right\rangle_P + \left\langle \rho^{a_1}(x_1)\rho^{b_2}(y_2)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_1}(y_1)\right\rangle_P$$



A



B



C

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$

$$\times \left\{ \mu^2(x_1,x_2)\,\mu^2(y_1,y_2) \left\langle \mathrm{tr}\left\{ [U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)][U(\bar{z}_2)-U(y_2)][U^\dagger(z_2)-U^\dagger(x_2)]\right\}\right\rangle_T \right. \quad \text{Q}$$

$$+\mu^2(x_1,y_1)\,\mu^2(x_2,y_2) \left\langle \mathrm{tr}\left\{ [U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)]\right\}\mathrm{tr}\left\{ [U(z_2)-U(x_2)][U^\dagger(\bar{z}_2)-U^\dagger(y_2)]\right\}\right\rangle_T \quad \text{dd}$$

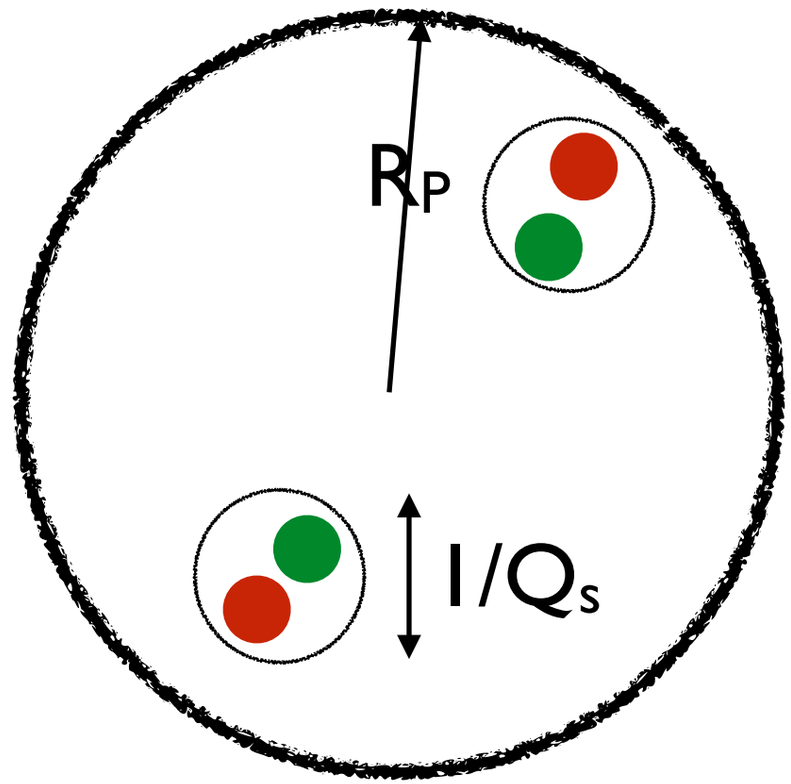
$$\left. +\mu^2(x_1,y_2)\,\mu^2(x_2,y_1) \left\langle \mathrm{tr}\left\{ [U(z_1)-U(x_1)][U^\dagger(\bar{z}_1)-U^\dagger(y_1)][U(z_2)-U(x_2)][U^\dagger(\bar{z}_2)-U^\dagger(y_2)]\right\}\right\rangle_T \right\} \quad \text{Q}$$

Target averaging:

- Cross section involves integration over all transverse coordinates ⇒ maximise the distance.

- But target ensemble must be colour neutral, and colour neutralisation takes place at distances $1/Q_s$, so S-matrix≠0 for colour singlets of size $>1/Q_s$.

⇒ the leading contribution (in $R_P Q_s$) comes from the 2n points in $\langle\ldots\rangle_T$ combined into n singlets at distances as large as possible (similar idea in Kovchegov-Wertepny, see also 1801.04875).



$$\left\langle U^{ab}(x)U^{cd}(y)\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{(N_c^2-1)^2}\left\langle \mathrm{tr}[U(x)U^\dagger(y)]\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{N_c^2-1}d(x,y)$$

$$d(x,y) \equiv \langle D(x,y)\rangle_T$$

$$\langle Q(x,y,z,v)\rangle_T \longrightarrow d(x,y)d(z,v) + d(x,v)d(z,y) + \frac{1}{N_c^2-1}d(x,z)d(y,v),$$

$$\langle D(x,y)D(z,v)\rangle_T \longrightarrow d(x,y)d(z,v) + \frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right]$$

Final result:

- Making the Fourier transform for a translationally invariant target:

$$d(x_1,x_2) \;=\; \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2} e^{-iq_1\cdot x_1+iq_2\cdot x_2}\; d\left(\frac{q_1+q_2}{2}\right)\delta^{(2)}(q_1-q_2)$$

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2(N_c^2-1)^2\int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2} d(q_1)d(q_2)\left\{ I_0 + \frac{1}{N_c^2-1}I_1 + \frac{1}{(N_c^2-1)^2}I_2\right\}$$

$$I_0 = \mu^2\Big[(k_1-q_1),(q_1-k_1)\Big]\mu^2\Big[(k_2-q_2),(q_2-k_2)\Big] L^i(k_1,q_1)L^i(k_1,q_1)L^j(k_2,q_2)L^j(k_2,q_2)$$

$$\begin{aligned} I_1 = {} & \mu^2\Big[(k_1-q_1),(q_2-k_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_1)\Big] L^i(k_1,q_1)L^i(k_1,q_1)L^j(k_2,q_2)L^j(k_2,q_2) \\ & + \mu^2\Big[(k_1-q_1),(q_1-k_2)\Big]\mu^2\Big[(k_2-q_2),-(k_1-q_2)\Big] L^i(k_1,q_1)L^i(k_1,q_2)L^j(k_2,q_1)L^j(k_2,q_2) \\ & + (k_2\to -k_2) \end{aligned}$$

Type A - Q
Type B - dd
Type C - Q

$$\begin{aligned} I_2 = {} & \mu^2\Big[(k_1-q_1),-(k_1-q_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_2)\Big] L^i(k_1,q_1)L^i(k_1,q_2)L^j(k_2,-q_1)L^j(k_2,-q_2) \\ & + (k_2\to -k_2) \\ & + \mu^2\Big[(k_1-q_1),-(k_2-q_2)\Big]\mu^2\Big[-(k_1+q_2),(q_1+k_2)\Big] L^i(k_1,q_1)L^i(k_1,-q_2)L^j(k_2,-q_1)L^j(k_2,q_2) \\ & + (k_2\to -k_2) \end{aligned}$$

Lipatov vertex $\quad L^i(k,q) \equiv \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}$

Interpretation:

- Taking a factorised distribution for the projectile colour charges:

$$\mu^2(k,p) = T\left(\frac{k-p}{2}\right) F[(k+p)R]$$

with R the projectile radius, T its transverse momentum dependent distribution of valence charges, F(q) a form factor peaked at q=0:

I_0 gives the uncorrelated emission of two gluons (coming from Type B diagrams);

I_1 gives:

➜ forward Bose enhancement in the projectile WF,

➜ forward HBT,

from Type C, with Type A ($k_2 \rightarrow -k_2$) giving the backward contributions;

I_2 gives:

➜ forward and backward Bose enhancement in the target WF (Type B),

➜ $1/N_c^2$-correction to forward (Type C) and backward (Type A) Bose enhancement in the projectile WF.

Interpretation:

- Taking a factorised distribution for the projectile colour charges:

$$\mu^2(k,p) = T\left(\frac{k-p}{2}\right) F[(k+p)R]$$

with R the projectile radius, T its transverse momentum dependent distribution of valence charges, F(q) a form factor peaked at q=0:

I_0 gives the uncorrelated emission of two gluons (coming from Type B diagrams);

I_1 gives:

➜ forward Bose enhancement in the projectile WF,

➜ forward HBT,

from Type C, with Type A ($k_2 \to -k_2$) giving the backward contributions;

➜ Fully correlated (Bose or HBT) production at leading N_c coming from the highest order correlator of Wilson lines in the target (quadrupole in this case).

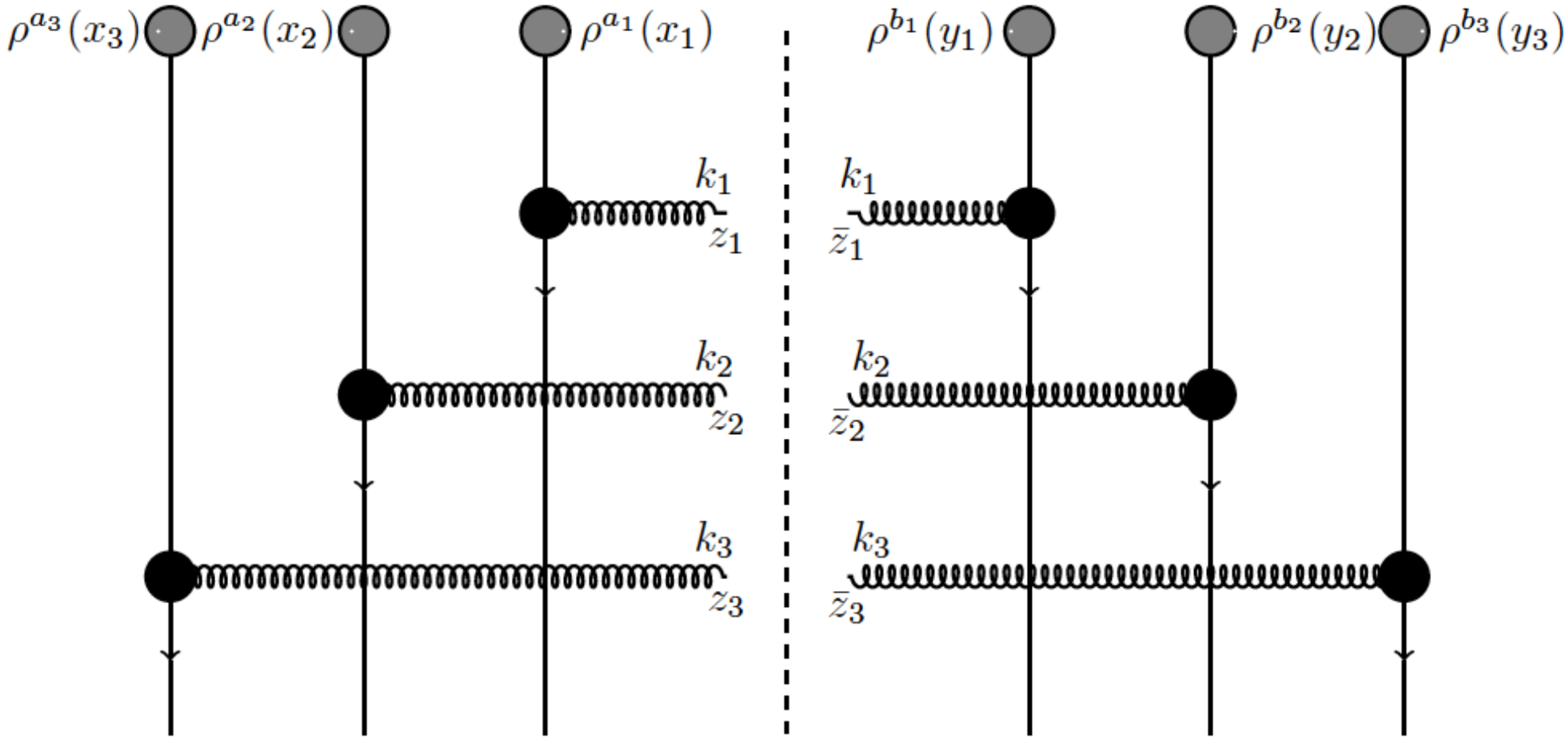
➜ Bose enhancement in the target suppressed by $1/N_c^2$ with respect to that in the projectile (different from Glasma graphs).

Contents:

Eur. Phys. J. C78 (2018) 702, arXiv:1805.07739 [hep-ph]

See the following talk by Mark Mace.

Triple inclusive cross section:



$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3 (4\pi)^3 \int_{z_1\bar{\mathbf{z}}_1 z_2\bar{\mathbf{z}}_2 z_3\bar{\mathbf{z}}_3} e^{ik_1\cdot(z_1-\bar{\mathbf{z}}_1)+ik_2\cdot(z_2-\bar{\mathbf{z}}_2)+ik_3\cdot(z_3-\bar{\mathbf{z}}_3)}$$

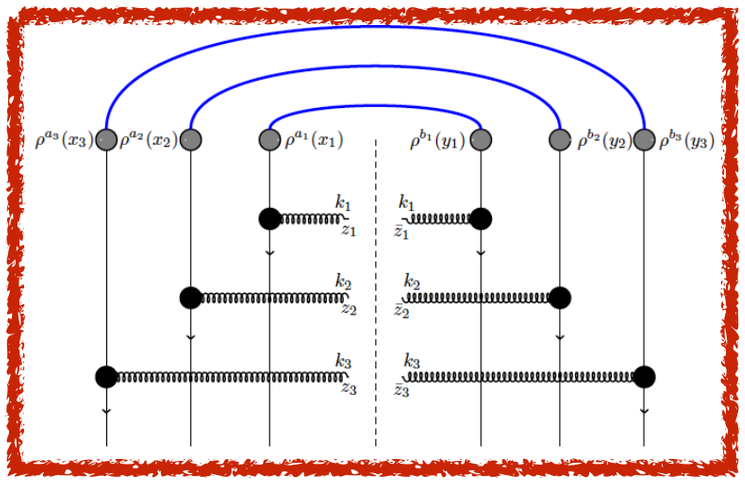
$$\times \int_{x_1y_1x_2y_2x_3y_3} A^i(x_1-z_1)A^i(\bar{\mathbf{z}}_1-y_1)A^j(x_2-z_2)A^j(\bar{\mathbf{z}}_2-y_2)\,A^k(x_3-z_3)A^k(\bar{\mathbf{z}}_3-y_3)$$

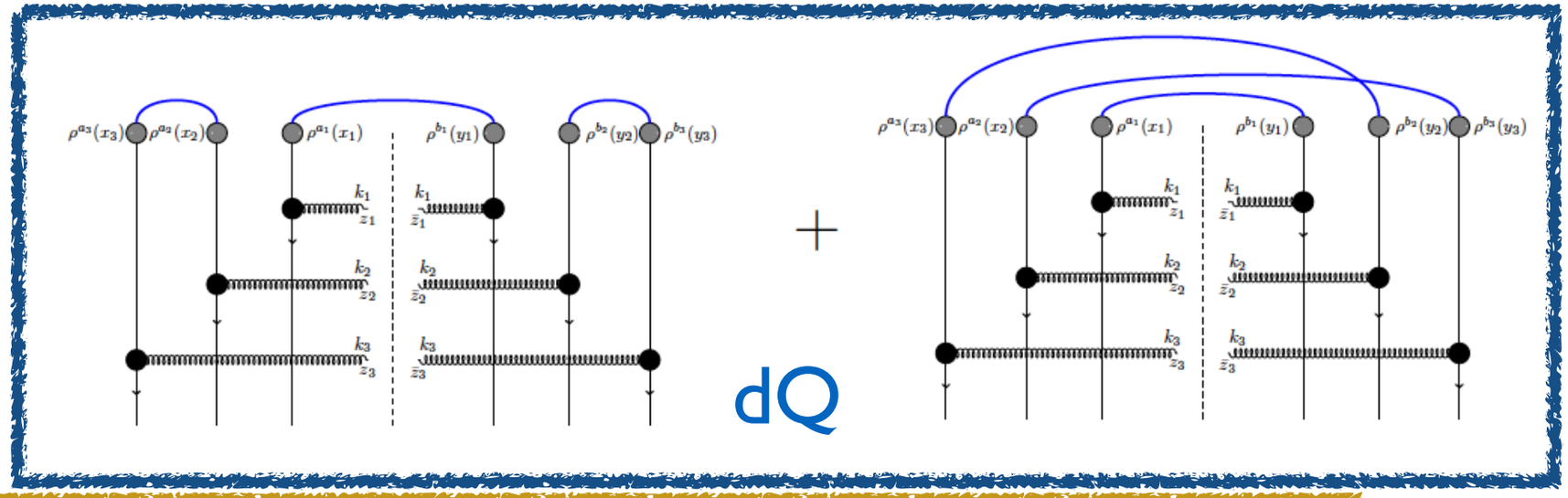
$$\times \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{a_3}(x_3)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\rho^{b_3}(y_3)\right\rangle_P$$

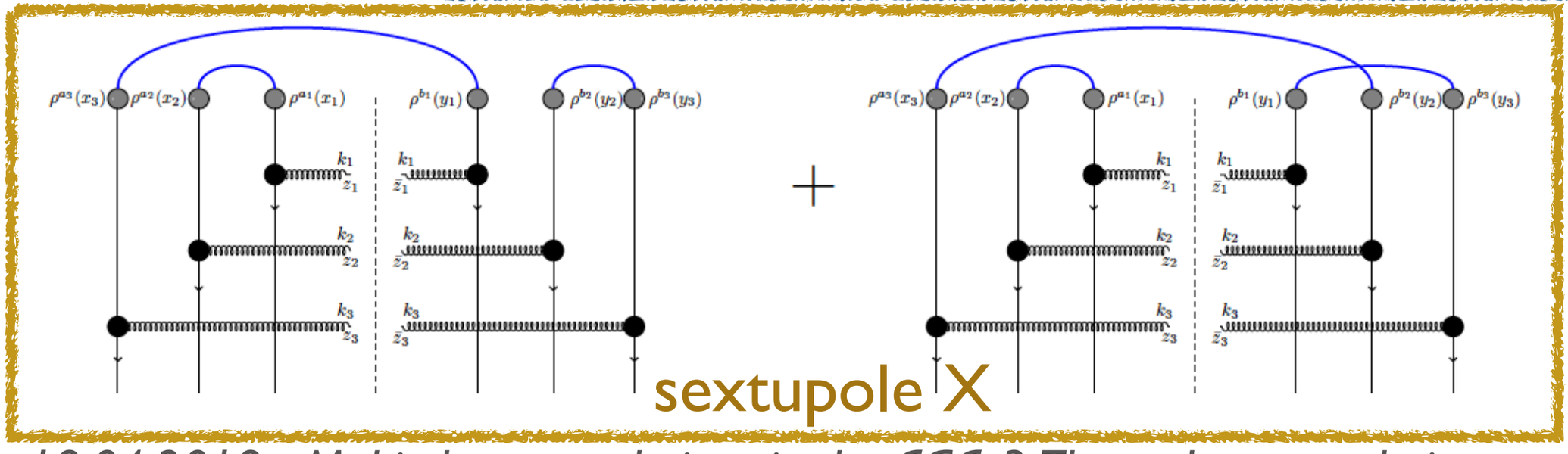
$$\times \left\langle \left(U_{z_1}-U_{x_1}\right)^{a_1c_1}\left(U^\dagger_{\bar{\mathbf{z}}_1}-U^\dagger_{y_1}\right)^{c_1b_1}\left(U_{z_2}-U_{x_2}\right)^{a_2c_2}\left(U^\dagger_{\bar{\mathbf{z}}_2}-U^\dagger_{y_2}\right)^{c_2b_2}\left(U_{z_3}-U_{x_3}\right)^{a_3c_3}\left(U^\dagger_{\bar{\mathbf{z}}_3}-U^\dagger_{y_3}\right)^{c_3b_3}\right\rangle_T$$

Projectile averaging:

$$\left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle_P = \left\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\right\rangle\left\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\right\rangle\left[\left\langle \rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\right\rangle\left\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_2}_{x_2}\rho^{b_3}_{y_3}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\right\rangle\right]$$
$$+ \left\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\right\rangle\left[\left\langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\right\rangle\right] + \left\langle \rho^{a_3}_{x_3}\rho^{b_3}_{y_3}\right\rangle\left[\left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\right\rangle\right]$$
$$+ \left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\right\rangle\left[\left\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\right\rangle\left\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\right\rangle\right] + \left\langle \rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\right\rangle\left[\left\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle\right]$$
$$+ \left\langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\right\rangle\left[\left\langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\right\rangle\left\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\right\rangle\right] + \left\langle \rho^{a_2}_{x_2}\rho^{b_3}_{y_3}\right\rangle\left[\left\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\right\rangle + \left\langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\right\rangle\left\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle\right]$$

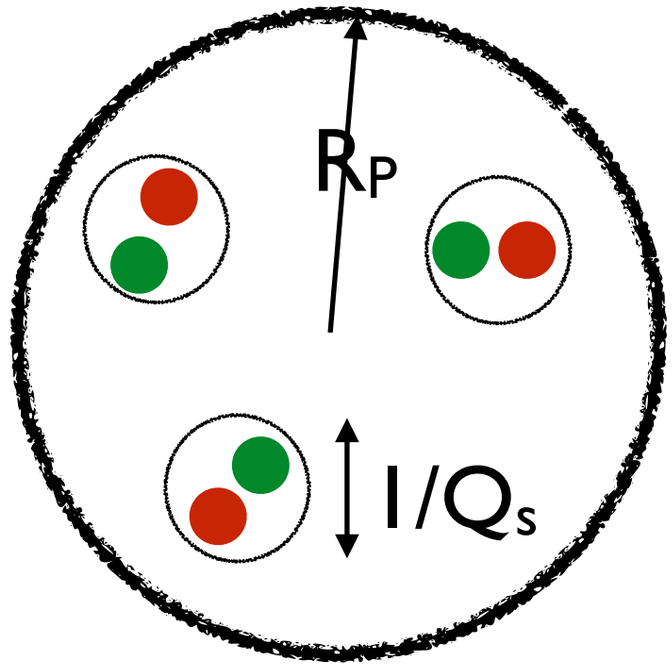






Target averaging:

- Using the argument for the pairwise contraction of target averages to get the leading contribution in $R_P Q_s$:



$$\left\langle D(x_1,x_1')D(x_2,x_2')D(x_3,x_3')\right\rangle_T \to d(x_1,x_1')d(x_2,x_2')d(x_3,x_3')$$

$$+\frac{1}{(N_c^2-1)^2}\Big\{d(x_1,x_1')\Big[d(x_2,x_3)d(x_2',x_3')+d(x_2,x_3')d(x_2',x_3)\Big]+d(x_2,x_2')\Big[d(x_1,x_3)d(x_1',x_3')+d(x_1,x_3')d(x_1',x_3)\Big]$$

$$+\,d(x_3,x_3')\Big[d(x_1,x_2)d(x_1',x_2')+d(x_1,x_2')d(x_1',x_2)\Big]\Big\}+O\left(\frac{1}{(N_c^2-1)^4}\right)$$

$$\left\langle D(x_1,x_1')Q(x_2,x_2',x_3,x_3')\right\rangle_T \to d(x_1,x_1')\Big[d(x_2,x_2')d(x_3,x_3')+d(x_2,x_3')d(x_3,x_2')\Big]+\frac{1}{N_c^2-1}d(x_1,x_1')d(x_2,x_3)d(x_2',x_3')$$

$$\left\langle X(x_1,x_1',x_2,x_2',x_3,x_3')\right\rangle_T \to d(x_1,x_1')d(x_2,x_2')d(x_3,x_3')+d(x_1,x_3')d(x_2,x_1')d(x_3,x_2')$$

$$+\,d(x_1,x_1')d(x_2,x_3')d(x_3,x_2')+d(x_2,x_2')d(x_3,x_1')d(x_1,x_3')+d(x_3,x_3')d(x_1,x_2')d(x_2,x_1')+O\left(\frac{1}{N_c^2-1}\right)$$

Final result:

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$

$$\times\Bigg\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\Big[I_{\mathrm{dQ},1} + I_{\mathrm{dQ},2} + I_{\mathrm{dQ},3}\Big]$$

$$+\frac{1}{(N_c^2-1)^2}\Bigg(\Big[I_{\mathrm{ddd},1} + I_{\mathrm{ddd},2} + I_{\mathrm{ddd},3}\Big] + \Big[I'_{\mathrm{dQ},1} + I'_{\mathrm{dQ},2} + I'_{\mathrm{dQ},3}\Big] + \Big[I_{\mathrm{X},1} + I_{\mathrm{X},2} + I_{\mathrm{X},3} + I_{\mathrm{X},4} + I_{\mathrm{X},5}\Big]\Bigg)$$

$$+\mathcal{O}\left(\frac{1}{(N_c^2-1)^3}\right) + \mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right)\Bigg\}\,.$$

Final result:

Independent emission of 3 gluons

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\, d(q_1)d(q_2)d(q_3)$$

$$\times\Bigg\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\Big[I_{\mathrm{dQ},1}+I_{\mathrm{dQ},2}+I_{\mathrm{dQ},3}\Big]$$

$$+\frac{1}{(N_c^2-1)^2}\Bigg(\Big[I_{\mathrm{ddd},1}+I_{\mathrm{ddd},2}+I_{\mathrm{ddd},3}\Big]+\Big[I'_{\mathrm{dQ},1}+I'_{\mathrm{dQ},2}+I'_{\mathrm{dQ},3}\Big]+\Big[I_{\mathrm{X},1}+I_{\mathrm{X},2}+I_{\mathrm{X},3}+I_{\mathrm{X},4}+I_{\mathrm{X},5}\Big]\Bigg)$$

$$+\mathcal{O}\left(\frac{1}{(N_c^2-1)^3}\right)+\mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right)\Bigg\}\,.$$

Final result:

Independent emission of 3 gluons

Independent emission of 1 gluon and projectile Bose/HBT of the other 2

Independent emission of 1 gluon and target Bose of the other 2

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\, d(q_1)d(q_2)d(q_3)$$

$$\times\left\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\left[I_{\mathrm{dQ},1} + I_{\mathrm{dQ},2} + I_{\mathrm{dQ},3} \right] \right.$$

$$+\frac{1}{(N_c^2-1)^2}\left(\left[I_{\mathrm{ddd},1} + I_{\mathrm{ddd},2} + I_{\mathrm{ddd},3} \right] + \left[I'_{\mathrm{dQ},1} + I'_{\mathrm{dQ},2} + I'_{\mathrm{dQ},3} \right] + \left[I_{\mathrm{X},1} + I_{\mathrm{X},2} + I_{\mathrm{X},3} + I_{\mathrm{X},4} + I_{\mathrm{X},5} \right] \right)$$

$$\left. +\mathcal{O}\left(\frac{1}{(N_c^2-1)^3}\right) + \mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right) \right\}.$$

N_c-suppressed independent emission of 1 gluon and projectile Bose of the other 2

Final result:

Independent emission of 3 gluons

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$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\, d(q_1)d(q_2)d(q_3)$$

$$\times\left\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\left[I_{\mathrm{dQ},1} + I_{\mathrm{dQ},2} + I_{\mathrm{dQ},3} \right] \right.$$

$$+\frac{1}{(N_c^2-1)^2}\left(\left[I_{\mathrm{ddd},1} + I_{\mathrm{ddd},2} + I_{\mathrm{ddd},3} \right] + \left[I'_{\mathrm{dQ},1} + I'_{\mathrm{dQ},2} + I'_{\mathrm{dQ},3} \right] + \left[I_{\mathrm{X},1} + I_{\mathrm{X},2} + I_{\mathrm{X},3} + I_{\mathrm{X},4} + I_{\mathrm{X},5} \right] \right)$$

$$\left. +\mathcal{O}\left(\frac{1}{(N_c^2-1)^3}\right) + \mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right) \right\}.$$

N_c-suppressed independent emission of 1 gluon and projectile Bose of the other 2

HBT of two gluons and projectile Bose of other two

Final result:

Independent emission of 3 gluons

Independent emission of 1 gluon and projectile Bose/HBT of the other 2

Independent emission of 1 gluon and target Bose of the other 2

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\, d(q_1)d(q_2)d(q_3)$$

$$\times\left\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\left[I_{\mathrm{dQ},1} + I_{\mathrm{dQ},2} + I_{\mathrm{dQ},3} \right] \right.$$

$$+\frac{1}{(N_c^2-1)^2}\left(\left[I_{\mathrm{ddd},1} + I_{\mathrm{ddd},2} + I_{\mathrm{ddd},3} \right] + \left[I'_{\mathrm{dQ},1} + I'_{\mathrm{dQ},2} + I'_{\mathrm{dQ},3} \right] + \left[I_{\mathrm{X},1} + I_{\mathrm{X},2} + I_{\mathrm{X},3} \right] + I_{\mathrm{X},4} + I_{\mathrm{X},5} \right)$$

$$\left. +\mathcal{O}\left(\frac{1}{(N_c^2-1)^3}\right) + \mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right) \right\}.$$

N_c-suppressed independent emission of 1 gluon and projectile Bose of the other 2

HBT of two gluons and projectile Bose of other two

Projectile Bose of the 3 gluons

HBT of the 3 gluons

Summary:

- We have computed the complete two-gluon and the leading N_c contribution to the 3-gluon cross section in pA collisions in the CGC, using the MV model, under the approximations:
  - ➜ Translational invariance of the nucleus;
  - ➜ Pairwise contraction of target averages giving the leading contribution in $R_P Q_s$.

- The totally correlated pieces appear only for the highest colour correlators of Wilson lines in the target (quadrupole and sextupole).

- The (Bose) correlations in the target wave function are N_c-suppressed with respect to those in the projectile.

- To do: investigate the implications on even azimuthal harmonics.

Summary:

- We have computed the complete two-gluon and the leading N_c contribution to the 3-gluon cross section in pA collisions in the CGC, using the MV model, under the approximations:
  - ➜ Translational invariance of the nucleus;
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Many thanks to:
➜ *Vladi Skokov for discussions.*
➜ *Mauricio Martínez, Matt Sievert and Doug Wertepny for pointing out an error in the first version of the manuscript.*
➜ *You all for your attention.*

- To do: investigate the implications on even azimuthal harmonics.

Backup:

The sextuple contribution:

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\right|_{\rm X} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)\int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$
$$\times\left\{\Big[I_{\rm X,1}+I_{\rm X,2}+I_{\rm X,3}+I_{\rm X,4}+I_{\rm X,5}\Big]+\mathcal{O}\left(\frac{1}{(N_c^2-1)}\right)+\mathcal{O}\left(\frac{1}{(N_c^2-1)^2}\right)\right\},$$

$$I_{\rm X,1}=\Big[\tilde{I}_{\rm X,1}+(k_3\to -k_3)\Big]+\Big[\tilde{I}'_{\rm X,1}+(k_1\to -k_1)\Big]$$

$$I_{\rm X,2}=\Big[\tilde{I}_{\rm X,1}(1\to 2,2\to 3,3\to 1)+(k_3\to -k_3)\Big]+\Big[\tilde{I}'_{\rm X,1}(1\to 2,2\to 3,3\to 1)+(k_1\to -k_1)\Big],$$
$$I_{\rm X,3}=\Big[\tilde{I}_{\rm X,1}(1\to 3,3\to 2,2\to 1)+(k_3\to -k_3)\Big]+\Big[\tilde{I}'_{\rm X,1}(1\to 3,3\to 2,2\to 1)+(k_1\to -k_1)\Big].$$

$$\begin{aligned}\tilde{I}_{\rm X,1} &= \mu^2(k_2-q_2,q_2-k_1)\,\mu^2(k_1-q_1,q_3-k_3)\,\mu^2(k_3-q_3,q_1-k_2)\;L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,q_1)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)\\ &\quad+\mu^2(k_2+q_2,k_1-q_2)\,\mu^2(k_3-q_3,q_1-k_1)\,\mu^2(q_3-k_3,-q_1-k_2)\;L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,-q_1)L^i(k_2,-q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3),\end{aligned}\tag{65}$$

$$\begin{aligned}\tilde{I}'_{\rm X,1} &= \mu^2(k_1-q_2,q_2-k_2)\,\mu^2(k_2-q_1,k_3-q_3)\,\mu^2(q_1-k_1,q_3-k_3)\;L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_1)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)\\ &\quad+\mu^2(-k_1-q_2,q_2-k_2)\,\mu^2(k_2+q_1,q_3-k_3)\,\mu^2(k_3-q_3,k_1-q_1)\;L^i(k_1,-q_2)L^i(k_1,q_1)\,L^j(k_2,-q_1)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3).\end{aligned}\tag{66}$$

$$\begin{aligned}I_{[\rm X,4]} &= \mu^2(k_2-q_2,q_1-k_1)\,\mu^2(k_1-q_1,q_3-k_3)\,\mu^2(k_3-q_3,q_2-k_2)\;L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)+(k_3\to -k_3)\\ &\quad+\mu^2(k_2-q_2,q_3-k_3)\,\mu^2(k_3-q_3,k_1-q_1)\,\mu^2(q_1-k_1,q_2-k_2)\;L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)+(k_1\to -k_1)\\ &\quad+\mu^2(k_2-q_2,k_1-q_1)\,\mu^2(k_3-q_3,q_1-k_1)\,\mu^2(q_3-k_3,q_2-k_2)\;L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)+(k_3\to -k_3)\\ &\quad+\mu^2(q_1-k_1,q_3-k_3)\,\mu^2(k_1-q_1,q_2-k_2)\,\mu^2(k_2-q_2,k_3-q_3)\;L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_3)+(k_1\to -k_1),\end{aligned}\tag{69}$$

$$\begin{aligned}I_{[\rm X,5]} &= \mu^2(k_2-q_2,q_2-k_1)\,\mu^2(k_1-q_1,q_1-k_3)\,\mu^2(k_3-q_3,q_3-k_2)\;L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,q_3)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_1)+(k_3\to -k_3)\\ &\quad+\mu^2(k_2-q_2,q_2-k_3)\,\mu^2(k_3-q_3,q_3+k_1)\,\mu^2(q_1-k_2,-k_1-q_1)\;L^i(k_1,-q_1)L^i(k_1,-q_3)\,L^j(k_2,q_2)L^j(k_2,q_1)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_2)+(k_1\to -k_1)\\ &\quad+\mu^2(k_2+q_1,k_1-q_1)\,\mu^2(k_3-q_3,q_3-k_1)\,\mu^2(q_2-k_3,-k_2-q_2)\;L^i(k_1,q_1)L^i(k_1,q_3)\,L^j(k_2,-q_2)L^j(k_2,-q_1)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_2)+(k_3\to -k_3)\\ &\quad+\mu^2(k_2+q_3,k_3-q_3)\,\mu^2(-k_1-q_1,q_1-k_3)\,\mu^2(k_1-q_2,q_2-k_2)\;L^i(k_1,-q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,-q_3)\\ &\quad\times L^k(k_3,q_3)L^k(k_3,q_1)+(k_1\to -k_1).\end{aligned}\tag{70}$$