Synchrotron radiation as a probe of confinement and QGP

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Motivation

The excess has been observed for the first time in CERN, QGP - ?
Motivation

Our explanation of this PHENIX and ALICE puzzle

Intensive radiation of magnetic bremsstrahlung type (synchrotron radiation) resulting from the interaction of escaping quarks with the collective confining colour field is discussed as a new possible mechanism of observed direct photon anisotropy.

Theoretically, the basic conditions to have such a radiation available are easily realized as:

- presence of relativistic light quarks (u and d quarks) in QGP;
- the semiclassical nature of their motion;
- confinement.

Then as a result, each quark (antiquark) at the boundary of the system volume moves along a curve trajectory and (as any classical charge undergoes an acceleration) emits photons.
Our explanation of this PHENIX and ALICE puzzle

The interaction of escaping quarks with the collective confining color field (in the chromo-electric flux tube model*):

Confinement $\rightarrow$ a constant restoring force $\sigma \approx 0.2 \text{ GeV}^2$, directed along the normal QGP surface.

Theoretical framework

A large value of the confining force $\sigma$ results in the large magnitude of characteristic parameter for $u$ and $d$ quarks:

$$\chi \approx \left( \frac{3 \sigma E}{2 \mu^3} \right)^{1/3}$$

[the strong field case]

In this regime the spectral distribution can be represented as *:

$$\frac{dN_{\nu}}{d\omega dt} = \frac{1}{2} e_q^2 \alpha \omega^{-2/3} \left( \frac{\sigma \sin \phi}{E} \right)^{2/3}, 0 < \omega < E,$$

(1)

$\alpha = 1/137$ is the fine structure constant;
$e_q$ is the quark charge in units of electron charge;
$\phi$ is the angle between the quark velocity and the direction of quark confining force.

This expression hold for all frequencies $\omega$ except those near $E$.

* [A. Sokolov and I. Ternov, Synchrotron Radiation, Nauka, Moscow, 1966]
Theoretical framework

If the confining force acts along the $z$-axis we have the equation of motion for a quark crossing a surface of QGP volume in the following form:

$$ p_z = \sigma t, \quad p_y = p_{y0}, \quad -p_{z0}/\sigma \leq t \leq p_{z0}/\sigma, $$

(2)

where $p_{z0} > 0, p_{y0}, p_{x0}$ are the initial values of the corresponding components of quark momentum.

From Eq. (1) we find the following spectral angular distribution of photons radiated at one quark “reflection” from the QGP surface:

$$ \frac{dN_y}{d\omega d\Omega} = \int_{-p_{z0}/\sigma}^{p_{z0}/\sigma} dt \delta(n - v(t)) \theta[\omega < p(t)] \times \frac{e^2 a \sigma^{2/3}}{2\omega^{2/3}} \frac{\sin^{2/3} \phi(t)}{p^{2/3}(t)}, $$

(3)

where $v(t)$ is the quark velocity vector, $n$ is the unit vector along the photon momentum and

$$ p(t) = \left( p_x^2 + p_y^2 + p_z^2 \right)^{1/2}, \quad \sin \phi(t) = \left( p_x^2 + p_y^2 \right)^{1/2} / p(t). $$
Folding (3) with the flux of quark reaching the boundary and integrating over initial quark momenta we have:

\[
\frac{dN_y}{dS dt \omega^2 d\Omega} = \frac{g \langle e_q^2 \rangle \alpha}{(2\pi)^3 \sigma^{1/3}} \frac{3}{7} \omega^{2/3} \sin^{2/3} \phi_0 \times \\
\times \int_1^\infty d\xi \exp \left( -\frac{\omega}{T} \xi \right) \left( \xi^{7/3} - 1 \right),
\]  

(4)

where \( \langle e_q^2 \rangle = e_u^2 + e_d^2 \), \( e_u \) and \( e_d \) are the u- and d-quark charges, \( g = \text{spin x color} = 6 \) is the number of quark degrees of freedom, \( T \) is the plasma temperature, \( \phi_0 \) is the angle between the normal to QGP surface and the direction of emitted photons.
Theoretical framework

The total number of radiated photons can be obtained from Eq.(4), integrating over $d\omega$ and $d\Omega$ in the following form:

$$\frac{dN_\nu}{dSdt} = \Lambda \langle e_q^2 \rangle \alpha T^{11/3} \sigma^{-1/3}, \quad (5)$$

where $\Lambda = 3.12 g^2 2^{5/3} \Gamma^2 (4/3)/(2\pi)^2 \approx 1.2, \Gamma$ is the gamma function.

In the picture of employing a hydrodynamical scaling solution*, one has a cylindrically symmetric plasma volume (for central collisions) expanding in the longitudinal directions. Taking for the QGP an ideal gas equations of state, we have:

$$T = T_0 \left( \frac{\tau_0}{\tau} \right)^{1/3}, \quad (6)$$

where $T_0$ is the temperature at the proper time $\tau_0$ of hydrodynamic stage.

Theoretical framework

Volume emission of photons:
- “compton scattering of gluons”, \( gq \rightarrow \gamma g \)
- annihilation quark-antiquark pairs, \( \bar{q}q \rightarrow \gamma g \)

The functional distinction between the proposed mechanism and the "standard" volumetric one is mainly determined by the parameter that is just the dimensionless combination as:

\[
\left( \frac{r T_c^{1/3} \sigma^{1/3}}{\sigma} \right)^{-1}.
\]  

(7)

where \( r \) is the cylinder radius, \( T_c \) is the phase - transition temperature.

The interaction of quarks with the collective color field results in an intensive radiation of the magnetic bremsstrahlung type (synchrotron radiation) to be observed in the total rate.
The spectrum of synchrotron photons

From our “master” Eq. (4) we obtain:

\[
\frac{dN_\nu}{dS dt k_\perp dk_\perp} = \frac{g \langle e^2 \rangle \alpha k_\perp^{5/3}}{(2\pi)^3 \sigma^{1/3}} \frac{6}{7} \int_0^{2\pi} d\alpha \int_0^\infty dx \int_0^1 d\xi \times
\]

\[
\times \left( x^2 + \sin^2 \alpha \right)^{1/3} \left( \xi^{7/3} - 1 \right) \times \exp \left[ -\frac{k_\perp \left( 1 + x^2 \right)^{1/2}}{T} \xi \right].
\]  

(8)

In the limit \( k_\perp \gg T \) Eq. (8) simplifies considerably and the spectrum can be written as:

\[
\frac{dN_\nu}{dS dt k_\perp dk_\perp} = \Xi \langle e^2 \rangle \alpha \sigma^{-1/3} k_\perp^{-5/6} T^{5/2} \exp \left( -\frac{k_\perp}{T} \right),
\]  

(9)

where \( \Xi = 0.52 g 2^{1/6} \Gamma^2 (5/6) \pi^{-5/2} / \Gamma(5/3) \approx 0.29 \) is the result of averaging over angles.
The spectrum of synchrotron photons

Integration of Eq.(9) over the QGP surface, taking into account evolution, gives:

\[
\frac{d^2N_y}{2\pi k_\perp dk_\perp dy} = \int \frac{dN_y}{dS dt k_\perp dk_\perp} r \tau d\tau =
\]


\[
= \Xi \langle e_q^2 \rangle \alpha \times 3 \left( \tau_0 T_0^3 \right)^2 r k_\perp^{-13/3} \sigma^{-1/3} \left[ \Gamma \left( \frac{7}{2}, \frac{k_\perp}{T_0} \right) - \Gamma \left( \frac{7}{2}, \frac{k_\perp}{T_c} \right) \right]
\]

with

\[
\Gamma \left( n, \alpha_1 \right) - \Gamma \left( n, \alpha_2 \right) = \int_{\alpha_1}^{\alpha_2} dt t^{-1} e^{-t}
\]

and \( y \) is the rapidity.
The spectrum of synchrotron photons

Spectrum (10) is presented for the different parameters $T_\circ$ and $\tau_\circ$ at the transverse size of QGP system fixed as $r=10$ fm ($T_c = 0.2$ GeV and $\sigma=0.2$ GeV$^2$).
Spectrum (10) is presented for the different parameters $T_0$ and $\tau_0$ at the transverse size of QGP system fixed as $r \approx 7$ fm ($T_c = 0.2$ GeV and $\sigma = 0.2$ GeV$^2$).
The peculiarities in angular distributions

The synchrotron radiation is characterized by a high degree of photon polarization.

When the photon polarization is taken into account, the equations corresponding to:

\[
\frac{dN_v}{d\omega dt} = \frac{1}{2} e_q^2 \alpha \omega^{-2/3} \left( \sigma \sin\phi / E \right)^{2/3}, 0 < \omega < E,
\]

have the form:

\[
\frac{dN_1}{d\omega dt} = \frac{1}{4} \frac{dN_v}{d\omega dt}, \quad \frac{dN_2}{d\omega dt} = \frac{3}{4} \frac{dN_v}{d\omega dt}, \quad \frac{dN_l}{d\omega dt} = \frac{1}{2} \frac{dN_v}{d\omega dt}
\]

\( l=1 \rightarrow \) right-handed circularly polarized photon, \( l=-1 \rightarrow \) left-handed circularly polarized photon; \( N_1 \rightarrow \) linear polarization of the photon along the vector \( e_1 = \frac{\sigma \times k}{|\sigma \times k|} \), \( N_2 \rightarrow \) linear polarization of the photon along the vector \( e_2 = \frac{e_1 \times k}{|e_1 \times k|} \).
The peculiarities in angular distributions

The presence of a photon polarization is closely related to the geometrical feature of the QGP volume, over whose surface we should integrate.

In a collision of relativistic heavy nuclei there is a special direction -- the collision axis.

- In the picture of employing a hydrodynamical scaling solution [J.D. Bjorken, Phys. Rev. D, 140, 1983], one has a cylindrically symmetric plasma volume (for central collisions) expanding in the longitudinal directions and the calculations for the final polarization can be done in the explicit form:

\[ P = \left( \frac{dN_2}{d\omega dt} - \frac{1}{2} \frac{dN_y}{d\omega dt} \right) \bigg/ \frac{dN_y}{2d\omega dt} = \frac{1}{2} \]  

The primary degree of polarization reduces to about 20% for a plasma with a cylindrically symmetric volume after the transparent, but laborious calculations [V.V. Goloviznin, G.M. Zinov'ev, and A.M. Snigirev, Yad Fiz., 1826 1988, Sov. J. Nucl. Phys., 1099, 1988].
The peculiarities in angular distributions

We have found that the lepton distribution in the radiation angle takes the form:

\[
\frac{dN}{dtd\Omega_1} = \frac{\alpha n}{2\pi k^0} \int \frac{p^2 dp}{p_1^0(k^0 - p_1^0)} \delta[f(p)]
\]

(13)

\[
x \left[ \frac{k^2 + 2\mu^2}{3} - \frac{2}{3} \delta p^2 \sin^2\theta_1 \cos 2\phi_1 \right]
\]

at the decay of massive photons with the four-momentum \( k \) into a lepton pair with the four-momenta of the lepton \( p_1 \) and antilepton \( p_2 \).

Deriving Eq. (13) we define:

- \( n(1+\delta)/3 \) as the photon number of the states with polarization \( e_1 \);
- \( n(1-\delta)/3 \) as the photon number of the states with polarization \( e_2 \);
- \( n/3 \) as the same with polarization \( e_3 \).
The peculiarities in angular distributions

Choosing the reference frame with the z axis directed along the three-vector $k$ and the x and y axes tallying with the directions of $e_1$ and $e_2$:

$$e_1 = \{0,1,0,0\}, \quad e_2 = \{0,0,1,0\}$$

$$e_3 = \{k^0, 0, 0, |k|\}, \quad k = \{k^0, 0, 0, |k|\},$$

$$p_1 = \{\sqrt{p^2 + \mu^2}, psin\theta_1 cos\phi_1, psin\theta_1 sin\phi_1, pcos\theta_1\}.$$  

- The photons are unpolarized or have longitudinal polarization (along the vector $e_3$) → the angular lepton distribution is independent of the azimuthal angle $\phi_1$;

- A massive photon has transverse (in the three-dimensional space) polarization ($\delta$ is not zero) → a characteristic dependence on the azimuthal angle $\phi_1$ takes place.
The peculiarities in angular distributions

In our case the intermediate photons could be considered up to the masses $\sqrt{k^2} \approx \sqrt{\sigma} = 0.45$ GeV as having a small virtuality and their properties are quite close to real photons [V.G. Zhulego, V.N. Rodionov, and A.I. Studenikin, Yad Fiz., 524, Sov. J. Nucl. Phys., 306, 1982].

It means these photons are transversely polarized with practically the same degree of polarization $\delta$ about 20% as calculated for real photons at a cylindrically symmetric geometry.

the “bremsstrahlung" leptons could be identified by measuring their angle anisotropy that is absent in the "standard" volumetric mechanism.
The peculiarities in angular distributions

*In the transverse (x-y) plane (the beam is running along (z)-axis) the direction of this normal (emitted photons) is determined by the spatial azimuthal angle $\phi_s = \tan^{-1}(y/x)$ as:

$$\tan(\phi_s) = (r_x/r_y)^2 \tan(\phi_s),$$  \hspace{1cm} (14)

where $r_x = r(1-\varepsilon)$, $r_y = r\sqrt{1-\varepsilon^2}$, $\varepsilon = b/2r$, $b$ is the impact parameter, $r$ is the radius of the colliding nuclei.

In this case the “mean normal” is not zero and is equal to:

$$\int_0^{2\pi} d\phi_s \cos(2\phi_s)/(2\pi) = \varepsilon.$$  \hspace{1cm} (15)

This means that the photon azimuthal anisotropy, characterized by the second Fourier component:

$$v_y^2 = \frac{\int d\phi_y \cos(2\phi_y)(dN^y/d\phi_y)}{\int d\phi_y (dN^y/d\phi_y)} \neq 0 \propto \varepsilon$$  \hspace{1cm} (16)

Conclusions

- the synchrotron radiation should be necessary taken into consideration at a description of global photon and dilepton data;
- for the most central collisions (where the relative effect due to the synchrotron radiation is minimal compared with other volume sources because of a size factor $1/r$) the boundary photons contribute to the experimentally measured rate of direct photons at the level of 10%;
- to select this new uncommon mechanism of radiation unambiguously we suggest to study the specific noticeable anisotropy in the angle distribution of leptons with respect to the three-momentum of the pair. The origin of this anisotropy is rooted in the existence of a characteristic direction in the field where the quarks are moving;
- besides as another distinctive feature of the synchrotron radiation will be nonisotropic for the noncentral collisions because the photons are dominantly emitted around the direction fixed by a surface normal, which "mean" value is not zero.
Thank you for attention!