Jet quenching parameter in an expanding QCD plasma

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Hard Probe 2018, Aix-Les-Bains, Savoie, France
Outline

1. Motivations

2. Rethink of $\hat{q}$ in heavy-ion collisions

3. Longitudinally boost-invariant $\hat{q}$

4. Summary and Perspectives
1. $\hat{q}$ measured as $p_\perp$-broadening

a dijet in a PbPb collision at $\sqrt{s_{NN}} = 2.76$ TeV

1. $\hat{q}$ measured as $p_\perp$-broadening

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involves both vacuum and medium-induced radiation

1. $\hat{q}$ measured as $p_\perp$-broadening

**Vacuum:** $-\frac{\alpha_s(C_F+C_F)}{\pi} \ln^2(p_T x_T)$

Sun, Yuan and Yuan, Phys. Rev. Lett. 113, no. 23, 232001 (2014)
1. $\hat{q}$ measured as $p_\perp$-broadening

Vacuum: $-\frac{\alpha_s(C_F+C_F)}{\pi} \ln^2(p_T x_T) - \frac{\alpha_s(C_F+C_F)}{\pi} \ln(p_T x_T) \ln(R^2)$

Sun, Yuan and Yuan, Phys. Rev. Lett. 113, no. 23, 232001 (2014)
1. $\hat{q}$ measured as $p_\perp$-broadening

Static medium (Brick):

$$-2\frac{1}{4} x_\perp^2 \hat{q} \left[ 1 + \frac{\alpha_s N_c}{2\pi} \ln^2 \left( L/l_0 \right) \right]$$

Liou, Mueller, BW (2013); Blaizot, Mehtar-Tani (2014); Iancu (2014)
1. \( \hat{q} \) measured as \( p_{\perp} \)-broadening

Factorization of vacuum & medium double logs 

*illustrated in DIS off big nuclei in*

1. $\hat{q}$ measured as $p_{\perp}$-broadening

$\hat{q}$ is encoded in the broadening of jets in $\phi$ (and $\eta$)

Shu-Yi Wei’s talk

For some more elaborated discussions: Chen, Qin, Wei, Xiao and Zhang (2017); Stasto, Wei, Xiao, Yuan (2018); Luo, Cao, He and Wang (2018); Gyulassy, Levai, Liao, Shi, Yuan, Wang (2018); Tannenbaum (2018) ···
2.1 From Brick to reality: mission complicated

Jet: collimated spray of “particles” in $\eta$-$\phi$ space
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2.1 From Brick to reality: mission complicated

The same event seen in space
2.1 From Brick to reality: mission complicated

\[ p_\perp^2 = \hat{q} L \text{ for each jet} \]

\( \perp \) directions are different for different jets

Complication: different \( \perp \) directions (and \( \hat{q} \)) to \((\phi, \eta)\)
2.1 From Brick to reality: mission complicated

9 jet event in pp

Things may get more complicated!

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2.2 A shortcut: choose new coordinates

Focus on final-state interaction based on factorization
2.2 A shortcut: choose new coordinates

Calculate $\Delta \phi$ and $\Delta \eta$ of each jet in medium.
2.2 A shortcut: choose new coordinates

Consider one parton product at $\tau \approx 0$
2.2 A shortcut: choose new coordinates

choose a new coordinate:

$p^\mu = p_0 \hat{n}^\mu + p_T \hat{\tau}^\mu - p_0 \hat{\theta}^\mu - p^\phi \hat{\phi}^\mu$
$x^\mu = \tau \hat{n}^\mu + x^T \hat{\tau}^\mu - x^\theta \hat{\theta}^\mu - x^\phi \hat{\phi}^\mu$

$p_0^\mu \rightarrow p^\mu \text{ at } \tau = L, \text{ decoupled from bulk matter}$
2.2 A shortcut: choose new coordinates

In new coordinates, \( p_\perp = p_T(\Delta \phi, \Delta \eta) \).
2.3 A byproduct: redefinition of $\hat{q}$

1. **QFT in new coordinates:**
   
   $$p \cdot x \approx p^n x^T + p^\tau x^n - p_\perp \cdot x_\perp$$

   $$\hat{n}^\mu = \frac{p_0^\mu}{p_T} = (\cosh \eta_0, \cos \phi_0, \sin \phi_0, \sinh \eta_0),$$
   $$\hat{\tau}^\mu = (\cosh \eta_0, 0, 0, \sinh \eta_0),$$
   $$\hat{\phi}^\mu \equiv (0, \sin \phi_0, -\cos \phi_0, 0),$$
   $$\hat{\eta}^\mu \equiv (-\sinh \eta_0, 0, 0, -\cosh \eta_0).$$

2. **Correspond to Brick model**

<table>
<thead>
<tr>
<th></th>
<th>$(x^n, x^T, x^\eta, x^\phi)$</th>
<th>$(t, x, y, z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy loss</td>
<td>$p^n = p_T$</td>
<td>$p^0 = E$</td>
</tr>
<tr>
<td>Time</td>
<td>$x^T = \tau = \sqrt{t^2 - z^2}$</td>
<td>$x^0 = t$</td>
</tr>
<tr>
<td>Transverse</td>
<td>$p_\perp = (p_\eta, p_\phi) = p_T(\Delta \eta, \Delta \phi)$</td>
<td>$(p^x, p^y)$</td>
</tr>
</tbody>
</table>

3. **$\hat{q}$ in new coordinates**

   $$\hat{q} = \frac{d}{d\tau} \left\langle p_\perp^2 \right\rangle = p_T^2 \frac{d}{d\tau} \left\langle \Delta \phi^2 + \Delta \eta^2 \right\rangle.$$

4. **Generality**

   Just like LC/Milne coordinates, all formalisms (BDMPSZ, GLV, high-twist) can be adapted into this new frame.

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2. Rethink of $\hat{q}$
3.1 Longitudinal boost-invariance

In a Longitudinally boost-invariant plasma:

\[
\frac{dN}{d^2 p_\perp} \approx \frac{1}{\pi Q_0^2(L)} e^{-\frac{p_\perp^2}{Q_0^2(L)}} \Leftrightarrow \frac{dN}{d\Delta \phi d\Delta \eta} \approx \frac{p_T^2}{\pi Q_0^2(L)} e^{-\frac{p_T^2(\Delta \phi^2 + \Delta \eta^2)}{Q_0^2(L)}}
\]

where \( \hat{q} \) depends only \( \tau \) and \( \Delta \phi \) and \( \Delta \eta \) are also longitudinally boost-invariant, that is,

\[
Q_0^2(L) \equiv \int_{\tau_0}^{L} d\tau \hat{q}_0(\tau)
\]

\[
\hat{q}_0(\tau) \sim \hat{q}_0(\tau_0) \left( \frac{T(\tau)}{T_0} \right)^3 \sim \hat{q}_0(\tau_0) \left( \frac{\tau_0}{\tau} \right)^\beta.
\]

Here, \( \frac{dN}{d^2 p_\perp} = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ip_\perp \cdot r_\perp} S(r_\perp) \).

3.2 Radiative correction to $\hat{q}$

At next leading order:

$$
\delta S(r) \simeq -\frac{\alpha_s N_c r^2}{4} \Re \int \frac{d\omega}{\omega^3} \int_{\tau_0}^L d\tau_2 \int_{\tau_0}^{\tau_2} d\tau_1 
\times (\nabla_{B_2} \cdot \nabla_{B_1})^2 [G(B_2, \tau_2; B_1, \tau_1) - G_0(B_2, \tau_2; B_1, \tau_1)] \bigg|_{B_1=B_2=0},
$$

where $\omega = k_T$, and

$$
G(B_2, \tau_2; B_1, \tau_1) \equiv \frac{i\omega}{2\pi D(\tau_2, \tau_1)} e^{\frac{-i\omega}{2D(\tau_2, \tau_1)}} [c_1 B_1^2 + c_2 B_2^2 - 2 B_2 \cdot B_1],
$$

with $c_1 \equiv c(\tau_2, \tau_1)$, $c_2 \equiv c(\tau_1, \tau_2)$ and the following definitions for the functions $D(\tau_2, \tau_1)$ and $c(\tau_2, \tau_1)$:

$$
D(\tau_2, \tau_1) = \pi \nu \sqrt{\tau_1 \tau_2} \left[ J_\nu (2\nu \Omega_1 \tau_1) Y_\nu (2\nu \Omega_2 \tau_2) - J_\nu (2\nu \Omega_2 \tau_2) Y_\nu (2\nu \Omega_1 \tau_1) \right],
$$

$$
c(\tau_2, \tau_1) = \frac{\pi \nu \sqrt{\tau_1 \tau_2 \Omega_2}}{\sin(\pi \nu)} \left[ J_{\nu-1} (2\nu \Omega_2 \tau_2) J_{-\nu} (2\nu \Omega_1 \tau_1) + J_{-\nu} (2\nu \Omega_2 \tau_2) J_{\nu} (2\nu \Omega_1 \tau_1) \right].
$$
3.2 Radiative correction $\hat{q}$

Resummation of medium double logs:

$$\hat{q}(\tau) = \hat{q}_0(\tau) \frac{I_1(2 \sqrt{\bar{\alpha}} \ Y)}{\sqrt{\bar{\alpha}} \ Y} \approx \hat{q}_0(\tau) \left\{ 1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{\tau}{\lambda(\tau)} \right\}$$

where $Y = \ln(\tau/\lambda(\tau))$ and $\lambda = 1/T(\tau)$ is the thermal wavelength.

In contrast with Brick, it is quasi-local.
Summary & Perspectives

**Summary:** \( \hat{q} \) can be defined as broadening in \( \phi-\eta \)

**New coordinates:**
a bridge between Brick and experiments

**Perspectives:** pQCD (EFT) calculation of \( \gamma/Z/W \)-jet correlation
(with Chien & Shao)