Effects of drag induced radiation and multi-stage evolution on heavy quark energy loss

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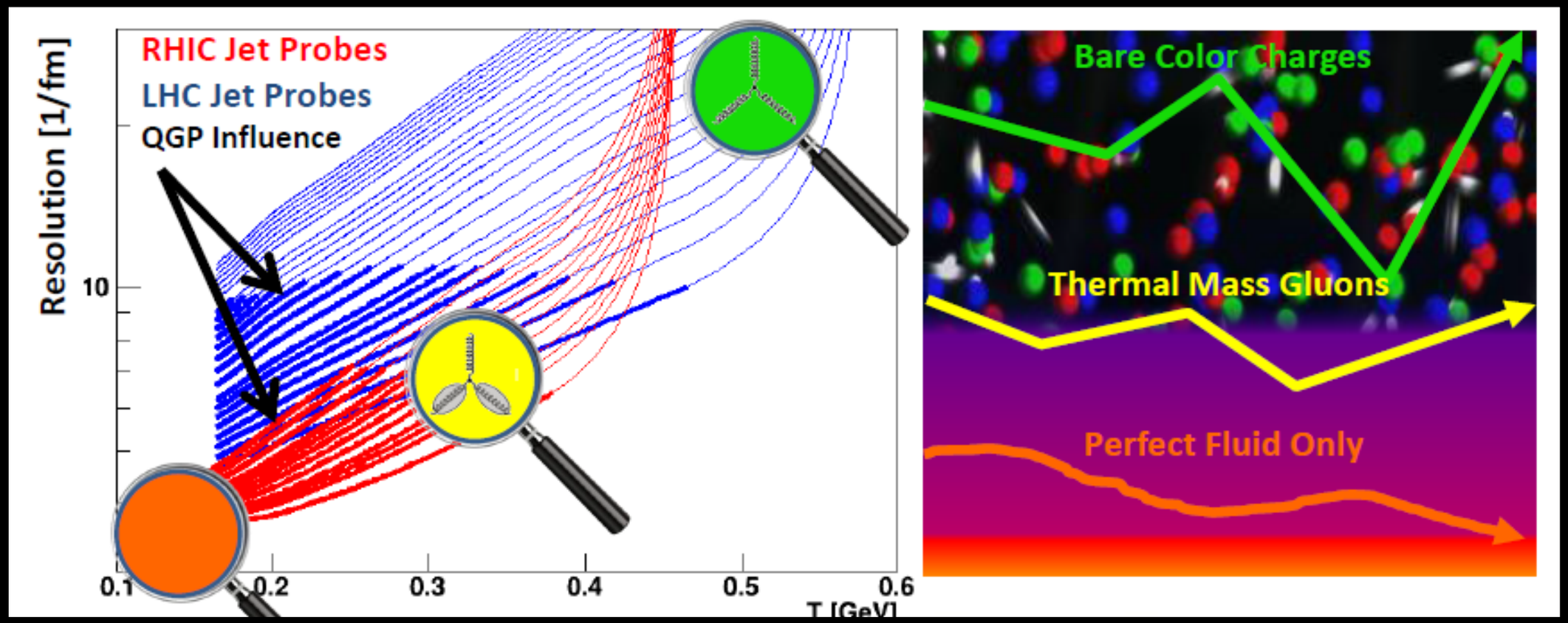
Scale dependence: of probe and target

The medium looks different at different length scales

The probe behaves differently at different scales

Need a comprehensive tool to study QGP with jets

From the 2015 LRP



Jets in a medium, grand picture

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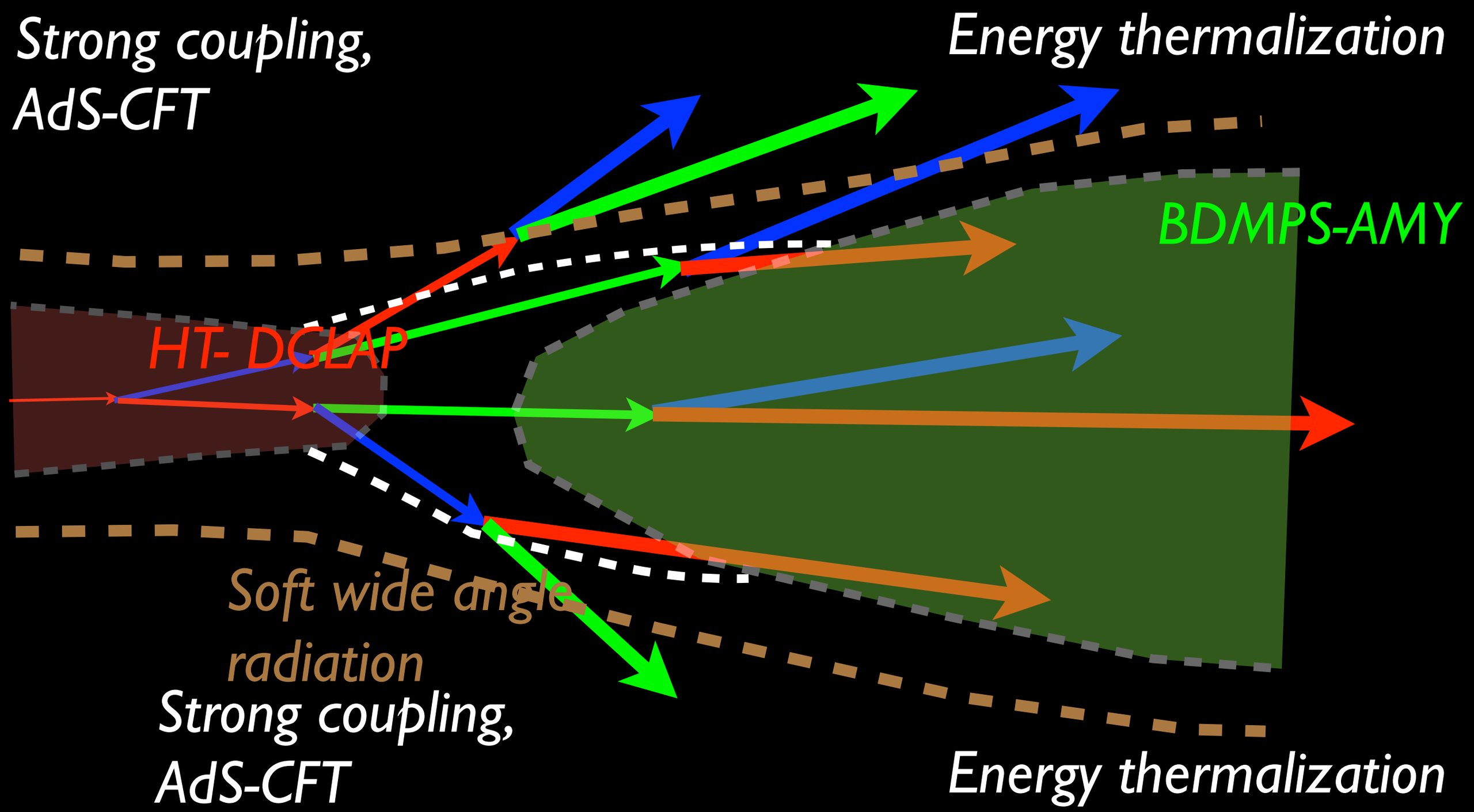
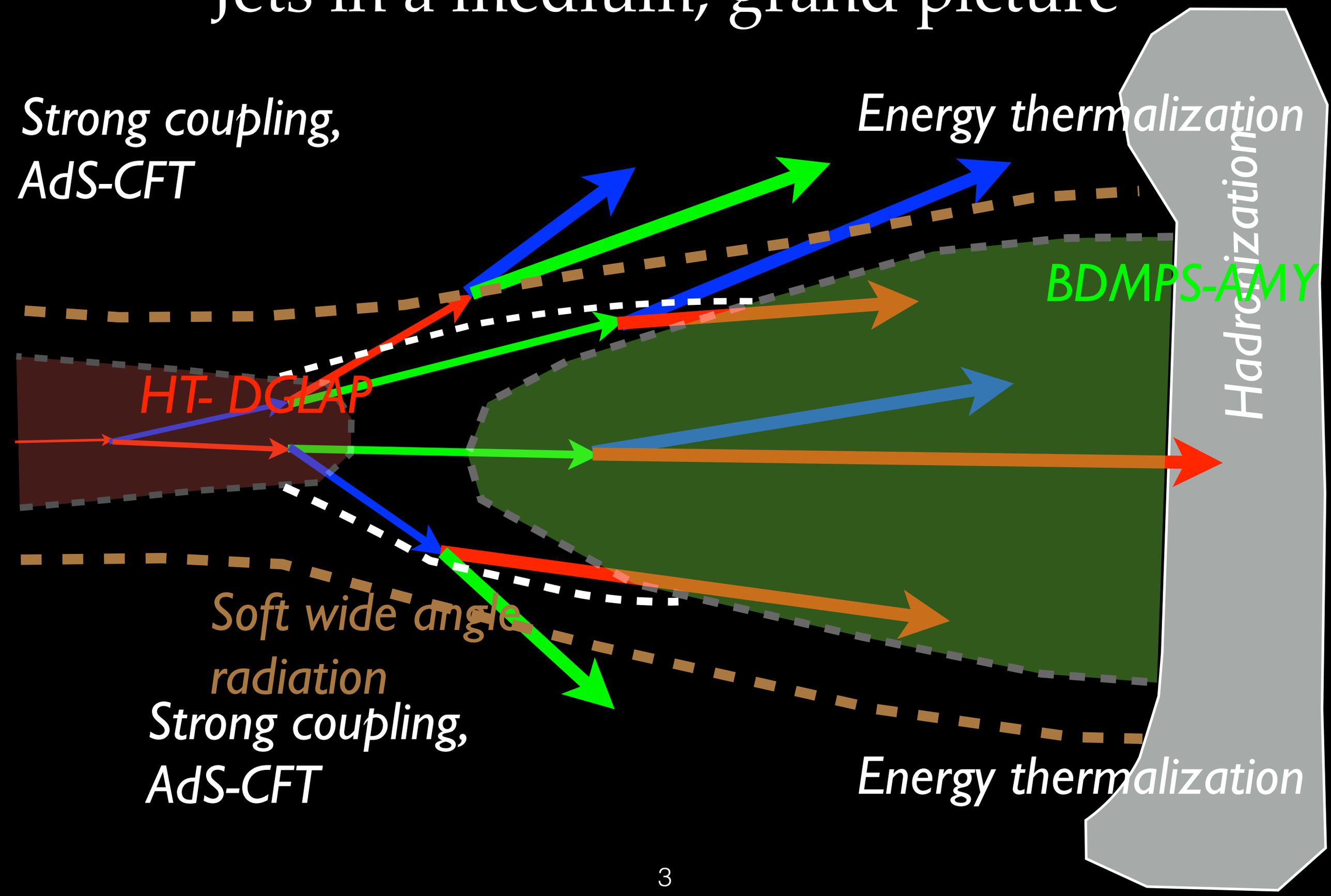


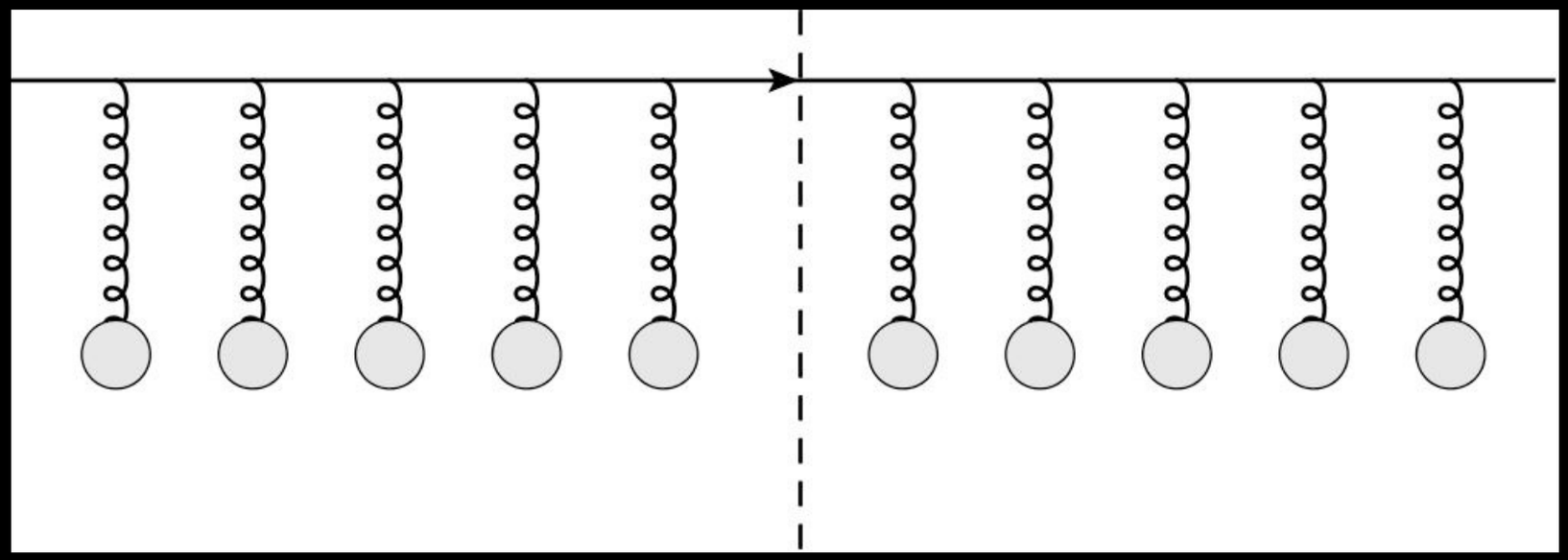
Jets in a medium, grand picture



Scales of a hard parton in a medium

A parton in a jet shower, has momentum components

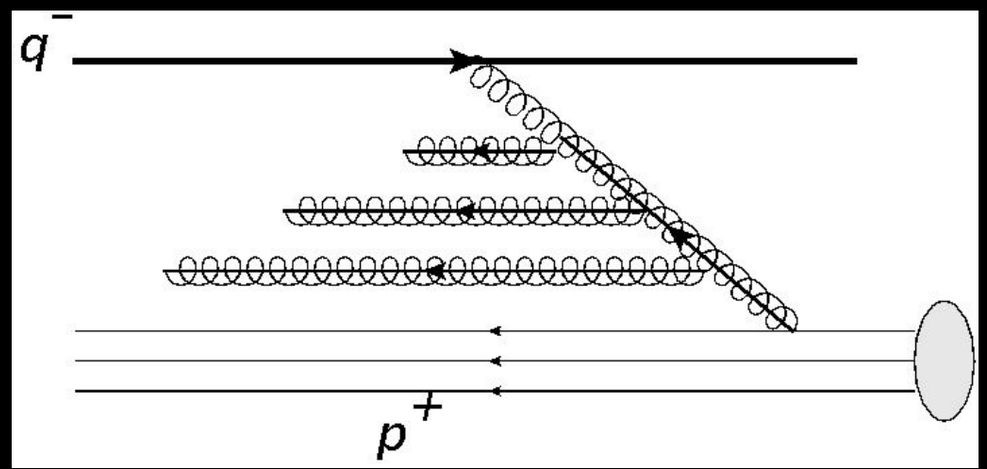
$q = (q^-, q^+, q_T) = (1, \lambda^2, \lambda)Q$, Q: Hard scale, $\lambda << 1$, $\lambda Q >> \Lambda_{QCD}$



hence, gluons have

$$k_\perp \sim \lambda Q, \qquad k^+ \sim \lambda^2 Q$$

Called Glauber gluons

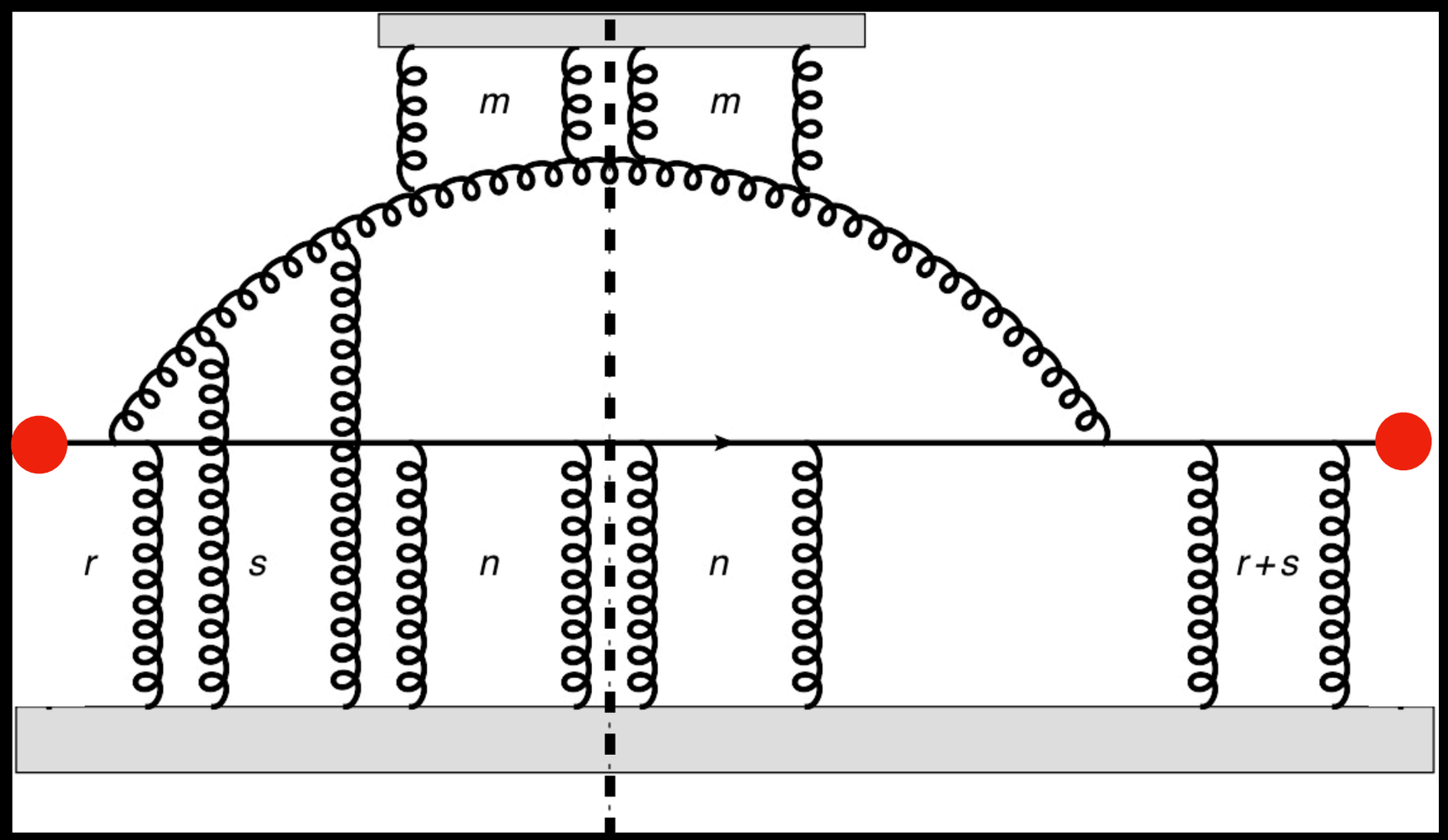


At large virtuality

The transverse momentum is resolved at the scale Q >> $\hat{q}$L
Rare hard scatterings.

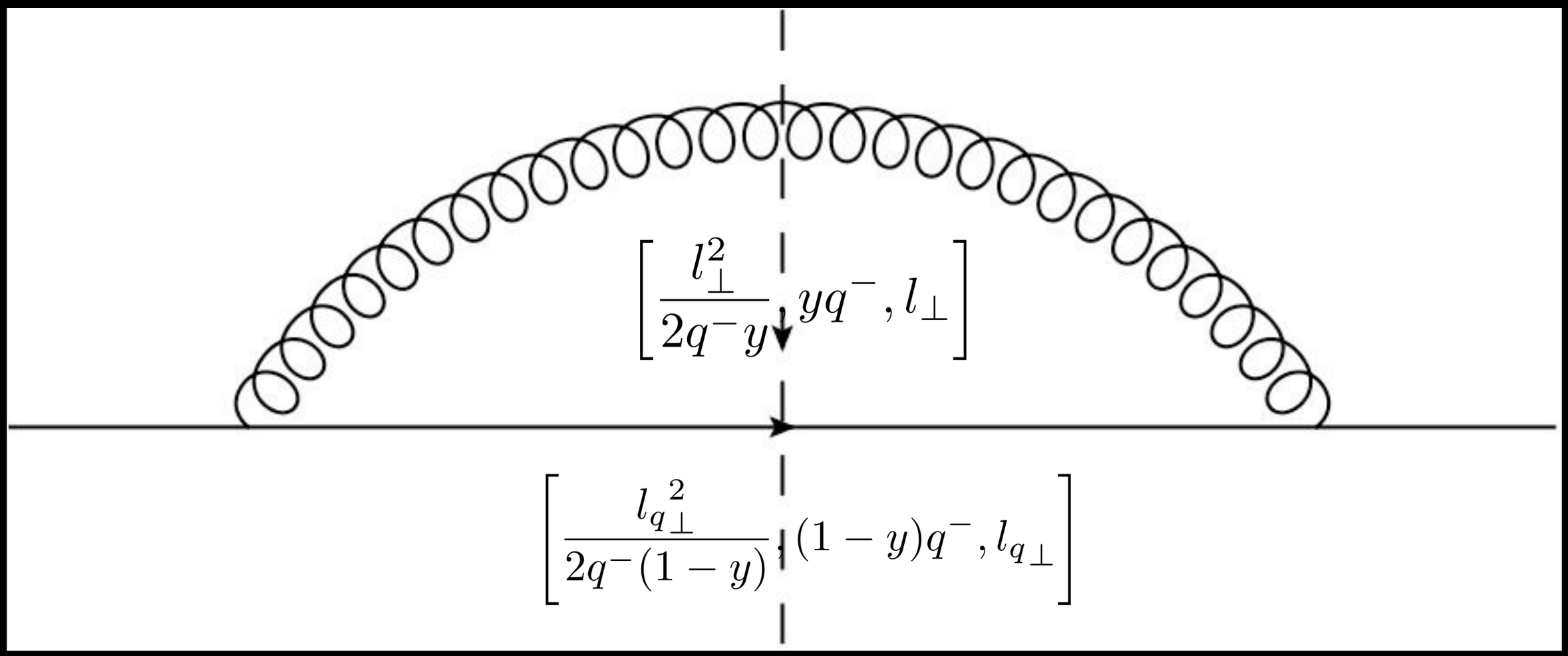
$$Q^2 \sim l_\perp^2 \sim k_\perp^2$$

$$l_\perp^2 \gg \langle k_\perp^2 \rangle \sim \hat{q} L$$



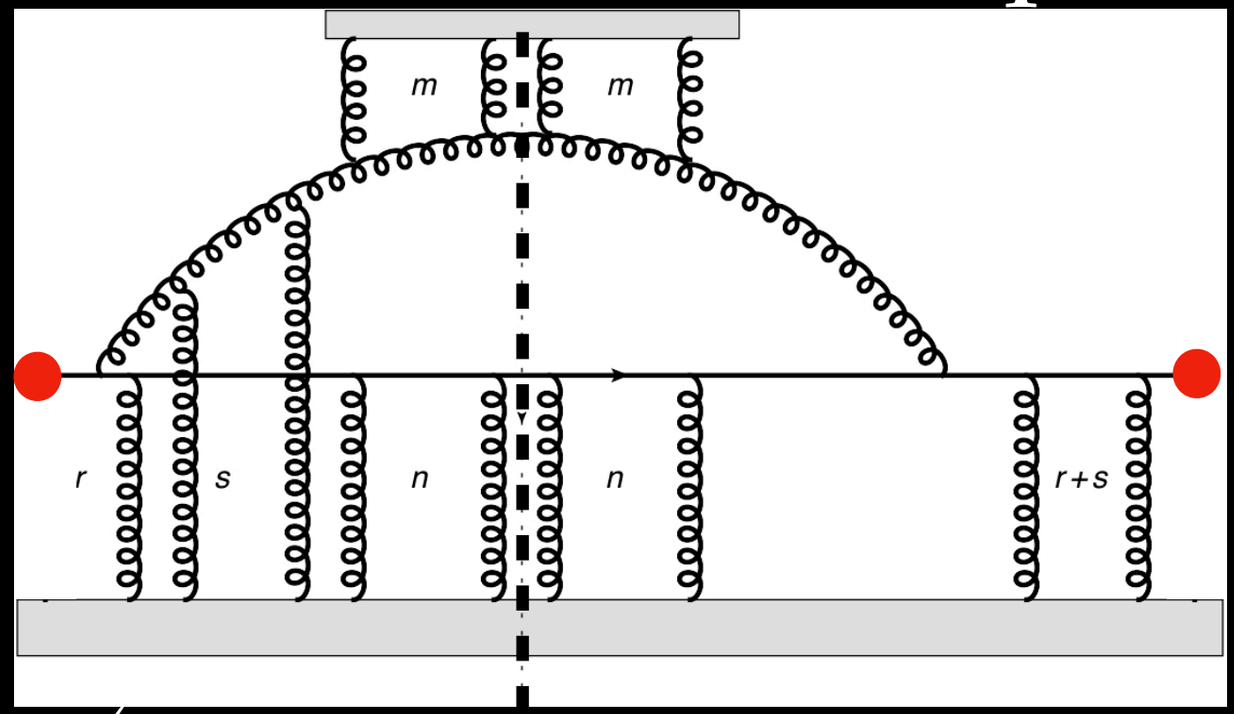
A.M. Phys.Rev. D85 (2012) 014023

Case of no scattering



$$\sim \frac{\alpha_s C_F}{2\pi} \int dy d^2 l_\perp d^2 l_{q\perp} P(y) \frac{l_\perp \cdot l_\perp}{l_\perp^2 l_\perp^2} \delta^2(l_\perp + l_{q\perp})$$

One emission from multiple scattering



$$\int dl_\perp dl_{q\perp} dy \text{ C.F. } \delta^2\left(q_\perp + l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{j=1}^{r} p_\perp^j - \sum_{l=1}^{m} k_\perp^l - \sum_{k=1}^{n} p_\perp^k\right)$$

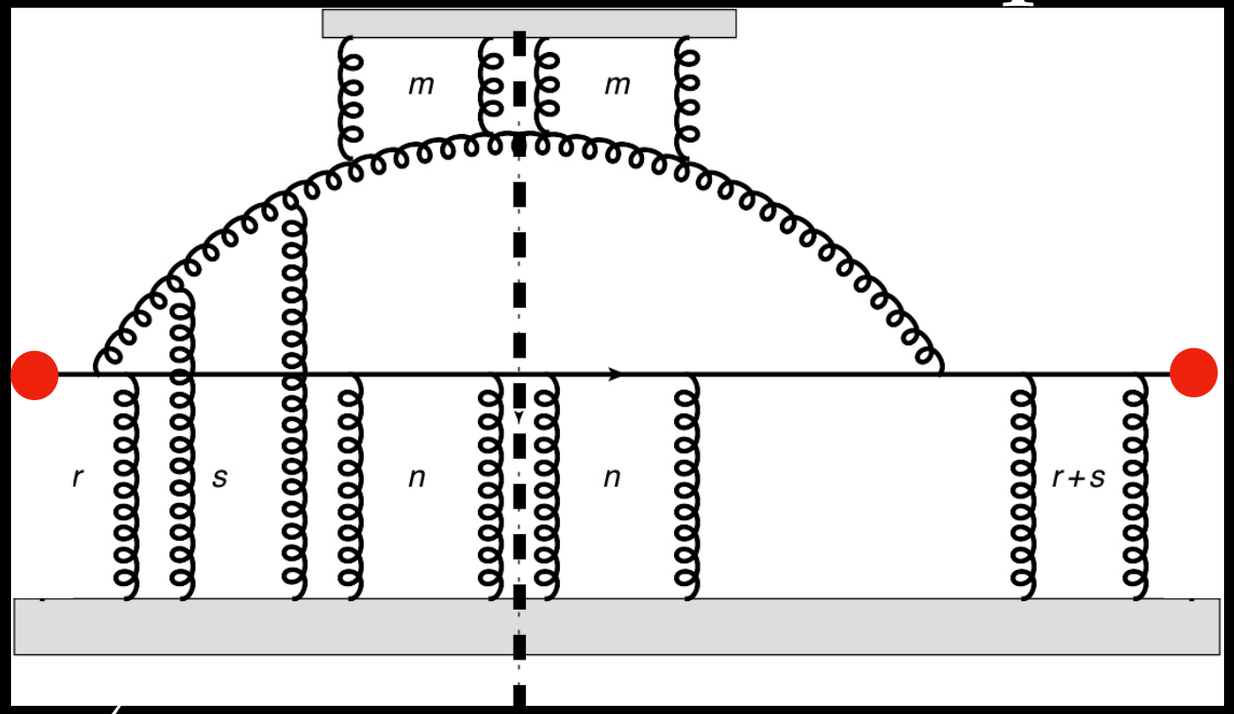
$$\frac{l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2} \cdot \frac{l_\perp - y\sum_{i=1}^{r+s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - y\sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2}$$

$$\prod_{i=1}^{N} \int dy_i^- \frac{\int d^3\delta y_i \rho \langle p|A^+(y_i^- + \delta y_i^-, 0)A^+(y_i^-, -\delta y_\perp^i)|p\rangle}{2p^+(N_c^2-1)} e^{ik_\perp^i \delta y_\perp^i}$$

$$\left[\theta(\zeta_I^- - y_E^-)\left\{e^{-ip^+x_L y_E^-} - e^{-ip^+x_L\zeta_I^-}\right\} - \theta(\zeta_I^- - y_I^-)e^{-ip^+x_L y_I^-} - \theta(y_I^- - \zeta_I^-)e^{-ip^+x_L\zeta_I^-}\right]$$

$$\left[\theta(\zeta_C^- - y_0^-)\left\{e^{ip^+x_L y_0^-} - e^{ip^+x_L\zeta_C^-}\right\} - \theta(\zeta_C^- - y_C^-)e^{ip^+x_L y_C^-} - \theta(y_C^- - \zeta_C^-)e^{ip^+x_L\zeta_C^-}\right] \quad \ldots$$

One emission from multiple scattering



$$\int dl_\perp dl_{q\perp} dy \text{ C.F. } \delta^2\left(q_\perp + l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{j=1}^{r} p_\perp^j - \sum_{l=1}^{m} k_\perp^l - \sum_{k=1}^{n} p_\perp^k\right)$$

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if l_T >> <k_T>, can expand in ratio

$$\frac{1}{l_\perp^2} - \frac{(1-y+y^2)\left(\sum_{i=1}^{s} k_\perp^i\right)^2}{l_\perp^4} - \frac{\left(\sum_{i=1}^{m} k_\perp^i\right)^2}{l_\perp^4} + 2(1+y^2)\frac{\left(l_\perp \cdot \sum_{i=1}^{s} k_\perp^i\right)^2}{l_\perp^6} + 4\frac{\left(l_\perp \cdot \sum_{i=1}^{m} k_\perp^i\right)^2}{l_\perp^6}.$$

$$\left[\theta(\zeta_I^- - y_E^-)\left\{e^{-ip^+x_L y_E^-} - e^{-ip^+x_L \zeta_I^-}\right\} - \theta(\zeta_I^- - y_I^-)e^{-ip^+x_L y_I^-} - \theta(y_I^- - \zeta_I^-)e^{-ip^+x_L \zeta_I^-}\right]$$
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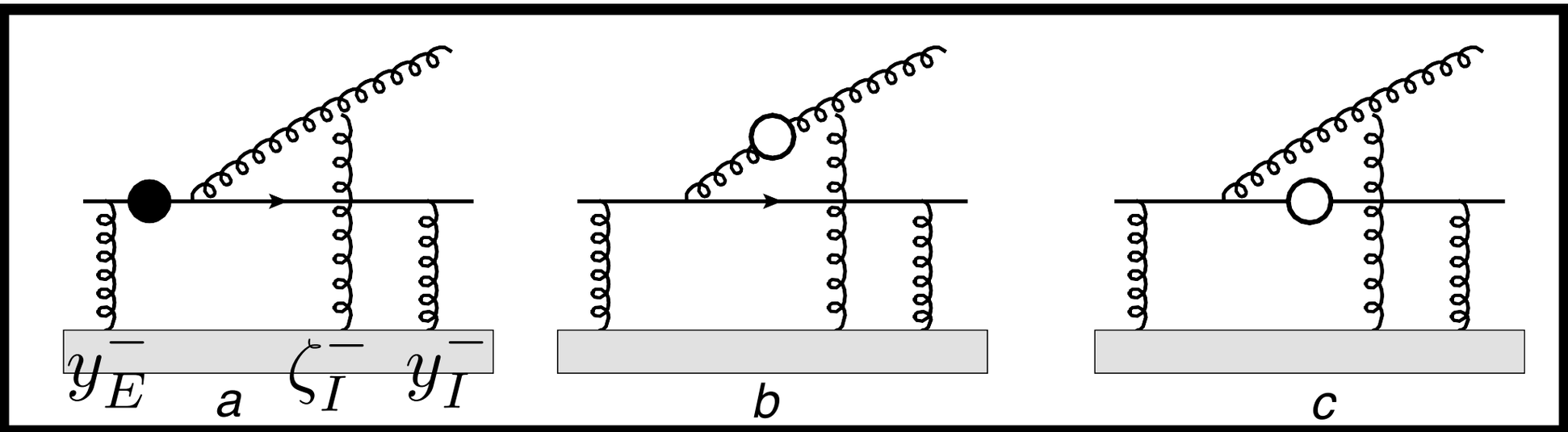
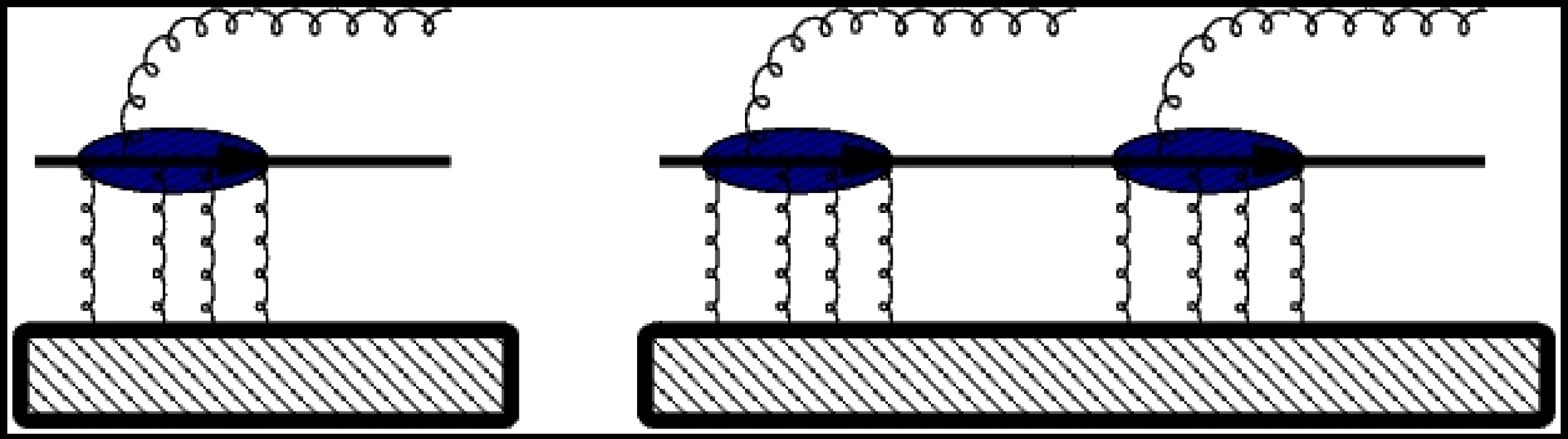


This is what we mean by vacuum medium interference

Can show that this reduces to the case of single scattering induced single emission as in Wang and Guo Nucl.Phys. A696 (2001) 788-832.

This needs to be repeated as long as $Q \gg \hat{q}L$



- Usual assumption, multiple emissions are independent!
- The reason for this depends on your approximation scheme
- The method of repeating/resumming multiple emissions depends on the scale involved.

Resum with DGLAP

Per radiation, with or without scattering

$$\frac{\alpha_S}{2\pi}\int\frac{dl_\perp^2}{l_\perp^2}\int dyP(y)\left[1+\int_0^{\tau_f}d\zeta\frac{\hat{q}}{l_\perp^2}\left(PhaseFactors\right)\right]$$

No divergence, yields finite term

$$\tau_f = \frac{2Ey(1-y)}{l_\perp^2}$$

Resum into Fragmentation function

$$\frac{\alpha_S}{2\pi}\int dyP(y)\left[\frac{1}{\epsilon}-\gamma+\log(\mu^2/Q^2)+\hat{q}\#\right]$$

Or, simulate with Sudakov.

As virtuality comes down

- High energy partons with virtuality $Q = \hat{q}\,\tau$
- Now partons with virtuality $\mu^2 = \lambda^2 Q^2$ have lifetime $1/(\lambda^2 Q)$
- Over this long lifetime, the parton "can" endure several scatterings
- BDMPS regime
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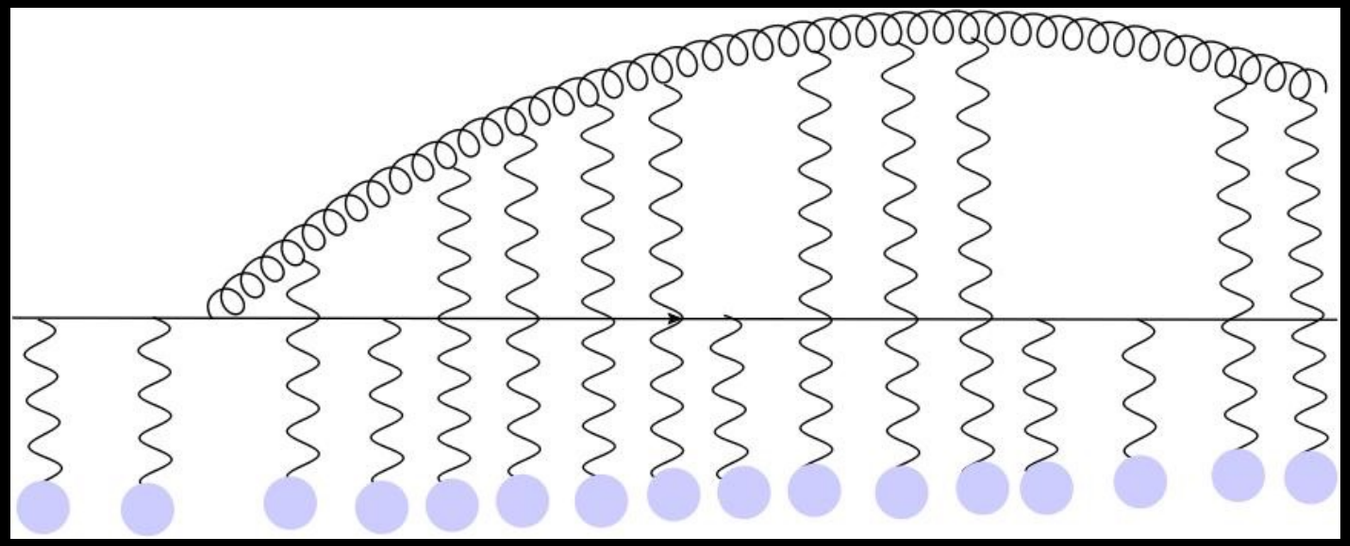


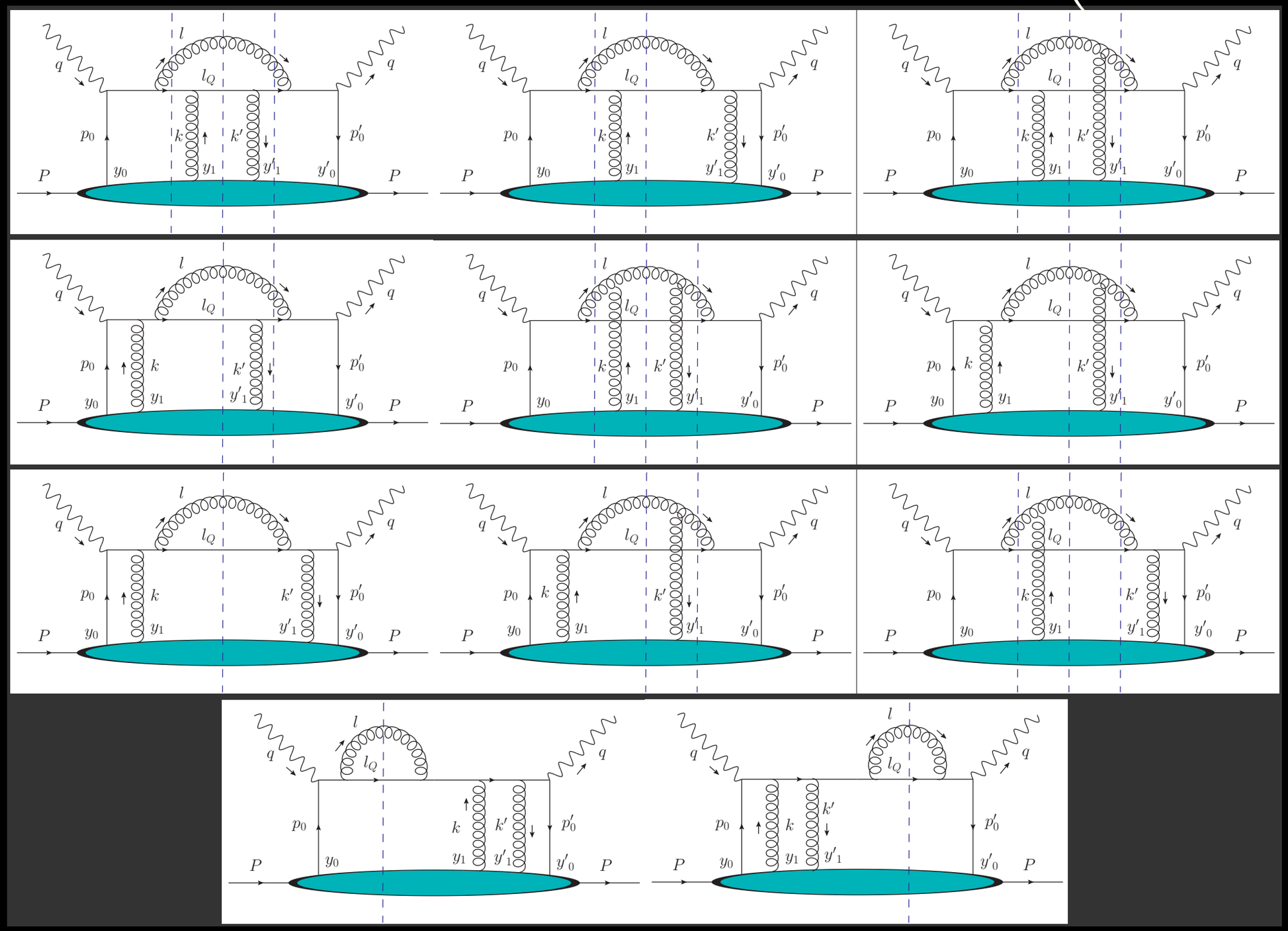
Heavy quark at intermediate energy

- Heavy quarks where p >> M behave like light quarks
- Mass effect is evident only when p ~ M.
- Incorporate this in the same power counting
- We assume $p, M \sim \sqrt{\lambda} Q$
- $\sqrt{\lambda} Q$ implies smaller than Q, but not small enough to neglect
- Typical numbers Q ~ 100GeV, λQ ~ 1 GeV, $\sqrt{\lambda} Q$ ~ 10 GeV

Raktim Abir, Gagan Deep Kaur, A.M. PRD 90, 114026 (2014)

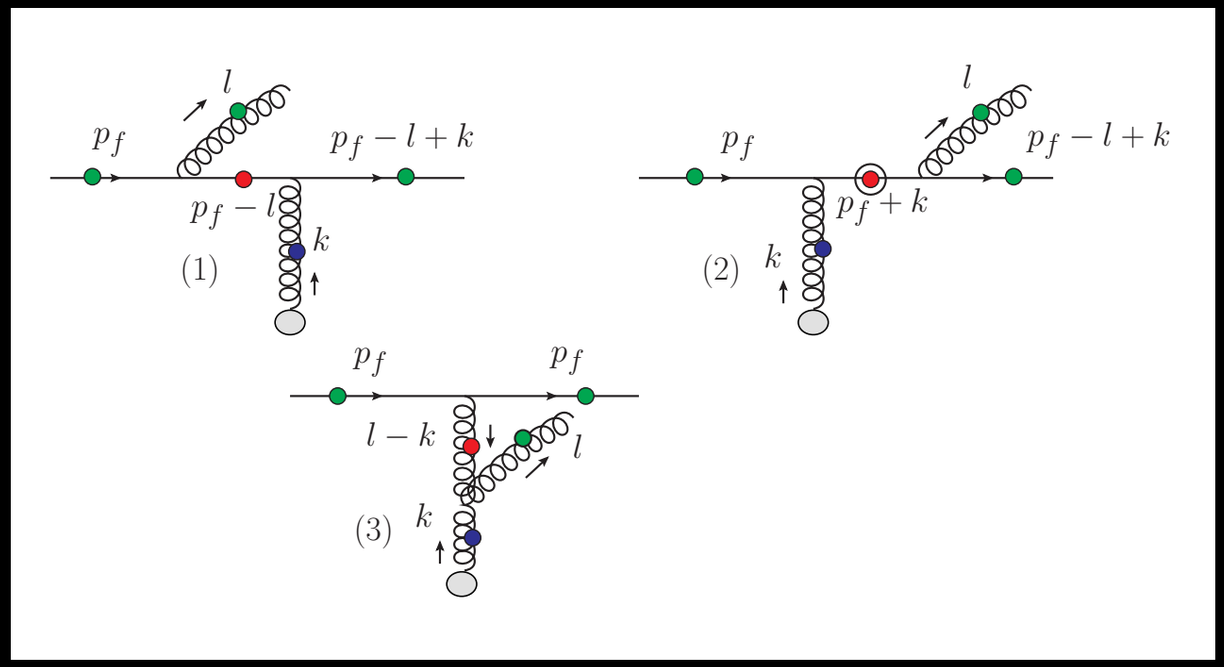
Intermediate energy heavy quark e-loss

Heavy quark momentum $p \equiv (p^+, p^-, p_\perp) \sim \left(\sqrt{\lambda}Q, \sqrt{\lambda}Q, 0\right)$



Raktim Abir, A.M. PRC **94**, 054902 (2016)

Why are these diagrams sufficient?

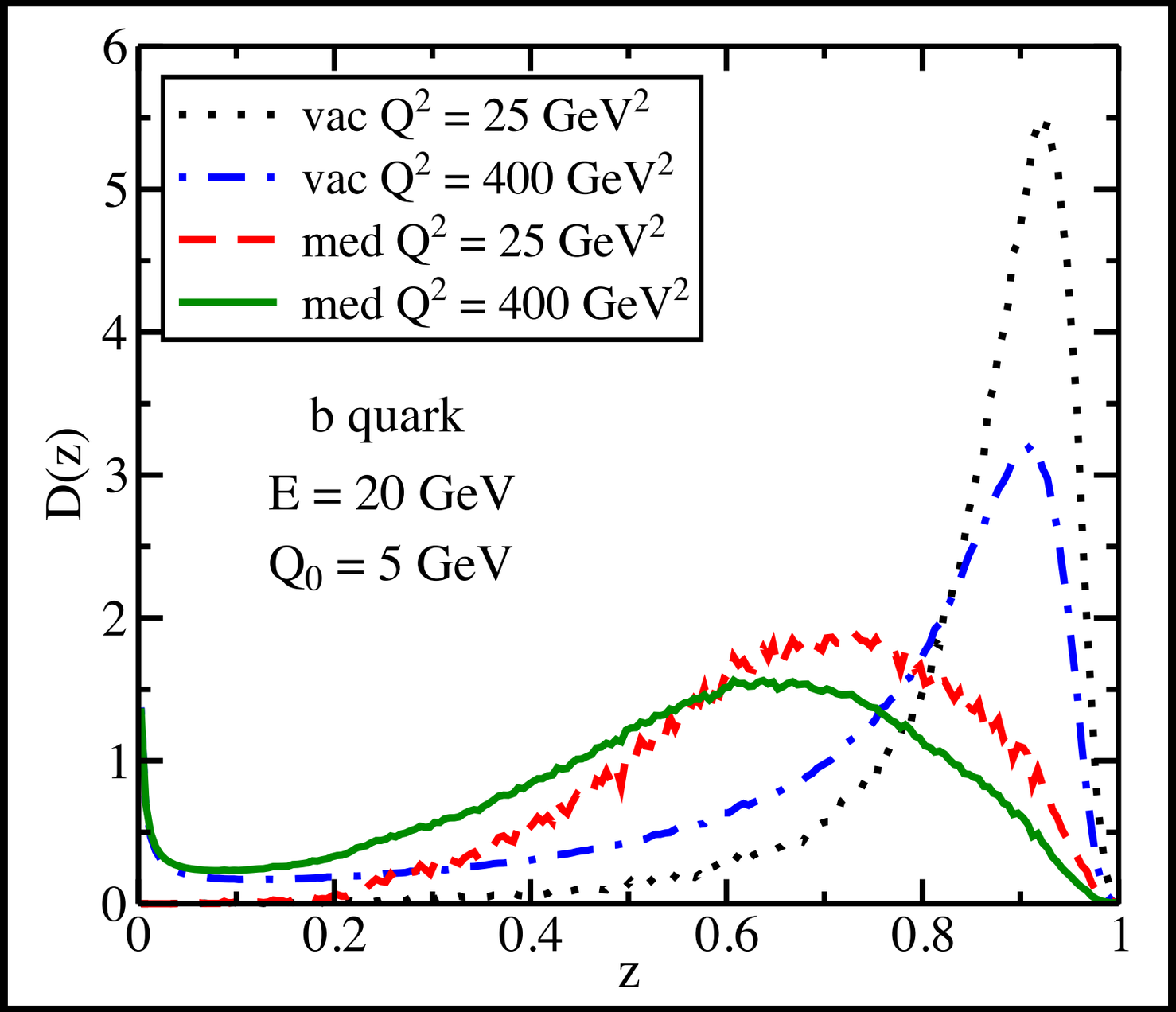


- Dominant contribution from radiated gluon $l \sim (\lambda Q, \lambda Q, \lambda Q)$
- Glauber gluon scales differently $k \sim \left(\lambda^{3/2} Q, \lambda^{3/2} Q, \lambda Q\right)$
- Formation length $\tau = \dfrac{E_Q y(1-y)}{\mu^2} \simeq \dfrac{l^-}{\mu^2} = \dfrac{\lambda Q}{\lambda^2 Q^2} \sim \dfrac{1}{\lambda Q}$
- Parametrically shorter formation length, higher Glauber p
- Order of magnitude fewer scatterings

A consistent picture of few scatterings

- A posteriori justification of only considering one scattering
- No BDMPS phase for non-collinear emission
- A Gunion-Bertsch phase of single scattering and single emission.
- Rate equation with GB kernel for intermediate energy Qs
- combine with DGLAP for high energy, high virtuality Qs

Multistage for heavy flavor



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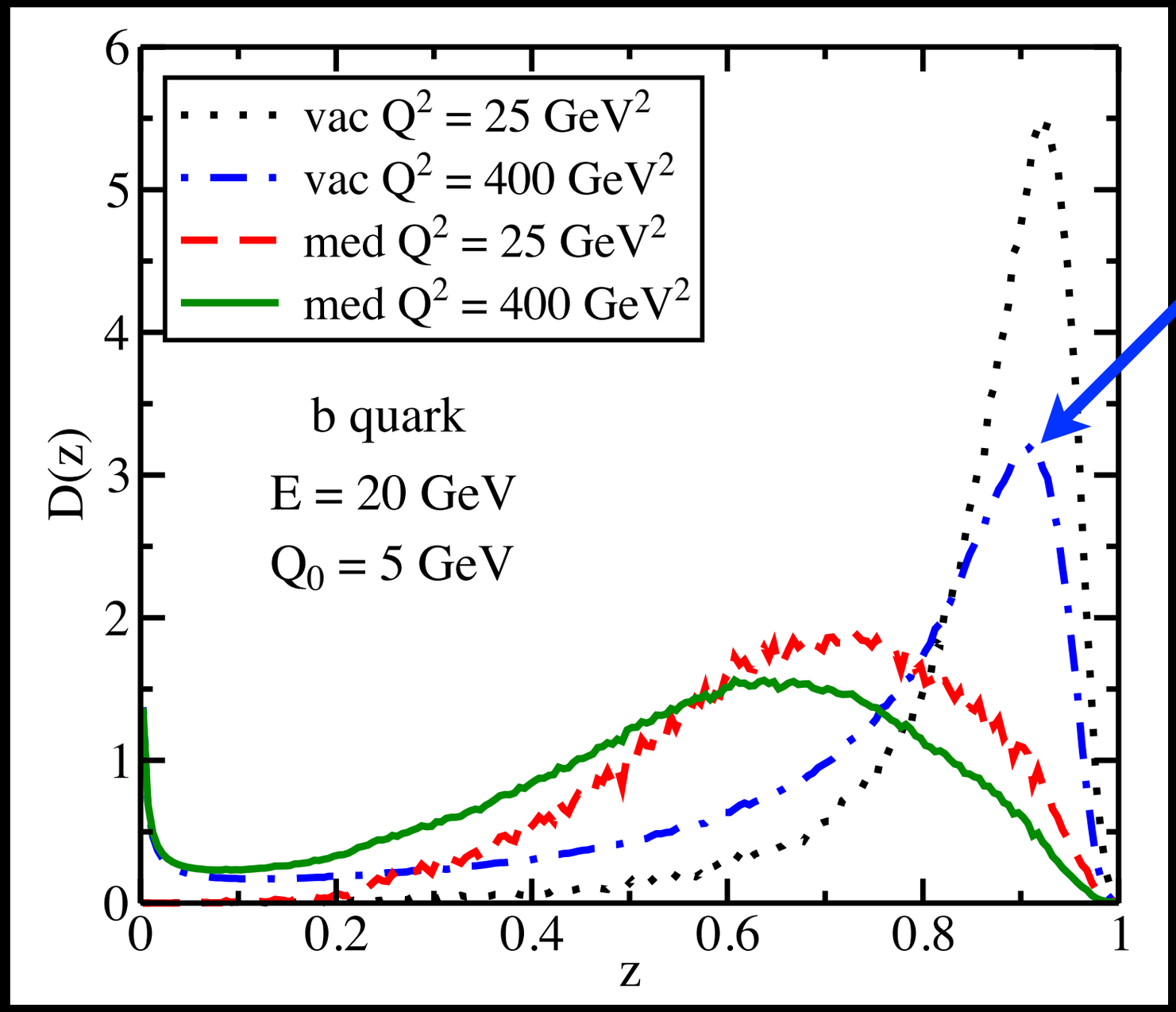
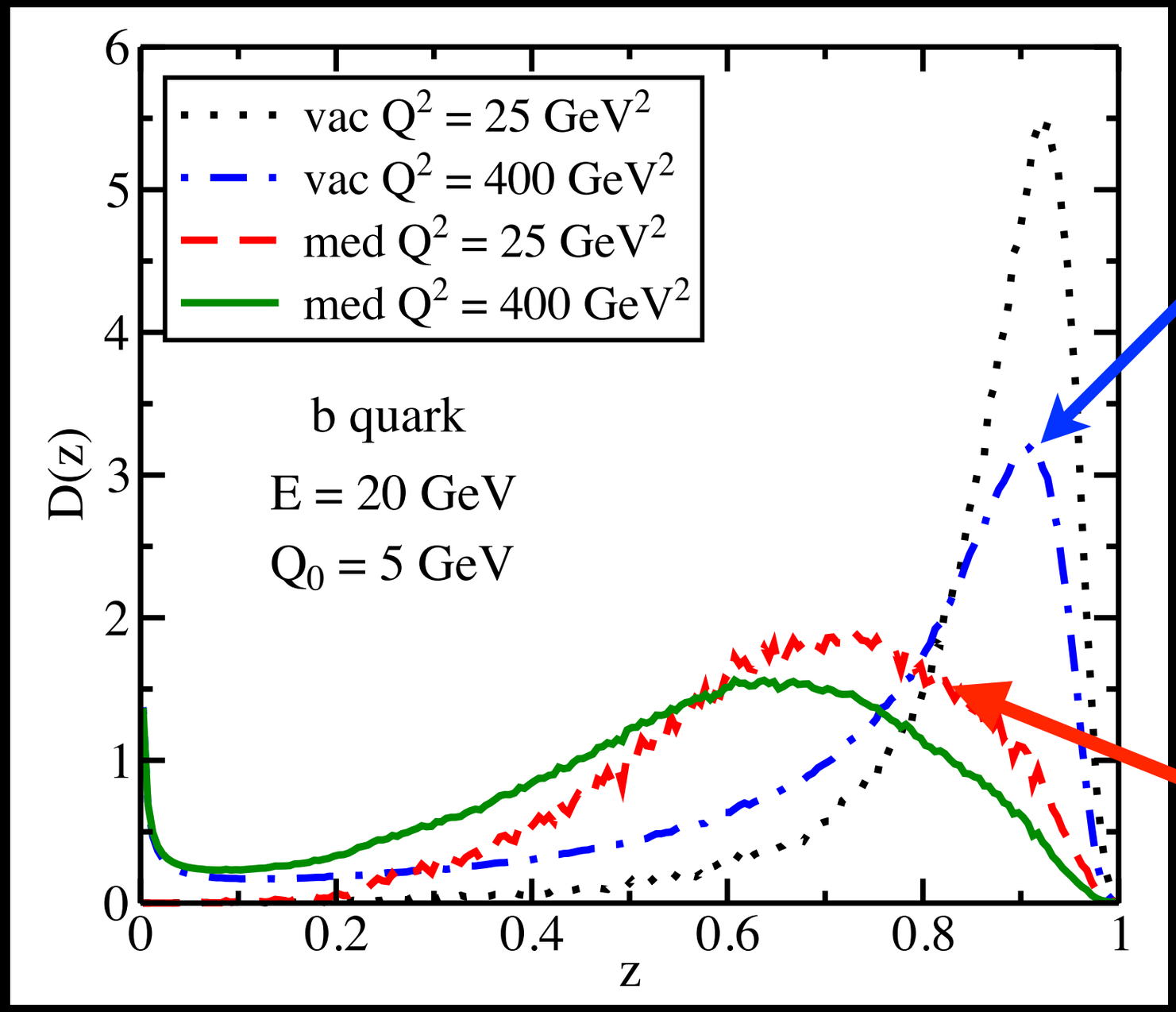


Evolved Vacuum Fragmentation Function

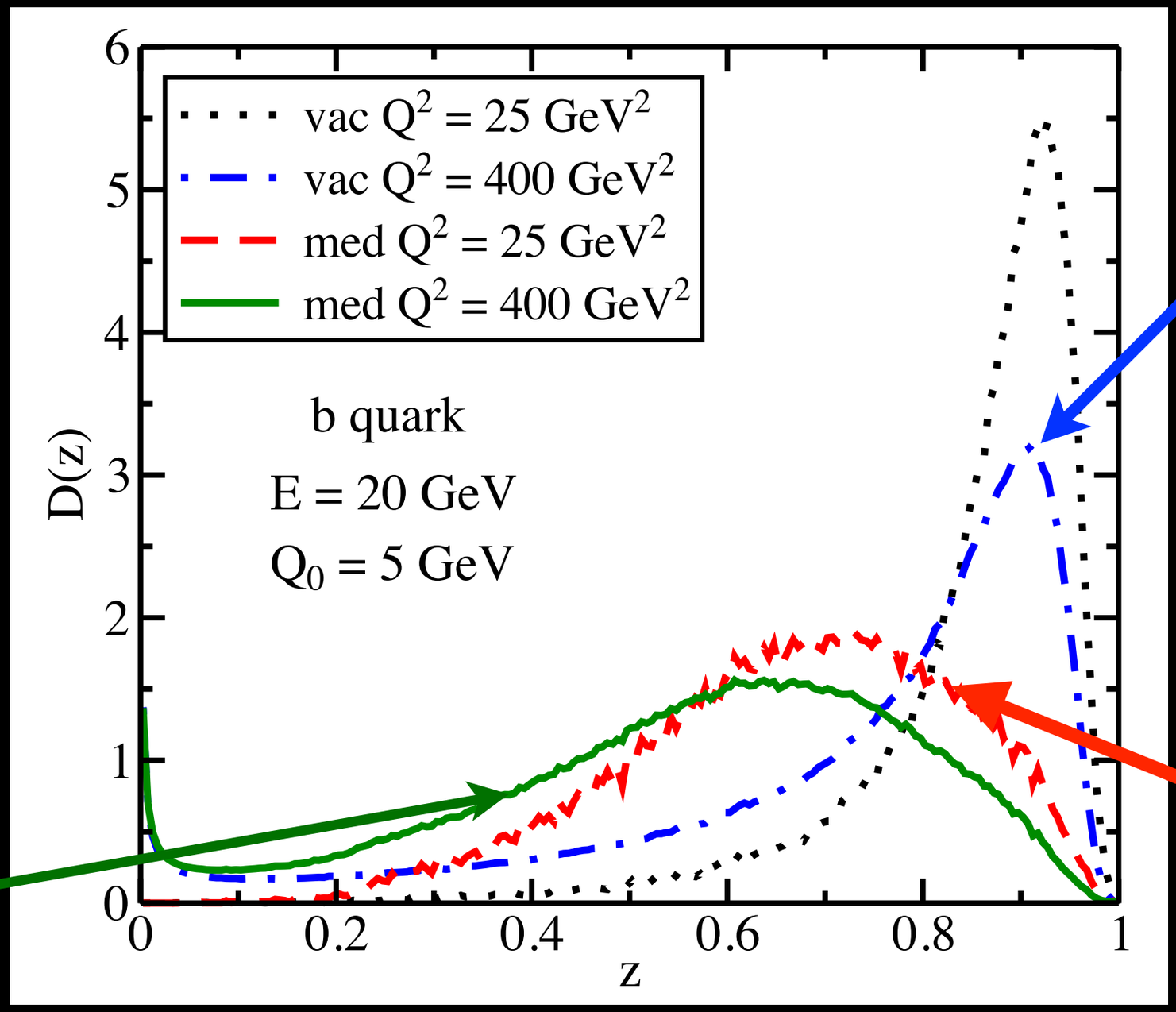
Multistage for heavy flavor



Evolved Vacuum Fragmentation Function

Transport shifted

Multistage for heavy flavor



Evolved Vacuum Fragmentation Function

Transport shifted

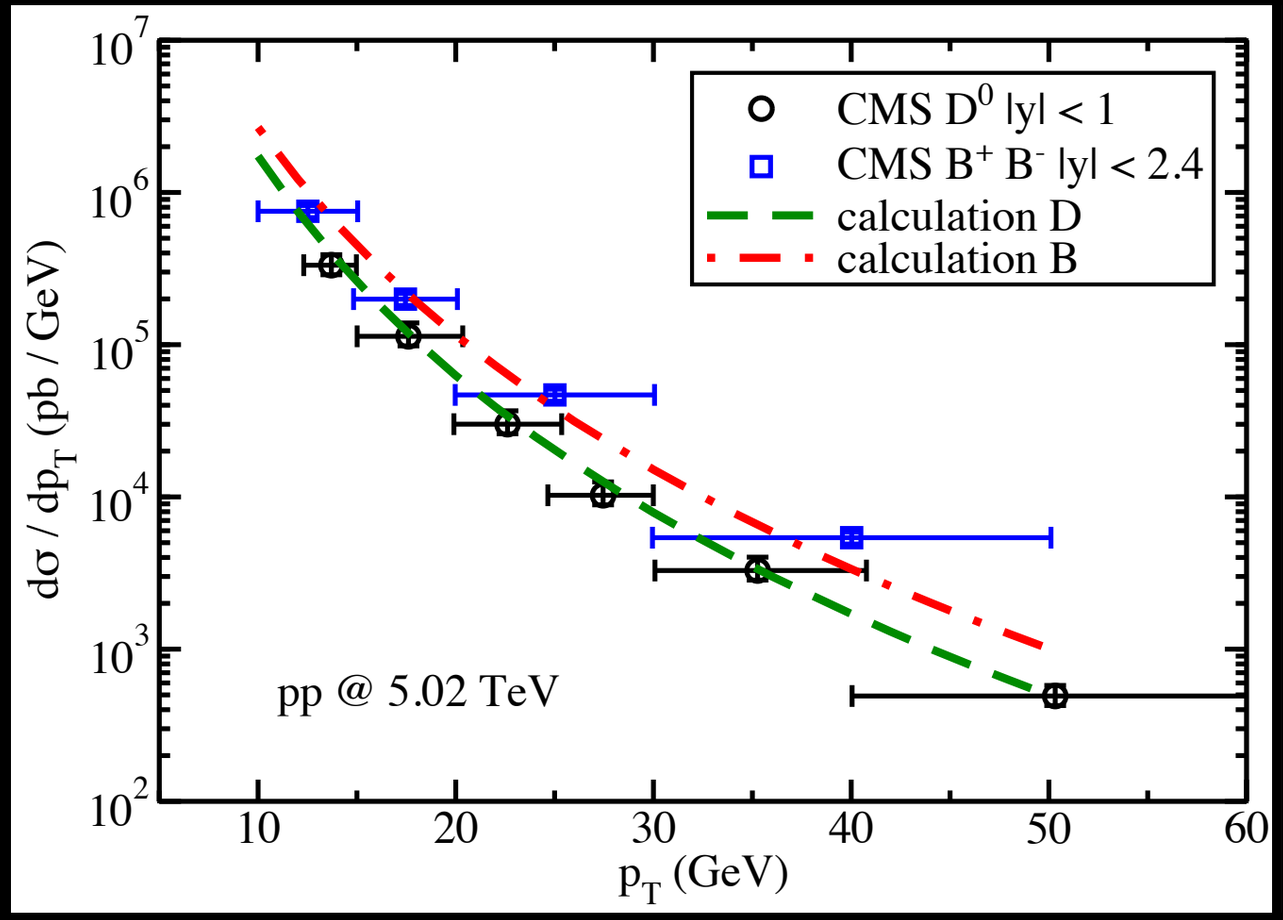
Evolved

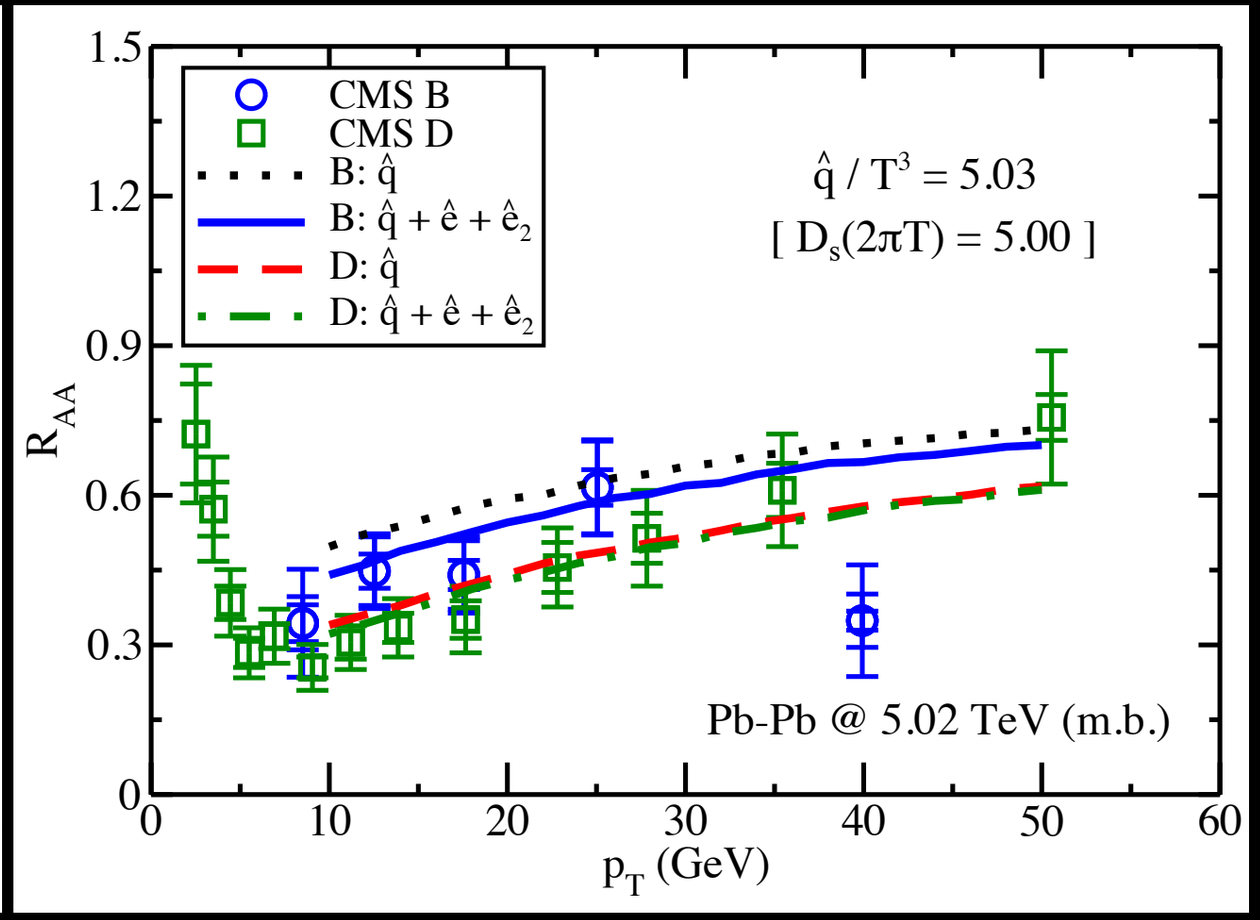
Comparing with data (drag and diffusion)

$$\frac{dN_g}{dy dl_\perp^2 d\tau} = 2\,\frac{\alpha}{\pi}\,P(y)\,\frac{1}{l_\perp^4}\left(\frac{1}{1+\chi}\right)^4 \sin^2\left(\frac{l_\perp^2}{4l^-(1-y)}(1+\chi)\,\tau\right)$$
$$\times\ \left[\left\{\left(1-\frac{y}{2}\right)-\chi+\left(1-\frac{y}{2}\right)\chi^2\right\}\,\hat{q}\right.$$
$$\left.+\frac{l_\perp^2}{l^-}\chi\,(1+\chi)^2\,\hat{e}+\frac{l_\perp^2}{(l^-)^2}\chi\left(\frac{1}{2}-\frac{11}{4}\chi\right)\hat{e}_2\right].$$

$$\chi = \frac{y^2 M^2}{l_\perp^2}$$

On a 2+1D hydro simulation





Is $\hat{q}$ for a heavy-quark the same?

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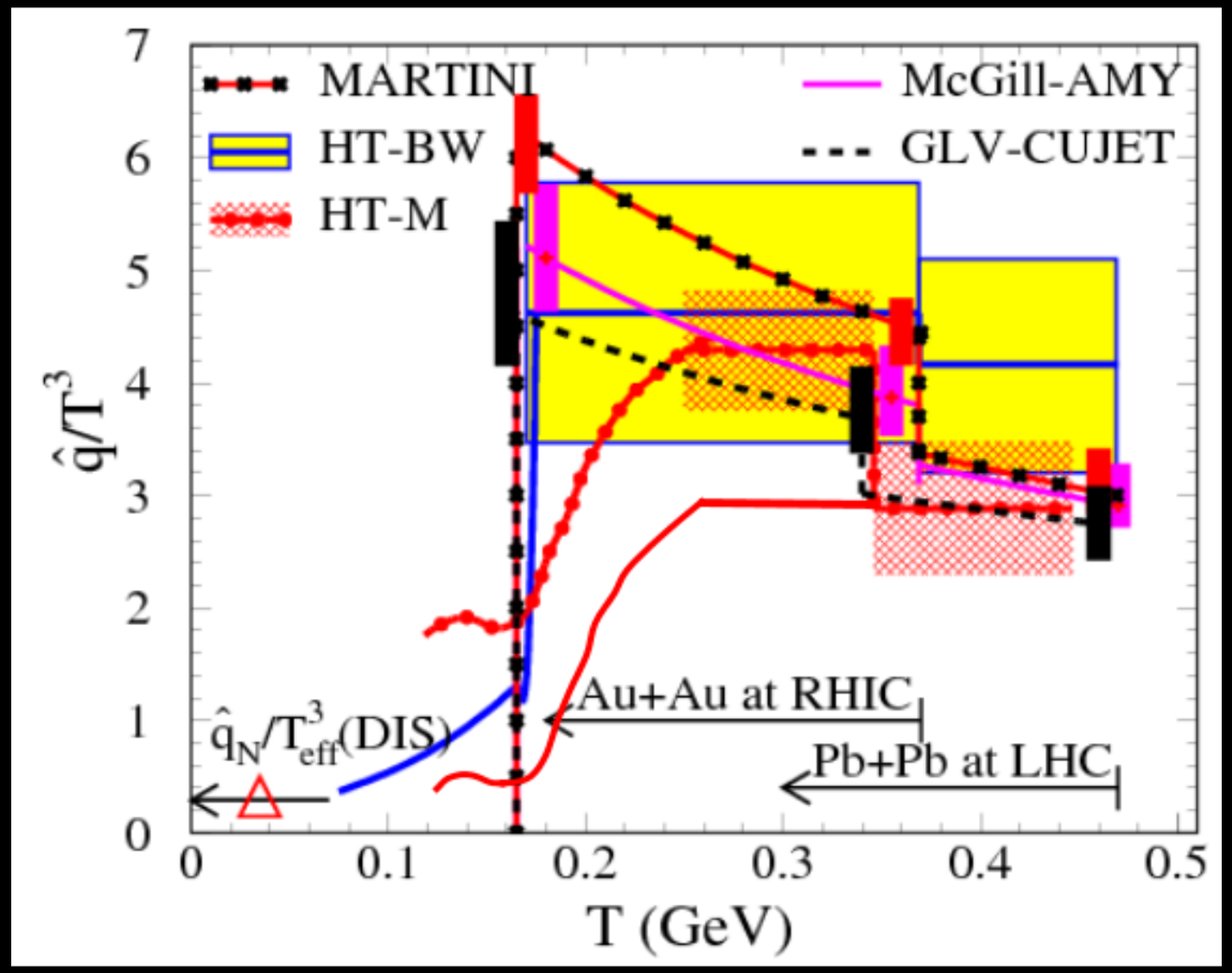

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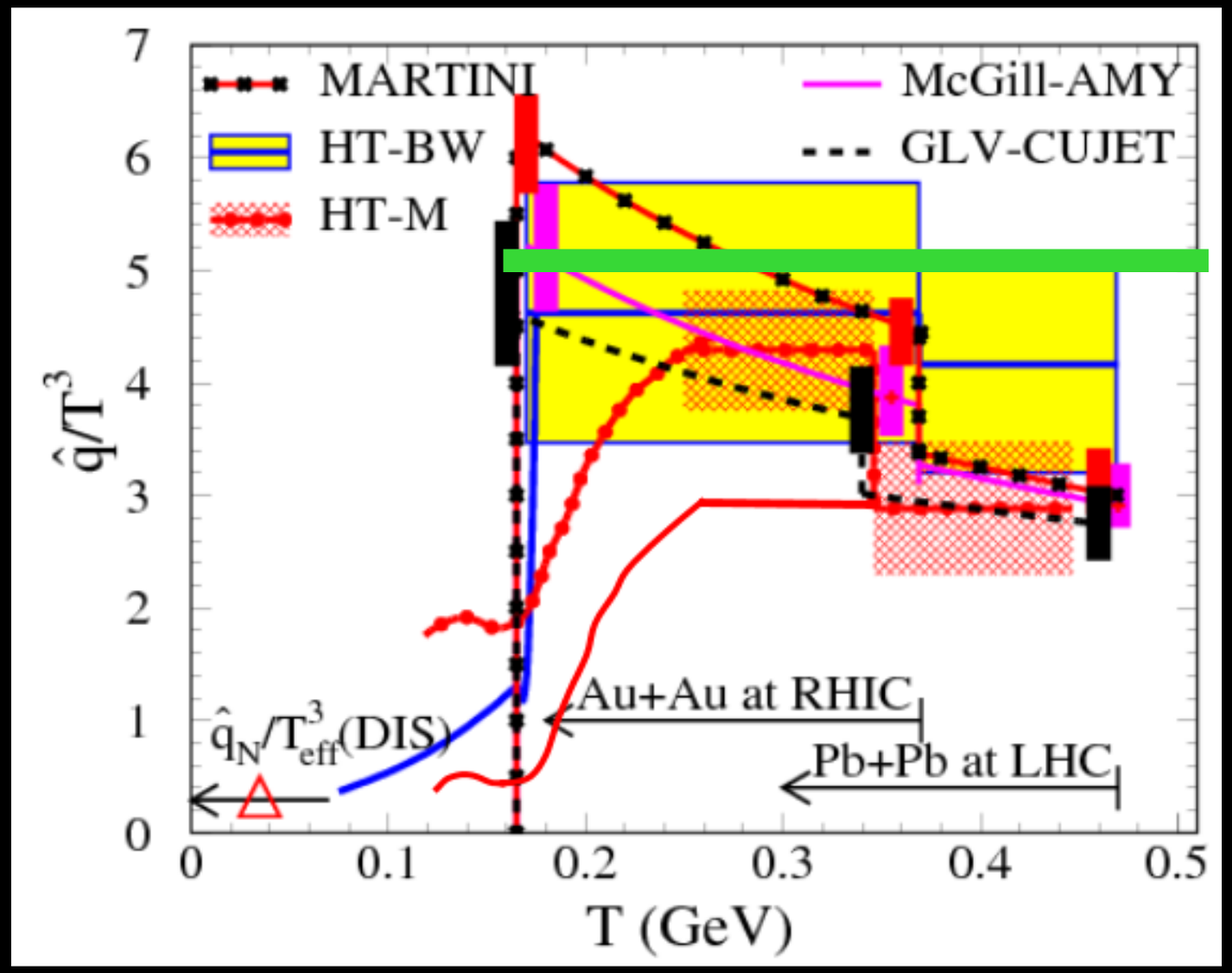

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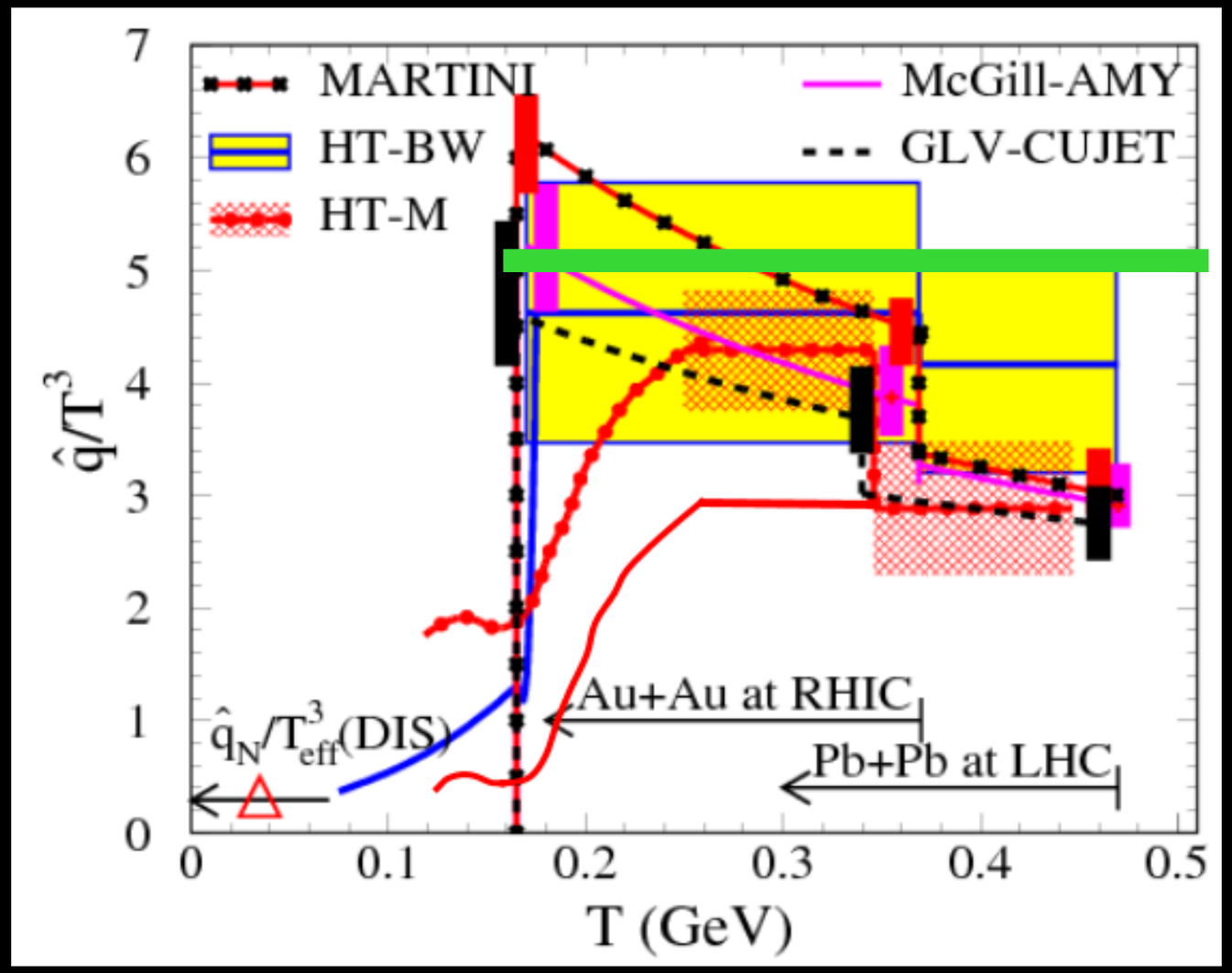

$$\hat{q} = \frac{4\pi^2 C_R \alpha_s}{N_C^2 - 1} \int \frac{dy^-}{\pi} \frac{\rho}{2p^+} \langle A | F_\perp{}^+(y^-) F^{\perp +}(0) | A \rangle \; e^{-i\bar{\Delta} P^+ y^-}$$

Summary

- Jets and heavy flavors are multi-scale objects
- Hard heavy quarks behave differently from intermediate energy heavy-quarks, from slow heavy quarks
- The SCET_G based power counting can be extended to heavy flavors
- No BDMPS phase for heavy quarks, replaced by a Gunion Bertsch phase
- Drag and diffusion also trigger additional radiation for heavy-quarks
- A successful description seems to require a slightly higher q than light flavors

Assuming the medium has a large length.

Or, the parton has a long life time, $1/(\lambda^2 Q)$

Multiple independent scattering dominates over multiple correlated scattering

Re-summing gives a diffusion equation for the p_T distribution

$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$

$$\langle p_\perp^2 \rangle = 4Dt$$

$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dt \langle F^{\mu\alpha}(t) v_\alpha F^\beta_\mu(0) v_\beta \rangle$$

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Michael Ber