\( \hat{q} \) analysis in a hybrid Boltzmann-Langevin approach with an improved LPM treatment

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Recent efforts to extract heavy quark $\hat{q}$

Extraction of $\hat{q}$ from systematic model-to-data comparison

- Radiation-improved Langevin Eq. (Cao et al PRC 92 024907), coupled to a tuned 2+1D viscous hydro evolution (Bernhard arXiv 1804.06469).
- Input: functional forms of $\hat{q}(E, T)$.
- Compare to $R_{AA}$ and $v_2$ measurements at the LHC.

Xu et al, PRC 97 014907 and work in progress, ALICE PRL 120 102301,
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**Issue 1: different approaches lead to different results**

- Compare improved Langevin Eq to a recent linearized Boltzmann Eq. ([Ke et al, arXiv:1806.08848](https://arxiv.org/abs/1806.08848)).
- Use same medium evolution. Compare to the same set of observables.
- However, different 95% credible region of $\hat{q}$.
- **Motivate a model that is more inclusive on assumptions.**

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**Graphical Representation:**

![Graph showing comparison between Linear Boltzmann and Improved Langevin models](image-url)
Recent efforts to extract heavy quark $\hat{q}$

Issue 2: how to implement a kinetic theory with coherence effect?

- Landau-Pomeranchuk-Migdal effect suppresses incoherent radiation.
  - Qualitative features of coherence ✓
  - Quantitative agreement with theory ?
- This affects the interpretation of theory-to-data comparison.
- Need an improved implementation of the LPM effect.
Roadmap of this presentation

Interpolate diffusion approximation to the scattering picture

- Absorb small $\hat{t}$ processes of the rate equation into a diffusion equation
  Ghiglieri et al, JHEP 03 (2016) 095 → introduce a separation scale $\hat{t}_{\text{cut}}$.
  First implementation for light sector (Dai, today Parallel 1, 17:45).

- Elastic processes $\propto 1/\hat{t}^2$
  Inelastic processes

  $\{ \}$ Rate equation. Sensitive to $\hat{t}$ cut-off.

- Small-$|t|$ interactions are frequent and soft $\approx$ a diffusion process.
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- Elastic processes $\propto 1/\hat{t}^2$
- Inelastic processes
- Small-$|t|$ interactions are frequent and soft $\approx$ a diffusion process.

Implement and validate an improved treatment of LPM effect.

- Fine tune MC to match theory calculation ($dl/d\omega$) in special cases.
- Achieve less ambiguous statement when compare to data.
Build the model: start with a light quark

- Large momentum-transfer elastic scattering solved by a rate equation.
- Restrict momenta transfer to $|t| > |\hat{t}_{\text{cut}}|$.

$$\frac{d f_Q}{d t} = C^{2\leftrightarrow 2}$$

Large-$|t|$ scattering
$|t| > |\hat{t}_{\text{cut}}|$
Build the model: start with a light quark

- Small momentum-transfer processes solved by a Langevin equation.
- Transport coefficient $\hat{q}_< = \alpha_s C_F T m_D^2 \ln(1 + |\hat{t}_{\text{cut}}|/m_D^2)$.

$$\frac{df_Q}{dt} = C_2^{\leftrightarrow 2} + \partial_p (Ap + \partial_p B)_< f_Q$$
Build the model: start with a light quark

- Improved Gunion-Bertsch matrix-element \[ \text{Fochler et al, PRD 88 014018} \]
- Again, require \(|t| > |\hat{t}_{\text{cut}}|\).

\[
df_Q/dt = C^{\leftrightarrow2}_{>2} + \partial_p (A_p + \partial_p B) f_Q + C^{2\to3}_{>3}
\]
Build the model: start with a light quark

- Take small momenta transfer limit of Gunion Bertsch matrix-element.
- Diffusion induced radiation $d\Gamma/dxdk^2_\perp \sim \alpha_s \hat{q}_g,/<(\pi xk^4_\perp)$

\[
df_Q/dt = C^{2\leftrightarrow2}_> + \partial_p (A\rho + \partial_p B)_< f_Q + C^{2\rightarrow3}_> + C^{1\rightarrow2}_<
\]
Build the model: start with a light quark

- Allow gluon elastic processes until it is “fully formed” when $\tau_f < \Delta t$
  
  Zapp et al, JHEP 07 (2011) 118.

$$\frac{df_g}{dt} = C_{2\leftrightarrow 2}^2 + \partial_p (A_g p + \partial_p B_g)_g f_g$$

- Gluon formation time $\tau_f \sim 2k/k_\perp^2$ changes as a function of $\Delta t$. 

Build the model: start with a light quark

- LPM effect: accept incoherent radiation with probability $p \sim \frac{m_D^2}{\hat{q} \tau_f}$.

\[
\frac{df_g}{dt} = C_2^{2\leftrightarrow 2} + \partial_p (A_g p + \partial_p B_g) f_g
\]
\[
\frac{df_Q}{dt} = C_2^{2\leftrightarrow 2} + \partial_p (A p + \partial_p B) f_Q + C_2^{2\leftrightarrow 3} + C_1^{1\leftrightarrow 2}
\]

Accept with $p \sim \frac{m_D^2}{\hat{q} \tau_f}$

Diagram:

- Diffusion induced rad.
- Scattering induced rad.
- Large-$|t|$ scattering
- Multiple scattering & LPM suppression
Validate the LPM treatment: infinite medium limit

- Theoretical spectra $dI/d\omega$ from AMY, NLL Arnold and Dogan, PRD 78 065008.
- $\omega < 2\pi T$, goes back to incoherent simulation (blue lines)
- $\omega > 2\pi T$, agree with theoretical results within $\pm 15\%$

$$\alpha_s = 0.3$$
Validate the LPM treatment: finite medium

- Path-length \((L)\) dependent \(dI/d\omega\) Caron-Huot and Gale, PRC 82 064902.
- Achieve similar level of accuracy as the previous case.

\[ E = 16 \text{ GeV}, \ \alpha_s = 0.3 \]

\[ dP/dtd\omega \]

\[ \omega = 3 \text{ GeV}, \ T = 0.2 \text{ GeV} \]

\[ \omega = 3 \text{ GeV}, \ T = 0.4 \text{ GeV} \]

\[ \omega = 8 \text{ GeV}, \ T = 0.2 \text{ GeV} \]

\[ \omega = 8 \text{ GeV}, \ T = 0.4 \text{ GeV} \]
Validate the LPM treatment: expanding medium

- Spectra in an expanding medium Baier et al, PRC 58 1706.
- \( (T/T_0)^3 = (\tau_0/\tau)^{2-1/\nu} \). Static: \( \nu = 1/2 \). Bjorken: \( \nu = 1 \).
Running coupling and dead-cone effect

Running coupling constant

- $\alpha_{s}^{\text{el}} = \alpha_{s}(Q^{2} = \hat{t})$, $\alpha_{s}^{\text{rad}} = \alpha_{s}(Q^{2} = k_{\perp}^{2}(\Delta t))$.
- The running is cut-off at a medium scale $\mu \sim T$, $\alpha_{s} = \alpha_{s}(\max\{Q, \mu\})$.

Mass (Dead-cone) effect

- Accept gluon according to $\left(\frac{\theta^{2}}{\theta^{2} + \theta_{D}^{2}}\right)^{n}$, $\theta_{D} = M/E$.
- $\theta = k_{\perp}/\omega$ evolves with $\Delta t$ due to gluon reinteraction.
Benchmark result:

- Fixed coupling calculation $\alpha_s = 0.3, 0.4$; $|\hat{t}_{\text{cut}}| = m_D^2$.
- Reasonable description above $p_T = 10$ GeV with large $\alpha_s$.

![Graphs showing $R_{AA}$ and $v_2$ as a function of $p_T$ for different Pb+Pb collision centrality.](image)
Benchmark result:

- **Running coupling** $\alpha_s(\max\{Q, \mu\})$, $\pi T < \mu < 2\pi T$, $|\hat{t}_{\text{cut}}| = m_D^2$.
- **Shape of $R_{AA}$ and $v_2$** slightly improved.
Benchmark result:

- $|\hat{t}_{\text{cut}}| \to \infty$, approximate all interactions by diffusion+radiation.
- Intermediate $p_T$ range suppressed more compared to high $p_T$.

\[ \ln \left( \frac{\Lambda^2}{\mu^2 (1 + \frac{\mu^2}{m_D^2})} \right) - \frac{\ln^2 (\frac{\mu^2}{\Lambda^2})}{\ln (6ET/\Lambda^2)} \]

Application to future Bayesian analysis

More parameters: $\hat{q}_{\text{LO, pQCD}} + \Delta \hat{q}$

$\Delta \hat{q}$ may be
1. Higher order.
2. Non-perturbative.
3. Parametric.
Application to future Bayesian analysis

More parameters:
\( \hat{q}_{\text{LO, pQCD}} + \Delta \hat{q} \)

Diffusion approximation

How does data constrain the parameter space?

\( \pi T \)

\( 4\pi T \)

\( B m_D^2 \)

\( A m_D^2 \)

\( \mu \)

\( \Delta \hat{q} \) may be
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Weiyao Ke (Duke University)

Hard Probe 2018, Aix-Les-Bains, France

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The development of the Lido model

- Small-\(|t|\) processes are absorbed into a diffusion equation.
- Large-\(|t|\) processes are solved by a rate equation.

Improving the LPM implementation

- Gluon reinteraction is included.
- Validate in infinite/finite, static/expanding medium.

Future plan

- Perform a Bayesian parameter extraction of $\hat{q}$.
- Couple to quarkonium transport (Yao, Wed Session 3, 09:40).
- Integrate the model into the JetScape framework.
In our old approach, LPM effect is introduced as a coherence factor in the $2 \rightarrow 3$ matrix-element, but exclude multiple scatterings.

\[ \int \frac{d\sigma_{23}}{d\hat{t}dk^3} \frac{d\hat{t}dk^3}{2k} \rightarrow \int \frac{d\sigma_{23}}{d\hat{t}dk^3} \frac{d\hat{t}dk^3}{2k} \left[ 1 - \cos \left( \frac{\Delta t}{\tau_f} \right) \right] \frac{d\hat{t}dk^3}{2k}, \tau_f \sim \frac{2k}{k_\perp} \]
Back-up: detailed comparison

\( \alpha_s = 0.1 \)

\( \alpha_s = 0.3 \)
Back-up: energy loss in an infinite medium

\[ \frac{dE}{dx} \alpha_s^2 \sqrt{ET^3} \]

\begin{align*}
\alpha_s &= 0.075 \quad T = 0.2 \text{ GeV} \\
\alpha_s &= 0.15 \quad T = 0.2 \text{ GeV} \\
\alpha_s &= 0.3 \quad T = 0.2 \text{ GeV} \\
\alpha_s &= 0.6 \quad T = 0.2 \text{ GeV} \\
\alpha_s &= 0.075 \quad T = 0.4 \text{ GeV} \\
\alpha_s &= 0.15 \quad T = 0.4 \text{ GeV} \\
\alpha_s &= 0.3 \quad T = 0.4 \text{ GeV} \\
\alpha_s &= 0.6 \quad T = 0.4 \text{ GeV} \\
\alpha_s &= 0.075 \quad T = 0.6 \text{ GeV} \\
\alpha_s &= 0.15 \quad T = 0.6 \text{ GeV} \\
\alpha_s &= 0.3 \quad T = 0.6 \text{ GeV} \\
\alpha_s &= 0.6 \quad T = 0.6 \text{ GeV}
\end{align*}
Back-up: mass dependence

\( T = 0.4 \text{ GeV} \)

\( \frac{dE}{dx}/\alpha_s^2 \sqrt{ET^3} \)

- Light
- Charm
- Bottom

\( \frac{\Delta E}{E} \)

\( L = 5 \text{ [fm]} \)