Finite formation time effects for in-medium parton splittings

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Motivation: jet substructure

- Jets have substructure:
  - Dynamics of an energetic parton in matter.
  - We need theoretical tools to compute multiparticle scenarios from first principles.
  - Keys: Coherence, medium-induced radiation, jet fragmentation...
Motivation: unifying existing knowledge

- Existing knowledge well understood separately:
  - Vacuum jets.
  - BDMPS-Z spectrum and energy loss.
  - Color coherence in original antenna setups: singlet antenna plus soft gluon emission spectrum in vacuum, in-medium antenna propagation case...

  [C. A. Salgado’s Jet Physics Lecture - Student Lectures Day]

- How can we combine them?

- Further studies towards an unified description:
  - (1) color coherence in multiple emissions setups.
    [Salgado’s Lecture]
  - (2) finite formation time corrections.
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In-medium finite formation time antenna

$[zE, p_1]$

$[(1 - z)E, p_2]$
Motivation
Antenna scenario
Time-scales
Numerics
Conclusions

\( \gamma \rightarrow q\bar{q}: \) amplitude

\[
\mathcal{M}^{in}_{\gamma \rightarrow q\bar{q}} = \frac{1}{2E} e^{i \frac{\vec{p}^2}{2zE} L + i \frac{\vec{p}^2}{2(1-z)E} L} \int_0^\infty dt \int_{\vec{k}_1, \vec{k}_2} \left[ \mathcal{G}(\vec{p}_1, L; \vec{k}_1, t|zE) \mathcal{G}(\vec{p}_2, L; \vec{k}_2, t|(1-z)E) \right]_{ij} \\
\times \gamma^{\rightarrow q\bar{q}}(\vec{k}, z) \mathcal{G}_0(\vec{k}_1 + \vec{k}_2, t|E)
\]

- \( \mathcal{G} (\mathcal{G}) \rightarrow \) quark (antiquark) propagators.
- \( \Gamma(\vec{k}, z) \rightarrow \) splitting vertex.
- \( \vec{k}_1, \vec{k}_2 \rightarrow \) transverse momentum after splitting.
Eikonal expansion of the propagator

- The propagator in the momentum space:
  \[ G(\vec{p}_1, t_1; \vec{p}_0, t_0) = \int_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}_1 \cdot \vec{x}_1 + i\vec{p}_0 \cdot \vec{x}_0} G(\vec{x}_1, \vec{x}_0) \]  
  (1)

- The propagator in the configuration space described by a path integral:
  \[ G(\vec{x}_1, \vec{x}_0) = \int_{\vec{r}(t_0)=\vec{x}_0}^{\vec{r}(t_1)=\vec{x}_1} D\vec{r} \exp\left[i \frac{E}{2} \int_{t_0}^{t_1} ds \vec{r}^2\right] V(t_1, t_0; \vec{r}[s]) \]  
  (2)

- The eikonal expansion of the propagator:
  \[ G^{(0)}(\vec{x}_1, \vec{x}_0) = G_0(\vec{x}_1 - \vec{x}_2, \tau) V(t_1, t_0; [\vec{x}_{cl}(s)]) \]  
  (3)
Derivation of the spectrum

\[ n_1 = \frac{p_1}{E_1} \]

\[ n_{12} = n_1 - n_2 \]

\[ n_2 = \frac{p_2}{E_2} \]
Derivation of the spectrum

- **Region I:**
  - $q$ and $\bar{q}$ phases: $\exp\left\{i \frac{p^2_{i1}}{2E_i} (t_2 - t_1)\right\}$.
  - Average of the Wilson lines: $\exp\left\{- \frac{1}{12} \hat{q} n_{12}^2 (t_2 - t_1)^3\right\}$.

- **Region II:**
  - Average of a trace of four Wilson lines:
    \[Q(t_L, t_2) = \frac{1}{N_c} \left\langle Tr \left[ W_1(t_L, t_2) W_2^\dagger(t_L, t_2) W_2(t_L, t_2) W_1^\dagger(t_L, t_2) \right] \right\rangle\]
    - It accounts for the accumulated effects of medium interactions over long distances.
The time-scales

- Kinematical formation time:

\[ t_f = \frac{z(1 - z)E}{p^2} \]  \hspace{1cm} (4)

- Decoherence time:

\[ t_d \sim \left( \frac{1}{\hat{q}\theta^2} \right)^{1/3} \]  \hspace{1cm} (5)

- Broadening time:

\[ t_{broad} \sim \left( \frac{1}{\hat{q}\theta^2 L} \right)^{1/2} \]  \hspace{1cm} (6)

- Length of the medium: \( L \)
The \textit{real} formation time

\[ \tau \lesssim \min[t_f, t_d, t_{br}] \]

- The \textbf{real formation time} is governed by the \textbf{smallest} of the three \textbf{physical time scales} of the problem: either the kinematical formation time, the decoherence time or the broadening time.
The Lund diagram
• (A.1) $t_f < t_{\text{broad}} < t_d < L$: particles are created early in the medium and the dipole will decohere at a finite distance.
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(A.2) $t_{\text{broad}} < t_f < t_d < L$: deviations from pure vacuum-like behaviour.
(A.3) $t_{\text{broad}} < t_d < t_f < L$: the formation of the dipole is strongly suppressed.
(A.4) \( t_f < L < t_d < t_{broad} \): the created partons will never decohere in color and the splittings should follow a vacuum emission pattern.
(B) \( t_f > L \): no medium modification is expected.
(A.1) and (A.4) are regions of vacuum-like emissions inside the medium.
The medium modification factor

\[ \frac{dl^{med}}{dz \, dp^2} = \frac{dl^{vac}}{dz \, dp^2} (1 + F_{med}) \] (7)

- The medium modifications factor out into \( F_{med} \).

\[
F_{med} = 2 \int_0^{\zeta_L} ds \left[ \int_s^{\zeta_L} ds' \cos(s' - s) \, S_{12}(s', s) \, Q(\zeta_L, s, s') - \sin(\zeta_L - s) \, S_{12}(\zeta_L, s) \right]
\] (8)

\[
\cos(s' - s) \, S_{12}(s', s) \, Q(\zeta_L, s, s') \propto \left\langle \left| \tilde{M}_{qq}^{in} \right| \right\rangle^2
\]

\[
\sin(\zeta_L - s) \, S_{12}(\zeta_L, s) \propto \left\langle \tilde{M}_{qq}^{in} \tilde{M}_{qq}^{\dagger, out} \right\rangle
\]
Numerics: $F_{med}$ in the Lund plane

$F_{med}$

$p_T=250$ GeV

$q\hat{=}1.5$ GeV$^2$/fm

$L=2$ fm
In spite of the singlet antenna limitations it turns out to be a very convenient *laboratory*.

The in-medium finite formation time antenna setup shows two regimes of vacuum-like emissions, lending support to the notion of purely vacuum-like emissions that are emitted inside the medium.

The finite formation time scenario shows us an interesting theoretical guidance for MC.

These computations go a step forward to obtain a complete description of a QCD cascade.
Thanks for your attention