Finite formation time effects for in-medium parton splittings

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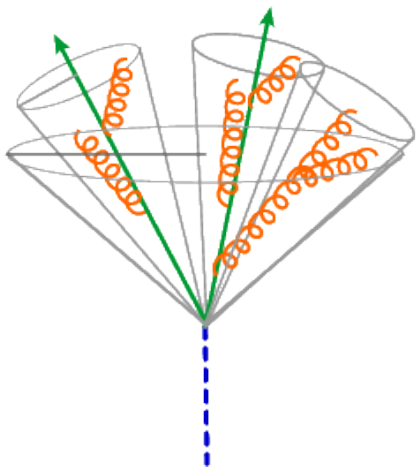
Motivation: jet substructure



- Jets have substructure:
  - Dynamics of an energetic parton in matter.
  - We need theoretical tools to compute multiparticle scenarios from first principles.
  - Keys: Coherence, medium-induced radiation, jet fragmentation...

Motivation: unifying existing knowledge

- Existing knowledge well understood separately:
  - Vacuum jets.
  - BDMPS-Z spectrum and energy loss.
  - Color coherence in original antenna setups: singlet antenna plus soft gluon emission spectrum in vacuum, in-medium antenna propagation case...

  [C. A. Salgado's Jet Physics Lecture - Student Lectures Day]

- How can we combine them?

- Further studies towards an unified description:
  - (1) color coherence in multiple emissions setups. [Salgado's Lecture]
  - (2) finite formation time corrections.

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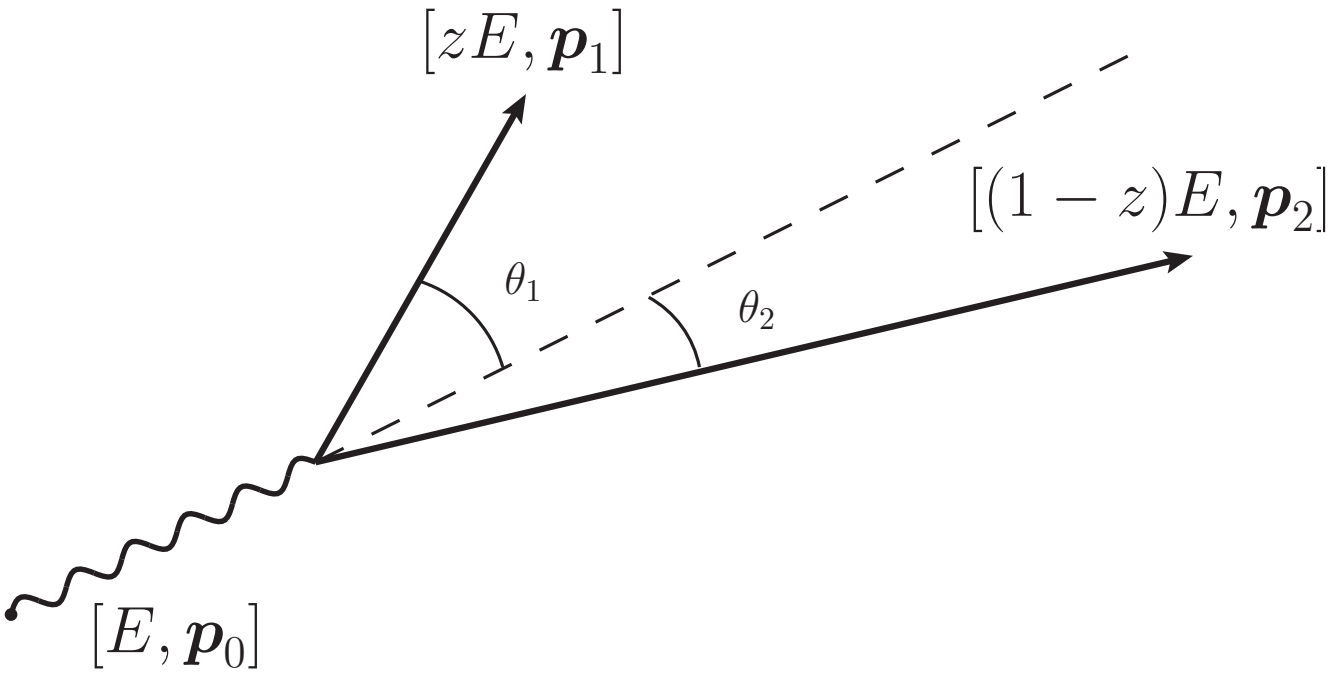
In-medium finite formation time antenna



[In preparation: Mapping collinear in-medium parton splittings, F. Domínguez, G. Milhano, C. A. Salgado, K. Tywoniuk and V. Vila]

$\gamma \to q\bar{q}$: amplitude

$$\bar{\mathcal{M}}^{in}_{\gamma\to q\bar{q}} = \frac{1}{2E}\, e^{i\frac{\vec{p}_1^{\,2}}{2zE}L + i\frac{\vec{p}_2^{\,2}}{2(1-z)E}L} \int_0^\infty dt \int_{\vec{k}_1,\vec{k}_2} \left[\mathcal{G}(\vec{p}_1, L; \vec{k}_1, t|zE)\, \bar{\mathcal{G}}(\vec{p}_2, L; \vec{k}_2, t|(1-z)E)\right]_{ij}$$
$$\times\, \Gamma^{\gamma\to q\bar{q}}_{\lambda,s,s'}(\vec{k}, z)\, \mathcal{G}_0(\vec{k}_1 + \vec{k}_2, t|E)$$

- $\mathcal{G}$ ($\bar{\mathcal{G}}$) $\to$ quark (antiquark) propagators.
- $\Gamma(\vec{k}, z)$ $\to$ splitting vertex.
- $\vec{k}_1$, $\vec{k}_2$ $\to$ transverse momentum after splitting.

Eikonal expansion of the propagator

- The propagator in the momentum space:

$$\mathcal{G}(\vec{p}_1, t_1; \vec{p}_0, t_0) = \int_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}_1 \cdot \vec{x}_1 + i\vec{p}_0 \cdot \vec{x}_0} \; \mathcal{G}(\vec{x}_1, \vec{x}_0) \tag{1}$$

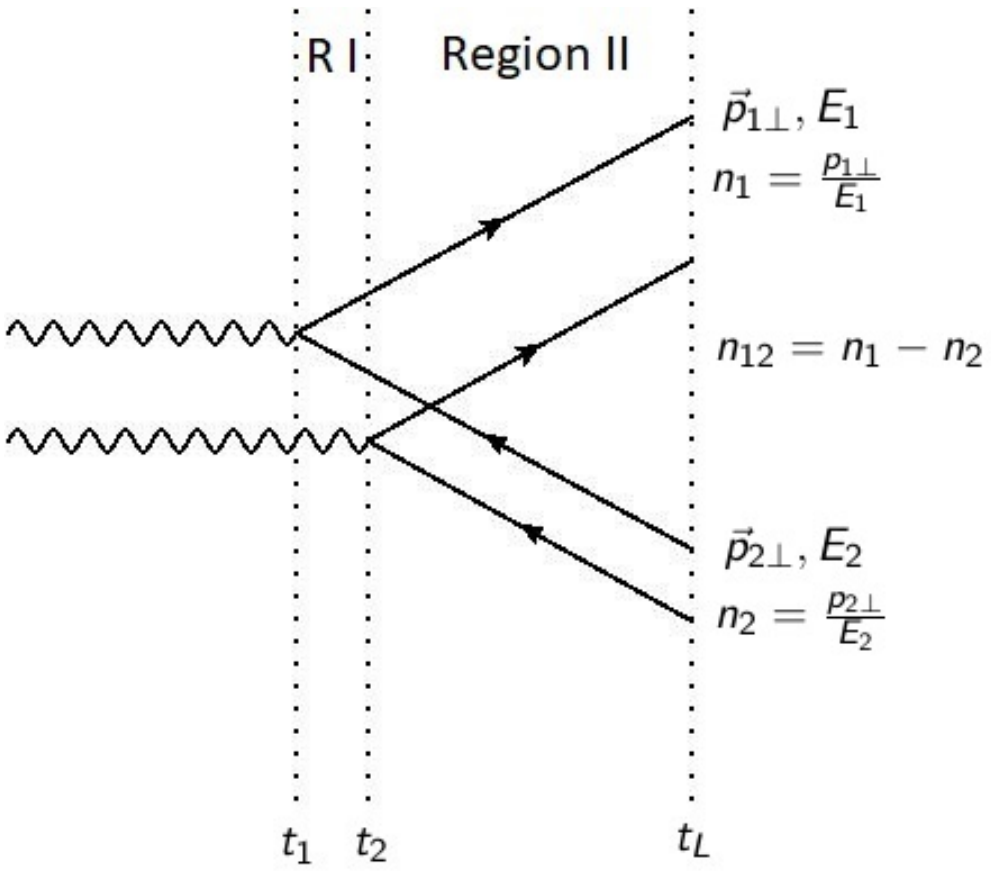
- The propagator in the configuration space described by a path integral:

$$\mathcal{G}(\vec{x}_1, \vec{x}_0) = \int_{\vec{r}(t_0)=\vec{x}_0}^{\vec{r}(t_1)=\vec{x}_1} \mathcal{D}\vec{r} \; exp\Big[i \frac{E}{2} \int_{t_0}^{t_1} ds \; \dot{\vec{r}}^2 \Big] \; V(t_1, t_0; \vec{r}[s]) \tag{2}$$

- The *eikonal* expansion of the propagator:

$$\mathcal{G}^{(0)}(\vec{x}_1, \vec{x}_0) = \mathcal{G}_0(\vec{x}_1 - \vec{x}_2, \tau) \; V(t_1, t_0; [\vec{x}_{cl}(s)]) \tag{3}$$

Derivation of the spectrum



Derivation of the spectrum

- **Region I**:
  - q and $\bar{q}$ phases: $exp\left\{ i\frac{p_{i\perp}^2}{2E_i}(t_2 - t_1)\right\}$.
  - Average of the Wilson lines: $exp\left\{ -\frac{1}{12}\hat{q}n_{12}^2(t_2 - t_1)^3\right\}$.
- **Region II**:
  - Average of a trace of four Wilson lines:

$$Q(t_L, t_2) = \frac{1}{N_c}\left\langle Tr\left[W_1(t_L, t_2)W_2^\dagger(t_L, t_2)W_{\bar{2}}(t_L, t_2)W_{\bar{1}}^\dagger(t_L, t_2)\right]\right\rangle$$

  - It accounts for the accumulated effects of medium interactions over long distances.

The time-scales

- Kinematical formation time:

$$t_f = \frac{z(1-z)E}{\vec{p}^2} \tag{4}$$

- Decoherence time:

$$t_d \sim \left(\frac{1}{\hat{q}\theta^2}\right)^{1/3}, \tag{5}$$

- Broadening time:

$$t_{broad} \sim \left(\frac{1}{\hat{q}\theta^2 L}\right)^{1/2} \tag{6}$$

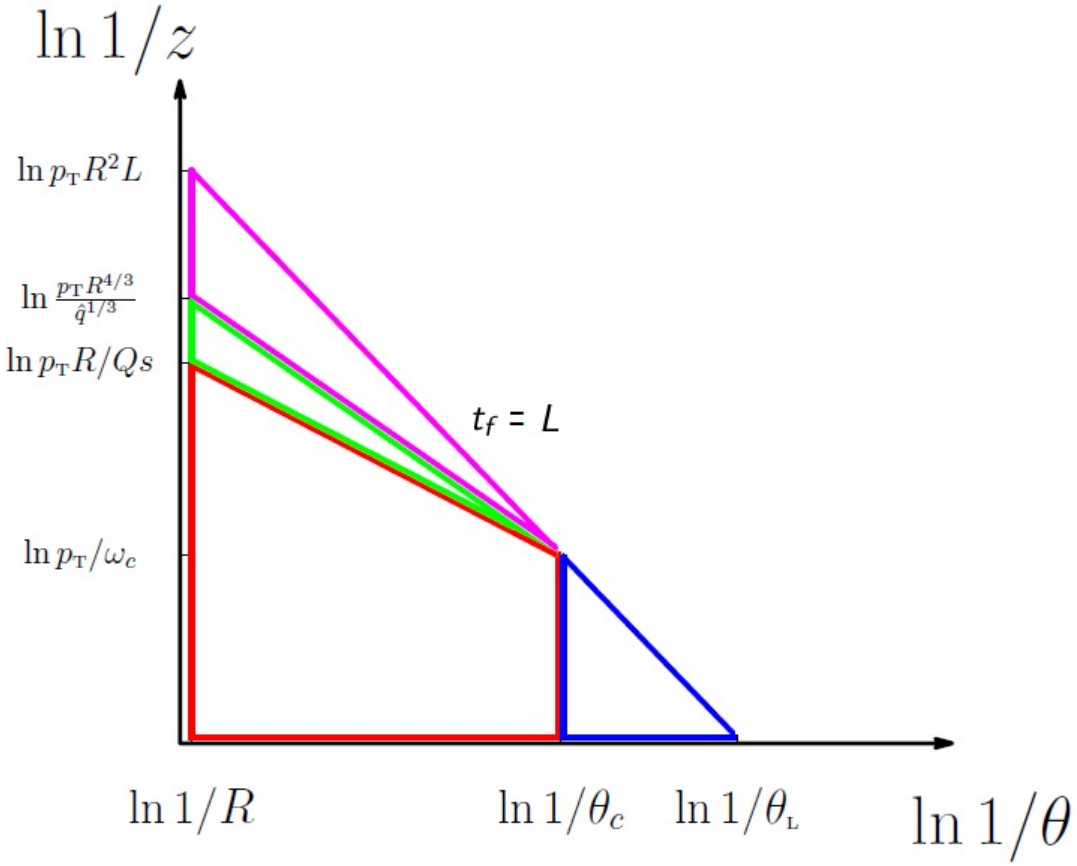
- Length of the medium: L.

The *real* formation time

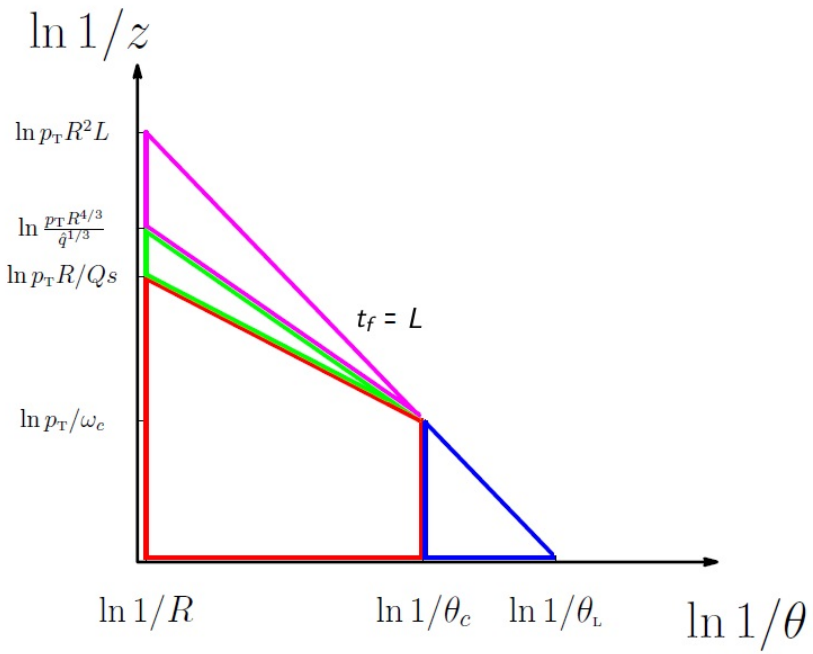
$$\tau \lesssim min[t_f, t_d, t_{br}]$$

- The **real formation time** is governed by **the smallest** of the three **physical time scales** of the problem: either the kinematical formation time, the decoherence time or the broadening time.

The Lund diagram

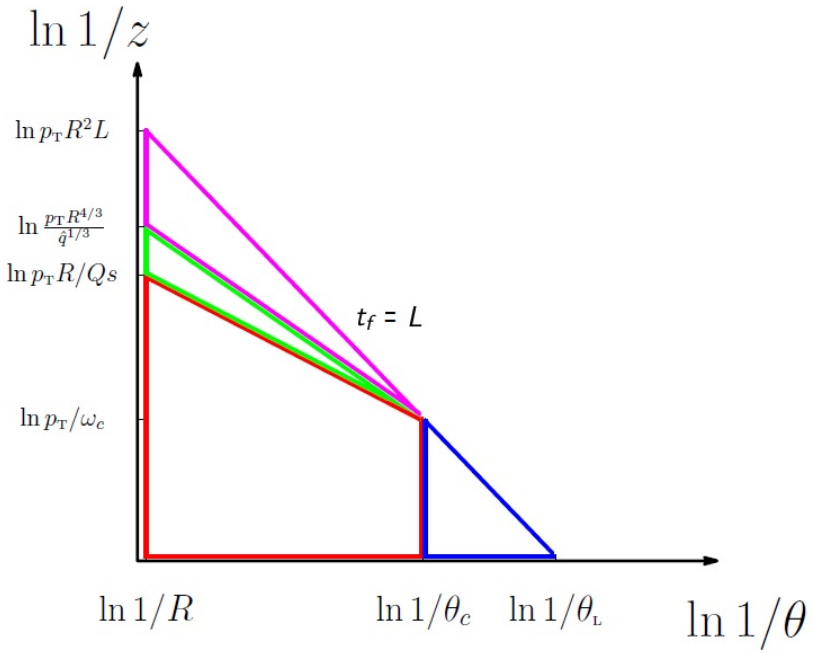


Discussion of time-scales



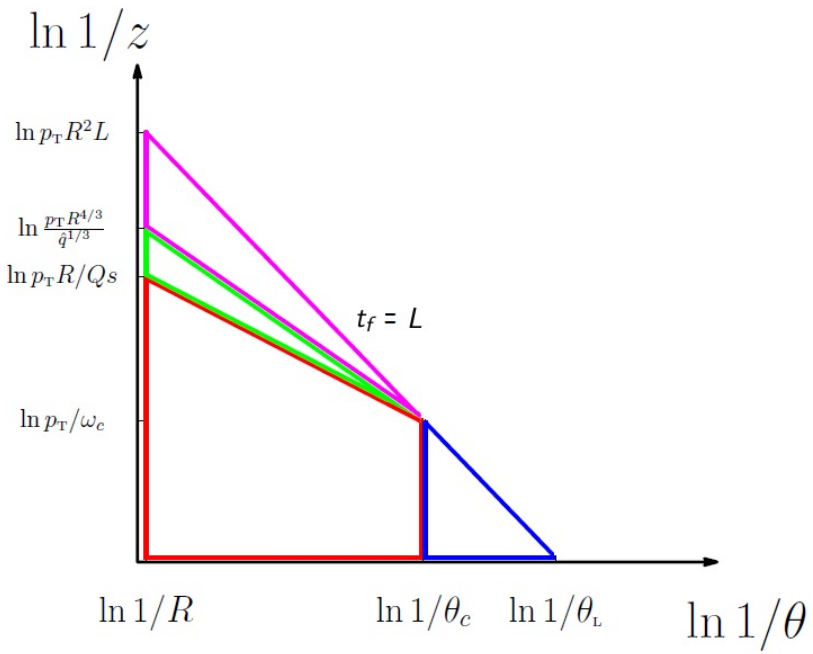
- (A.1) $t_f < t_{broad} < t_d < L$: particles are created early in the medium and the dipole will decohere at a finite distance.

Discussion of time-scales



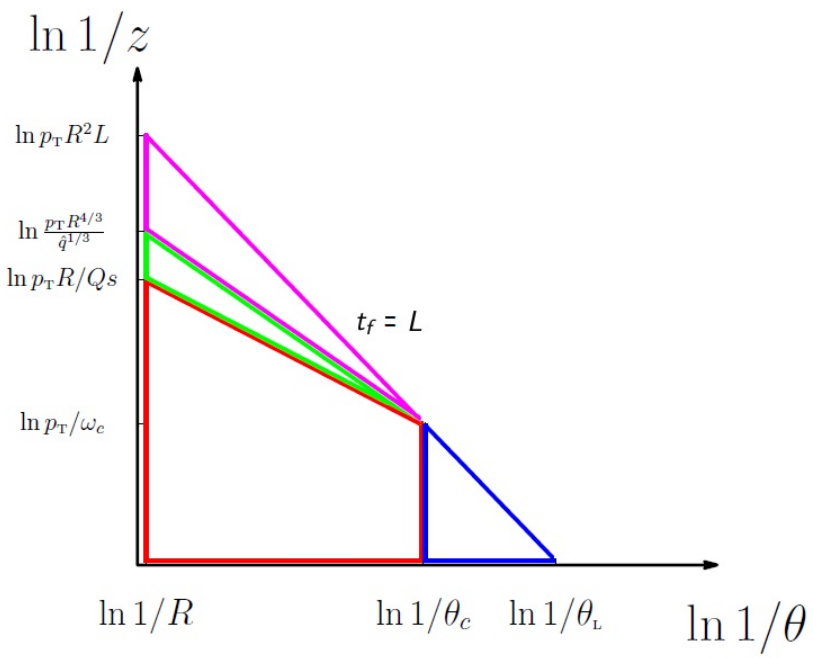
- (A.2) $t_{broad} < t_f < t_d < L$: deviations from pure vacuum-like behaviour.

Discussion of time-scales



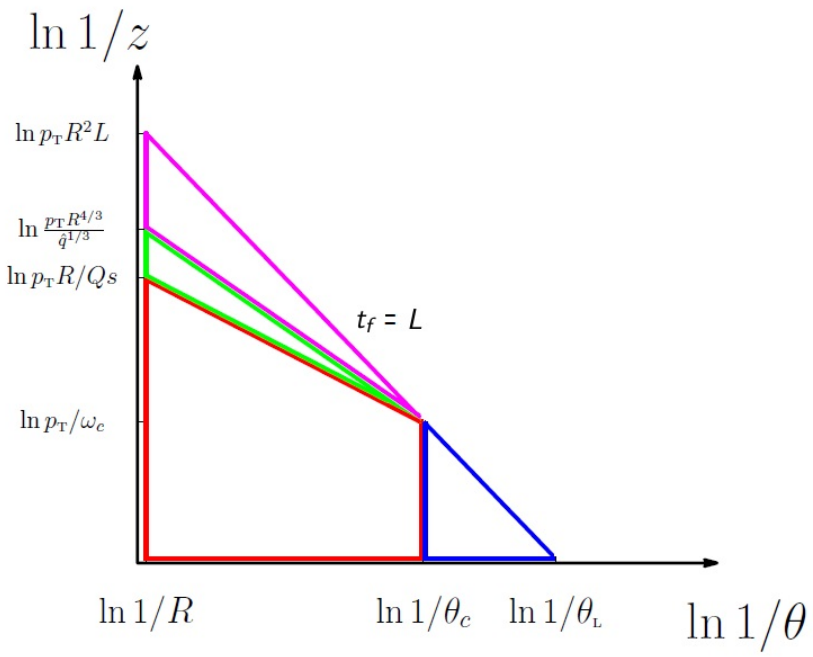
- (A.3) $t_{broad} < t_d < t_f < L$: the formation of the dipole is strongly suppressed.

Discussion of time-scales



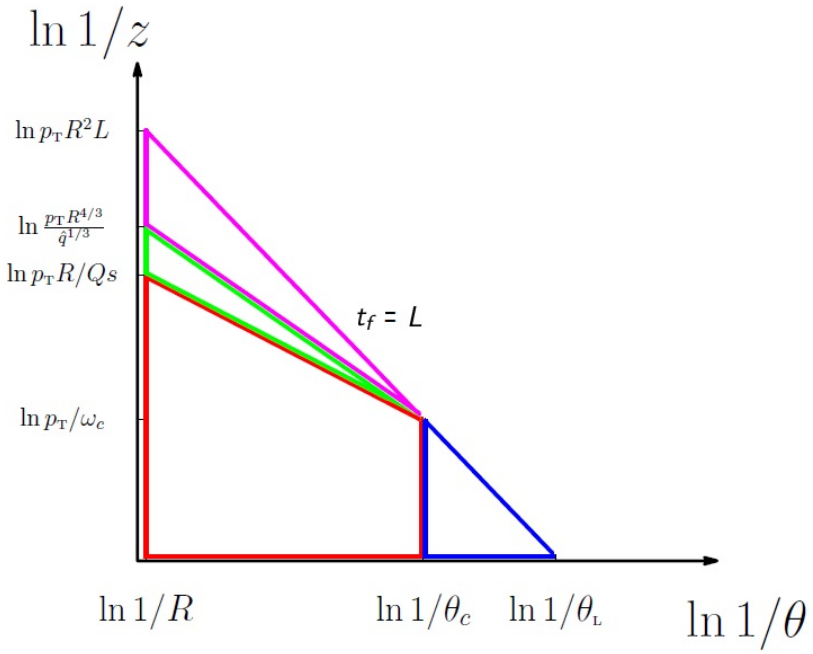
- (A.4) $t_f < L < t_d < t_{broad}$: the created partons will never decohere in color and the splittings should follow a vacuum emission pattern.

Discussion of time-scales



- (B) $t_f > L$: no medium modification is expected.

Discussion of time-scales



- (A.1) and (A.4) are regions of vacuum-like emissions inside the medium.

The medium modification factor

$$\frac{dI^{med}}{dz\ d\vec{p}^2} = \frac{dI^{vac}}{dz\ d\vec{p}^2}\ (1 + F_{med}) \tag{7}$$

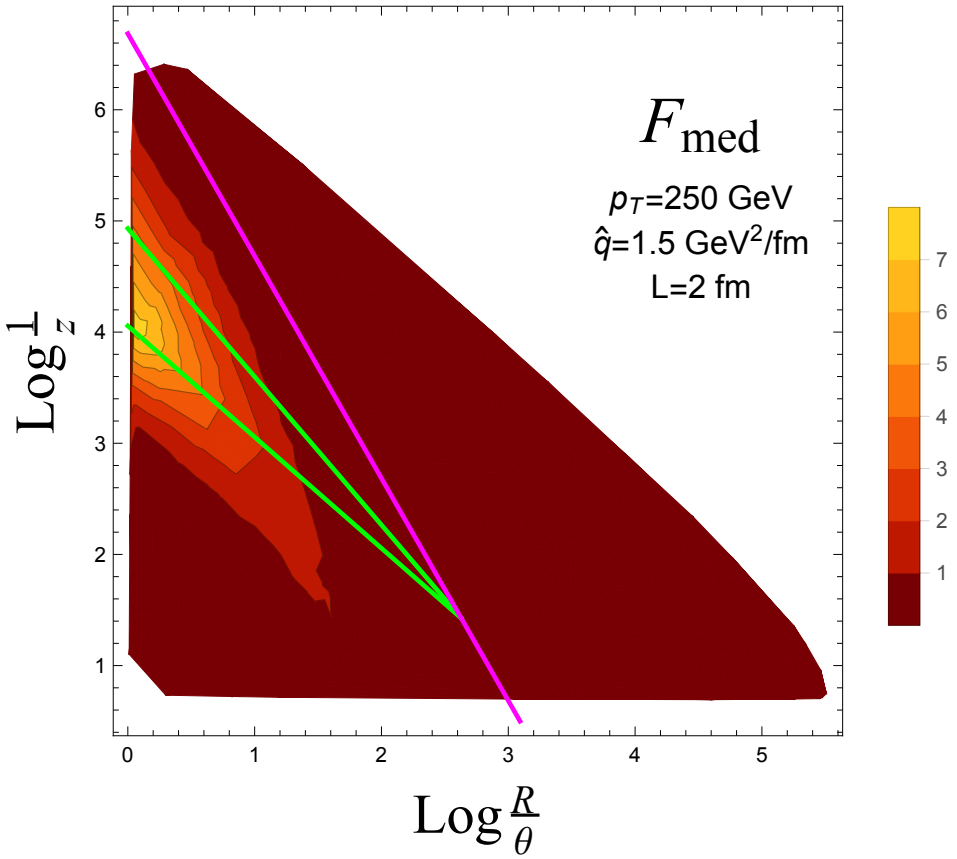
- The medium modifications factor out into F_{med}.

$$F_{med} = 2\int_0^{\zeta_L} ds\ \left[\int_s^{\zeta_L}\ ds'\ cos(s'-s)\ S_{12}(s',s)\ Q(\zeta_L,s,s') - sin(\zeta_L - s)\ S_{12}(\zeta_L,s)\right] \tag{8}$$

$$cos(s'-s)\ S_{12}(s',s)\ Q(\zeta_L,s,s')\ \propto \left\langle \left|\bar{\mathcal{M}}^{in}_{q\bar{q}}\right| \right\rangle^2$$

$$sin(\zeta_L - s)\ S_{12}(\zeta_L,s)\ \propto \left\langle \bar{\mathcal{M}}^{in}_{q\bar{q}} \bar{\mathcal{M}}^{\dagger,out}_{q\bar{q}} \right\rangle$$

Numerics: F_{med} in the Lund plane



Conclusions

- In spite of the singlet antenna limitations it turns out to be a very convenient *laboratory*.
- The in-medium finite formation time antenna setup shows two regimes of vacuum-like emissions, lending support to the notion of purely vacuum-like emissions that are emitted inside the medium.
- The finite formation time scenario shows us an interesting theoretical guidance for MC.
- These computations go a step forward to obtain a complete description of a QCD cascade.

Thanks for your attention