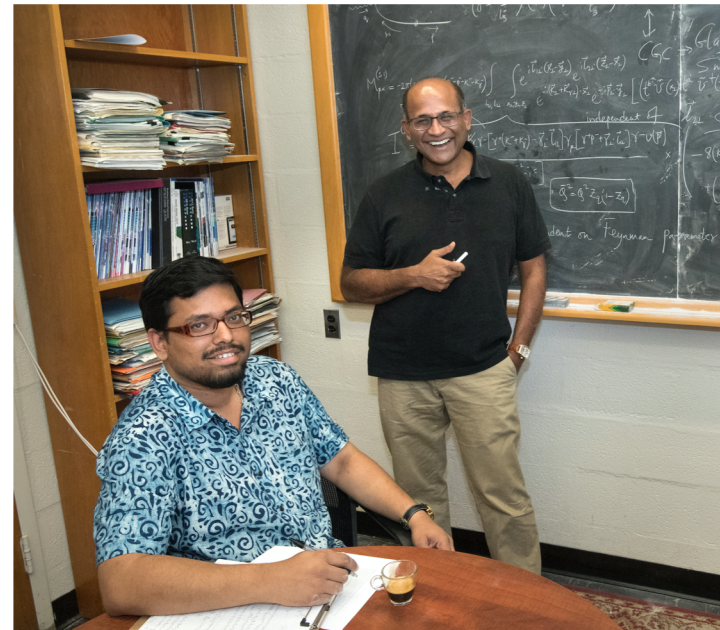
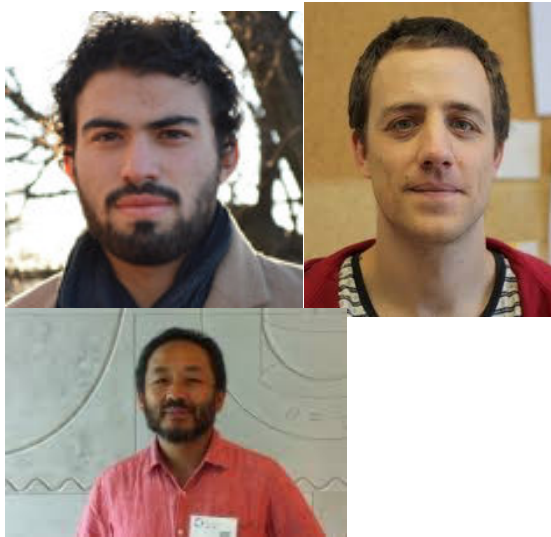


Inclusive photon+di-jet production in p+A and e+A collisions at small x

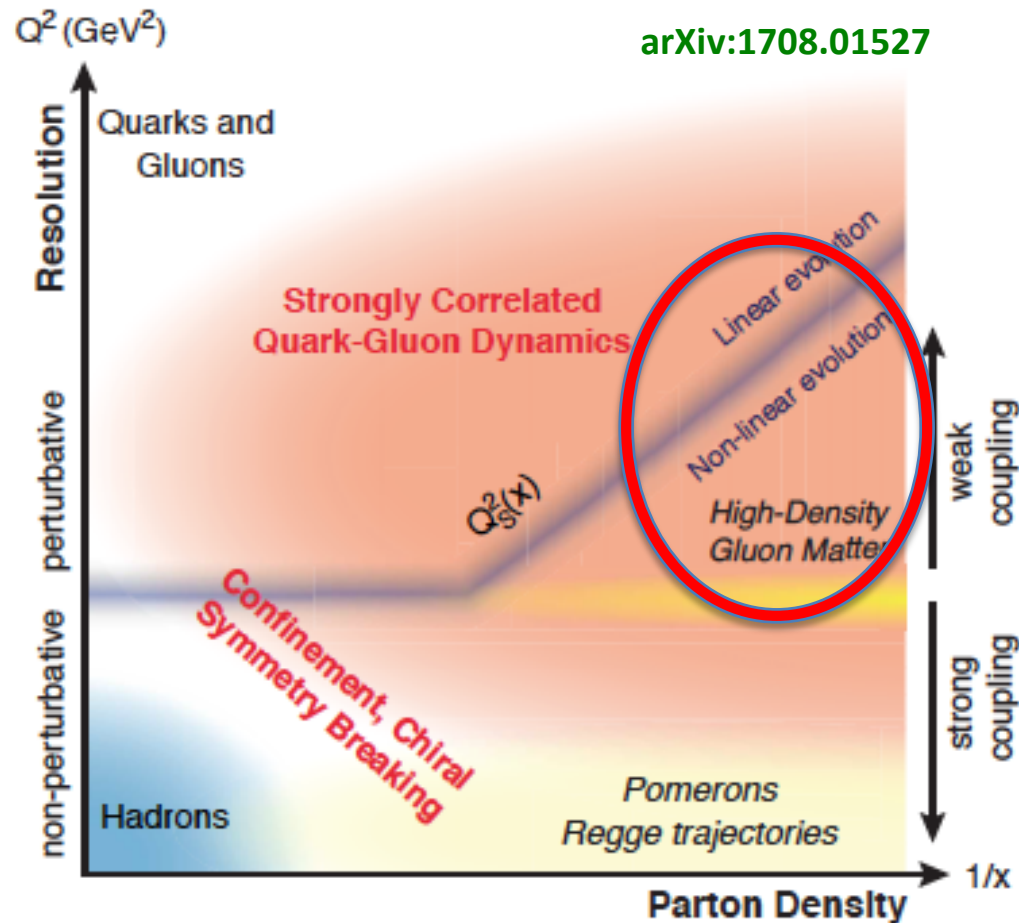
Raju Venugopalan

Brookhaven National Laboratory



Hard Probes 2018, Aix-les-Bains, October 1-5, 2018

The CGC EFT



EFT in the Regge limit of $Q^2 \gg \Lambda_{\text{QCD}}^2$ to study QCD matter at high parton densities

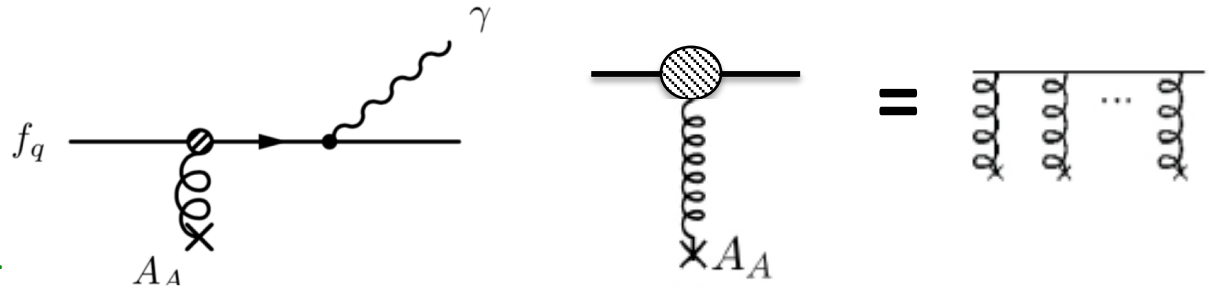
Photons are clean probes of dynamical many-body gluon correlations

Uncover universal structures from p+A and e+A collisions

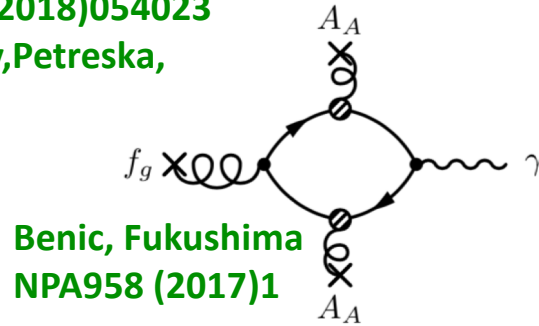
Dilute-dense CGC framework for p+p/p+A collisions

LO contribution:
computed previously

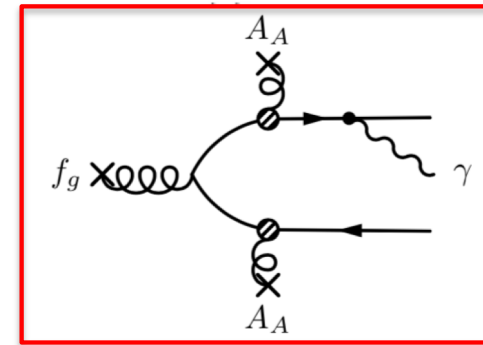
Gelis, Jalilian-Marian, PRD66 (2002) 014021
 Ducloue, Lappi, Mantysaari, PRD97 (2018) 054023
 Altinoluk, Armesto, Kovner, Lublinsky, Petreska, JHEP1804(2018)063



NLO contributions:



Benic, Fukushima
 NPA958 (2017)1



Benic, Fukushima,
 Garcia-Montero, RV,
 JHEP 1701(2017) 115

QCD Yang-Mills Eqns: $[D_\mu, F^{\mu\nu}](x) = g\delta^{\nu+}\delta(x^-)\rho_p(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_A(\mathbf{x}_\perp)$

Solve for gauge fields and propagators in the dilute-dense approximation: $\frac{\rho_p}{k_{\perp 1}^2} \ll 1$ $\frac{\rho_A}{k_{\perp 2}^2} \sim 1$ Blaizot, Gelis, RV, hep-ph/0402257

Inclusive photon+di-jet amplitude at NLO

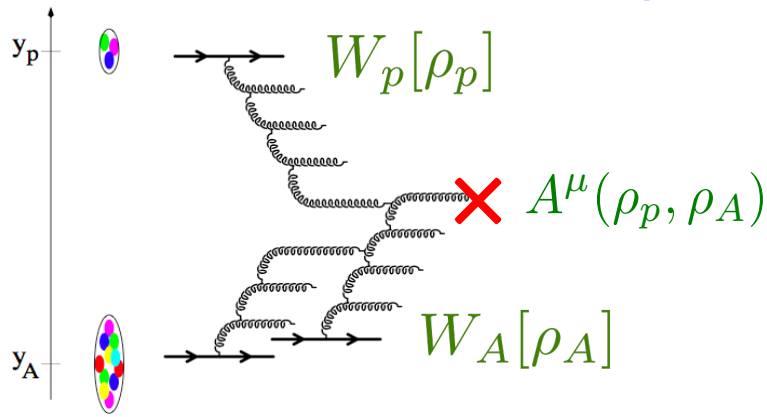
$$\mathcal{M}^\mu(\mathbf{p}, \mathbf{q}, \mathbf{k}_\gamma) = -q_f e g^2 \int_{\mathbf{k}_\perp \mathbf{k}_{1\perp}} \int_{\mathbf{x}_\perp \mathbf{y}_\perp} \frac{\rho_p^a(\mathbf{k}_{1\perp})}{k_{1\perp}^2} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp + i(\mathbf{P}_\perp - \mathbf{k}_\perp - \mathbf{k}_{1\perp}) \cdot \mathbf{y}_\perp} \times \bar{u}(\mathbf{q}) \{ T_g^\mu(\mathbf{k}_{1\perp}) U(\mathbf{x}_\perp)^{ba} t^b + T_{q\bar{q}}^\mu(\mathbf{k}_\perp, \mathbf{k}_{1\perp}) \tilde{U}(\mathbf{x}_\perp) t^a \tilde{U}^\dagger(\mathbf{y}_\perp) \} v(\mathbf{p})$$

Lightlike Wilson lines

$$\tilde{U}(\mathbf{x}_\perp) = \mathcal{P}_+ \exp \left[-ig^2 \int_{-\infty}^{\infty} dz^+ \frac{1}{\nabla_\perp^2} \rho_A(z^+, \mathbf{x}_\perp) \cdot t \right]$$

Dirac structures $T_g^\mu(\mathbf{k}_{1\perp})$ & $T_{q\bar{q}}^\mu$

Inclusive photon cross-section



Color distributions evolved in rapidity via the JIMWLK functional RG equation

$$\langle \mathcal{O} \rangle = \int [d\rho_A][d\rho_B] W_p[\rho_p] W_A[\rho_A] \mathcal{O}[\rho_p, \rho_A]$$

unintegrated gluon distribution

Target forward dipole amplitude

$$\frac{d\sigma^{\text{NLO}}}{d^2\mathbf{k}_{\gamma\perp} d\eta_\gamma} = S_\perp \sum_f \frac{\alpha_e \alpha_S N_c^2 q_f^2}{64\pi^4 (N_c^2 - 1)} \int_{\eta_q \eta_p} \int_{\mathbf{q}_\perp \mathbf{p}_\perp \mathbf{k}_{1\perp} \mathbf{k}_\perp} \frac{\varphi_p(Y_p, \mathbf{k}_{1\perp})}{k_{1\perp}^2} \tilde{\mathcal{N}}_{t, Y_t}(\mathbf{k}_\perp) \tilde{\mathcal{N}}_{t, Y_t}(\mathbf{P}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_\perp) \times [2\tau_{g,g}(\mathbf{k}_{1\perp}; \mathbf{k}_{1\perp}) + 4\tau_{g,q\bar{q}}(\mathbf{k}_{1\perp}; \mathbf{k}_\perp, \mathbf{k}_{1\perp}) + 2\tau_{q\bar{q},q\bar{q}}(\mathbf{k}_\perp, \mathbf{k}_{1\perp}; \mathbf{k}_\perp, \mathbf{k}_{1\perp})],$$

Results obtained by solving the rcBK equation

Benic, Fukushima, Garcia-Montero, RV, JHEP 1701(2017) 115

For $k_{\perp\gamma} \gg Q_S$ recover kt-factorization

$$\frac{d\sigma^\gamma}{d^6K_\perp d^3\eta_K} = \frac{\alpha_e \alpha_S^2 q_f^2}{256\pi^8 N_c (N_c^2 - 1)} \int_{\mathbf{k}_{1\perp} \mathbf{k}_{2\perp}} (2\pi)^2 \delta^{(2)}(\mathbf{P}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \frac{\varphi_p(Y_p, \mathbf{k}_{1\perp}) \varphi_A(Y_A, \mathbf{k}_{2\perp})}{k_{1\perp}^2 k_{2\perp}^2} \Theta(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp})$$

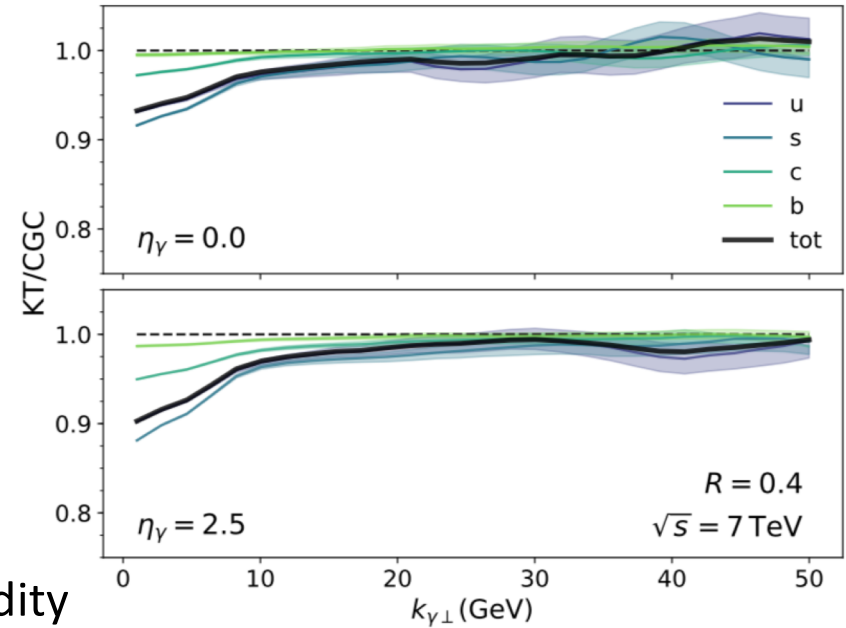
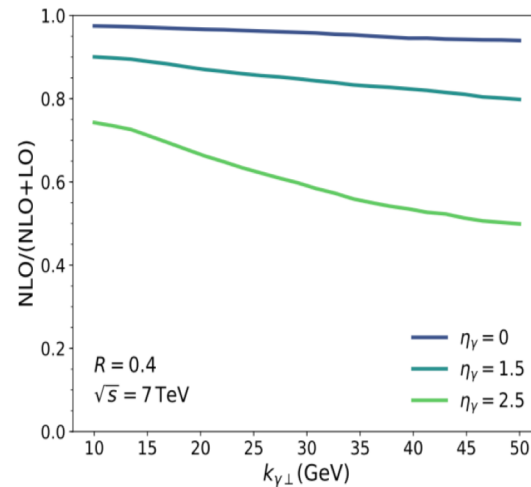
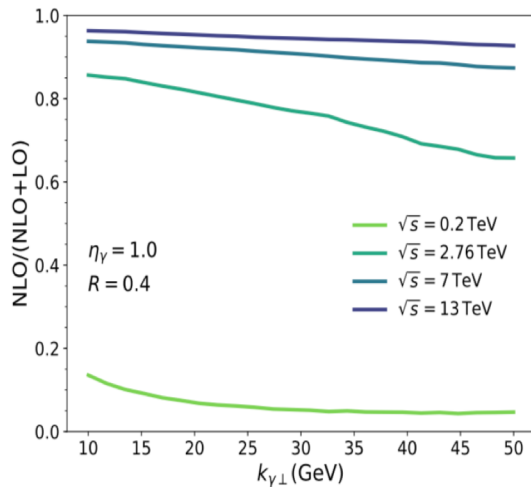
... and collinear factorization

$$\frac{d\sigma^\gamma}{d^2\mathbf{k}_\gamma d\eta_{k_\gamma}} = \frac{1}{16} \int_0^\infty \frac{dq^+}{q^+} \frac{dp^+}{p^+} \int_{\mathbf{q}_\perp \mathbf{p}_\perp} (2\pi)^2 \delta^{(2)}(\mathbf{p}_\perp + \mathbf{q}_\perp + \mathbf{k}_{\gamma\perp}) x_p f_{g,p}(x_p, Q^2) x_A f_{g,A}(x_A, Q^2) |\mathcal{M}_{gg \rightarrow q\bar{q}\gamma}|^2$$

$$|\mathcal{M}_{gg \rightarrow q\bar{q}\gamma}|^2 \equiv \lim_{\substack{\mathbf{k}_{1\perp} \rightarrow 0 \\ \mathbf{k}_{2\perp} \rightarrow 0}} \frac{q_f^2 \alpha_e \alpha_S^2}{N_c (N_c^2 - 1)} \frac{\Theta(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp})}{k_{1\perp}^2 k_{2\perp}^2}$$

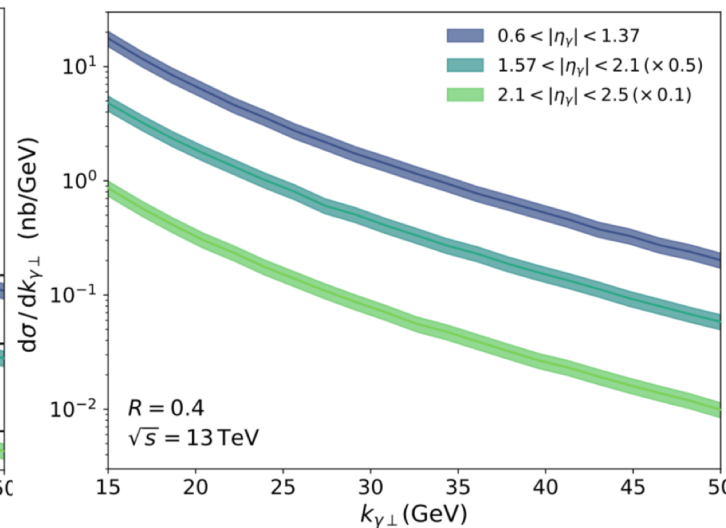
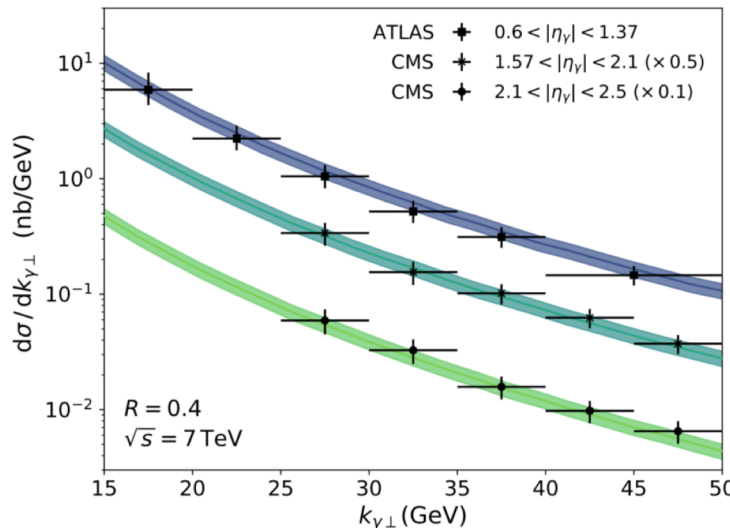
Small x nuclear gluon distribution

Results for inclusive photon production in p+p



- NLO dominates with increasing energy for fixed rapidity
- At LHC, LO fraction increases towards fragmentation region
- Dilute-dense framework includes **both** small x –resummation and saturation effects. However latter in p+p at most $\sim 10\%$

Benic, Fukushima,
Garcia-Montero, Venugopalan,
arXiv:1807.03806

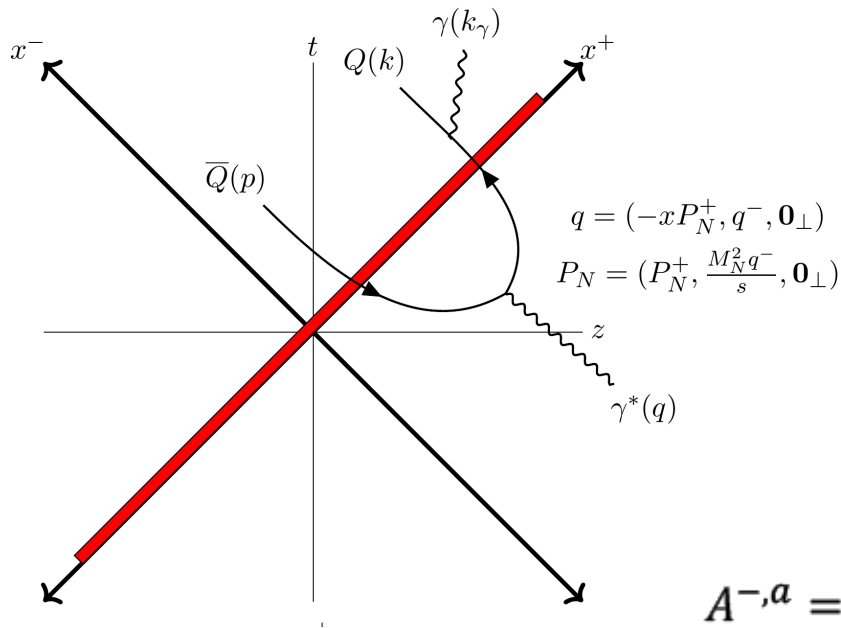


Expect p+A to be more
sensitive to saturation effects

K factor of 2.4 - need to go to
“NNLO” in this framework

Easier to first look at DIS...

Inclusive photon+dijet production in e+A DIS

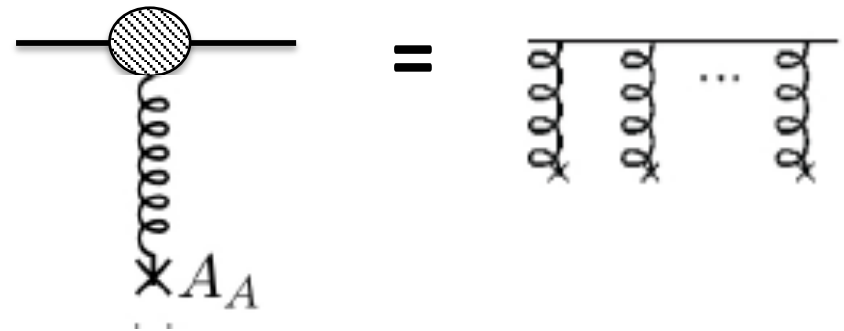
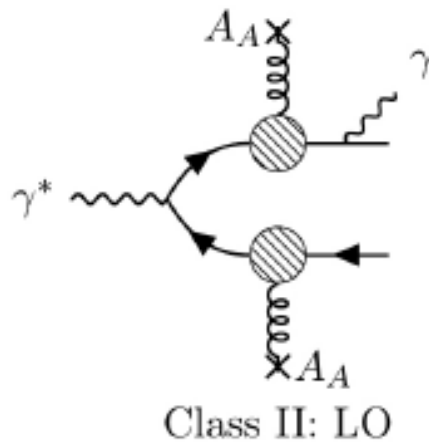
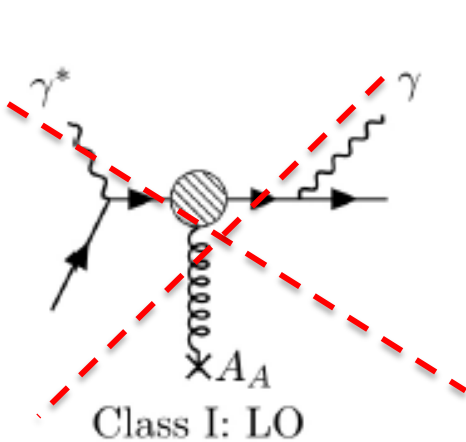


Right moving nucleus with momentum P_N^+ is Lorentz contracted in x direction

Glue fields satisfy Yang-Mills eqns.

$$[D_\mu, F^{\mu\nu}](x) = g \delta^{\nu+} \delta(x^-) \rho_A(x_\perp)$$

$$A^{-,a} = 0, F_{ij}^a = 0 \text{ with } A^{+,a}, A^{i,a} \text{ static (independent of } x^+)$$



Suppressed at small x

Inclusive photon+dijet production in DIS at LO

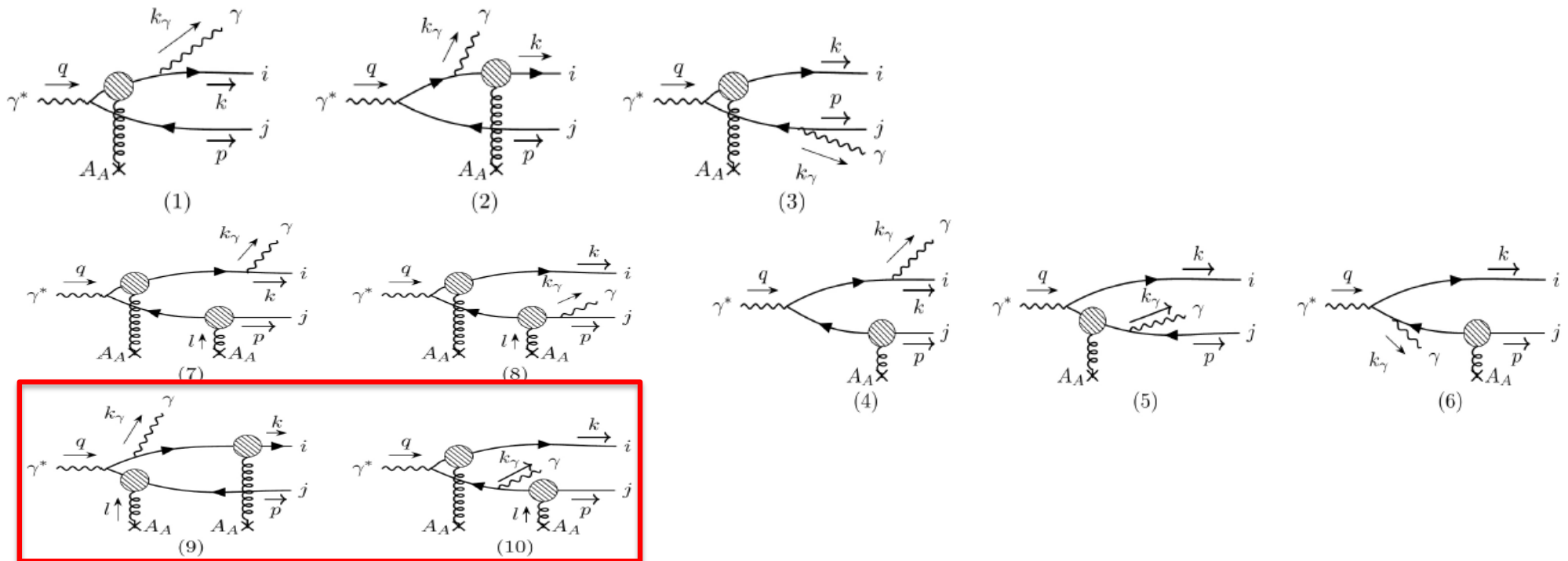
$$e(\tilde{l}) + A(P) \longrightarrow e(\tilde{l}') + Q(k) + \bar{Q}(p) + \gamma(k_\gamma) + X$$

Roy, RV; JHEP 1805 (2018) 013

$$\frac{d\sigma}{dx dQ^2} = \frac{2\pi y^2}{64\pi^3 Q^2} \frac{d^3k}{(2\pi)^3 2E_k} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3k_\gamma}{(2\pi)^3 2E_{k_\gamma}} \frac{1}{2q^-} \left(\frac{1}{2} \sum_{\text{spins}, \lambda} \langle |\tilde{\mathcal{M}}|^2 \rangle_{Y_A} \right) (2\pi) \delta(P^- - q^-)$$

$$\frac{1}{2} \sum_{\text{spins}, \lambda} \langle |\tilde{\mathcal{M}}|^2 \rangle_{Y_A} = L_{\mu\nu} X^{\mu\nu}$$

$L_{\mu\nu}$ is well known-the lepton tensor
 $X^{\mu\nu}$ - the hadron tensor for inclusive photon +dijet production is what we compute



DIS inclusive cross-section at LO

Roy, RV; JHEP 1805 (2018) 013

$$\frac{d\sigma}{dx dQ^2 d^2\mathbf{k}_{\gamma\perp} d\eta_{k_\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2} \frac{1}{2q^-} \int_0^{+\infty} \frac{dk^-}{k^-} \int_0^{+\infty} \frac{dp^-}{p^-} \int_{\mathbf{k}_\perp, \mathbf{p}_\perp} L^{\mu\nu} \tilde{X}_{\mu\nu} (2\pi) \delta(P^- - q^-)$$

$$L^{\mu\nu} = \frac{2e^2}{Q^4} \left[(\tilde{l}^\mu \tilde{l}'^\nu + \tilde{l}'^\nu \tilde{l}^\mu) - \frac{Q^2}{2} g^{\mu\nu} \right]$$

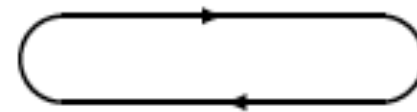
$$\tilde{X}_{\mu\nu} = \int_{\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{l}_\perp, \mathbf{l}'_\perp} e^{-i(\mathbf{P}_\perp - \mathbf{l}_\perp) \cdot \mathbf{x}_\perp - i\mathbf{l}_\perp \cdot \mathbf{y}_\perp + i(\mathbf{P}_\perp - \mathbf{l}'_\perp) \cdot \mathbf{x}'_\perp + i\mathbf{l}'_\perp \cdot \mathbf{y}'_\perp} \tau_{\mu\nu}^{q\bar{q}, q\bar{q}}(\mathbf{l}_\perp, \mathbf{l}'_\perp | \mathbf{P}_\perp) \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp)$$

Includes Dirac trace

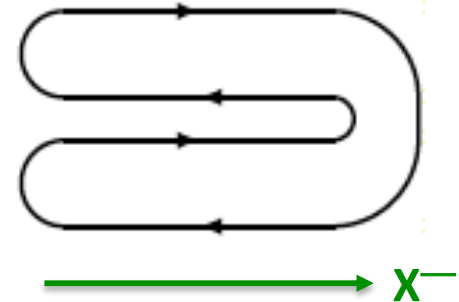
All the nonperturbative info about strongly correlated gluons is in

$$\Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) = 1 - D(\mathbf{x}_\perp, \mathbf{y}_\perp) - D(\mathbf{y}'_\perp, \mathbf{x}'_\perp) + Q(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)$$

Dipoles: $D(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$



Quadrupoles: $Q(x_\perp, y_\perp) = \frac{1}{N_c} \langle \text{Tr} \left(\tilde{U}(y'_\perp) \tilde{U}^\dagger(x'_\perp) \tilde{U}(x_\perp) \tilde{U}^\dagger(y_\perp) \right) \rangle_{Y_A}$



Interesting limits

When $k_\gamma \rightarrow 0$, the amplitude satisfies the Low-Burnett-Kroll theorem:

$$\mathcal{M}_\mu(\mathbf{q}, \mathbf{k}, \mathbf{p}, \mathbf{k}_\gamma) \rightarrow -(eq_f)\epsilon_\alpha^*(\mathbf{k}_\gamma, \lambda) \left(\frac{p^\alpha}{p \cdot k_\gamma} - \frac{k^\alpha}{k \cdot k_\gamma} \right) \mathcal{M}_\mu^{NR}(\mathbf{q}, \mathbf{k}, \mathbf{p})$$

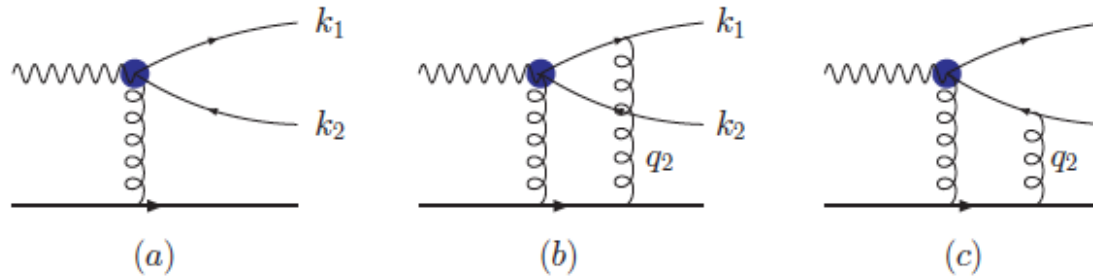
Polarization vector

×

Vectorial structure depending only on momenta of emitted particles

×

Non-radiative DIS amplitude

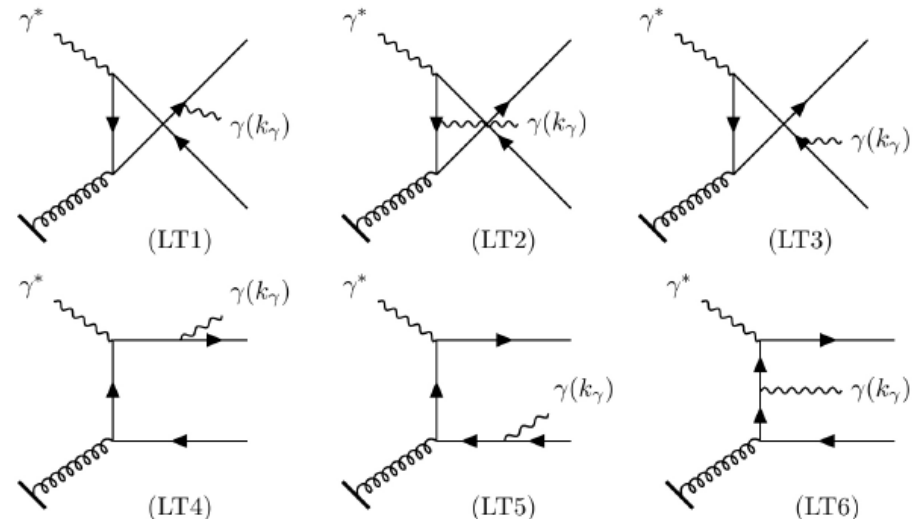


Recover results in soft photon limit for di-jet production
 - sensitive to the gluon Weizsäcker-Williams distribution
 for large pair momenta

Dominguez, Marquet, Xiao, Yuan,
 PRD83 (2011)105005

As for p+A, recover in DIS
 kt-factorization & collinear factorization
 small x limits
 (sensitivity to leading twist gluon distribution)

Aurenche et al., Z. Phys. C24, 309 (1984)



Structure of higher order computations: Shockwave propagators

Convenient to work in the **wrong** light cone gauge $A^- = 0$ for this problem
(Gauge links in pdf definitions are unity in the **right** LC gauge $A^+ = 0$)

Dressed quark and gluon propagators: remarkably simple forms in $A^- = 0$ gauge

$$S(p, p') = (2\pi)^4 \delta^{(4)}(p - p') S_0(p) + S_0(p) \mathcal{T}(p, p') S_0(p')$$

$$G^{\mu\nu; ab}(p, p') = (2\pi)^4 \delta^{(4)}(p - p') G_0^{\mu\nu; ab} + G_0^{\mu\rho; ac} \mathcal{T}_{\rho\sigma; cd} G_0^{\sigma\nu; db}(p')$$

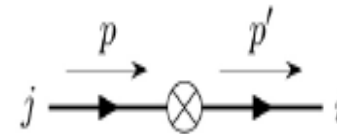
McLerran, RV; hep-ph/9402335

Ayala, Jalilian-Marian, McLerran, RV;
hep-ph/9508302

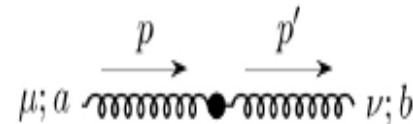
Balitsky, Belitsky, hep-ph/0110158

Structure of vertices identical
to quark-quark-reggion and
gluon-gluon-reggeon in
Lipatov's Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov,
Prygarin, arXiv:1708.05183
Hentschinski, arXiv:1802.06755

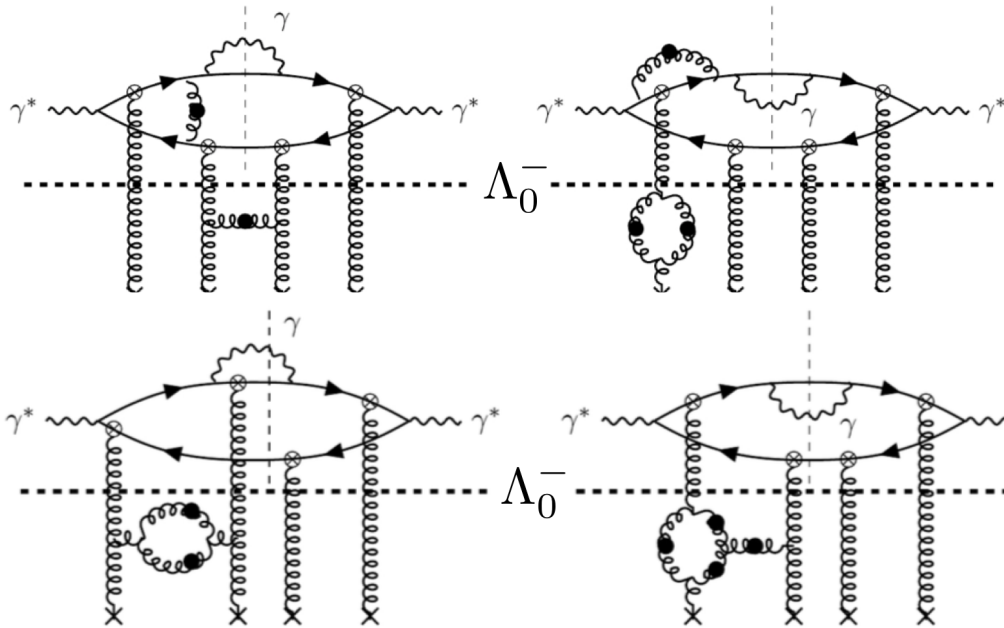


$$\mathcal{T}_{ij}(p, p') = (2\pi) \delta(p^- - p'^-) \gamma^- \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} \tilde{U}^{\text{sign}(p^-)}(\mathbf{z}_\perp)_{ij}$$



$$\mathcal{T}_{\mu\nu; ab}(p, p') = -2\pi \delta(p^- - p'^-) \times (2p^-) g_{\mu\nu} \text{sign}(p^-) \int d^2 \mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp) \cdot \mathbf{z}_\perp} U_{ab}^{\text{sign}(p^-)}(\mathbf{z}_\perp)$$

DIS inclusive photo+dijet production at NLO+NLLx



Roy, RV: in preparation

Formally NNLO: but collect NLO pieces in the photon+dijet impact factor + leading log pieces $\alpha_s \log(\Lambda_1^- / \Lambda_0^-)$

Formally NNLO: but collect LO pieces in the photon+dijet impact factor + NLL leading log pieces $\alpha_s^2 \log(\Lambda_1^- / \Lambda_0^-)$

$$\begin{aligned}
 \langle d\sigma_{NLO+NLLx} \rangle &= \int [\mathcal{D}\rho_A] \left\{ W^{NLLx}[\rho_A] d\hat{\sigma}_{LO}[\rho_A] + W^{LLx}[\rho_A] d\hat{\sigma}_{NLO}[\rho_A] \right\} \\
 &= \int [\mathcal{D}\rho_A] \left(W^{NLLx}[\rho_A] \left\{ d\hat{\sigma}_{LO}[\rho_A] + \boxed{d\hat{\sigma}_{NLO}[\rho_A]} \right\} + O(\alpha_s^3 \ln(\Lambda_1^- / \Lambda_0^-)) \right)
 \end{aligned}$$

$\ln(\Lambda_1^- / \Lambda_0^-) (\mathcal{H}_{LO} + \mathcal{H}_{NLO})$
NLO photon+dijet impact factor

Leading order JIMWLK Hamiltonian computed 20 years ago: 1997-2001

NLO JIMWLK Hamiltonian: 2013-2016

Balitsky, Chirilli, arXiv:1309.7644, Grabovsky, arXiv:1307.5414

Caron-Huot, arXiv:1309.6521, Kovner, Lublinsky, Mulian, arXiv:1310.0378, Lublinsky, Mulian, arXiv:1610.03453

The NLO inclusive photon impact factor

Several computations exist for inclusive DIS – subtleties in choice of scheme, etc.

Balitsky,Chirilli, arXiv:1009.4729

Beuf, arXiv:1606.00777, 1708.06557

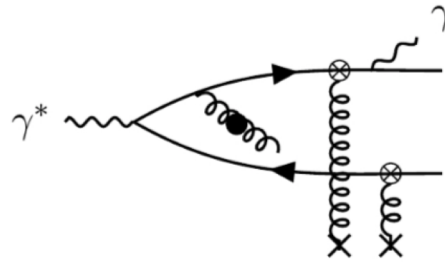
Hanninen, Lappi, Paatelainen, 1711.08207

Dijet: Boussarie, Grabovsky, Szymanowski, Wallon,1606.00419

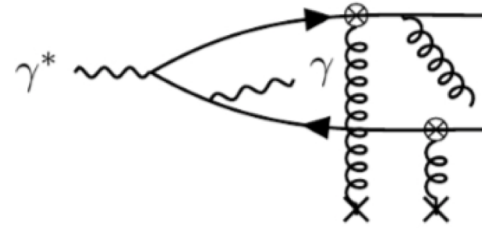
First computation discussed here of photon+dijet:

Roy, RV, in preparation

I) Real contributions:
(20 × 20 diagrams)



Gluon rescatters (or not) along with quarks

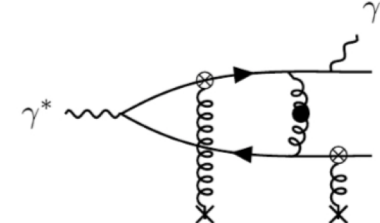
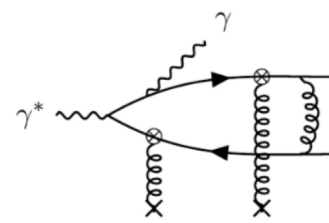
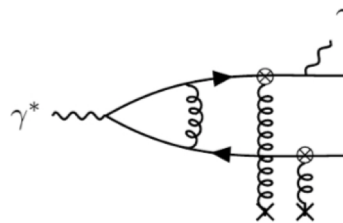


Gluon emitted after rescattering

II) Interference contributions:

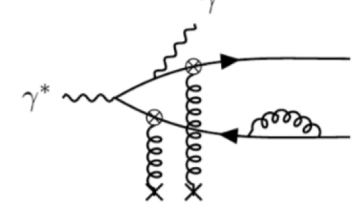
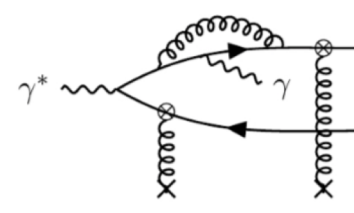
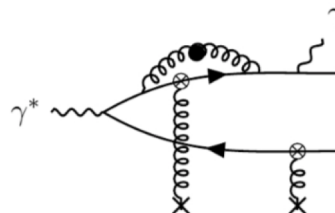
A) Vertex corrections

●●● and 21 more permutations



B) Self-energy corrections:

●●● and likewise, 21 more



Coda: A nontrivial derivation of JIMWLK evolution

coda : something that serves to round out, conclude, or summarize and usually has its own interest.

Roy, RV, in preparation

$$\text{Recall } X_{\mu\nu}^{\text{LO}} = C_{\mu\nu}^{\text{LO}} \otimes \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-) \quad \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-) = 1 - D_{xy} - D_{y'x'} + Q_{y'x';xy}$$

In the "soft gluon limit" that generates logs in x, our NLO hadron tensor gives

$$\begin{aligned} X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln(\Lambda_1^- / \Lambda_0^-) & \left[\frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) D_{xy} + \left(\begin{array}{c} \mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp \\ \mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp \end{array} \right) \right\} - \frac{\alpha_S N_c}{(2\pi)^2} \mathcal{K}_1(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{y}'_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) Q_{xy;y'x'} \right. \\ & - \frac{\alpha_S N_c}{2\pi^2} \left\{ \mathcal{K}_B(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{z}_\perp) D_{xz} D_{zy} + \left(\begin{array}{c} \mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp \\ \mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp \end{array} \right) \right\} + \frac{\alpha_S N_c}{(2\pi)^2} \left(\left\{ \mathcal{A}(\mathbf{x}_\perp, \mathbf{y}'_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) D_{xx'} D_{y'y} + \mathbf{x}_\perp \leftrightarrow \mathbf{y}'_\perp \right\} \right. \\ & \left. \left. + \left\{ \mathcal{K}_2(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp; \mathbf{z}_\perp) D_{xz} Q_{zy;y'x'} + \left(\begin{array}{c} \mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp \\ \mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp \end{array} \right) \right\} + \left\{ \mathcal{K}_2(\mathbf{x}'_\perp, \mathbf{x}_\perp, \mathbf{y}'_\perp; \mathbf{z}_\perp) D_{zx'} Q_{y'z;xy'} + \left(\begin{array}{c} \mathbf{x}_\perp \rightarrow \mathbf{y}'_\perp \\ \mathbf{y}_\perp \rightarrow \mathbf{x}'_\perp \end{array} \right) \right\} \right) \right], \end{aligned}$$

Nontrivial combinations of dipole and quadrupole operators

Dominguez,Mueller,Munier,Xiao, PLB705(2011)106

Remarkably, this can be reexpressed as $X_{\mu\nu;LLx}^{\text{NLO}} = C_{\mu\nu}^{\text{LO}} \otimes \ln\left(\frac{\Lambda_1^-}{\Lambda_0^-}\right) H_{\text{JIMWLK}}^{\text{LO}} \Xi(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp | \Lambda_0^-)$

This immediately leads to the JIMWLK RG equation for the rapidity evolution of many-body gluon correlators

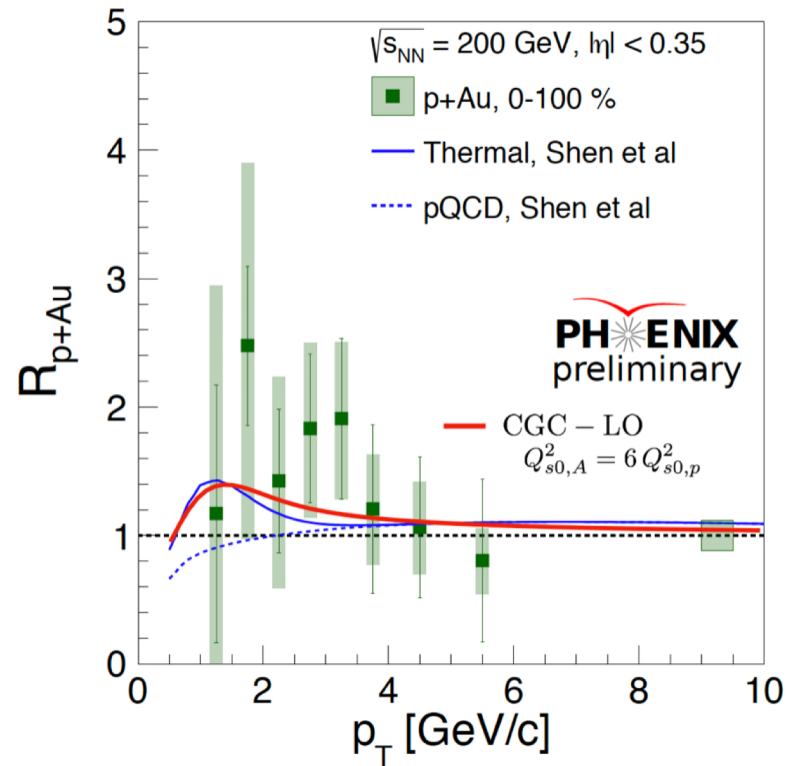
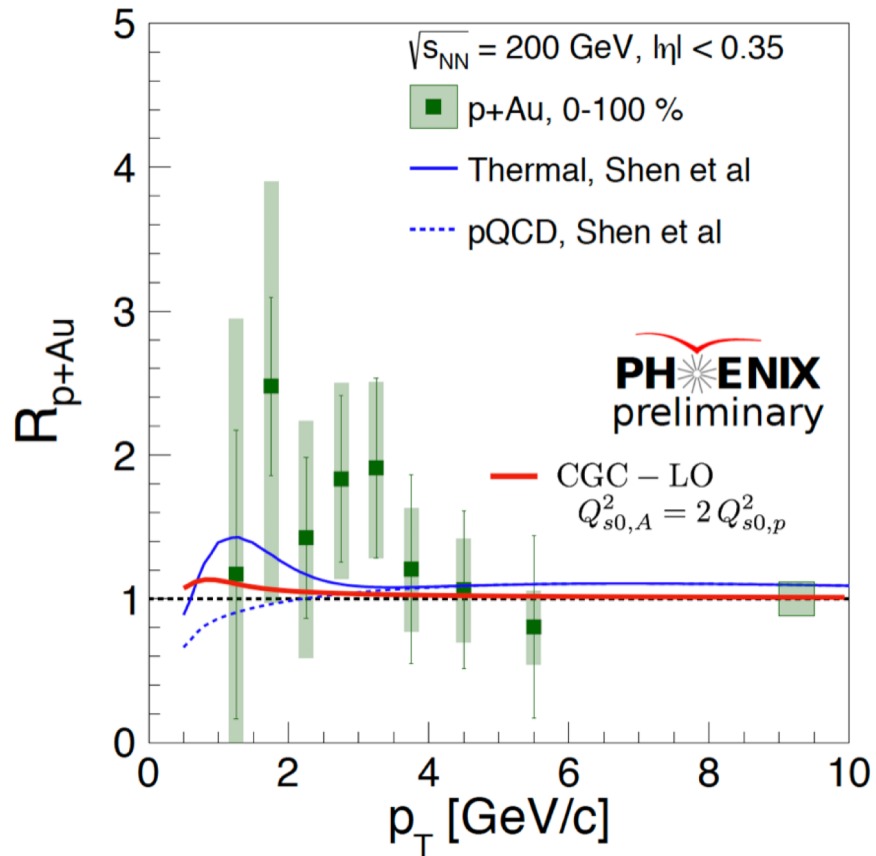
$$W_{\Lambda_1^-}[\rho_A] = \left(1 + \ln\left(\frac{\Lambda_1^-}{\Lambda_0^-}\right) H_{\text{JIMWLK}}^{\text{LO}}\right) W_{\Lambda_0^-}[\rho_A]$$

Summary and Outlook

- ◆ Discussed inclusive photon production in p+p/A collisions. Good agreement with data in p+p but sensitivity to saturation effects small in min. bias. Compute p+A. Need NNLO to improve accuracy
- ◆ First computation of inclusive photon+dijet in e+A collisions. Rich structure in terms of 2-point and 4-point Wilson line correlators. Well-known result for Weizsäcker-Williams gluon distribution recovered in the soft photon limit. In leading twist, the cross-section is directly proportional to the nuclear gluon distribution
- ◆ The structure of dressed quark and gluon propagators in the “wrong” light cone gauge $A^- = 0$ is remarkably simple and *facilitates higher order computations in momentum space* (using standard techniques in pQCD)
- ◆ Outlined first computation of the NLO inclusive photon+dijet impact factor. Nontrivial derivation of JIMWLK functional RG equation

Backup slides

Inclusive photon production in p+A



Preliminary (theory) results courtesy of Oscar Garcia-Montero

The NLO inclusive photon impact factor

Inclusive photon cross-section: NLO-real * NLO*real + LO * NLO virtual

Wilson line factor	Real emission	Virtual: Vertex	Virtual: Self-energy
$\frac{N_c^2}{2} \left(1 - D_{xz} D_{zy} - D_{y'z} D_{zx'} + D_{y'y} D_{xx'} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(1)*} T_R^{(1)}$		
$C_F N_c \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(2)*} T_R^{(2)} + T_R^{(3)*} T_R^{(3)}$	$T_{LO}^* T_V^{(3)} + c.c$	$T_{LO}^* T_S^{(3)} + c.c$
$\frac{N_c^2}{2} [(1 - D_{xy})(1 - D_{y'x'})] - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(2)*} T_R^{(3)} + c.c$	$T_{LO}^* T_V^{(4)} + c.c$	
$\frac{N_c^2}{2} \left(1 + (Q_{zy;y'x'} - D_{zy}) D_{xz} - D_{y'x'} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(2)*} T_R^{(1)}$	$T_{LO}^* T_V^{(1)}$	$T_{LO}^* T_S^{(1)}$
$\frac{N_c^2}{2} \left(1 + (Q_{xy;y'z} - D_{y'z}) D_{zx'} - D_{xy} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(1)*} T_R^{(2)}$	$T_V^{(1)*} T_{LO}$	$T_S^{(1)*} T_{LO}$
$\frac{N_c^2}{2} \left(1 + (Q_{y'x';xz} - D_{xz}) D_{zy} - D_{y'x'} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(3)*} T_R^{(1)}$	$T_{LO}^* T_V^{(2)}$	$T_{LO}^* T_S^{(2)}$
$\frac{N_c^2}{2} \left(1 + (Q_{xy;zx'} - D_{zx'}) D_{y'z} - D_{xy} \right) - \frac{1}{2} \Xi(x_\perp, y_\perp; y'_\perp, x'_\perp)$	$T_R^{(1)*} T_R^{(3)}$	$T_V^{(2)*} T_{LO}$	$T_S^{(2)*} T_{LO}$

For each Wilson line structure, collinear divergences cancel between real and Interference contributions

Rapidity and UV divergent pieces: these can be absorbed, in a subtraction scheme, into the NLLx JIMWLK expressions.