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Charm, bottom, and quarkonia cross sections for double and triple-parton scatterings in high-energy proton-nucleus and nucleus-nucleus collisions

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based on (mainly):
Phys. Rev. Lett. 118, 122001 (2017);
Eur. Phys. J C 78, 359 (2018);
arXiv:1708.07519 [hep-ph], MPI at the LHC, World Scientific, 2018

PARTON MODEL (PM)

Elastic scattering : electron — proton
——–> **proton (hadron) is NOT point-like**

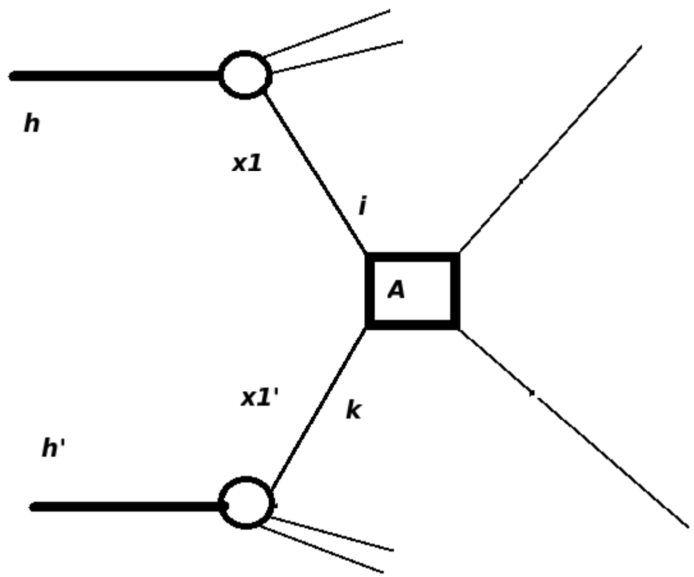
Deep inelastic scattering : electron — proton
——–> **proton (hadron) consists of point-like particles-partons**

Cross section (hadron) $=$ Σ cross section (parton) $\times$ weights

Weights — probabilities (densities) in the system of infinite momentum

(Bjorken, Feynman)

IN QCD weights depend on Q of hard processes
(SCALING VIOLATION, improved PM)



$$\sigma^A_{\mathrm{SPS}} = \sum_{i,k} \int D^i_h(x_1; Q_1^2)\hat{\sigma}^A_{ik}(x_1, x_1^{'})D^k_{h'}(x_1^{'}; Q_1^2)dx_1 dx_1^{'}$$

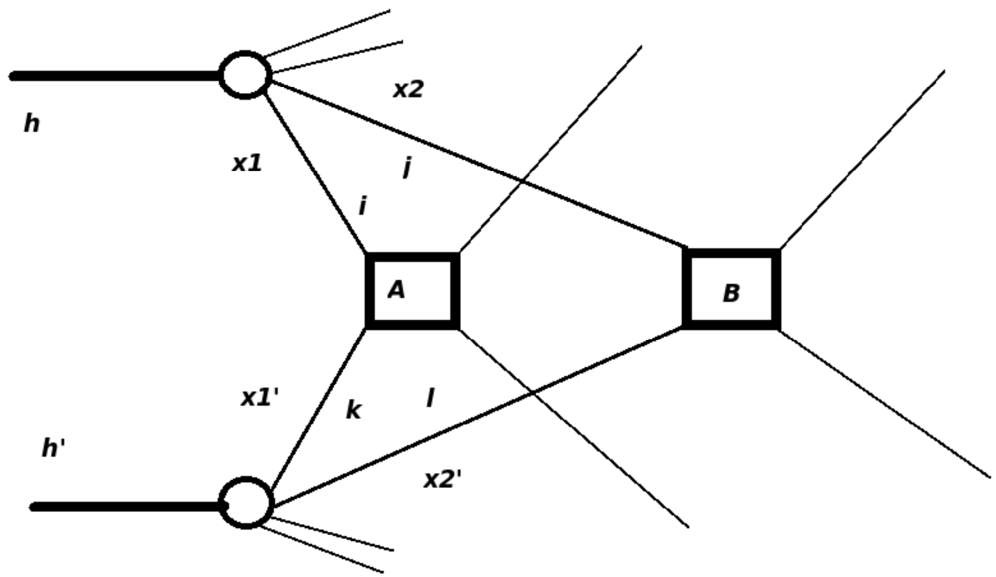
Scaling violation (dependence on Q) from DGLAP (*Dokshitzer-Gribov-Lipatov-Altarelli-Parisi* **) equations:**

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x',t) P_{j' \to j}\left(\frac{x}{x'}\right)$$

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}{\ln\left(\frac{\mu^2}{\Lambda^2_{QCD}}\right)}\right], \quad b = \frac{33 - 2n_f}{12\pi},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2,
n_f is the number of active flavours,
Λ_{QCD} is the dimensional QCD parameter.

It is possible (BUT less likely): hard double parton scattering (subprocesses A and B)



The inclusive cross section of a double parton scattering process in a hadron collision is written in the following form (with only the assumption of factorization of the two hard parton subprocesses A and B)

(Paver, Treleani,..., Blok,...., Diehl,...).

$$\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}^B_{jl}(x_2, x_2^{'}, Q_2^2)$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2b_1 d^2b_2 d^2b,$$

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two parton undergoing hard processes A and B at the scales Q_1 and Q_2.

$\hat{\sigma}^A_{ik}$ and $\hat{\sigma}^B_{jl}$ are the parton-level subprocess cross sections.

The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain all the information when probing the hadron in two different points simultaneously, through the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2),$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1,$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (not calculated in pQCD).

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$\boldsymbol{D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2)},$$

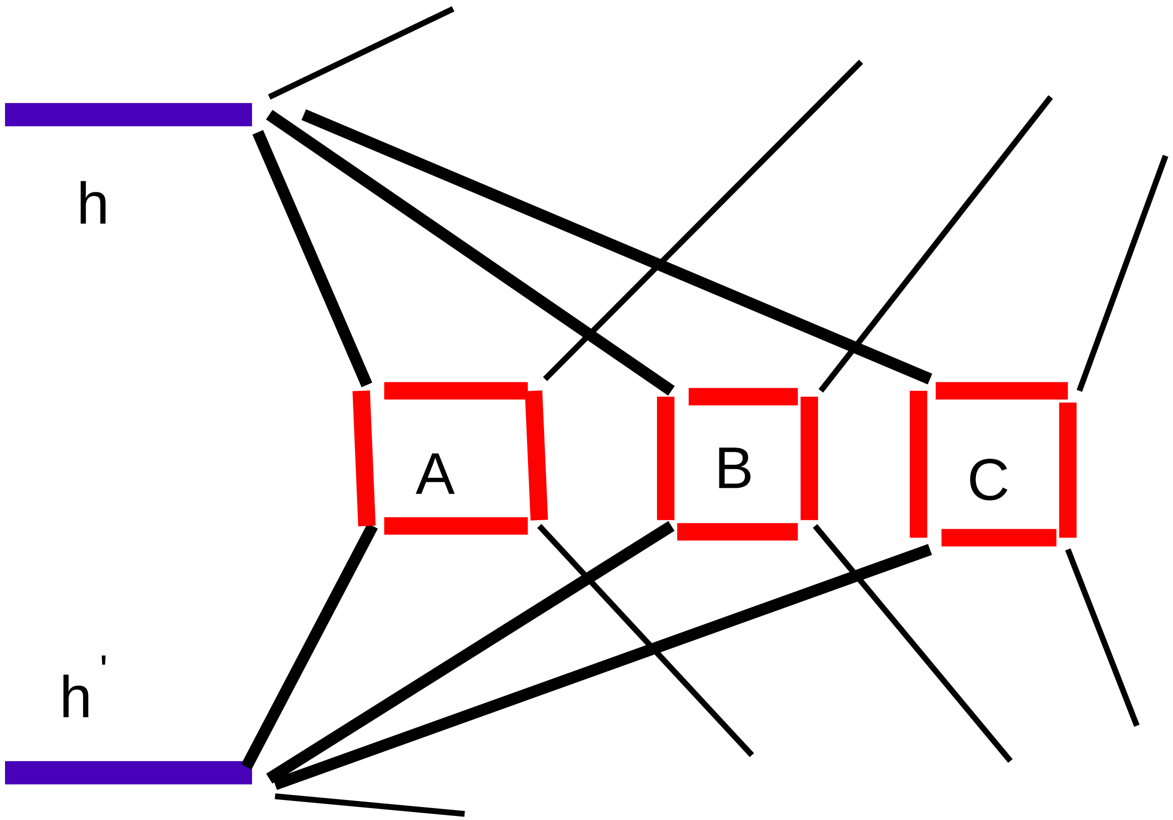
the cross section of double parton scattering can be expressed in the simple form

$$\boldsymbol{\sigma_{\mathrm{DPS}}^{\mathrm{AB}} = \frac{m}{2} \frac{\sigma_{\mathrm{SPS}}^{A} \sigma_{\mathrm{SPS}}^{B}}{\sigma_{\mathrm{eff}}}},$$

$$\boldsymbol{\pi R_{\mathrm{eff}}^2 = \sigma_{\mathrm{eff}} = [\int d^2 b (T(\mathrm{b}))^2]^{-1}}$$

is the effective interaction transverse area (effective cross section). R_{eff} is an estimate of the size of the hadron.

It is **possible** **(BUT very rarely): hard TRIPLE parton scattering (subprocesses** A**,** B **and** C**)**



Similar to DPS with only the assumption of factorization of the three hard parton subprocesses A, B and C, the inclusive cross section of a TPS process in a hadron collision may be written in the following form

$$\sigma_{\mathrm{TPS}}^{(A,B,C)} = \sum_{i,j,k,l,m,n} \int \Gamma_{ijk}(x_1, x_2, x_3; \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3; Q_1^2, Q_2^2, Q_3^2)$$

$$\times \hat{\sigma}_{il}^{A}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}_{jm}^{B}(x_2, x_2^{'}, Q_2^2) \hat{\sigma}_{kn}^{C}(x_3, x_3^{'}, Q_3^2)$$

$$\times \Gamma_{lmn}(x_1^{'}, x_2^{'}, x_3^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}, \mathrm{b}_3 - \mathrm{b}; Q_1^2, Q_2^2, Q_3^2)$$

$$\times dx_1 dx_2 dx_3 dx_1^{'} dx_2^{'} dx_3^{'} d^2b_1 d^2b_2 d^2b_3 d^2b.$$

Here $\Gamma_{ijk}(x_1, x_2, x_3; \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}; Q_1^2, Q_2^2, Q_3^2)$ are the triple parton distribution functions, which depend on the longitudinal momentum fractions x_1, x_2 and x_3 and on the transverse position $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{b_3}$ of the three partons i, j and k undergoing the hard subprocesses A, B and C at the scales Q_1, Q_2 and Q_3. $\hat{\sigma}_{il}^{A}$, $\hat{\sigma}_{jm}^{B}$ and $\hat{\sigma}_{kn}^{C}$ are the parton-level subprocess cross sections.
The appropriate combinatorial factor $(m/3!)$ should be taken into account in the case of the indistinguishable final states.

As in the case of DPS it is typically taken that the triple parton distribution functions may be decomposed in terms of the longitudinal and transverse components as follows:

$$\Gamma_{ijk}(x_1, x_2, x_3; \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3; Q_1^2, Q_2^2, Q_3^2)$$

$$= D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) f(\mathrm{b}_1) f(\mathrm{b}_2) f(\mathrm{b}_3),$$

where $f(\mathrm{b}_1)$ is supposed to be an universal function for all kind of partons as before.

If one makes the further assumption that the longitudinal components $D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2)$ reduce to the product of three independent single parton distributions,

$$D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2) D_h^k(x_3; Q_3^2)$$

the cross section of TPS can be expressed in the simple form

$$\sigma_{\mathrm{TPS}}^{(A,B,C)} = \frac{\sigma_{\mathrm{SPS}}^{A} \sigma_{\mathrm{SPS}}^{B} \sigma_{\mathrm{SPS}}^{C}}{\sigma_{\mathrm{eff,TPS}}^{2}}$$

$$\sigma^2_{\text{eff,TPS}} = [\int d^2b(T(\text{b}))^3]^{-1}.$$

$$\sigma_{\text{eff}} = \sigma_{\text{eff,DPS}} = [\int d^2b(T(\text{b}))^2]^{-1}$$

$$\sigma_{\text{eff,TPS}} = k \cdot \sigma_{\text{eff}}$$

with $k = 0.82 \pm 0.11$ as the average of all typical parton transverse profiles usually used in the literature (Gaussian, dipole fit, PYTHIA, HERWIG,....)

In numerical examples:

$$\sigma_{\text{eff,DPS}} = \sigma_{\text{eff}} = 15 \pm 5mb$$

extracted from a wide range of DPS measurements at Tevatron and LHC and:

$$\sigma_{\text{eff,TPS}} = 12.5 \pm 4.5mb$$

The cross section of NPS can also be expressed in the simple form

$$\boldsymbol{\sigma}^{\text{NPS}}_{hh'\to a_1\ldots a_n} = \left(\frac{m}{n!}\right)\frac{\boldsymbol{\sigma}^{\text{SPS}}_{hh'\to a_1}\ldots\boldsymbol{\sigma}^{\text{SPS}}_{hh'\to a_n}}{\boldsymbol{\sigma}^{n-1}_{\text{eff,NPS}}}$$

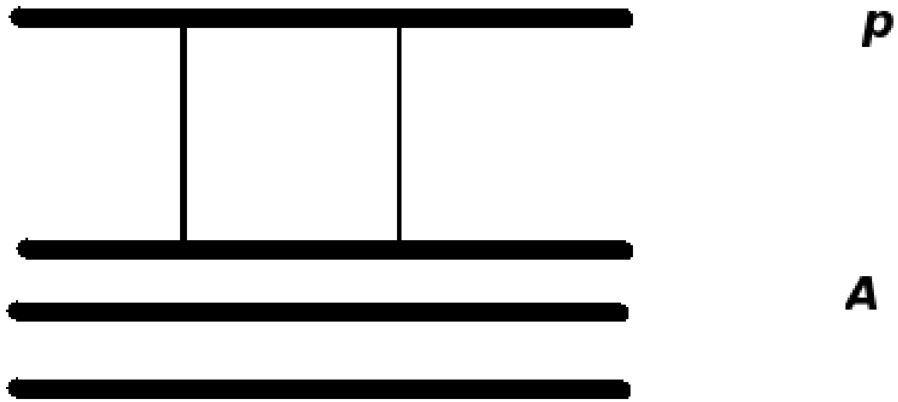
with

$$\boldsymbol{\sigma}^{n-1}_{\text{eff,NPS}} = \left[\int d^2b(T(\mathrm{b}))^n\right]^{-1}.$$

DPS in pA

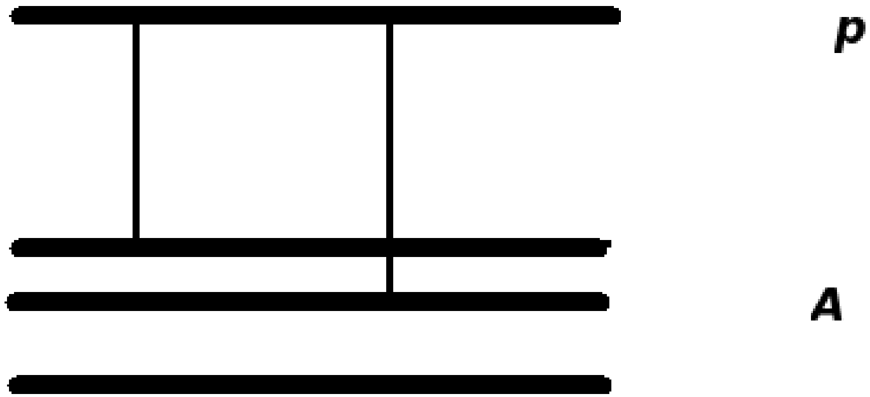
(Strikman, Treleani; Blok, Strikman, Wiedemann; d'Enterria, Snigirev,.....) **:**

1. The two partons of the nucleus belong to the same nucleon



Nuclear enhancement factor A as for SPS

2. The two partons of the nucleus belong to the different nucleons



Nuclear enhancement factor: $\propto A^2/A^{2/3} = A^{1+1/3}$
($A^{2/3}$ **due to the difference of the transverse sizes between p and A)**

The final DPS cross section "pocket formula" in pA collisions:

$$\sigma^{\mathrm{DPS}}_{(pA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{\mathrm{SPS}}_{(NN\to a)} \cdot \sigma^{\mathrm{SPS}}_{(NN\to b)}}{\sigma_{\mathrm{eff,DPS,pA}}},$$

where

$$\sigma_{\mathrm{eff,DPS,pA}} = \frac{1}{A\left[\sigma^{-1}_{\mathrm{eff,DPS,pp}} + \frac{1}{A}\,\mathrm{T}_{\mathrm{AA}}(0)\right]} = 21.5\mu\mathrm{b}$$

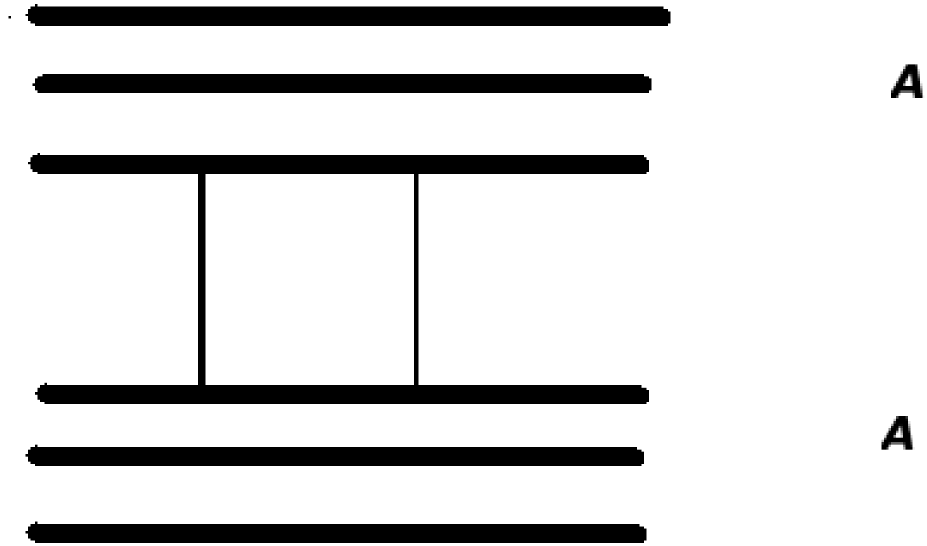
for p-Pb at $\sigma_{\mathrm{eff,DPS,pp}} = 15$ mb and $\mathrm{T}_{\mathrm{AA}}(0) = 30.25$ 1/mb for the standard nuclear overlap function normalized to A^2.

The relative contribution of the two terms are approximately 1 : 2

The overall increase of DPS cross sections in pA compared to pp collisions is $\sigma_{\mathrm{eff,DPS,pp}}/\sigma_{\mathrm{eff,DPS,pA}} \approx [A + A^{4/3}/\pi]$ which, in the case of pPb implies a factor of ∼600 relative to pp (ignoring nuclear PDF effects here), i.e. a factor of $[1+A^{1/3}/\pi] \approx 3$ higher than the naive expectation assuming the same A-scaling of the single-parton cross sections

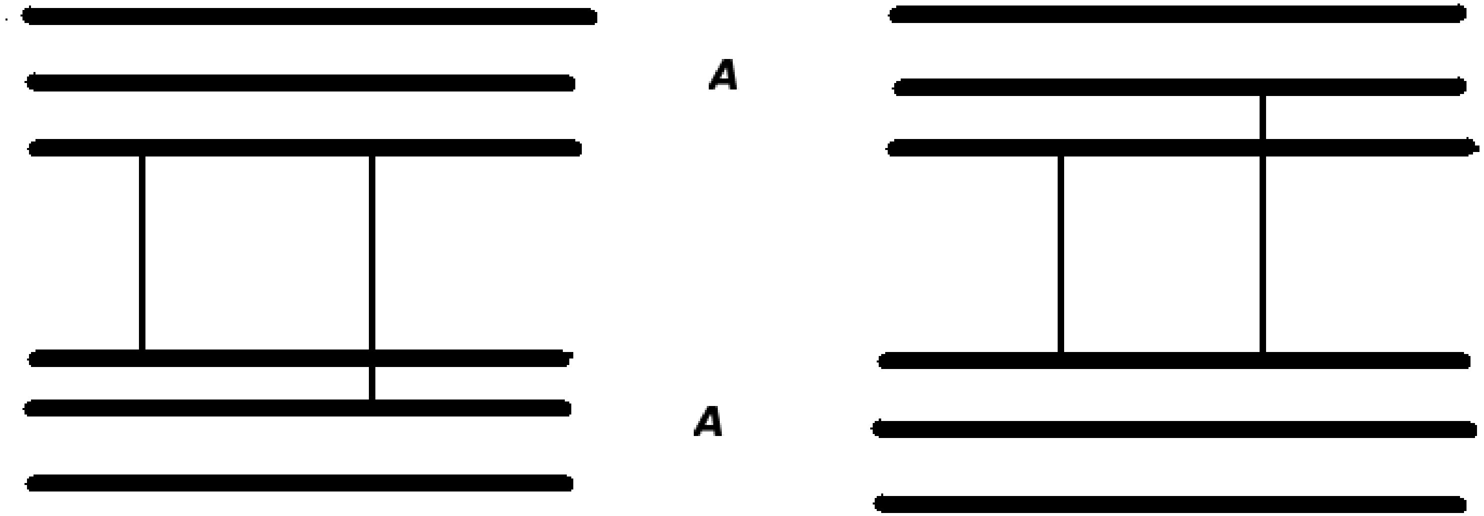
DPS in AA :

1. The two colliding partons belong to the same pair of nucleons



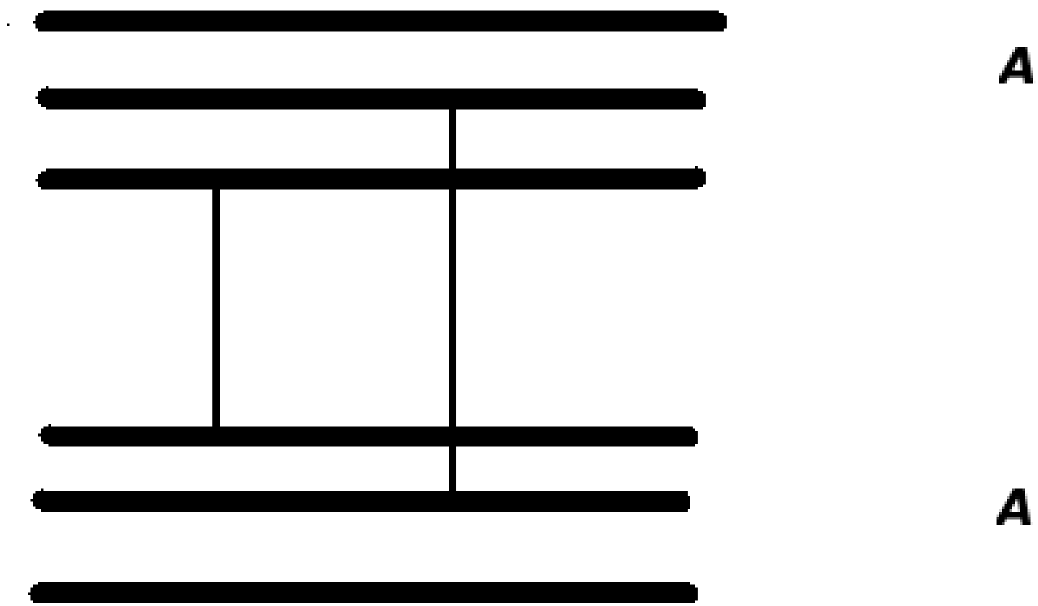
Nuclear enhancement factor A^2 as for SPS

2. Partons from one nucleon in one nucleus collide with partons from two different nucleons in the other nucleus



Nuclear enhancement factor: $\propto A^3/A^{2/3} = A^{2+1/3}$
($A^{2/3}$ **due to the difference of the transverse sizes between p and A)**

3. The two colliding partons belong to two different nucleons from both nuclei (in fact, double nucleon scattering)



Nuclear enhancement factor: $\propto A^4/A^{2/3} = A^{2+4/3}$
($A^{2/3}$ due to the difference of the transverse sizes between p and A)

The final DPS cross section "pocket formula" in AA collisions:

$$\sigma^{\rm DPS}_{(AA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{\rm SPS}_{(NN\to a)} \cdot \sigma^{\rm SPS}_{(NN\to b)}}{\sigma_{\rm eff,DPS,AA}},$$

where

$$\sigma_{\rm eff,DPS,AA} = \frac{1}{A^2\left[\sigma^{-1}_{\rm eff,DPS,pp} + \frac{2}{A}\,\mathrm{T_{AA}}(0) + \frac{1}{2}\,\mathrm{T_{AA}}(0)\right]} = 1.5 \text{ nb}$$

for Pb-Pb at $\sigma_{\rm eff,DPS,pp} = 15$ mb and $\mathrm{T_{AA}}(0) = 30.25$ 1/mb for the standard nuclear overlap function normalized to A^2.

The relative contribution of 3 terms are approximately 1 : 4 : 200

Clearly, the "pure" DPS contributions arising from partonic collisions within a single nucleon are much smaller than the last term from double particle production coming from two independent *nucleon-nucleon* collisions. The DPS cross sections in AA are practically unaffected by the value of $\sigma_{\rm eff,DPS,pp}$, but dominated instead by double-parton interactions from *different nucleons* in both nuclei.

TPS in pA
(Strikman, Treleani, d'Enterria, Snigirev) **:**

Three contributions:

1. The three partons of the nucleus belong to the same nucleon.
Nuclear enhancement factor A as for SPS

2. The interactions of parton from two different nucleons in the nucleus.
Nuclear enhancement factor: $\propto A^2/A^{2/3} = A^{1+1/3}$
($A^{2/3}$ due to the difference of the transverse sizes between p and A)

3. The interactions among parton from three different nucleons.
Nuclear enhancement factor: $\propto A^3/(A^{2/3})^2 = A^{1+2/3}$
($(A^{2/3})^2$ due to the difference of the transverse sizes between p and A)

The final TPS cross section "pocket formula" in pA collisions:

$$\sigma^{\mathrm{TPS}}_{(pA\to abc)} = \left(\frac{m}{6}\right) \frac{\sigma^{\mathrm{SPS}}_{(pN\to a)} \cdot \sigma^{\mathrm{SPS}}_{(pN\to b)} \cdot \sigma^{\mathrm{SPS}}_{(pN\to b)}}{\sigma^2_{\mathrm{eff,TPS,pA}}},$$

where

$$\sigma_{\mathrm{eff,TPS,pA}} \simeq \frac{A}{\sigma^2_{\mathrm{eff,TPS}}} + \frac{A^{4/3}}{5.7[\mathrm{mb}]\pi\sigma_{\mathrm{eff,TPS}}} + \frac{A^{5/3}}{160[\mathrm{mb}^2]\pi^2} = 0.29\mathrm{mb}$$

for p-Pb at $\sigma_{\mathrm{eff,TPS}} = 12.5$ mb and $\mathrm{T_{AA}}(0) = 30.4$ 1/mb for the standard nuclear overlap function normalized to A^2.

The relative contribution of the three terms are approximately

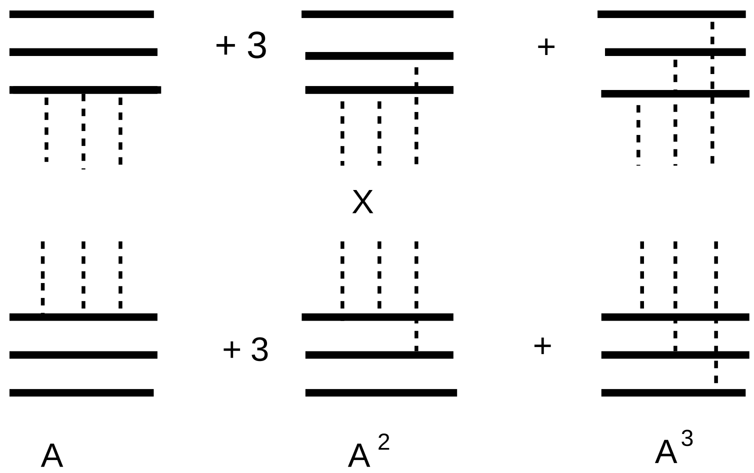
1 : 4.54 : 3.56

Namely, in pPb collisions, 10% of the TPS yields come from partonic interactions within just one nucleon of the lead nucleus, 50% involve scatterings within two nucleons, and 40% come from partonic interactions in three different Pb nucleons.

TPS in AA

(d'Enterria, Snigirev) **:**

TPS cross section in AA collisions results from the sum of nine terms, schematically represented in Fig., generated by three independent structures appearing in triple parton scatterings in pA:



$$\sigma^{\mathrm{TPS}}_{\mathrm{AA}\to\mathrm{abc}} \propto A\cdot A + 3A\cdot A^2 + A\cdot A^3$$

$$+3A^2\cdot A + 9A^2\cdot A^2 + 3A^2\cdot A^3$$

$$+A^3\cdot A + 3A^3\cdot A^2 + A^3\cdot A^3.$$

These nine terms have different prefactors that can be expressed as a function of the nuclear thickness function, and the effective TPS and DPS cross sections, as done previously for the simpler pA case. For instance, the first $A \cdot A$ term is just the TPS cross section in NN collisions scaled by A^2:

$$\boldsymbol{\sigma}^{\mathrm{TPS},1}_{\mathrm{AA}\to\mathrm{abc}} = \boldsymbol{A}^2 \boldsymbol{\sigma}^{\mathrm{TPS}}_{\mathrm{NN}\to\mathrm{abc}}$$

whereas the last $A^3 \cdot A^3$ contribution arises from interactions of partons from three different nucleons in one nucleus with partons from three different nucleons in the other nucleus (i.e., they result from triple *nucleon-nucleon* scatterings). The ratio

$$\boldsymbol{\sigma}^{\mathrm{TPS},1}_{\mathrm{AA}\to\mathrm{abc}} / \boldsymbol{\sigma}^{\mathrm{TPS},9}_{\mathrm{AA}\to\mathrm{abc}} \simeq [2/\boldsymbol{\sigma}_{\mathrm{eff,DPS}} \boldsymbol{T}_{\mathrm{AA}}(0)]^2$$

shows that the "pure" TPS contributions arising from partonic collisions within a single nucleon (which scale as A^2) are negligible compared to triple particle production coming from three independent nucleon-nucleon collisions which scale as $A^6 (r_p/R_{\mathbf{a}})^4 \propto A^{14/3}$.

In the PbPb case, the relative weights of these two "limiting" TPS contributions are $1 : 40000$, to be compared with $1 : 200$ for the similar DPS weights.

The many other intermediate terms correspond to the various "mixed" parton-nucleon contributions, which **can be also written in analytical form** in this approach but, however, are suppressed by additional powers of A compared to the dominant nucleon-nucleon triple scattering.

Thus, as found in the DPS case, TPS processes in AA collisions **are not so useful** to derive $\sigma_{\text{eff,DPS}}$ or $\sigma_{\text{eff,TPS}}$ and thereby study the intranucleon partonic structure as in pp or pA collisions. The estimates presented here demonstrate that double- and triple- (hard) nucleon-nucleon scatterings represent a significant fraction of the inelastic hard AA cross section, and the standard Glauber MC provides a simper approach to compute their occurrence in a given heavy-ion collision.

The formalism of DPS was applied to study:

same-sign W-boson pair production in pPb collisions at LHC energies
(*Phys. Lett. B 718, 1395 (2013)*)

J/ψ**-pair production in Pb-Pb collisions at LHC energies**
(*Phys. Lett. B 727, 157 (2013)*)

Specification in calculations, results and plots
— in original papers (+ nice presentations (*d'Enterria*) on Hard Probes 2013, Quark Matter 2014)

DPS production cross sections of
double-J/ψ, $J/\psi+\Upsilon$, J/ψ+W, J/ψ+Z,
double-Υ, Υ+W, Υ+Z, and same-sign WW
in Pb-Pb and p-Pb at the LHC:

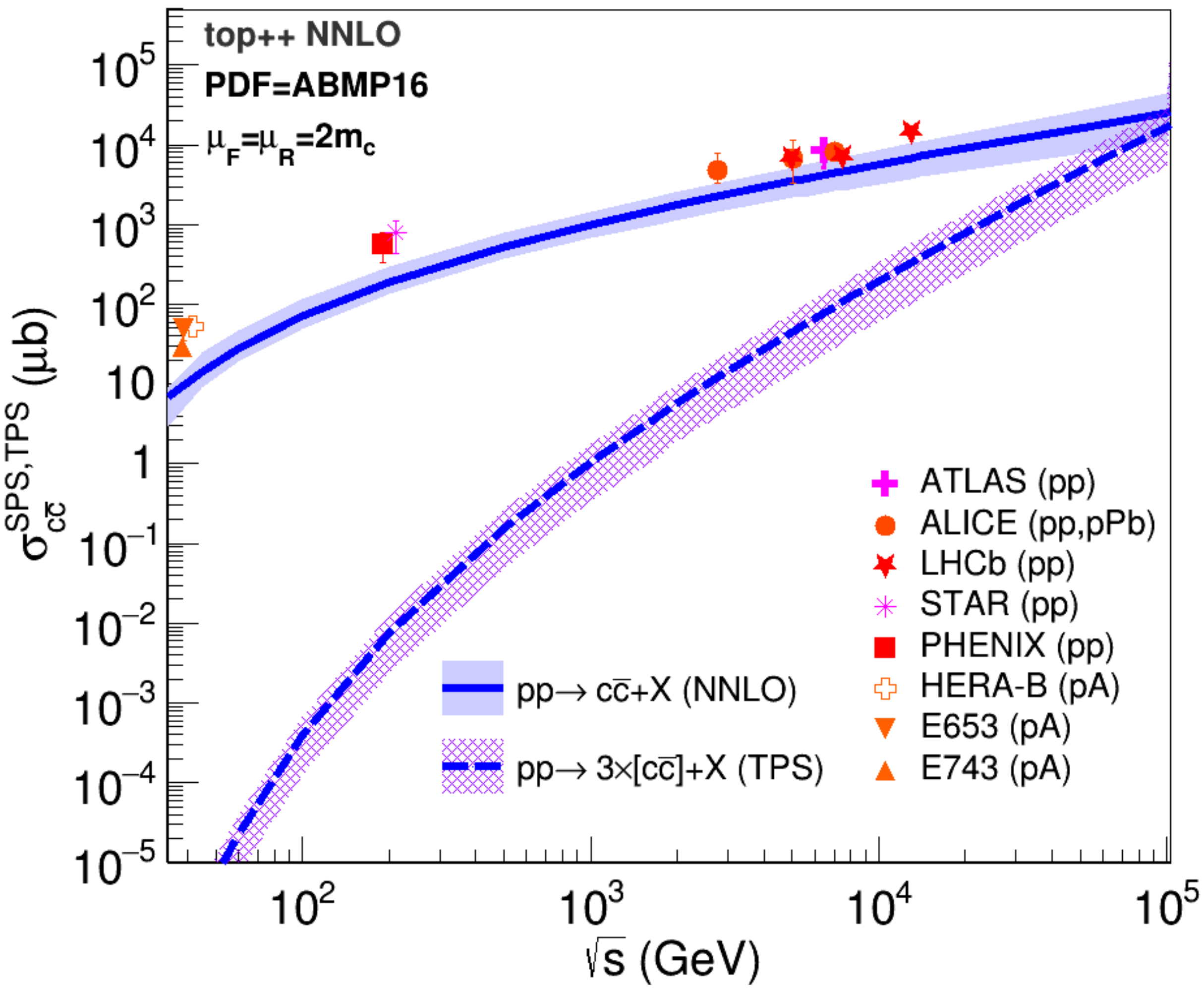
System		$J/\psi+J/\psi$	$J/\psi+\Upsilon$	J/ψ+W	J/ψ+Z	$\Upsilon+\Upsilon$	Υ+W	Υ+Z	ss WW
Pb-Pb	$\sigma^{\rm DPS}$	210 mb	28 mb	500 μb	330 μb	960 μb	34 μb	23 μb	630 nb
5.5 TeV	$\rm N^{DPS}$ (1 nb^{-1})	∼250	∼340	∼65	∼14	∼95	∼35	∼8	∼15
p-Pb	$\sigma^{\rm DPS}$	45 μb	5.2 μb	120 nb	70 nb	150 nb	7 nb	4 nb	150 pb
8.8 TeV	$\rm N^{DPS}$ (1 pb^{-1})	∼65	∼60	∼15	∼3	∼15	∼8	∼1.5	∼4

(Nucl. Phys. A 931, 303 (2014); A 932, 296 (2014))

The corresponding DPS yields, after (di)lepton decays and acceptance+efficiency losses, are given for 1 nb^{-1} and 1 pb^{-1} respectively.

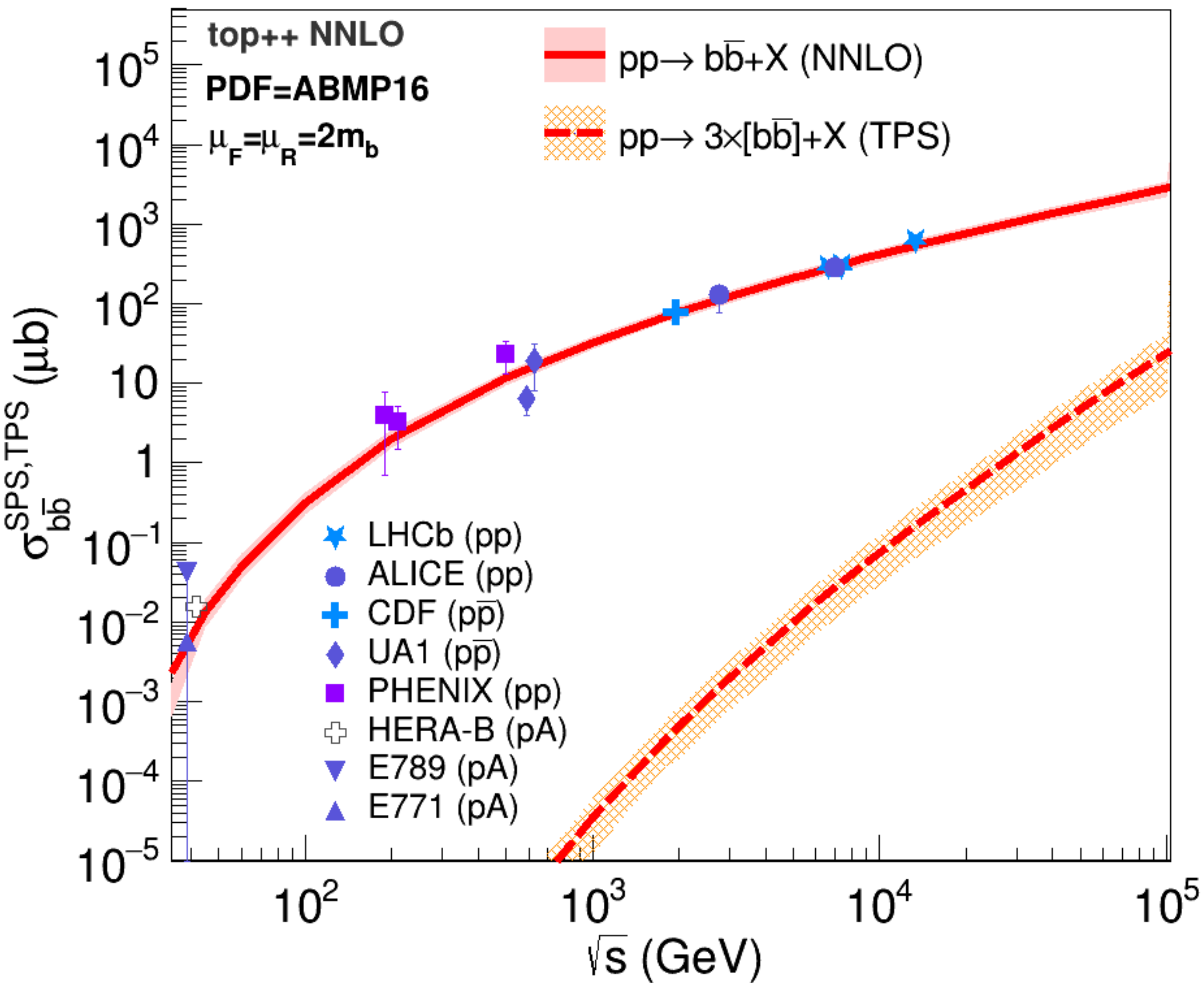
Thus, the simultaneous production of quarkonia and/or electroweak bosons from DPS processes have large visible cross sections and are open to study in p-Pb and Pb-Pb at the LHC.

TPS in pp:



Cross sections for charm production in SPS (NNLO) and TPS processes in pp collisions at LHC and FCC energies. The quoted uncertainties include scales (sc), PDF, and total (quadratic, including $\sigma_{\rm eff,TPS}$ values).

TPS in pp:



Cross sections for bottom production in SPS (NNLO) and TPS processes in pp collisions at LHC and FCC energies. The quoted uncertainties include scales (sc), PDF, and total (quadratic, including $\sigma_{\mathrm{eff,TPS}}$ values).

TPS in pp:

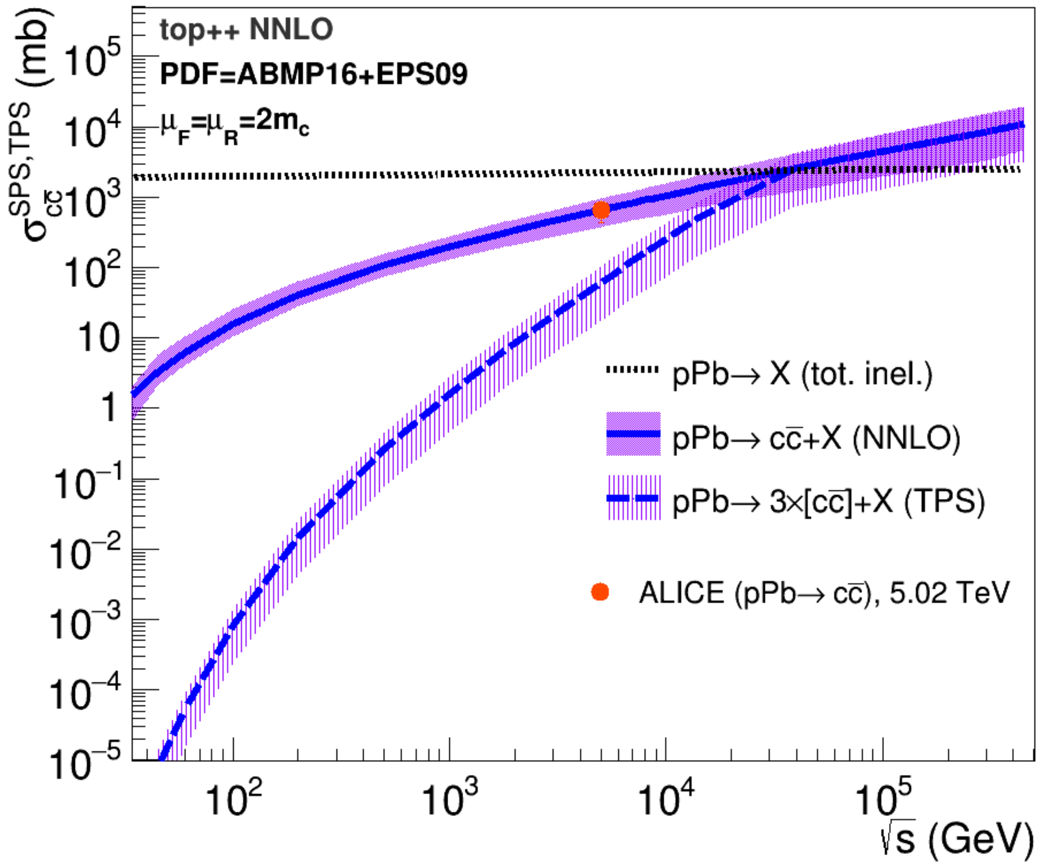
Cross sections for charm and bottom production in SPS (NNLO) and TPS processes in pp collisions at LHC and FCC energies. The quoted uncertainties include scales (sc), PDF, and total (quadratic, including $\sigma_{\mathrm{eff,TPS}}$) values

Final state	$\sqrt{s}$ =**14 TeV**	$\sqrt{s}$ =**100 TeV**
$\sigma^{\mathrm{SPS}}_{c\bar{c}+X}$	$\mathbf{7.1}\pm 3.5_{\mathrm{sc}} \pm 0.3_{\mathrm{pdf}}$ **mb**	$\mathbf{25.0}\pm 16.0_{\mathrm{sc}} \pm 1.3_{\mathrm{pdf}}$ **mb**
$\sigma^{\mathrm{TPS}}_{c\bar{c},c\bar{c},c\bar{c}+X}$	$\mathbf{0.39}\pm 0.28_{\mathrm{tot}}$ **mb**	$\mathbf{16.7}\pm 11.8_{\mathrm{tot}}$ **mb**
$\sigma^{\mathrm{SPS}}_{b\bar{b}+X}$	$\mathbf{0.56}\pm 0.09_{\mathrm{sc}} \pm 0.01_{\mathrm{pdf}}$ **mb**	$\mathbf{2.8}\pm 0.6_{\mathrm{sc}} \pm 0.1_{\mathrm{pdf}}$ **mb**
$\sigma^{\mathrm{TPS}}_{b\bar{b},b\bar{b},b\bar{b}+X}$	$\mathbf{0.19}\pm 0.12_{\mathrm{tot}}$ μ**b**	$\mathbf{24}\pm 17_{\mathrm{tot}}$ μ**b**

The possibility of detecting triple charm-meson production in pp collisions at the LHC has been discussed in:

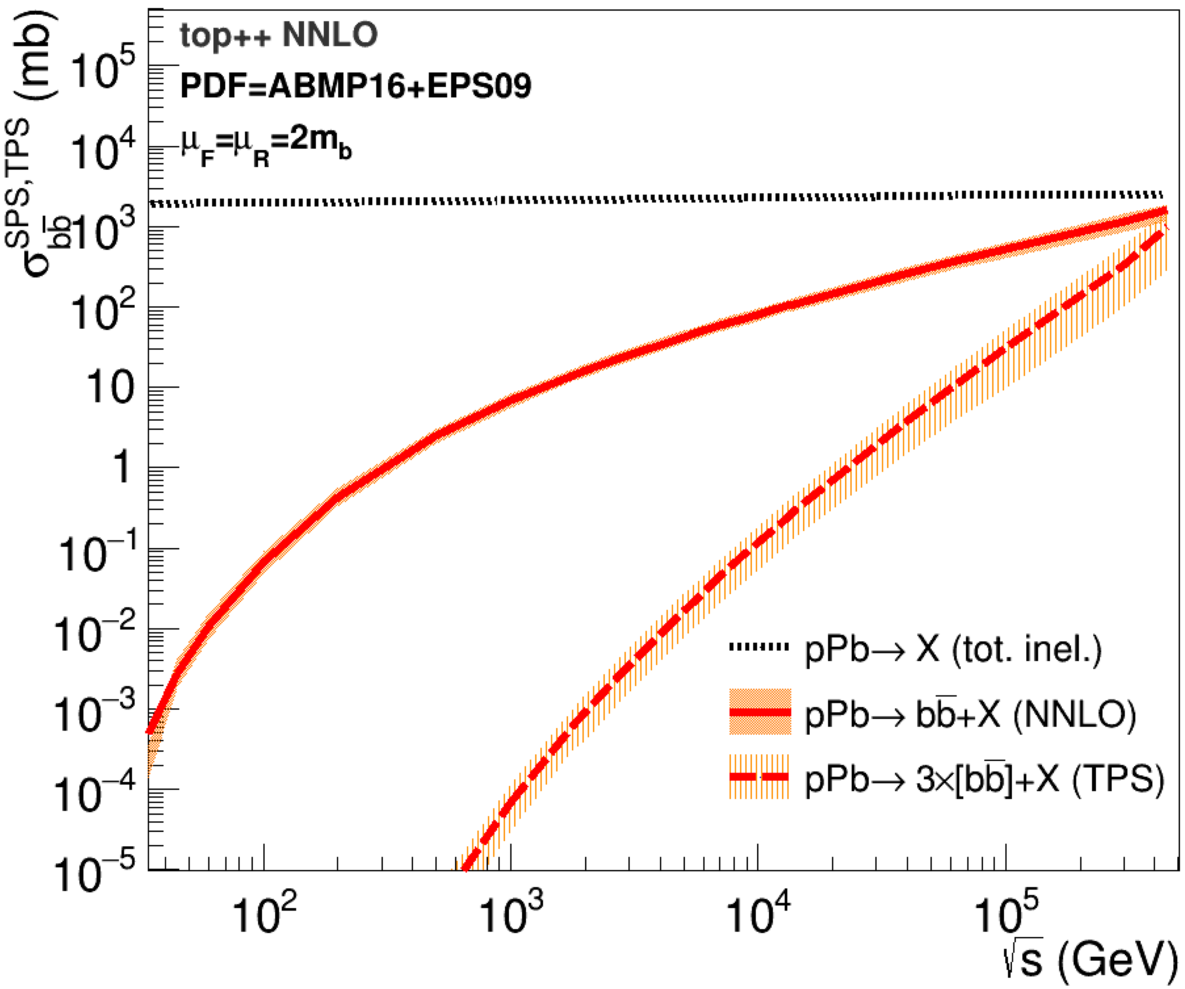
R.Maciula, A. Szczurek, Phys. Lett. B 772, 849 (2017).

TPS in pA: Charm cross sections in pPb collisions as a function of



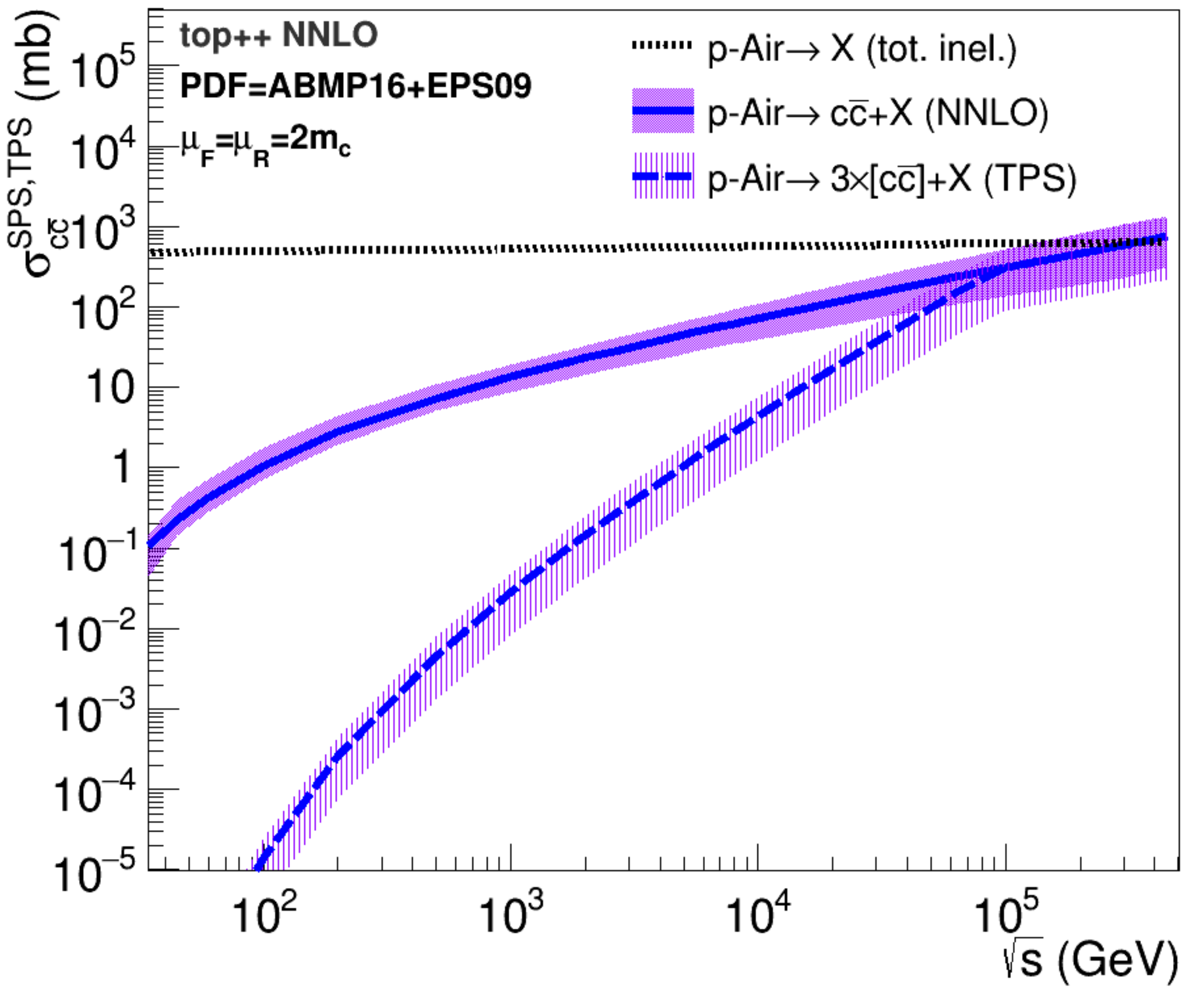
c.m. energy, in single-parton (solid band) and triple-parton (dashed band) scatterings, compared to the total inelastic pPb cross sections (dotted line). Bands around curves indicate scale, PDF (and $\sigma_{\rm eff,TPS}$, in the TPS case) uncertainties added in quadrature. The $\mathrm{pPb} \to c\bar{c}+X$ charm data point has been derived from the ALICE D-meson data

TPS in pA: Bottom cross sections in pPb collisions as a function of



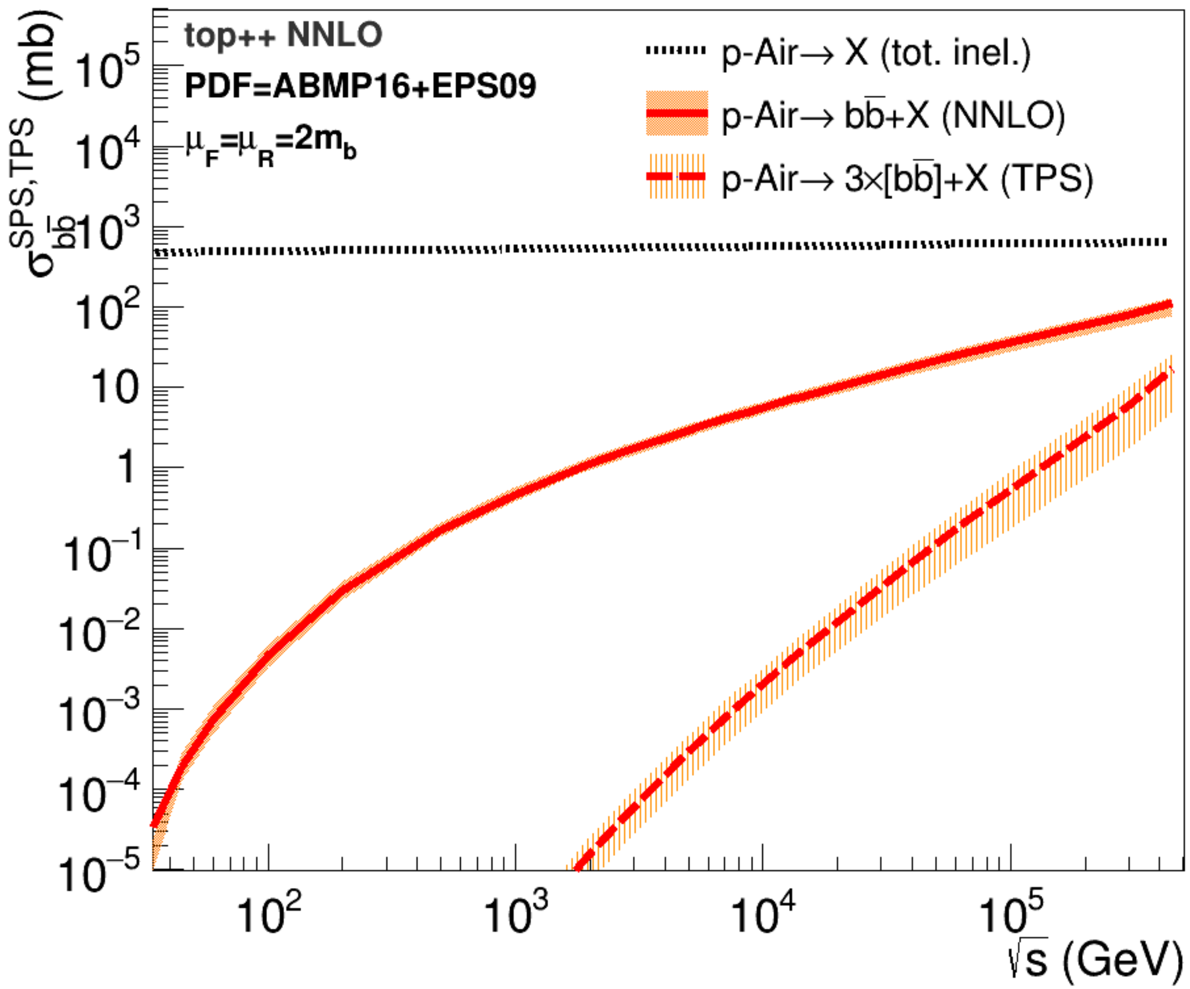
c.m. energy, in single-parton (solid band) and triple-parton (dashed band) scatterings, compared to the total inelastic pPb cross sections (dotted line). Bands around curves indicate scale, PDF (and $\sigma_{\mathrm{eff,TPS}}$, in the TPS case) uncertainties added in quadrature.

TPS in pA: Charm cross sections in pAir collisions as a function of



c.m. energy, in single-parton (solid band) and triple-parton (dashed band) scatterings, compared to the total inelastic pAir cross sections (dotted line). Bands around curves indicate scale, PDF (and $\sigma_{\mathrm{eff,TPS}}$, in the TPS case) uncertainties added in quadrature.

TPS in pA: Bottom cross sections in pAir collisions as a function of



c.m. energy, in single-parton (solid band) and triple-parton (dashed band) scatterings, compared to the total inelastic pAir cross sections (dotted line). Bands around curves indicate scale, PDF (and $\sigma_{\mathrm{eff,TPS}}$, in the TPS case) uncertainties added in quadrature.

TPS in pA:

Cross sections for inclusive inelastic, and for SPS and TPS charm and bottom production in pPb (at LHC and FCC energies) and p-Air (at GZK-cutoff c.m. energies) collisions. For the SPS and TPS cross sections the quoted values include scales, PDF, and total (quadratically added, including $\sigma_{eff,TPS}$) uncertainties.

Process	pPb(8.8 TeV)	pPb(63 TeV)	p-Air(430 TeV)
$\sigma_{\rm pA}^{\rm inel}$	**2.2±0.4 b**	**2.4±0.4 b**	**0.61±0.10 b**
$\sigma_{c\bar{c}+X}^{SPS}$	$\mathbf{0.96}\pm 0.45_{\rm sc}\pm 0.10_{\rm pdf}$ **b**	$\mathbf{3.4}\pm 1.9_{\rm sc}\pm 0.4_{\rm pdf}$ **b**	$\mathbf{0.75}\pm 0.5_{\rm sc}\pm 0.1_{\rm pdf}$ **b**
$\sigma_{c\bar{c},c\bar{c},c\bar{c}+X}^{TPS}$	$\mathbf{200}\pm 140_{\rm tot}$ **mb**	$\mathbf{8.7}\pm 6.2_{\rm tot}$ **b**	$\mathbf{5.0}\pm 3.6_{\rm tot}$ **b**
$\sigma_{b\bar{b}+X}^{SPS}$	$\mathbf{72}\pm 12_{\rm sc}\pm 5_{\rm pdf}$ **mb**	$\mathbf{370}\pm 75_{\rm sc}\pm 30_{\rm pdf}$ **mb**	$\mathbf{110}\pm 25_{\rm sc}\pm 5_{\rm pdf}$ **mb**
$\sigma_{b\bar{b},b\bar{b},b\bar{b}+X}^{TPS}$	$\mathbf{0.084}\pm 0.045_{\rm tot}$ μ**b**	$\mathbf{11}\pm 7_{\rm tot}$ μ**b**	$\mathbf{17}\pm 11_{\rm tot}$ μ**b**

Conclusions

The factorized framework that allows one to compute the cross sections for the simultaneous perturbative production of particles in DPS, TPS scatterings, from the corresponding single-parton scattering (SPS) cross sections in proton-proton, proton-nucleus, and nucleus-nucleus collisions is reviewed.

Numerical examples for the cross sections and visible yields expected for the concurrent DPS and TPS production of heavy-quarks, quarkonia, and/or gauge bosons in proton and nuclear collisions at LHC, FCC, and at ultra-high cosmic-ray energies have been provided.

Processes such as double-J/ψ, $J/\psi\Upsilon$, J/ψ W, J/ψ Z, double-Υ, Υ W, ΥZ, and same-sign W W production have large cross sections and visible event rates for the nominal LHC and FCC luminosities. The study of such processes in proton-nucleus collisions provides an independent means to extract the effective $\sigma_{\rm eff,DPS}$ parameter characterising the transverse parton distribution in the nucleon. In addition, we have shown that double-J/ψ and double-Υ final states have to be explicitly taken into account in any event-by-event analysis of quarkonia production in heavy-ion collisions.

The TPS processes, although not observed so far, have visible cross sections for charm and bottom in pp and pA collisions at LHC and FCC energies. At the highest c.m. energies reached in collisions of cosmic rays with the nuclei in the upper atmosphere, the TPS cross section for triple charm-pair production equals the total p-Air inelastic cross section, indicating that the average number of $c\bar{c}$-pairs produced in multiple partonic interactions is above unity.

The results presented here emphasize the importance of having a good understanding of the NPS dynamics in hadronic collisions at current and future colliders, both as genuine probes of QCD phenomena and as backgrounds for searches of new physics in rare final-states with multiple heavy-particles, and their relevance in our comprehension of ultrarelativistic cosmic-ray interactions with the atmosphere.